## Index

$\theta$-schemes, 151, 172, 228
$n \mathrm{D}$ wave equation, see wave equation, $n \mathrm{D}$

A-stability, 20
abstract sound synthesis, $1-8$
difficulties with, 8
physical modeling sound synthesis and, 16, 18-19, 45
accuracy, ix-x
finite difference schemes and, 155, 321
modal methods and, 155, 321
of difference operators, 29-30, 104
of numerical boundary conditions, 138, 312
of schemes for the 1D wave equation, 131, 136-137
of schemes for the SHO, 52-53, 60-61
of schemes for Webster's equation, 257-258
perceptual effects of, 155
spectral methods and, 388
acoustic tube, 249-250
1 D wave equation and, 117,118
admittance of, 262
boundary conditions and, 124, 126, 253-254
coupling to reed model, 267
formants and, 259
glottal excitation and, 258-259
scattering methods and, 261-263
time-varying, 264
transmission line analogy and, 258
wall vibration and, 259-260
adaptor, 59
additive synthesis, 3-4
modal synthesis and, 2, 18-19
Adrien, J.-M., 2
aliasing, 21
grid interpolation and, 104
nonlinear systems and, 231
all-pass interpolation, 5, 102
AM synthesis, 6
Ames, W., 383
amplification factor, 32, 105
amplification polynomial, 33, 106, 295
analysis-synthesis methods, 2, 4, 178
anisotropy
numerical, 290, 295, 307, 313
of thin plates, 292, 349-352
ansatz, 32, 96, 105, 290, 294
arched bars, 211-213
artificial reverberation
computational complexity and, 169, 381
digital waveguides and, 13
finite difference schemes and, 311
plates and, 345
psychoacoustic model reduction techniques and, 22
room acoustics and, 305
springs and, 199
aspect ratio, 289
auto-oscillations
bow and, 86, 180
reed-bore system and, 269
averaging operators, $27,99,100,302$
as used in finite difference schemes, 55-56
bandwidth
finite difference schemes for
spatially-varying systems and, 207-208, 212
schemes in radial coordinates and, 320-321, 353, 354
stability conditions and, 134-136, 170, 186, 225
Bank, B., 239
bar
ideal, see ideal bar
nonlinear vibration of, 232
of variable cross-section, 210-213
thick, see Timoshenko beam theory
bassoon, 278
beating, 185
Berger's equation, 361-362
Bernoulli's law, 266
bi-Laplacian, 289, 293
biharmonic operator, 289, 293
bilinear interpolation, 293
bilinear transformation, 38
BlockCompiler, 15
boundary conditions
for Webster's equation, 253-254
digital waveguides and, 147-148
energy analysis and, 97
energy-storing, 126, 254, 322
for 1D wave equation, 124-126
for 2D wave equation, 307-308, 318
for helical springs, 201-202
for ideal bar, 165-166
for Kirchhoff-Carrier equation, 222-223
for stiff string, 175
for thin plate model, 334-335, 352-353
for von Kármán system, 365
integration by parts and, 98, 290-291
lossy, 125-126
modal density and, 167, 381
modes and, 128, 166
numerical, see numerical boundary conditions
radiation, see radiation boundary condition
traveling waves and, 126
boundary element methods, 390
bow, 82-84
finite width of, 183
interaction with mass-spring system, 84-86
interaction with plate, 344
interaction with string, 180-183
bow force, 84, 180
bow velocity, 84,180
BRASS, 278
brass instruments, 11, 82, 252, 258, 278
digital waveguides and, 13
brightness, 184, 342, 374

