

Experimental Particle Physics Seminar

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Semileptonic B decays

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What? Cracking the Triangle

Framework: the CKM Matrix

- B⁺ and B⁰ are the most accessible 3rd-generation particles
- Their decays allow detailed studies of the CKM matrix

$$\mathbf{L} = -\frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u}_L & \overline{c}_L & \overline{t}_L \end{pmatrix} \gamma^{\mu} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_{\mu} + h.c.$$

- Unitary matrix V_{CKM} translates mass and weak basis
- 3 real parameters + 1 complex phase



- Is this the complete description of the CP violation?
 - Is everything consistent with a single unitary matrix?

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This is neatly represented by the familiar Unitarity Triangle



Angles α , β , γ can be measured with CPV of B decays 18 May 2006 F. Di Lodovico, QMUL

Test: the Consistency

• Compare the measurements (contours) on the (ρ , η) plane

- If the SM is the whole story, they must all overlap
- The <u>tells</u> us this is true as of today
 - Still large enough for New Physics to hide
- Precision of $\sin 2\beta$ outstripped the other measurements
 - Must improve the others to make more stringent test



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Next Step: $|V_{ub}/V_{cb}|$

Zoom in to see the overlap of "the other contours



Goal: Accurate determination of both $|V_{ub}/V_{cb}|$ and sin2 β

 $\sin\beta$ (all charmonium) = 0.687 ± 0.032 ~ percentage error: 4.6% (HFAG)



The B-Factory concept





- High luminosity B-factories @ Y(4S)
- CLEO: symmetric factory
- BaBar (BELLE) asymmetric factory
 - 9 (8) GeV e on 3.1 (3.5) GeV e+
 - (4S) boost: $\beta\gamma \sim 0.56 (0.43I)$

Luminosity: just spectacular!



How? The Semileptonic Decays

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- Extracting b \rightarrow u $\ell \nu$ signal challenging
- Sensitive to hadronic effects

Inclusive vs. Exclusive



Inclusive Decays

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Inclusive SL Decays

tree level, short distance:



+ long distance:



Decay properties depends directly on $|V_{cb}|$, $|V_{ub}|$, m_b, perturbative regime (α_s^n)

 \rightarrow short distance is calculable

- But quarks are bound by soft gluons: non-
- perturbative long distance ($\Lambda_{\rm QCD}$) interactions of

b-quark with light quark

Long distance leading order and short distance
 contribution are cleanly separated and probability
 to hadronize is 1.

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V_{cb}:

most accurate determination from the inclusive decays

9% precision limited by theory error

Precise Heavy Quarks parameters, tests of OPE

V_{ub}:

•7.5% precision shared between experimental and theoretical errors

- small rate and large $b \rightarrow c\ell v$ background

- space cuts to remove b $\rightarrow c\ell \nu$ background which introduce dependence on non-perturbative b-quark distribution function





 $\Gamma(b \rightarrow c\ell \nu)$ described by Heavy Quark Expansion in (1/m_b)ⁿ and α_s^k $\Gamma(B \rightarrow X_c l \nu) = G_F^2 m_b^5 / 192 \pi^3 |V_{cb}|^2 [[1+A_{ew}]A_{popert}A_{pert}]$

non-perturbative parameters need to be measured

The expansion depends on the m_b definition: non-perturbative terms are expansion dependent

Theory error was dominated by 1/m³ terms and above



Moments are related to non-perturbative parameters

Moments evaluated on the full lepton/mass spectrum or part of it: $p_{\ell} > p_{min}$ in the B rest frame

$$\left\langle E_{\ell}^{n}\right\rangle = \frac{1}{\Gamma_{c}} \left(E_{\ell} - \left\langle E_{\ell}\right\rangle\right)^{n} \frac{d\Gamma_{c}}{dE_{\ell}} dE_{\ell} \qquad \left\langle m_{X}^{n}\right\rangle = \frac{1}{\Gamma_{c}} m_{X}^{n} \frac{d\Gamma_{c}}{dm_{X}} dm_{X}$$

Higher moments are sensitive to 1/m $_3$ terms \rightarrow reduce theory error on $|V_{_{\rm cb}}|$ and HQ parameters

