Space-time reduction in large-N gauge theories: the view from the lattice

Barak Bringoltz
University of Washington

Numerical results obtained with S. Sharpe `08-`09:

“Non-perturbative volume-reduction of large-N QCD with adjoint fermions.”, PRD, **arXiv:0906.3538**

“Breakdown of large-N quenched reduction in SU(N) lattice gauge theories.”, PRD, **arXiv:0805.2146**

Outline

I. What is space-time reduction.

II. Space-time reduction with adjoint fermions:

A) Weak coupling perturbative analysis.

B) Non-perturbative lattice Monte-Carlo studies with S. Sharpe `09

III. Conclusions + future prospects.

I. What is the space-time reduction?

Given a torus ($L_1 x L_2 x L_3 x L_4$), with an SU(N) lattice gauge theory $(g^2 N, am, a\mu, \ldots)$

Then if:

- Translation symmetry is intact.
- Z_N center symmetry is intact.
- large-N factorization holds.

$\xrightarrow{\text{at } N=\infty}$ Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

- Reduce cost of large-N lattice studies.
- Leads to analytic weak-coupling small-volume methods (Unsal `07).

I. What is the space-time reduction?

Given a torus ($L_1 x L_2 x L_3 x L_4$), with an SU(N) lattice gauge theory $(g^2 N, am, a\mu, \ldots)$

Then if:

- Translation symmetry is intact.
- Z_N center symmetry is intact.
- large-N factorization holds.

Conditions derived by:

- Orbifold projections.
  Neuberger `02,
  Kovtun-Unsal-Yaffe `06
- Dyson-Schwinger Eqs.
  Eguchi-Kawai `82
- Perturbation theory.
  Bhanot-Heller-Neuberger`82,
  Gross-Kitazawa `82, Parisi-Zhang `82, ...

$\xrightarrow{\text{at } N=\infty}$ Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

- Reduce cost of large-N lattice studies.
- Leads to analytic weak-coupling small-volume methods (Unsal `07).

How to argue it can be valid? can derive Dyson-Schwinger Eqs.

Eguchi
Kawai `82

Show that these Eqs' are independent of $L_{1,2,3,4}$

How to argue it can be valid? can derive Dyson-Schwinger Eqs.

Eguchi
Kawai `82

Show that these Eqs' are independent of L1,2,3,4

$$A_\mu(x) \to U_{n,\mu} \in SU(N) \qquad @ \quad \text{periodic BC}$$

$$S_{\text{gauge}} = Nb \sum_{\substack{n \\ \mu<\nu}} 2\text{Re}\,\text{Tr}\left(U_{n,\mu} U_{n+\mu,\nu} U^\dagger_{n+\nu,\mu} U^\dagger_{n,\nu}\right)$$

$$b = (g^2 N)^{-1}$$

$$W_C = \frac{1}{N} \text{tr}\; U_{x,\hat\mu} U_{x+\hat\mu,\hat\nu} \cdots U_{x-\hat\nu-\hat\rho,\hat\rho} U_{x-\hat\nu,\hat\nu}\,,$$

$$U_{n\mu} \to \Omega_n\, U_{n\mu}\, \Omega^\dagger_{n+\mu} \quad ; \quad \Omega_n \in SU(N)$$

$$U_{[(\vec n,\tau),\mu]} \to U_{[(\vec n,\tau),\mu]}\, z_\mu \quad ; \quad z_\mu \in Z_N$$

How to argue it can be valid? can derive Dyson-Schwinger Eqs.