Cadoz, C., 2, 9
Cage, J., 187
Cartesian coordinates, 288
Cauchy-Schwartz inequality, 97, 109, 290, 295
cent, 57, 69
CFL condition, see Courant-Friedrichs-Lewy condition
Chaigne, A., 2, 17
Chebyshev norm, 128
Chebyshev polynomials, 8, 389
Chowning, J., 2, 6
circulant delay networks, 14
clarinet, 252, 269
collisions, 78-79
brightness and, 78, 342
hammer-string interaction and, 183
mallet-plate interaction and, 342-343
reed model and, 266, 269-270
collocation methods, 20, 389
commuted synthesis, 13, 18, 346, 349
compact schemes, 61, 150-152, 314
computational complexity, 20-22, 379-382
acoustic tubes and, 254
implicit schemes and, 151, 173
modal density and, 130-131, 167, 254, 314, 337, 381-382
modal methods and, 321-322
numerical stability and, 133-134, 169
of scattering methods vs. finite difference schemes, 263, 316, 321
schemes for the 1D wave equation and, 145-146
schemes for the 2 D wave equation and, 314, 321-322
schemes for the ideal bar equation and, 169
wave speeds and, 236, 252
conical tubes, 252-253
coordinate changes, 112-113
spatially-varying systems and, 208, 212, 208-213
CORDIS, 9, 382
Courant number, 131, 310
Courant-Friedrichs-Lewy condition, 117, 133, 139
as bound on computational complexity, 133-134
geometrical interpretation of, 133, 208, 225
crash, 368, 374-376
critical band, 22
cutoff
frequency, numerical, see numerical cutoff frequency
wavenumber, 154
cylindrical tubes, 129, 252-253
d'Alembert, 123
damper
connection between bars, 197
connection with string, 188-190
decay time
1D wave equation with loss and, 154
SHO with loss and, 65-66
stiff membrane and, 341
stiff string and, 178
degeneracy, 309
degrees of freedom
computational complexity and, 130-131, 169-170, 322, 379-382
finite difference schemes and, 133, 169-170, 380-381
for 1 D wave equation, $130-131$
for 2D wave equation, 309
for ideal bar equation, 167
for stiff string, 176
for thin plate equation, 336
memory requirements and, 155
modal density and, 130-131, 167, 309, 336, 381
detuning
acoustic tubes and, 258
due to stiffness, 176
numerical boundary conditions and, 143
numerical dispersion and, 142, 257, 321
of sets of strings, 186
string preparation and, 193
difference operators
accuracy of, 29-30, 104
anisotropy of, 295
combining of, 29, 100
digital filter interpretation of, 35-37
energy analysis and, 39-40, 110-111, 296, 301
frequency domain analysis and, 31-33, 105-106, 294-295
in transformed coordinates, 112
matrix forms of, 107-108, 296-299
mixed spatial-temporal, 101
parameterized, 100, 293
spatial
in 1D, 100-101
in 2D, Cartesian, 291-293
in 2D, radial, 299-301
in 3D, 324-325
stencils of, 100, 292, 299
temporal, 27-28, 99-100
temporal width of, 28-29
differential operators
frequency domain analysis and, 96, 289-290
in $1 \mathrm{D}, 93-94$
in 2D, 289-291
in transformed coordinates, 112
digital filters, 35-38
digital waveguides, 11-13
1D wave equation and, 117, 146-148
boundary termination of, 147-148
cylindrical and conical tubes and, 252
efficiency of, 21, 156, 384-385
finite difference schemes and, 19-20, 146, 261
in multiple dimensions, see waveguide mesh
Karplus-Strong algorithm and, 12
Kelly-Lochbaum speech synthesis model and, 14, 258
multiple channel output and, 144
numerical stability and, 157
scattering methods and, 261
toneholes and, 272
transmission lines and, 118
traveling wave decompositions and, 123
wavetable synthesis and, 6, 18
wind instruments and, 265
direct numerical simulation, 16-18
digital waveguide synthesis and, 19-20
lumped mass-spring networks and, 19
scattering methods and, 20
Dirichlet condition
in $1 \mathrm{D}, 124$
in 2D, 297, 308
losslessness of, 125, 207, 222
modes and, 129, 317
numerical, 108, 138, 312, 319
wave reflection and, 126
Webster's equation and, 253
dispersion
acoustic tubes and, 251-252, 254
helical springs and, 200-201
ideal bar and, 164, 165
inharmonicity and, $163,165,168$
numerical, see numerical dispersion
thin plates and, 332
dispersion relation, 96
for 1 D wave equation, 122
for 2 D wave equation, 307
for an anisotropic plate equation, 351
for helical spring, 200-201
for ideal bar, 164
for stiff string, 175
for thin plates, 332
for Webster's equation, 251
numerical, 134-135
dissipation, see loss
domains
in 1D, 93-94, 98
in 2D, 288-289, 291, 299
double reed instruments, 278
drift, 127
drums, 310, 318, 342
Duffing's equation, 75
energy analysis, $x-x i, 38-42$
boundary conditions and, 93, 97, 125-126, 137-141, 165-166, 171-172, 202, 222, 226, 253-256, 307-308, 312, 318, 334-335, 338-339, 365
modular connections and, 189, 192
nonlinear systems and, xi, 38, 73
numerical stability conditions and, xi, $38,39,42,54,77,83,86,93$, 139, 171, 198, 207, 209, 212, 226, 231, 238, 256, 311-312, 339
of a bar of variable cross-section, 211
of a string of variable density, 207 and schemes for, 207, 209
of bow mechanism, 83 and schemes for, 83
of collisions, 79
of coupled ideal bars, 196-197
and schemes for, 198
of helical springs, 201-202
of nonlinear oscillator, 74-75 and schemes for, 77
of nonlinear string vibration, 234-236 and schemes for, 237-238
of plate-string connection, 347-348
of prepared strings, 188-189 and schemes for, $189,191-192$
of the 1 D wave equation, $123-126$ and schemes for, $137-141,152$
of the 1 D wave equation with loss, 154 and schemes for, 155
of the 2D wave equation, 307-308, 318 and schemes for, $311-312,319$
of the bow-mass-spring interaction, 85 and schemes for, 86, 181-182
of the bow-string interaction, 180-181 and schemes for, 181-182
of the hammer-string interaction, 184
of the ideal bar equation, 165-166 and schemes for, 170-172
of the Kirchhoff-Carrier equation, 222-223
and modal decompositions for, 230
and schemes for, 226
of the $\mathrm{SHO}, 48$
and schemes for, 53-54, 56-57
of the SHO with loss, 64
and schemes for, 65
of the single reed model, 267-268
of the stiff string, 175-176
of the stiff string with loss, 179
of the thin plate equation, 333-335 and schemes for, 338-339
of the tonehole, 276-277
of the von Kármán system, 364-365
and schemes for, 370-371
of Webster's equation, 253-254 and schemes for, 255-256
vs. frequency domain analysis, $\mathrm{x}-\mathrm{xi}$
wave digital filters and, 59, 60
energy method, 48, 54
Erkut, C., 2
Euler-Bernoulli beam model, see ideal bar
fast Fourier transform, 2
modal synthesis and, 153, 156
spectral methods and, 389
FDTD, see finite difference time domain method
feedback delay networks, 14
FEM, see finite element methods
Fender-Rhodes electric piano, 195
Fettweis, A., 2, 15, 58
FFT, see fast Fourier transform
finite difference schemes, $x, 16-17$
bandwidth of, see bandwidth
boundary conditions for, see numerical boundary conditions
causality and, 37
digital waveguides and, 19, 146-147, 156, 261
for a bar of variable cross-section, 211-213
for a nonlinear oscillator, 75-78
for a reed model, 268-269
for a string of variable density, 207-209
for an anisotropic plate equation, 351-352
for Berger's equation, 362
for collisions, 79
for helical springs, 202-204
for multiple strings, 186
for nonlinear string vibration, 236-241
for non-planar string vibration, 243
for plate excitation, 341-342
for shell system, 373
for the 1D wave equation, 131-143, 148-152
for the 1 D wave equation with loss, 154-155
for the 2 D wave equation, $310-315$, 318-321
for the bow mechanism, 83-84
for the bow-mass-spring interaction, 85-86
for the bow-plate interaction, 344
for the bow-string interaction, 181-182
for the hammer-mass-spring interaction, 81
for the hammer-string interaction, 184-185
for the ideal bar equation, 168-174
for the Kirchhoff-Carrier equation, 224-229
for the plate-string interaction, 348
for the SHO, 49-61
for the SHO with a source term, 68
for the SHO with loss, 64-67
for the stiff membrane, 341
for the stiff string, 176-177
for the thin plate equation, 337-340
for the time-varying vocal tract, 264
for the von Kármán system, 367-371
for toneholes, 277
for Webster's equation, 255-258, 260
implicit, see implicit schemes
lumped mass-spring networks and, 19, $119,156,316,321,382-383$
matrix representations of, see matrix representations, of finite difference schemes
multiple channel output and, 144
numerical dispersion in, see numerical dispersion
parameterized, 56-57, 148-152, $172-174,176-177,202$, $228-229,312-314,320-321$, 339-340, 353, 373
scattering methods and, 146-147, 253, 257, 258, 261, 263, 316
spatial variation and, 207, 209, 211-213, 255
stability of, see numerical stability
wave digital filters and, 59, 269
waveguide mesh and, 321
finite difference time domain method, 16,49 , 258
finite element methods, $17-18,20,35,46,48$, $49,62,258,288,299,333,346$, 386-388
finite precision, 54, 137
finite volume methods, 390
flexural rigidity, 331
Florens, J.-L., 9
flutes, 278
FM synthesis, 6-7, 45
waveshaping and, 8
FOF, 8
formants, 254, 259
transitions of, 264-265
wall vibration and, 260