Moments in $B \rightarrow X_c \ell \nu$ decays

- E_{ℓ} : lepton energy spectrum in $B \rightarrow X_c \ell \nu$ (BaBar Belle CLEO Delphi)
- M_{x}^{2} : hadronic mass spectrum in $B \rightarrow X_{c} \ell \nu$ (BaBar CDF CLEO Delphi)
- Recent example for lepton and hadron mass moments from Belle:



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Global OPE Fit

- OPE predicts total rate Γ_c and moments $\langle E_c^n \rangle$, $\langle m_x^n \rangle$ as functions of $|V_{cb}|$, m_b , m_c , and several non-perturbative params
 - Each observable has different dependence
 Can determine all parameters from a global fit
- E_{γ} spectrum in $B \rightarrow X_{S}\gamma$ decays connected directly to the Shape Function (= what the b-quark is doing inside the B meson) so it is used in the global fit

OPE Fit Results

- Buchmüller & Flächer (hep-ph/0507253) fit data from 10 measurements with an OPE calculation by Gambino & Uraltsev (Eur. Phys. J. C34 (2004) 181)
 - Fit parameters: $|V_{cb}|$, m_b , m_c , μ_{π}^2 , μ_G^2 , ρ_D^3 , ρ_{LS}^3 , BR($B \rightarrow X_c \ell \nu$)

$$\begin{array}{c} \pm 2\% \quad |V_{cb}| = (41.96 \pm 0.23_{exp} \pm 0.35_{OPE} \pm 0.59_{\Gamma_{sl}}) \quad 10^{-3} \\ \pm 1\% \quad m_{b} = 4.590 \pm 0.025_{exp} \pm 0.030_{OPE} \text{ GeV} \\ m_{c} = 1.142 \pm 0.037_{exp} \pm 0.045_{OPE} \text{ GeV} \\ \mu_{\pi}^{2} = 0.401 \pm 0.019_{exp} \pm 0.035_{OPE} \text{ GeV}^{2} \\ BR = 10.71 \pm 0.10_{exp} \pm 0.08_{OPE}\% \end{array}$$

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0.2

4.4

4.5

4.6

m_b (GeV)

4.7



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$B \rightarrow X_u \ell v$ rate





Kinematical Cuts

•Three independent kinematic variables in $B \rightarrow X \ell v$



•Neasure partial rates in favorable regions of the phase space minimizing the effect of the Shape Function

•Caveat: Spectra more sensitive to non-perturbative effects than the total rate $\rightarrow O(1/m_b)$ instead of $O(1/m_b^2)$

Shape Function

Limited phase space to reduce the $B \rightarrow X_c \ell v$ background:

OPE doesn't work everywhere in the phase space \rightarrow non-perturbative Shape Function F(K⁺) to extrapolate to the full phase space



Shape Function need to be determined from experimental data

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Lepton Endpoint BABAR PRD 73:012006 Belle PLB 621:28 CLEO PRL 83:231803

Find leptons with large E_{e}

- Push below the charm threshold
 Larger signal acceptance
 Smaller theoretical error
- S/B~ 1/15 (E_{ℓ} > 2 GeV) → Accurate

subtraction of background is crucial!

	E_ℓ (GeV) $ V_{ub} $ (10)	
BABAR 80fb ⁻¹	2.0-2.6	$4.41\pm0.29_{\mathrm{exp}}\pm0.31_{\mathrm{SF+theo}}$
Belle 27fb ⁻¹	1.9–2.6	$4.82\pm0.45_{\rm exp}\pm0.30_{\rm SF+theo}$
CLEO 9fb ⁻¹	2.2-2.6	$4.09 \pm 0.48_{exp} \pm 0.36_{s_{F+theo}}$

Shape Function: determined from the OPE fit

18 Ma Theory errors: Lange et al. PRD72:073006



$q^2 E_{\ell}$ analysis with v reconstruction

• High energy electron $E_{e} > 2.1 \text{ GeV}$



- Missing momentum used for neutrino parameters estimation
- Cuts on missing momentum magnitude and direction and event shape
- Suppress background using: $s_h^{max} = m_B^2 + q^2 2m_B(E_e + \frac{q^2}{4E})$