Eguchi
Kawai `82

Show that these Eqs' are independent of L1,2,3,4

$A_\mu(x) \to U_{n,\mu} \in SU(N)$ @ periodic BC	$U_\mu \in SU(N)$
$S_{\text{gauge}} = Nb \sum_{\substack{n \\ \mu<\nu}} 2\text{Re}\,\text{Tr}\left(U_{n,\mu}U_{n+\mu,\nu}U^\dagger_{n+\nu,\mu}U^\dagger_{n,\nu}\right)$ $b = (g^2N)^{-1}$	$S_{EK} = Nb \sum_{\mu<\nu} 2\text{Re}\,\text{Tr}\,(U_\mu U_\nu U^\dagger_\mu U^\dagger_\nu)$ $b = (g^2N)^{-1}$
$W_C = \frac{1}{N}\,\text{tr}\; U_{x,\hat\mu}U_{x+\hat\mu,\hat\nu}\cdots U_{x-\hat\nu-\hat\rho,\hat\rho}U_{x-\hat\nu,\hat\nu}\,,$	$W_C^{\text{reduced}} = \frac{1}{N}\,\text{tr}\; U_\mu U_\nu \cdots U_\rho U_\nu\,.$
$U_{n\mu} \to \Omega_n\, U_{n\mu}\, \Omega^\dagger_{n+\mu} \quad ; \quad \Omega_n \in SU(N)$ $U_{[(\vec{n},\tau),\mu]} \to U_{[(\vec{n},\tau),\mu]}\, z_\mu \quad ; \quad z_\mu \in Z_N$	$U_\mu \to \Omega\, U_\mu\, \Omega^\dagger \quad ; \quad \Omega \in SU(N)$ $U_\mu \to U_\mu z_\mu \quad ; \quad z_\mu \in Z_N$

How to argue it can be valid? can derive Dyson-Schwinger Eqs.

Eguchi
Kawai `82

- Like χS Ward identities :

$$U_{n,\mu} \to U_{n\mu}\,(1 + i\epsilon t^a) \qquad \text{gauge}$$

$$U_{\mu} \to U_{\mu}\,(1 + i\epsilon t^a) \qquad \text{reduced}$$

- Crucial difference :

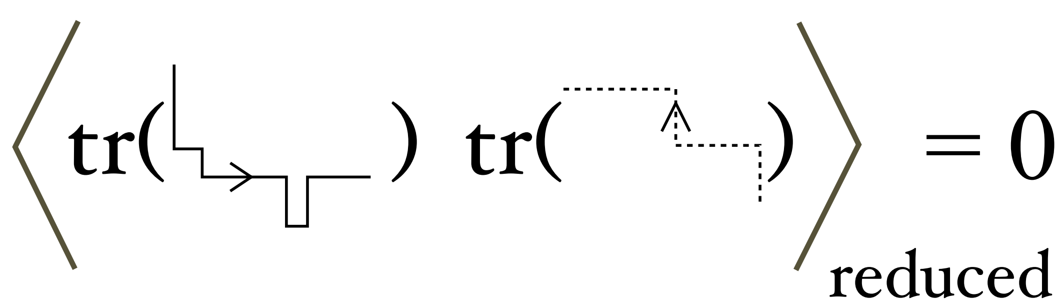
gauge

$$\mathrm{Tr}\,(\cdots U_{n\mu} U_{n+\mu,\nu} \cdots U_{m,\mu} U_{m+\mu,\rho} \cdots)$$

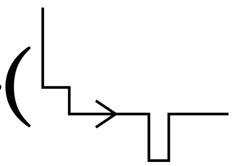
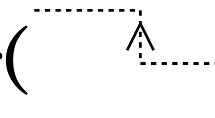
reduced

$$\mathrm{Tr}\,(\cdots U_{\mu} U_{\nu} \cdots U_{\mu} U_{\rho} \cdots)$$

- Get extra terms on the reduced side, so for EK reduction to hold :



e.g. $$\left\langle \mathrm{tr}\left(U_{\mu} U_{\nu}^{\dagger}\right)\ \mathrm{tr}\left(U_{\mu}^{\dagger} U_{\nu}\right)\right\rangle_{\text{reduced}} = 0$$

- Reduction holds if $\left\langle \mathrm{tr}(\right.$  $)\ \mathrm{tr}($  $\left.)\right\rangle_{\text{reduced}} = 0$

$$\textcolor{red}{1.}\quad \langle W_{C_1}\, W_{C_2}\rangle_{\text{reduced}} \;=\; \langle W_{C_1}\rangle_{\text{reduced}}\, \langle W_{C_2}\rangle_{\text{reduced}} + O(1/N^2),$$

$$\textcolor{red}{2.}\quad \langle W_{\text{open}}\rangle_{\text{reduced}} \;=\; 0. \qquad \text{or} \quad U_\mu \to U_\mu z_\mu \quad ; \quad z_\mu \in Z_N \quad \text{intact}$$

e.g. $W_{\text{open}} = \mathrm{tr}\, U_\mu$

$$\langle W_C\rangle_{\text{gauge theory}} = \langle W_C^{\text{reduced}}\rangle_{\text{reduced}} + O(1/N^2).$$

$$\text{Lattice } SU(N) \text{ on } L^d \;\overset{N=\infty}{\equiv}\; \text{Lattice } SU(N) \text{ on } 1^d$$

I. What is the space-time reduction

Given SU(N) gauge theory on an $L_1 \times L_2 \times L_3 \times L_4$ lattice, defined by $g^2 N, am, a\mu, \ldots$

Then if:

- Translation symmetry is intact.
- Z_N center symmetry is intact.
- large-N factorization holds.