## Fourier series

modal synthesis and, 96, 129, 309
representations of nonlinear strings, 229
spectral methods and, 388, 389
Fourier transform
discrete spatial, 105
spatial, 96
fractional delay, 5, 145, 263
frequency domain analysis
limitations of, $\mathrm{x}, 38,73$
modal analysis and, 128-129, 166-167, 176, 254, 309, 336
of an anisotropic plate equation, 351
of difference operators, 31-33, 105-106, 294-295
of differential operators, 96-97, 289-290
of finite difference schemes, see von Neumann analysis
of the 1 D wave equation, 122
of the 1 D wave equation with loss, 153-154
of the 2D wave equation, 306-307
of the ideal bar equation, 164-165
of the SHO, 47
and schemes for, 51-52, 55-56, 60
of the SHO with loss, 63-64
and schemes for, 64-67
of the stiff string, 175
of the stiff string with frequency-dependent loss, 178
of the thin plate equation, 332
of toneholes, 273-275
of Webster's equation, 251-252
vs. energy analysis, $\mathrm{x}-\mathrm{xi}, 38$
frequency warping, 9
in difference schemes for the SHO, 52, 57
in schemes for coupled systems of oscillators, 62
wave digital filters and, 59
frequency-dependent loss
in stiff membranes, 341
in strings, 177-180, 241
mixed spatial-temporal derivatives and, 101
FTM, see functional transformation method functional transformation method, 10
scattering methods and, 15