 ΔB (2.0,3.5) = (3.54 ± 0.33 ± 0.34) x 10⁴

$$|V_{ub}| = (4.10 \pm 0.27_{exp} \pm 0.36_{SF+theo}) \times 10^{-3}$$

Hadronic BTag

 \mathbf{P}_+

=

 E_X

 $-P_{x}$

Fully reconstruct one B in hadronic decays

- Use the recoiling B with known charge and momentum
- Access to all kinematic variables





BABARhep-ex/0507017

Belle PRL 95:241801

S/B~2Eff~0.1%

~			
	Region	<i>V_{ub}</i> (10 ³)	
	m_{χ} < 1.7 GeV, q^2 > 8 GeV ²	$4.70 \pm 0.37_{exp} \pm 0.31_{sr+theo}$	
Belle 253M <i>B</i> B	m_{χ} < 1.7 GeV	$4.09\pm0.28_{\rm exp}\pm0.24_{\rm SF+theo}$	
	$P_{+} > 0.66 \text{GeV}$	$4.19 \pm 0.36_{exp} \pm 0.28_{sr+theo}$	
BABAR 210M BB	m_{χ} < 1.7 GeV, q^2 > 8 GeV ²	$4.75 \pm 0.35_{exp} \pm 0.32_{sr+theo}$	Preli

$|V_{ub}|$ from Inclusive $B \rightarrow X_u \ell v$



Theory Errors (BLNP)

Quark-hadron duality is not considered (cut dependent)

• $b \rightarrow dv$ and $b \rightarrow s\gamma$ data fit well HQ predictions

Weak annihilation $\rightarrow \pm 1.9\%$ error • Expected to be <2% of the total rate • $\Gamma_{w.a}/\Gamma(b \rightarrow u) < 7.4\%$ from CLEO



HQ parameters $\rightarrow \pm 4.1\%$ mainly m; kinematics cuts depend on $m_b!$

Sub-leading shape function $\rightarrow \pm 3.8\%$ dominated by the lepton endpoint measurements

Inclusive $|V_{ub}|$ (DGE framework)

Dressed Gluon Exponentiation (DGE)

on-shell b-quark calculation converted into hadronic variables used as approximation to the meson decay spectrum

 $|V_{ub}|^{DGE} = (4.41 \pm 0.20 \pm 0.20)10^{-3}$



 $m_{\rm m}$ (MS) $\rightarrow \pm 1.3\%$ on event fraction $m_{\rm m}$ (MS)=4.20 \pm 0.04 GeV

 $\alpha_{s} \rightarrow \pm 1.0\%$ on event fraction

total $\Gamma_{\rm SL}$ $\rightarrow \pm 3.0\%$

Dealing with Shape Function

Solution \rightarrow use directly the b \rightarrow sy spectrum



Possible to combine $b \to u\ell v$ and $b \to s\gamma$ so that the SF cancels $\Gamma(B \to X_u | v) = \frac{|V_{ub}|^2}{|V_{ts}|^2} W(E_{\gamma}) \frac{d\Gamma(B \to X_s\gamma)}{dE_{\gamma}} dE_{\gamma}$ 18 May 2006

Weight function

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SF-Free V_{ub} Measurement BABAR applied Leibovich, Low, Rothstein (PLB 486:86) to 80 fb⁻¹ data Full $|V_{ub}| \times 10^3$ 8 Theory error rate $|V_{ub}|$ (10³) $m_{\rm x}$ cut Expt. error $4.43 \pm 0.45_{\text{exp}} \pm 0.29_{\text{theo}}$ 1.67 GeV $3.84 \pm 0.76_{exc} \pm 0.10_{theo}$ 2.5 GeV 2 1.67 2 m_{χ} cut (GeV)

- Trade SF error → Stat. error
- m_x < 2.5 GeV is almost (96%) fully inclusive → Theory error reduces to ±2.6%





Exclusive Measurements

- Exclusive rates determined by $|V_{xb}|$ and Form Factors (FF)
 - Theoretically calculable at kinematical limits
 - Lattice QCD works if D^{*} or π is
 ~ at rest relative to B
 - Empirical extrapolation is necessary to extract $|V_{xb}|$ from measurements
- Measure differential rates to constrain the FF shape, then use FF normalization from the theory for $|V_{ub}|$