$\xrightarrow{\text{at } N=\infty}$ Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

I. What is the space-time reduction

Given SU(N) gauge theory on an $L_1 \times L_2 \times L_3 \times L_4$ lattice, defined by $g^2 N, am, a\mu, \ldots$

Then if:

- Translation symmetry is intact.
- Z_N center symmetry is intact.
- large-N factorization holds.

at N=oo $\longrightarrow$ Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

This talk: fate of Z_N. First at weak coupling, then non-perturbatively

I. What is the space-time reduction

Given SU(N) gauge theory on an $L_1 x L_2 x L_3 x L_4$ lattice, defined by $g^2 N, am, a\mu, \ldots$

Then if:

- Translation symmetry is intact.
- Z_N center symmetry is intact.
- large-N factorization holds.

Not "academic" requirements:

breakdown of EK equivalence by formation of a baryon crystal
BB `08, BB `09

QEK model
BB+Sharpe `08

at N=oo → Wilson loops, Hadron spectra, condensates, etc. are independent of $L_{1,2,3,4}$.

This talk: fate of Z_N. First at weak coupling, then non-perturbatively

I. Some History

- Original Eguchi-Kawai. Jan `82. But Bhanot-Heller-Neuberger Feb `82.
- Quenched Eguchi-Kawai. (Also, Parisi, Gross-Kitazawa, Das-Wadia, Migdal-Kazajov) Bhanot Heller Neuberger, Feb `82. But BB-Sharpe, 2008.
- Twisted Eguchi-Kawai. Gonzalez-Arroyo Okawa, July `82. But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

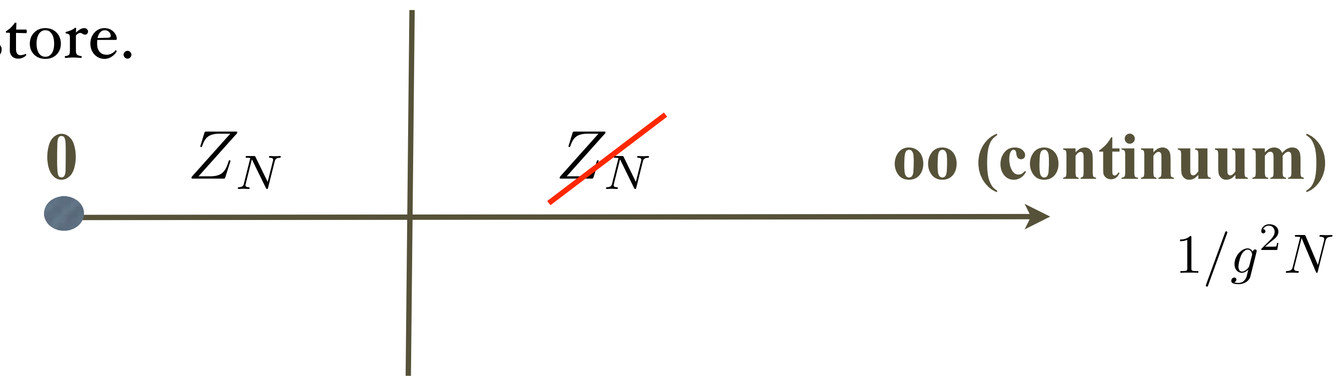
- Adjoint Eguchi-Kawai. Kovtun-Unsal-Yaffe, 2007. Most of this talk.
- Deformed Eguchi-Kawai. Unsal-Yaffe, 2008. BB, Vairinhos, 2008

I. Some History

- Original Eguchi-Kawai. Jan `82. But Bhanot-Heller-Neuberger Feb `82.
- Quenched Eguchi-Kawai. (Also, Parisi, Gross-Kitazawa, Das-Wadia, Migdal-Kazajov) Bhanot Heller Neuberger, Feb `82 But BB-Sharpe, 2008.
- Twisted Eguchi-Kawai. Gonzalez-Arroyo Okawa, July `82 But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

Remarks **original EK**:

- The Bhanot-Heller-Neubgerger `82 is suggestive but not conclusive.
- At moderate coupling symmetry may restore.