Giordano, N., 17, 351
glottal excitation, 258-259
granular synthesis, 8
grid function, $98,291,299$
grid spacing, 98, 291-292, 299
group velocity, see phase and group velocity
half plane, 289, 291
Hall, D., 81
hammer (mallet), 78-80
finite difference schemes for, 81
interaction with mass-spring system, 80-81
interaction with multiple strings, 186-187
interaction with plate, 342-343
interaction with string, 183-185
helical spring, 199-206
Helmholtz motion, 180
Helmholtz, H., 180
Hélie, T., 10
ideal bar
boundary conditions and, 165-166
connected to another, 194-199
dispersion and, 164-165
energy analysis and, 165-166
equation of motion for, 163-164
finite difference schemes and, 168-174
longitudinal motion of, 118, 119
modes and, 166-167
of varying cross-sectional area, 210-213
phase and group velocity for, 164-165
implicit schemes
averaging operators and, 27
boundary conditions and, 152
causality and, 37
computational complexity and, 151-152
for a nonlinear oscillator, 77
for shell vibration, 373
for the 1 D wave equation, $150-152$
for the 2 D wave equation, 314,320
for the bow mechanism, 84
for the ideal bar equation, 172-174
for the stiff string, 176-177
for the thin plate equation, 339-340, 353
for the von Kármán system, 367
matrix methods and, 107, 109, 141, 238-239
mixed derivatives and, 101
modularity and, 192
nonlinear systems and, 226, 240, 367
reduction of numerical dispersion and, $174,177,202,321,340,353$
suppression of numerical oscillations and, 185, 228-229
uniqueness of solutions to, $77,84,182$
vs. explicit, 106, 151-152, 179, 202, 353
inharmonicity
computational complexity and, 21
frequency warping and, 57
ideal bar and, 165,168
in prepared strings, 190
nonlinear string vibration and, 233
numerical, 97, 142-143
psychoacoustic significance of, 22
stiff strings and, 175
inharmonicity factor, 176
initial conditions
corresponding to strikes and plucks, 120-121
digital waveguides and, 123, 147
for 1 D wave equation, $120-121$
for 2 D wave equation, 306
for a nonlinear oscillator, 74
for difference schemes, see initialization
for ideal bar, 164
for SHO, 46
for stiff string, 175
for traveling waves, 123
initialization
digital waveguides and, 147
of schemes for the 1 D wave equation, 131-132
of schemes for the ideal bar equation, 169
of schemes for the SHO, 50
inner product
at boundary, 291
in 1D, definition of, 97, 109
in 2D, definition of, 290, 295, 300
vector form of, 98
instantaneous frequency, 4, 7
integration by parts, 98,290
in transformed coordinates, 113
interpolation
accuracy of, 104
all-pass, 5, 102
bilinear, 293
digital waveguides and, 21
excitation and, 181, 185, 342
in 1D, 101-102
in 2D, 293-294
Karplus-Strong algorithm and, 12
modular connections and, 189
output and, 143, 345
perceptual aspects of, 182
schemes for nonlinear string vibration and, 240-241
vocal tract configurations and, 265
wavetable synthesis and, 5
iterative methods
determination of modal frequencies and, 167
implicit schemes and, 109
lumped mass-spring networks and, 62
Newton-Raphson, 90-91

K variables, 15, 265
Karjalainen, M., 2
Karplus, K., 2, 12
Karplus-Strong algorithm, 12, 16
Kelly, J., 2, 9, 258
Kelly-Lochbaum speech synthesis model, 14, 261-263
finite difference schemes and, 257, 263
scattering methods and, 15, 258
transmission line analogy and, 250
kettledrum, 323, 325
Kirchhoff plate vibration model, see thin plate, Kirchhoff model of
Kirchhoff-Carrier equation, 221-222
boundary conditions and, 222
energy analysis and, 222-223
loss and, 223
modal analysis of, 229-231
pitch glides and, 223-224
schemes for, 224-229
tension modulation and, 221

Lagrange interpolation, 5, 102
Laplace transform, 31-32, 96
frequency domain ansatz and, 32, 96
Laplacian, 289
five-point approximations to, 293
matrix forms of, 297-299
in radial coordinates and approximations to, 300-301
integration by parts and, 290, 291