- F(1) = 1 in the heavy-quark limit; lattice QCD: $F(1) = 0.919^{+0.030}_{-0.035}$ Hashimoto et al,
- $\underline{F}(w)$ shape expressed by ρ^2 (slope at w=1) and R_1, R_2 (form factor ratios)
 - Curvature constrained by analyticity Caprini,

Caprini, Lellouch, Neubert NPB530 (1998) 153

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PRD 66 (2002) 014503

BaBar Exclusive |V_{cb}| Measurement

- Measure decay angles θ_{ℓ} , θ_{v} , χ
 - Fit 3-D distribution in bins of w to extract p², R₁, R₂



• Multi-Dimensional fit to helicity amplitudes for ρ^2 (slope parameter) and R_1 , R_2 (FF ratios), all functions $\frac{d\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_V d\chi} = \frac{3G_F^2 |V_{cb}|^2 \wp_D \cdot q^2}{8(4\pi)^4 M_B^2} \mathcal{B}_{D^*D} \times$

• of $H_{+,-,0}(w)$.

$$\frac{d\Gamma}{l\cos\theta_{\ell}\,d\cos\theta_{V}\,d\chi} = \frac{3G_{F}^{2}|V_{cb}|^{2}\wp_{D}\cdot q^{2}}{8(4\pi)^{4}M_{B}^{2}}\mathcal{B}_{D}\cdot_{D}\times \\ \left[H_{+}^{2}(1-\cos\theta_{\ell})^{2}\sin^{2}\theta_{V}+\right. \\ \left.H_{-}^{2}(1+\cos\theta_{\ell})^{2}\sin^{2}\theta_{V}+\right. \\ \left.H_{0}^{2}\sin^{2}\theta_{\ell}\cos^{2}\theta_{V}-2H_{+}H_{-}\sin^{2}\theta_{\ell}\sin^{2}\theta_{V}\cos2\chi\right. \\ \left.-2H_{+}H_{0}\sin\theta_{\ell}(1-\cos\theta_{\ell})\sin\theta_{V}\cos\varphi_{V}\cos\chi\right. \\ \left.+4H_{-}H_{0}\sin\theta_{\ell}(1+\cos\theta_{\ell})\sin\theta_{V}\cos\varphi_{V}\cos\chi\right.$$



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Syst error from 1.7% to 1.3%

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$|V_{cb}|$ from $B \rightarrow D^* \ell v$

HFAG average still uses FF from CLEO





Exclusive $B \rightarrow \pi \ell v$

• $B \rightarrow \pi \ell \nu$ rate is given by

$$\frac{d\Gamma(B \to \pi 1 v)}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 p_\pi^3 |f_+(q^2)|^2$$
 One FF for $B \to \pi \ell v$
with massless lepton

Form factor $f_+(q^2)$ has been calculated using

Lattice QCD

- Unquenched calculations by Fermilab (hep-lat/0409116) and HPQCD (PRD73:074502)
- $\pm 12\%$ for $q^2 > 16$ GeV²
- Light Cone Sum Rules
 - Ball & Zwicky (PRD71:014015)

■ ±13% for q² < 16 GeV² 18 May 2006 F. D



Approaches	s to Measurir	ng $B \rightarrow \pi \ell v$
Untagged• initial 4-momentum known• missing 4-momentum = v • Reconstruct $B \rightarrow \pi \ell v$ • Reconstruct $B \rightarrow \pi \ell v$ using m_B (beam-constrained)and $\Delta E = E_B - E_{beam}$	 Pros High efficiency Cons y resolution problematic Rel. high backgrounds (rel. low purity) 	Y(4S) F ⁰ B ⁰ B ⁰ π ⁻ Restused to reconstruct ν
 Semileptonic (SL) Tag One B reconstructed in a selection of D^(*) t v modes Two missing v in event Use kinematic constraints 	 Pros Lower backgrounds (higher purity) Cons Rel. low efficiency 	$ \begin{array}{c} \pi^{+} & e^{+} \\ \overline{\mu}^{0} & p^{*} + & Y(4S) \\ \overline{\mu}^{+} & \overline{B}^{0} & B^{0} \\ \mu^{-} & \underline{Signal} \\ \end{array} $ Signal Signal
 Full Recon Tag One B reconstructed completely in known b → c mode. Many modes used. 	 Pros Very good y resolution Very low backgrounds Cons Very low efficiency 	π^{+} π^{+} π^{+} π^{+} π^{-} B^{0} B^{0} π^{-} $ExampleTag$ $sile$