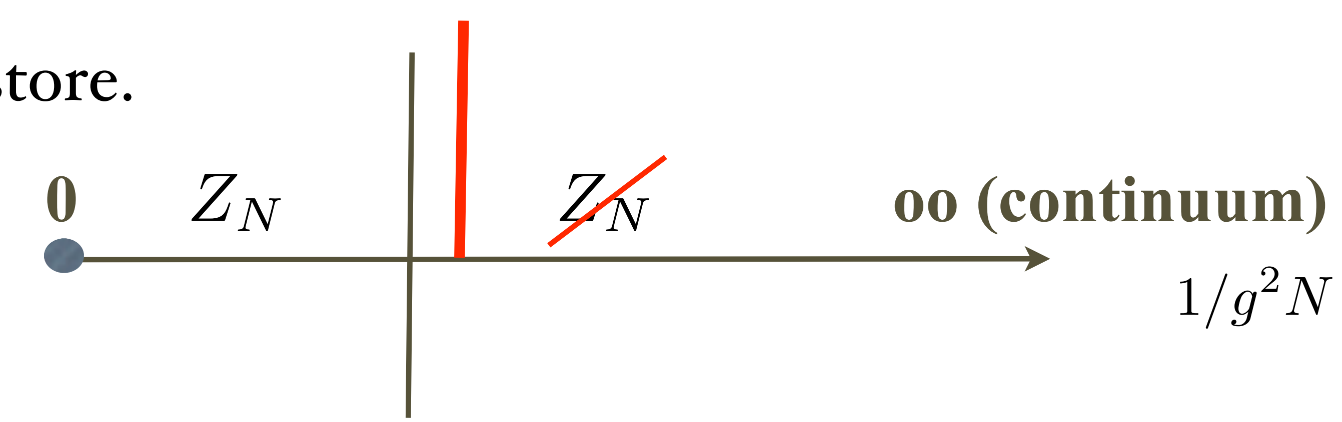


I. Some History

- Original Eguchi-Kawai. Jan `82. But Bhanot-Heller-Neuberger Feb `82.
- Quenched Eguchi-Kawai. Bhanot Heller Neuberger, Feb `82 But BB-Sharpe, 2008.
  (Also, Parisi, Gross-Kitazawa, Das-Wadia, Migdal-Kazajov)
- Twisted Eguchi-Kawai. Gonzalez-Arroyo Okawa, July `82 But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

Remarks **original EK**:

- The Bhanot-Heller-Neubgerger `82 is suggestive but not conclusive.
- At moderate coupling symmetry may restore.



I. Some History

- Original Eguchi-Kawai. Jan `82. But Bhanot-Heller-Neuberger Feb `82.

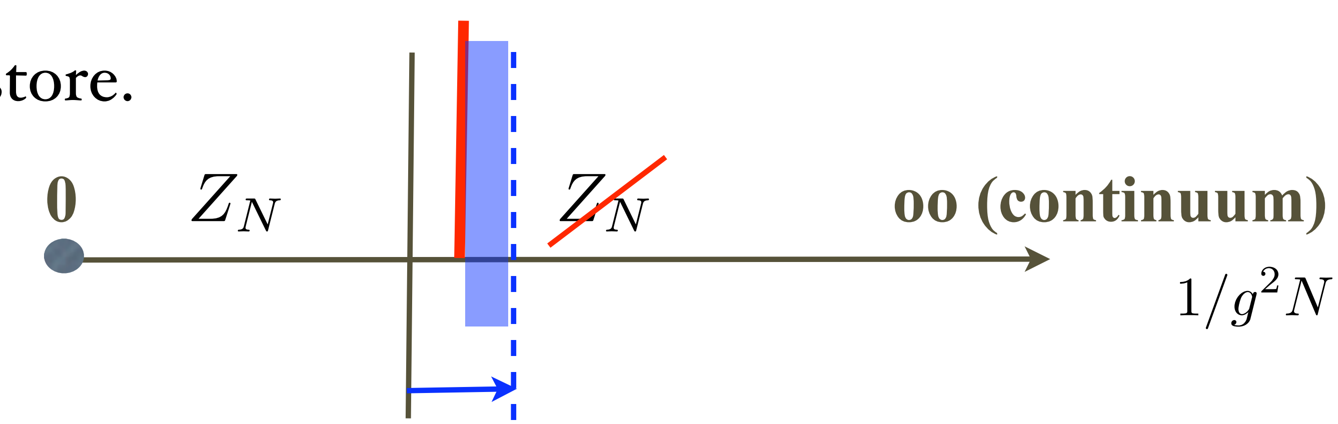
- Quenched Eguchi-Kawai. Bhanot Heller Neuberger, Feb `82 But BB-Sharpe, 2008.
  (Also, Parisi, Gross-Kitazawa, Das-Wadia, Migdal-Kazajov)