## Laplacian (continued)

nine-point approximations to, 293
summation by parts and, 296, 301
lattice-Boltzmann method, 390
LC circuit, 46
wave digital filters and, 58, 59
Leslie speaker, 199
linear and shift-invariant systems, 94-95, 206 anisotropy and, 349
frequency domain analysis and, 33, 95
modal analysis and, 167
musical systems and, 379-380
phase and group velocities and, 97
linear and time-invariant systems, 94-95
computational complexity and, 20-22
frequency domain analysis and, 38, 95
matrix method and, 140
modal synthesis and, 19, 157, 384
musical systems and, 38
linear systems, 94
Lochbaum, C., 2, 9, 258
loss
1D wave equation with, 153-155
acoustic tubes and, 259-260
boundary conditions and, see boundary conditions, lossy
computational complexity and, 379
damper and, 190
decay time and, 65-66, 154, 178
frequency-dependent, see frequency-dependent loss
in nonlinear string models, 223, 242
nonlinear oscillators and, 82
phase and group velocity and, 97
pitch glides and, 223, 362
plate models and, 341
reed models and, 267
SHO with, 63-64
stability conditions and, 64, 155
stiff string and, 177-180
toneholes and, 273, 274
LSI, see linear and shift-invariant systems
LTI, see linear and time-invariant systems
Luciani, A., 9
lumped mass-spring networks, $9-10$, 382-383
1D wave equation and, 118-119
2D wave equation and, 316
coupled SHOs and, 61-62
difficulties with, 382-383
direct numerical simulation and, 19
finite difference schemes and, 119, 321, 382-383
glottis modeling and, 258
SHO and, 46
wave digital filters and, 58
Lyapunov stability, 42
mallet, see hammer (mallet)
marimba, 210
masking, 22
mass-spring networks, see lumped mass-spring networks
Mathews, M., 2, 6, 258
matrix method, 140
matrix representations
of difference operators, 107-109, 296-299
of finite difference schemes, 141-142, 151, 172, 177, 180, 227, 238-239, 314
of interpolation operators, 104
membrane
2D wave equation and, 305
boundary conditions and, 308
drum modeling and, 310, 311
finite difference schemes and, 17
lumped mass-spring networks and, 9, 316
modal synthesis and, 10, 317
stiff, 341
vs. plate, 331, 337
waveguide mesh and, 14
memory requirements
for general simulations of LTI systems, 21, 380
modal synthesis and, 11, 22, 62, 156, 321-322, 381, 384
of difference schemes vs. scattering structures, 316
sampling synthesis and, 23
schemes for the 1 D wave equation and, 133, 146, 155-156
schemes for the 2 D wave equation and, 321-322
schemes for the ideal bar equation and, 170
schemes for the SHO and, 55
schemes for Webster's equation and, 263
simulation of nonlinear systems and, 22
sparse matrices and, 108
time domain methods and, 22
Mindlin-Reissner thick plate theory, 332, 333
modal density
1D wave equation and, 130-131
2D wave equation and, 309-310
boundary conditions and, 156, 381
computational complexity and, 310, 337
degeneracy and, 381
general LTI systems and, 381
ideal bar and, 167-168
problem geometry and, 381
thin plate equation and, 336-337
Webster's equation and, 254
modal synthesis, 10-11, 383-384
1D wave equation and, 152-153
2D wave equation and, 317-318
accuracy and, 321, 384
additive synthesis and, 2, 18-19
analysis-synthesis methods and, 178
loss and, 178
memory requirements and, $11,22,62$, $131,156,167,321-322,384$
multiple channel output and, 144
nonlinear string vibration and, 229-231, 242
numerical stability and, 157
output and, 157
range of applicability of, $17,19,384$
spatial variation and, 207
spectral methods and, 18, 20, 389
stability and, 157
Modalys/MOSAIC, 10, 317
modes
acoustic tubes and, 254
finite difference schemes and, 142, 257-258, 313
for 1D wave equation, 128-129
for 2D wave equation, 309-310
for general LTI systems, 381-382
for ideal bar, 166-167
for stiff string, 176
for thin plate equation, 336
of coupled systems, 349
modified equation methods, 61, 137
modular connections, 187
among distributed objects, 194-199
energy analysis and, 189, 196, 198
modal synthesis and, 384
numerical stability and, 190-192
rigid, 196
scattering methods and, 192
modular synthesis, 15, 187
CORDIS and, 9
finite difference schemes and, 20
scattering methods and, 269
wave digital filters and, 60
moment of inertia, 164, 210
Moog synthesizer, 83
multiphonics, 271, 277