- Boundaries indicative only
- Full recon tag will ultimately become method of choice

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¹⁸ May 2006



$|V_{ub}|$ from $B \rightarrow \pi \ell \nu$

Average BF measurements and apply FF calculations

$\Delta B(q^2 < 16) (10^4)$	$\Delta B(q^2 > 16) (10^4)$	Total B (104)
$0.94\pm0.06_{\mathrm{stat}}\pm0.06_{\mathrm{syst}}$	$0.39\pm0.04_{\text{stat}}\pm0.04_{\text{syst}}$	$1.34\pm0.08_{\text{stat}}\pm0.08_{\text{syst}}$



V_{ub}: CKM consistency



Other $B \rightarrow X_u \ell v$ Exclusive Modes

Expt	Made	Tag	BF [10 ⁴]	Reference
CLEO	$B^{o} \rightarrow \rho f \nu$	Untagged	2.17±0.34 ^{+0.47} ±0.41	PRD 68 (2003) 072003
CLEO	$B^{0} \rightarrow \rho - \ell \nu$	Untagged	2.69±0.41 ^{+0.35} -047±0.50	PRD 61 (2000) 052001
BaBar	$B^{0} \rightarrow \rho f \nu$	Full	2.57±0.52±0.59	hep-ex/0408068
BaBar	$B^{0} \rightarrow \rho - \ell \nu$	Untagged	3.29±0.42±0.47±0.60	PRL 90 (2003) 181801
BaBar	$B^{0} \rightarrow \rho - \ell \nu$	Untagged	2.14±0.21±0.51±0.28	PRD 72 (2005) 051102
Belle	$B^{0} \rightarrow \rho - \ell \nu$	SL	2.17±0.54±0.31±0.08	hep-ex/0604024
CLEO	$B^+ \rightarrow \eta \ell \nu$	Untagged	0.84±0.31±0.16±0.09	PRD 68 (2003) 072003
Belle	$B^+ \rightarrow \rho^o \ell \nu$	SL	1.33±0.23±0.17±0.05	hep-ex/06хоохх
Belle	$B^+ \rightarrow \omega \ell \nu$	Untagged	1.3±0.4±0.2±0.3	PRL 93 (2004) 131803

HFAG Compilation, except with updated Belle SL result included

Weak Annihilation

Weak Annihilation

- WA turns B^+ into ℓv + soft hadrons
- Size and shape of WA poorly known
- Minimize the impact
 - Measure $X_u \ell v$ with v. loose cuts
 - Cut away large q² region
- Measure WA contribution
 - $\blacksquare \ \Gamma_{\rm sl}(D) \ {\rm vs.} \ \Gamma_{\rm sl}(D_{\rm s})$
 - CLEO-c
 - **Distortion** in q^2
 - CLEO hep-ex/0601027
 - $\ \ \, \Gamma(B^{\scriptscriptstyle \bullet} \to X_{\!_{\!\!\!\!\!\!\!}}^{}\ell\nu) \text{ vs. } \Gamma(B^{\scriptscriptstyle \bullet} \to X_{\!_{\!\!\!\!\!\!\!\!\!}}^{}\ell\nu)$

Work in progress in BaBar/BELLE 18 May 2006 F. Di Lodovico, QMUL



Summary

- Semileptonic B decays offer exciting physics opportunities
 - $|V_{ub}/V_{cb}|$ complements sin2 β to test (in)completeness of the SM
- Challenge of hadronic physics met by close collaboration between theory and experiment
 - Inclusive $B \rightarrow X_c \ell v \& X_s \gamma$ fit precisely determines $|V_{cb}|$, m_b , etc.
 - Dramatic progress in both measurement and interpretation of inclusive $B \rightarrow X_{\mu} \ell \nu$ in the last 2 years
- Inclusive $|V_{ub}|$ achieved $\pm 7.4\%$ accuracy
 - Room for improvements with additional data statistics
- $B \rightarrow D\ell v$ form factors have improved by a factor 5
- Measurements of $B \rightarrow \pi \ell \nu$ becoming precise (better FF needed!) ¹⁸ May 2006 F. Di Lodovico, QMUL