- Twisted Eguchi-Kawai. Gonzalez-Arroyo Okawa, July `82 But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

Remarks **original EK**:

- The Bhanot-Heller-Neubgerger `82 is suggestive but not conclusive.

- At moderate coupling symmetry may restore.



- Indeed, Neuberger-Naraynan-et-al. `02: increase $1^d \rightarrow L^d$.

I. Some History

- Original Eguchi-Kawai. Jan `82. But Bhanot-Heller-Neuberger Feb `82.

- Quenched Eguchi-Kawai. Bhanot Heller Neuberger, Feb `82 But BB-Sharpe, 2008.
  (Also, Parisi, Gross-Kitazawa, Das-Wadia, Migdal-Kazajov)

- Twisted Eguchi-Kawai. Gonzalez-Arroyo Okawa, July `82 But, Teper-Vairinhos, Hanada-et-al, Bietenholtz et al. 2006.

- Adjoint Eguchi-Kawai. Kovtun-Unsal-Yaffe, 2007 Most of this talk.

- Deformed Eguchi-Kawai. Unsal-Yaffe, 2008 BB, Vairinhos, 2008

II. Space-time reduction with adjoint fermions

Adjoint fermions are interesting:

- N_f=1/2 is softly broken $\mathcal{N}$=1 SUSY.
- N_f=2 is in (or close by to) the conformal window.
- Any value of N_f and heavy enough quarks is YM.

Can study large-N limit of all these with method I will describe.

Main motivation: study the N_f=1 theory

II. Space-time reduction with adjoint fermions

A route to the orientifold limit

Study a large-N limit of QCD where quarks are back-reacting on gauge fields

→ Natural to put quarks in two-antisymmetric :

QCD(AS)
Corrigan-Ramond,
Armoni-Shifman-Veneziano,
Sannino et al.

Armoni-Veneziano-Shifman Planar equivalence `03:

"orientifold equivalence"

QCD(AS)
infinite volume
$2N_f$ fermions

↔

QCD(Adj)
infinite volume
N_f fermions

→ Study 1-flavor adjoint QCD gives physical QCD

Want to study this on a single site: But is the Z_N symmetry intact for this theory ?

II.A. Weak coupling analysis, continuum

Kovtun-Unsal-Yaffe, 2007

- Work on $R^3 x S^1$ with PBC

- Calculate V_{eff} for Polyakov loops Ω in continuum perturbation theory.

$$V(\Omega) = V^{1-\mathrm{loop}}_{\mathrm{Glue}}(\Omega) - 2N_f\, V^{1-\mathrm{loop}}_{\mathrm{Fermi}}(\Omega) = (1-2N_f)\, V^{1-\mathrm{loop}}_{\mathrm{Glue}}(\Omega) \quad ; \quad \mathrm{mass} = 0$$

causes eigenvalue attraction $\mathrm{tr}(\Omega) \neq 0$

- Z_N broken if $N_f = 0$ (failure of EK model, or deconfinement)

- Z_N unbroken for for $N_f = 1,2$.