Neumann condition
in 1D, 124
in 2D, 308
losslessness of, 125, 207, 222
modes and, 129, 317
numerical, 108, 138-139, 298, 312, 319
Webster's equation and, 253
non-dimensionalization, 94
1D domains and, 120
2D domains and, 305
of dependent variables, 222, 251, 267
nonlinear bar vibration, 222
nonlinear string vibration, see string, nonlinear vibration of
nonlinear systems, 94
computational complexity and, 22, 381-382
energy analysis and, 25, 38
finite difference schemes and, 17
frequency domain analysis and, 5, 38, 95
nonlinearity, 221, 223, 361
bow and, 82-83
center-limited, 81-82
cubic, 75
in plates, 361
in reed, 86-87, 266
in strings, 221
one-sided, 78
perceptual effects of, 76, 78
power law, 78
norm
in 1D, definition of, 97, 109
in 2D, definition of, 290, 295
numerical boundary conditions
accuracy and, 138
choice of inner product and, 109, 111, $138,171,312$
detuning of modes and, 143, 174
for plate-string connection, 348
numerical boundary conditions (continued) for schemes for the 1 D wave equation, 137-139
for schemes for the 2 D wave equation, 312, 319
for schemes for the ideal bar equation, 171-172
for schemes for the thin plate equation, 338-339, 353-354
for schemes for the von Kármán system, 371
for schemes for Webster's equation, 255-256
instability and, 125, 140-141
matrix representations of difference operators and, 107-108, 297-299
stencil width and, 28, 150
summation by parts and, 110, 296
virtual grid points and, 107-109
numerical cutoff frequency, 57, 170, 236, 239, 240
numerical dispersion
anisotropic, 313
computational complexity and, 156, 172-174
frequency warping and, 52, 57
implicit schemes and, 152, 172, 174, 340
in schemes for helical springs, 203-204
in schemes for nonlinear strings, 225, 236, 240
in schemes for the 1 D wave equation, 134-135
in schemes for the 2 D wave equation, 313, 321
in schemes for the ideal bar, 170
in schemes for Webster's equation, 257-258
inharmonicity and, 142, 155
modal methods and, 318
of schemes in radial coordinates, 320, 354
perceptual significance of, 93, 142-143, 257, 318, 388
phase and group velocities and, 106
spatially-varying systems and, 208, 257-258
stability conditions and, 136, 170, 229
vs. physical dispersion, 134
numerical instability
in schemes for the 1D wave equation, 133
in schemes for the Kirchhoff-Carrier equation, 225
in schemes for the SHO, 50-51
in schemes for the von Kármán system, 369
modular connections and, 190-192
nonlinear systems and, 77, 225, 369
numerical boundary conditions and, 125, 140-141
time-varying systems and, 262, 271
numerical oscillations
hammer-string interaction and, 185
nonlinear systems and, 227-229
suppression of, 185, 228-229
numerical phase and group velocity, 38, 106
for schemes for the 1 D wave equation, 134-135, 150
for schemes for the 2 D wave equation, 313
for schemes for the ideal bar, 170
for schemes for the stiff string, 177
for schemes for the thin plate equation, 339
numerical stability
bandwidth and, 134-136, 170
collisions and, 79
computational complexity and, 21 , $133-134,169,321,382$
digital waveguides and, 148, 157
energy analysis and, 38, 93
frequency domain analysis and, 31, 38, 140
implicit schemes and, 152
modal synthesis and, 157
modular connections and, 189, 192
of schemes for a bar of varying cross-section, 212
of schemes for a cubic nonlinear oscillator, 77
of schemes for a nonlinear oscillator, 75
of schemes for a string of varying density, 207, 209
of schemes for anisotropic plate vibration, 352
of schemes for coupled oscillators, 62
of schemes for nonlinear string vibration, 238
of schemes for helical spring vibration, 202-203
of schemes for plate-string connections, 348
of schemes for shell vibration, 374
of schemes for the 1 D wave equation, 132-133, 139, 149-151, 157
of schemes for the 1 D wave equation with loss, 155
of schemes for the 2D wave equation, 311-314, 319
of schemes for the bow mechanism, 83-84
of schemes for the bow-mass-spring system, 86
of schemes for the bow-string interaction, 181-182
of schemes for the ideal bar equation, 169, 171, 173
of schemes for the Kirchhoff-Carrier equation, 226, 231
of schemes for the SHO, 51-54, 56
of schemes for the SHO with loss, 64,65
of schemes for the stiff string, 176-177
of schemes for the thin plate equation, 338-340, 354
of schemes for the time-varying vocal tract, 264
of schemes for the von Kármán system, 370-371
of schemes for toneholes, 277
of schemes for Webster's equation, 256
polynomial root conditions and, 33-35, 106, 295
quadratic forms and, 39
sampling theory and, 52, 133
spatially-varying systems and, 163, 206
wave digital filters and, 60
waveguide meshes and, 316
oboe, 278
operation count
bounds on, 20
dependence on sample rate, 21
in schemes for general LTI systems, 380
in schemes for the 1 D wave equation, 156
in schemes for the 2 D wave equation, 322
in schemes for the SHO, 55
orthotropic plate vibration, 351
oscillator
additive synthesis and, 3
nonlinear, 74-75
nonlinear lossy, 82-83
simple harmonic, see simple harmonic oscillator
single reed model and, 265
output, 157
digital waveguides and, 13
from moving locations, 101, 144
from schemes for multiple strings, 187
from schemes for the 1D wave equation, 143-145
from schemes for the 2 D wave equation, 310-311
interpolation and, 101, 143, 310-311
modal synthesis and, 10
multiple channel, 11, 144-145
plate reverberation and, 345
time domain methods and, 11
parameter reduction, 120
Parseval's relation, 230
partial conservation, 79
partial differential equation
1D wave equation, 118, 120
with loss, 153
2D wave equation, 305
Berger's equation, 362
classification of, 94-95
coordinate changes and, 112
for anisotropic plate vibration, 351
for bar of variable cross-section, 210
for ideal bar, 163, 164
for nonlinear bar, 232
for stiff string, 175
with frequency-dependent loss, 177
for string of variable density, 206
for thin plate vibration, 331
for time-varying acoustic tube, 264
Kirchhoff-Carrier equation, 221
non-planar form of, 243
with loss, 223
musical instrument models and,