Tuning Δ into V_{ub}

Determining the Form Factor $f_+(q^2)$

- Earlier predictions made with quark models, e.g. ISGW2
- Lattice QCD
 - makes predictions at high q^2 ($q^2 > \sim 16 \text{ GeV}^2$)
 - unquenched calculations have become available in recent times
 - e.g. HPQCD
 FNAL
 hp-bt-0409116
- Light Cone Sum Rules
 - makes predictions at low q^2 ($q^2 < -14 \text{ GeV}^2$)
 - e.g. Ball & Zwicky PRD 71 (2005)014015
- Parametrization has traditionally been used to extend LQCD or LCSR to full q² range
- FF normalization main issue when extracting |V_{ub}|
- How much theory input needed for |V_{ub}| measurement?

hppph/0509090

Summary of $B \rightarrow \pi \ell \nu$

BABAR SL tag: $B^+ \rightarrow \pi^0 l^+ \nu \times 2\tau_0/\tau_+$ $3.31 \pm 0.68 \pm 0.42$ BABAR Breco tag: B⁺ $\rightarrow \pi^0 l^+ \gamma \times 2\tau_0/\tau_+$ $1.60 \pm 0.41 \pm 0.21$ BELLE SL tag: B $^+ \to \pi^0 \; l^+ \; \nu \times 2 \tau_{n} / \tau_{\scriptscriptstyle \perp}$ $1.37 \pm 0.24 \pm 0.17$ BABAR SL tag: $B^{0} \rightarrow \pi^{-} l^{+} v$ $1.02 \pm 0.25 \pm 0.13$ BELLE SL tag: $B^{0} \rightarrow \pi^{-} l^{+} \gamma$ $1.46 \pm 0.20 \pm 0.16$ BABAR Breco tag: $B^0 \rightarrow \pi^- l^+ \nu$ π $1.14 \pm 0.27 \pm 0.17$ CLEO untagged: B $\rightarrow \pi l^+ v$ $1.32 \pm 0.18 \pm 0.13$ BABAR untagged: B $\rightarrow \pi l^+ \nu$ $1.38 \pm 0.10 \pm 0.18$ Average: $B^0 \rightarrow \pi^- l^+ \nu$ $1.34 \pm 0.08 \pm 0.08$ $\chi^2/dof = 8.4/7 (CL = 30.1)$ HFAG Winter 2006 -2 0 $B(B^0 \rightarrow \pi^- l^+ \nu) [\times 10^{-4}]$

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$|V_{ub}|$ from Exclusive $B \rightarrow \pi V$

Ball-Zwicky q2 < 16 Ball-Zwicky full q2 $3.36 \pm 0.15 \pm 0.66 - 0.41$ $3.36 \pm 0.15 \pm 0.55 \pm 0.37$ HPQCD full q2 HPQCD q2 > 16 $3.81 \pm 0.16 \pm 0.82 \pm 0.50$ $4.20 \pm 0.29 \pm 0.63 \pm 0.43$ FNAL full q2 FNAL q2 > 16 $3.74 \pm 0.16 + 0.86 - 0.51$ $3.75 \pm 0.26 \pm 0.65 \pm 0.43$ APE full q2 APE q2 > 16 $3.53 \pm 0.15 + 1.08 - 0.56$ 3.78 ± 0.26 + 1.45 - 0.67 UsingFF calculated in region UsingFF extrapolated to fulldame ofaplicability HFAG HFAG Winter 2006 Winter 2006 2 4 2 4 $|V_{ub}| [\times 10^{-3}]$ $|V_{ub}| [\times 10^{-3}]$ I. LI LODOVICO, GIVIUL 10 IVERY 2000 UZ

Experimental of shape input not used (yet)