- Result is suggestive: not lattice, not single site, only perturbative

II.A. Weak coupling analysis, lattice, $L_{2,3,4}$ = oo, **L_1=1.** BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\, \delta_{ab})$

II.A. Weak coupling analysis, lattice, $L_{2,3,4}$ = oo, **L_1=1.** BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\, \delta_{ab})$

Get $$V(\theta) = \sum_{a\neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a - \theta^b}{2}\right)\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a - \theta^b\right) + m_W^2(\theta, p)\right] \right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3} \sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3} \sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + \frac{1}{2}\left[\hat{p}^2 + 4\sin^2\left((\theta^a - \theta^b)/2\right)\right]$$

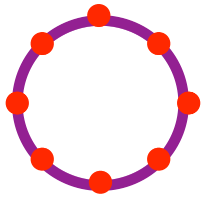
II.A. Weak coupling analysis, lattice, $L_{2,3,4} = \infty$, **$L_1=1$.** BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\,\delta_{ab})$

Get $$V(\theta) = \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3\left\{\log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] - 2N_f\log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a-\theta^b\right) + m_W^2(\theta,p)\right]\right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3}\sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3}\sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + \frac{1}{2}\left[\hat{p}^2 + 4\sin^2\left((\theta^a-\theta^b)/2\right)\right]$$

- What is the value of the one-loop potential at the ZN invariant g.s. ? θ^a :



$$\sum_{a\neq b} f(\theta_a-\theta_b) \xrightarrow{N\to\infty} N^2\int\frac{d\omega}{2\pi}\,f(\omega) \longrightarrow V(Z_N) = N^2\int\frac{d\omega}{2\pi}\int\left(\frac{dp}{2\pi}\right)^3\left\{\log\left[\hat{p}^2+\hat{\omega}^2\right] - 2N_f\log\left[\hat{\hat{p}}^2 + \hat{\hat{\omega}}^2 + m_W^2(\omega,p)\right]\right\}$$

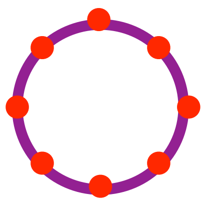
II.A. Weak coupling analysis, lattice, $L_{2,3,4} = \infty$, **$L_1=1$.** BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\,\delta_{ab})$

Get $$V(\theta) = \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3\left\{\log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] - 2N_f\log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a-\theta^b\right) + m_W^2(\theta,p)\right]\right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3}\sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3}\sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + \frac{1}{2}\left[\hat{p}^2 + 4\sin^2\left((\theta^a-\theta^b)/2\right)\right]$$

- What is the value of the one-loop potential at the ZN invariant g.s. ? θ^a :



$$\sum_{a\neq b} f(\theta_a-\theta_b) \xrightarrow{N\to\infty} N^2\int\frac{d\omega}{2\pi}f(\omega) \longrightarrow V(Z_N) = N^2\int\frac{d\omega}{2\pi}\int\left(\frac{dp}{2\pi}\right)^3\left\{\log\left[\hat{p}^2+\hat{\omega}^2\right] - 2N_f\log\left[\hat{\hat{p}}^2 + \hat{\hat{\omega}}^2 + m_W^2(\omega,p)\right]\right\}$$

- As if L1=oo : "Embedding of space-time in color space"

perturbation theory: Bhanot-Heller-Neuberger, Gross-Kitazawa, Parisi-Zhang `82, Neuberger `02

beyond perturbation? the soluble 1+1 case: Schon-Thies `01, BB `08.

II.A. Weak coupling analysis, lattice, $L_{2,3,4}$ = oo, **L_1=1.** BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\,\delta_{ab})$

Get $$V(\theta) = \sum_{a\neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a - \theta^b}{2}\right)\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a - \theta^b\right) + m_W^2(\theta, p)\right] \right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3} \sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3} \sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + \frac{1}{2}\left[\hat{p}^2 + 4\sin^2\left((\theta^a - \theta^b)/2\right)\right]$$

II.A. Weak coupling analysis, lattice, $L_{2,3,4}$ = oo, **L_1=1.** BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\,\delta_{ab})$

Get $$V(\theta) = \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3\left\{\log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] - 2N_f\log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a-\theta^b\right) + m_W^2(\theta,p)\right]\right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3}\sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3}\sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + \frac{1}{2}\left[\hat{p}^2 + 4\sin^2\left((\theta^a-\theta^b)/2\right)\right]$$

- Calculate $V(\theta)$ potential for different θ^a corresponding $Z_N,\, Z_N \to \emptyset,\, Z_N \to Z_2$

lattice units

$$\kappa = \frac{1}{8+2am_0}$$

massless

$$\kappa = 1/8$$

infinite mass

$$\kappa = 0$$

II.A. Weak coupling analysis, lattice, $L_{2,3,4}$ = oo, L_1=1. BB, `09

- Lattice one loop + Wilson fermions and axial gauge $(\Omega_{ab} = e^{i\theta_a}\,\delta_{ab})$