$$
8,379-380
$$

system for coupled ideal bars, 195
system for helical spring vibration, 200
system for nonlinear shell system, 372
partial differential equation (continued)
system for planar string vibration
general form of, 232
series-approximated form of, 236
von Kármán system, 363
Webster's equation, 250
PDE, see partial differential equation
phantom partials, 76, 221, 232-234, 242, 346
phase and group velocity, 97
for 1 D wave equation, 122
for 2D wave equation, 307
for ideal bar equation, 164
for stiff string, 175
for thin plates, 332
numerical, 106
Webster's equation and, 251
phase plane analysis, 42
phase vocoder, 4
physical modeling sound synthesis, $8-18$
abstract synthesis and, 1, 18-19
audio signal processing and, 26-27
modularity and, 15
motivations for, 22
subtractive synthesis and, 5
piano
bridge and, 234, 346
commuted synthesis and, 349
frequency-dependent loss in, 178
modal synthesis and, 242
multiple strings in, 81, 185, 186
phantom partials and, 221, 234
preparation of, 187
sampling synthesis and, 6, 23
soundboard and, 346
stiff string modeling and, 174
two-stage decay in, 187
pitch
arched bars and, 211
boundary conditions and, 129
delay lines and, 12
for vibrating bars, 167-168
numerical dispersion and, 143
pitch flattening, 86
pitch glides, 76
digital waveguides and, 13, 224
in bars, 232
in plates, $361,362,368$
in strings, 221, 223-224
plate, see thin plate
plate reverberation, 169, 199, 341, 345-346
plucks, 121
Poisson's ratio, 331
polynomials in $z, 33-35$
port resistance, 59
Portnoff, M., 17, 258
positive realness, 254
power law nonlinearity, 78
hammer (mallet) and, 78, 183, 342
mouthpiece flow and, 266
rattle and, 81
reed beating and, 266
precomputation, 156, 322, 346, 384
prepared piano, 187
quadratic forms, 41-42
quarter plane, 289, 291
Rabenstein, R., 2, 10
radial coordinates, 288
radiation boundary condition
1D wave equation and, 126
formant bandwidth and, 259
numerical, 140-141
Webster's equation and, 253-254
raised cosine distribution
in 1D, 121
in 2D, 306
Raman, C., 180
rattle, 81
connection with string, 192-193
reed beating, 87, 269-270
rigid body motion, $129,336,345$
ring modulation, 6
Risset, J.-C., 2, 3
Rocchesso, D., 10
room acoustics, 143, 305, 310, 322, 379, 381
boundary conditions and, 308
waveguide mesh and, 14
Ruiz, P., 2, 9, 17, 178
Rule of Thumb \#1, 136
Rule of Thumb \#2, 174
Rule of Thumb \#3, 192
Rule of Thumb \#4, 229
Rule of Thumb \#5, 354
Runge-Kutta methods, 35, 49, 61
sample rate, see sampling frequency
sampling frequency, 26
sampling synthesis, 6
memory requirements and, 23
sampling theorem, 26
computational complexity and, 380-381
saxophone, 252, 271
scanned synthesis, 144, 327
scattering junction, 12, 262, 315
scattering methods, $15,47,58,59$
acoustic tubes and, 261-264
digital waveguides as, 157
finite difference schemes and, 146, 253, 258, 263, 316
in 2D, 315
modular connections and, 192
time-varying systems and, 262
transmission lines and, 258
scheme, see finite difference schemes
self-sustained oscillation, see auto-oscillations shape matrix, 156
SHO, see simple harmonic oscillator shock waves, 278
short-time Fourier transformation, 4
simple harmonic oscillator
coupled systems and, 49
definition of, 46
energy analysis of, 48
first-order system and, 48-49
frequency domain analysis of, 47
schemes for, 49-61
wave digital filters and, 58-60
with a source term, 67-68
with loss, 63-67
single reed, 86-87, 265-268
sitar, 193
Smith III, J. O., 2, 3, 11, 147, 265, 315
soundboard, 346, 351
source-filter models, 5
subtractive synthesis and, 3
vocal synthesis and, 259
sources, 67-68
spatial variation
acoustic tubes and, 251
bars and, 210-213
coordinate changes and, 112
dispersion and, 251
strings and, 206-209
spectral methods, 18, 20, 231, 388-390
modal synthesis and, 20, 389
spectral modeling synthesis, 4
speech synthesis, 258
finite difference methods and, 17
Kelly-Lochbaum model, 261-264
spreading operators, 103, 294
excitation and, 181, 185, 342
gain of, 112
modular connections and, 189, 198, 348
spring, 46, 118, 316
connection between bars, 197
connection with string, 188-190
helical, see helical spring
stiffening, 75
squeaks, 271-272
stability, see numerical stability
state space, 142
representations of finite difference schemes, see matrix representations, of finite difference schemes
sound synthesis, 18
stability conditions and, 140
stencil
of 1D difference operators, 99
of 1D schemes, 106
of 2D difference operators, 292-293, 299
of 2D schemes, 313
wide, $149,150,155,299$
width, 28
stick-slip interaction, 86
stiff systems, 236
strikes, 121
string
1D wave equation and, 118
boundary conditions and, 124
bow interaction and, 180-183
connection with thin plate, 346-349
coupled transverse-longitudinal motion in, 232
digital waveguide modeling and, 13
finite difference schemes for, 17
frequency-dependent loss and, 177-178
hammer interaction with, 183-185
Karplus-Strong algorithm and, 12
longitudinal vibration of, 232, 346
lumped mass-spring model of, 10
modal synthesis and, 10
multiple, 185-187
non-planar vibration of, 242-244
nonlinear vibration of, 221
preparation and, 81, 187-193
spatial variation in, 206-209
string (continued)
stiffness and, 174-175, 241-242
strikes and plucks and, 120
sympathetic vibration of, 349
Strong, A., 2, 12
Sturm-Liouville problem, 10
subharmonic generation, 361, 366
subtractive synthesis, 4-5, 45
summation by parts, 110-111, 296, 301
in transformed coordinates, 113
symbol, 96
sympathetic vibration, 349
symplectic methods, $42,54,75$