Get
$$V(\theta) = \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3\left\{\log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] - 2N_f\log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a-\theta^b\right) + m_W^2(\theta,p)\right]\right\}$$

$$\hat{p}^2 = 4\sum_{i=1}^{3}\sin^2 p_i/2 \xrightarrow{a\to 0} \vec{p}^2 \qquad \hat{\hat{p}}^2 = \sum_{i=1}^{3}\sin^2 p_i \xrightarrow{a\to 0} \vec{p}^2 \qquad m_W = am_0 + \frac{1}{2}\left[\hat{p}^2 + 4\sin^2\left((\theta^a-\theta^b)/2\right)\right]$$

- Calculate $V(\theta)$ potential for different θ^a corresponding $Z_N,\ Z_N \to \emptyset,\ Z_N \to Z_2$

lattice units

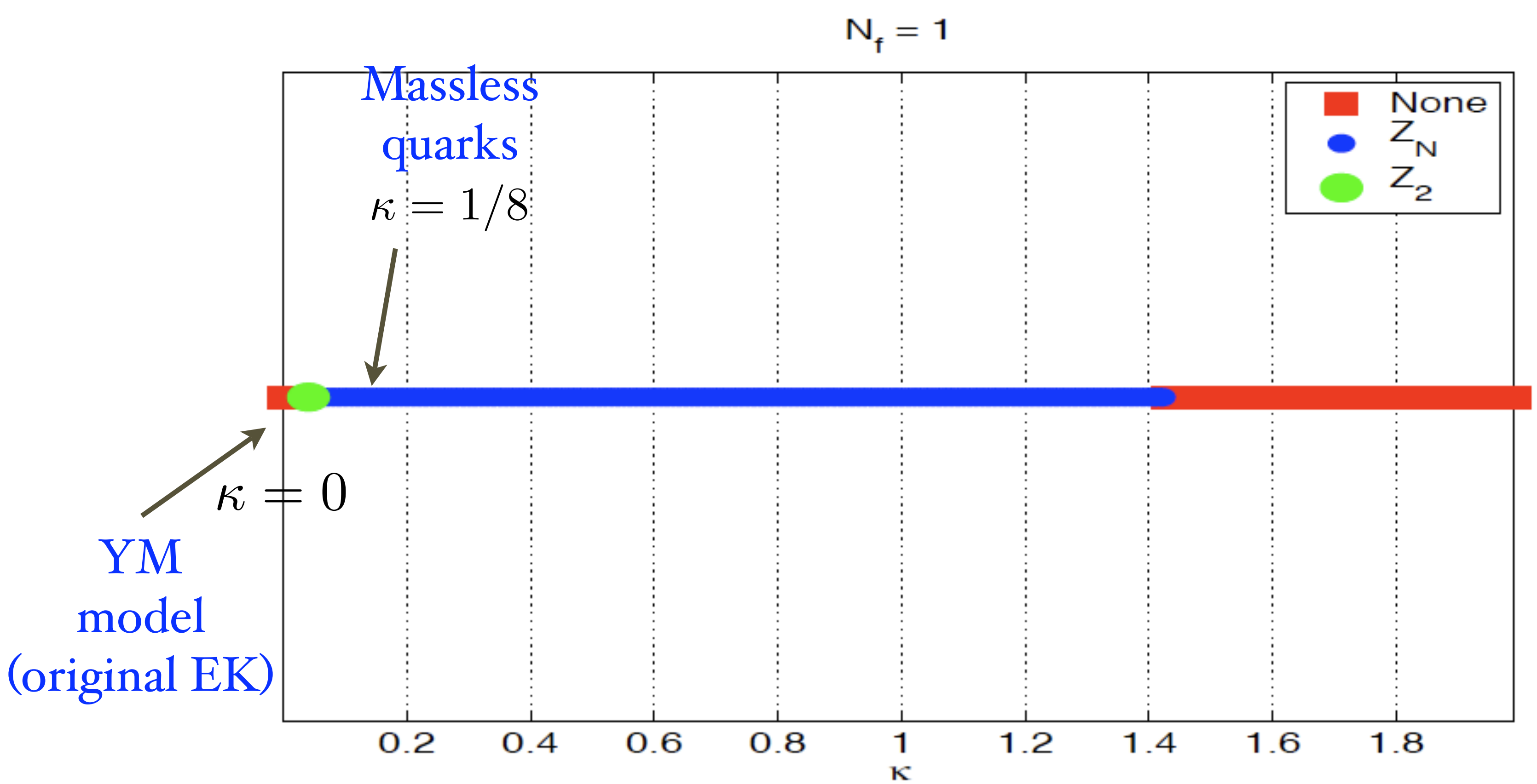
$$\kappa = \frac{1}{8+2am_0}$$

massless

$$\kappa = 1/8$$

infinite mass

$$\kappa = 0$$



These are good news:

Reduction works with **m=0** !

II.A. Weak coupling analysis: related developments

- Bedaque et al. `09: Treat $L_1=1$ as 3D theory in a spatial continuum: $Z_N \longrightarrow Z_2$ at $m=0$

II.A. Weak coupling analysis: related developments

- Bedaque et al. `09: Treat $L_1=1$ as 3D theory in a spatial continuum: $Z_N \longrightarrow Z_2$ at m=0
- But can show that this result is UV sensitive BB `09

$$S_{\text{gauge}} = \frac{2N}{\lambda}\,\text{Re}\sum_{\substack{x\\ i<j}} \text{Tr}\left(U_{x,i}U_{x+i,j}U^\dagger_{x+j,i}U^\dagger_{x,j}\right)$$

$$+\frac{2N}{\lambda}\,\text{Re}\sum_{\substack{x\\ i}} \text{Tr}\left(U_{x,i}\Omega_{x+i}U^\dagger_{x,i}\Omega^\dagger_{x}\right)$$

$$\xrightarrow{\text{“}a_s \to 0\text{”}}$$

$$S^{\text{one-site}} = \int d^3x \left(\frac{1}{g^2}\text{Tr}\sum_{i<j\in[1,3]} F_{ij}^2 + f^2\,\text{Tr}\sum_i |D_i\,\Omega|^2\right)$$