TAO synth, 316
telegrapher's equations, 250
tension modulation, 13, 22, 221, 242
plate vibration and, 361
thin plate
anisotropic, 349-352
boundary conditions and, 333-335
bow interaction and, 344
connection with string, 346-349
energy analysis and, 333-335
excitation and, 341-342
Kirchhoff model of, 331-332
loss in, 341
mallet interaction and, 342-343
modal analysis and, 336
nonlinear vibration of, see Berger's equation, von Kármán system
reverberation and, 345-346
schemes for, 337-340
vs. thick, 332, 333
Thomas algorithm, 109, 151
time series, 26-27
identities for, 31, 40
time step, 26
Timoshenko beam theory, 97, 164, 213, 333
Timoshenko, S., 165
TLM, see transmission line matrix method Toeplitz matrices, 108
torsional wave motion, 183
transmission line
1D wave equation and, 118
acoustic tubes and, 258
Webster's equation and, 250
transmission line matrix method, 15, 315, 316
trapezoid rule, 37-38
wave digital filters and, 58
Trautmann, L., 2, 10
traveling wave decomposition, 123
acoustic tubes and, 261
digital waveguides and, 12, 123, 146-147
dispersion relation for 1D wave equation and, 122
reflection at boundary and, 126
triangle inequality, 97, 109, 290, 295
tube, see acoustic tube
tuning fork, 195
two-stage decay, 187
van Duyne, S., 315
vibrato, 259
virtual analog, 199
virtual grid points, 108, 172
vocal synthesis, 258
digital waveguide models and, 13
finite difference methods and, 17,258
granular methods and, 8
source-filter models and, 5
waveguide mesh and, 14
vocal tract
2D models of, 13-14, 305
acoustic tube modeling and, 258
formants and, 259
time variation and, 264
Volterra series, 10, 73, 229
von Kármán system, 363-371
von Neumann analysis, 35, 38, 47
of schemes for the 1 D wave equation, 132-133, 149-151
with loss, $154-155$
of schemes for the 2 D wave equation, 311
of schemes for the ideal bar, 169
of schemes for the thin plate, 338
VOSIM, 8
vowels, 259
wave digital filters, $38,47,58,118$
computability and, 269
digital waveguides and, 11
finite difference schemes and, 20, 59-60, 269
hybrid methods and, 15

SHO and, 58-60
speech synthesis and, 14
toneholes and, 13, 14, 272
wave equation
1D
boundary conditions for, 124-126
definition of, 118
digital waveguides and, 11, 12, 146-148
dispersion relation for, 122
energy analysis and, 123-126
finite difference schemes for, 131-143, 148-152
initial conditions for, 120-121
loss and, 153-155
lumped mass-spring networks and, 118-119
modes and, 128-129
phase and group velocity for, 122
traveling wave solutions and, 123
Webster's equation and, 252
2D
boundary conditions for, 307-308, 318
definition of, 305
dispersion relation for, 307
energy analysis and, 307-308
finite difference schemes for, 310-315
initial conditions for, 306
lumped mass-spring networks and, 316
modes and, 309-310
phase and group velocity for, 307
waveguide mesh and, 315-316
3D
definition of, 323
finite difference schemes for, 311 , 324-325
modes and, 310, 323
wave speed, $118,120,305$
acoustic tubes and, 250, 251
amplitude-dependent, 225
computational complexity and, 236
numerical dispersion and, 258
spatially varying systems and, 207, 208
wave variables
acoustic tube modeling and, 262
digital waveguides and, 12, 146
power-normalized, 59
scattering methods and, 15
wave digital filters and, 58
waveguide mesh and, 315
waveguide, see digital waveguides
waveguide mesh, 14, 315-316
computational complexity and, 21, 321
finite difference schemes and, 19, 310, 316
vocal tract modeling and, 258
wavenumber, 96, 105
in 2D, 290
waveshaping, 8
wavetable synthesis, 5-6, 45
digital waveguides and, 12, 16, 18
Karplus-Strong algorithm and, 12, 16
Webster's equation, 250
boundary conditions and, 253-254
dispersion and, 251-252
energy analysis and, 253-254
finite difference schemes for, 255-258
modes and, 254
phase and group velocity and, 251
time-varying form of, 264
whirling, 221, 243
woodwind instruments, 11, 82, 258, 265
digital waveguides and, 13

Young's modulus, 118, 164, 222
$z$ transform, 32, 105