$$D_i\Omega(x) = \partial_i\Omega(x) + i[A_i,\Omega]$$

$$\Omega \in SU(N)$$

II.A. Weak coupling analysis: related developments

- Bedaque et al. `09: Treat $L_1=1$ as 3D theory in a spatial continuum: $Z_N \longrightarrow Z_2$ at $m=0$
- But can show that this result is UV sensitive BB `09

$$S_{\text{gauge}} = \frac{2N}{\lambda}\,\text{Re}\sum_{\substack{x\\ i<j}} \text{Tr}\left(U_{x,i}U_{x+i,j}U^{\dagger}_{x+j,i}U^{\dagger}_{x,j}\right)$$

$$+\frac{2N}{\lambda}\,\text{Re}\sum_{\substack{x\\ i}} \text{Tr}\left(U_{x,i}\Omega_{x+i}U^{\dagger}_{x,i}\Omega^{\dagger}_{x}\right)$$

$$\xrightarrow{\text{“}a_s \to 0\text{”}}$$

$$S^{\text{one-site}} = \int d^3x \left(\frac{1}{g^2}\text{Tr}\sum_{i<j\in[1,3]} F_{ij}^2 + f^2\,\text{Tr}\sum_i |D_i\,\Omega|^2\right)$$

$$D_i\Omega(x) = \partial_i\Omega(x) + i[A_i,\Omega]$$

$$\Omega \in SU(N)$$

$$V(\theta) = \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3 \log\left[a_t^2p^2 + \sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] = \Lambda^3 + \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3 \log\left[1 + \frac{1}{a_t^2p^2}\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right]$$

II.A. Weak coupling analysis: related developments

- Bedaque et al. `09: Treat $L_1=1$ as 3D theory in a spatial continuum: $Z_N \longrightarrow Z_2$ at $m=0$
- But can show that this result is UV sensitive BB `09

$$S_{\text{gauge}} = \frac{2N}{\lambda}\,\mathrm{Re}\sum_{\substack{x\\ i<j}} \mathrm{Tr}\left(U_{x,i}U_{x+i,j}U^{\dagger}_{x+j,i}U^{\dagger}_{x,j}\right) + \frac{2N}{\lambda}\,\mathrm{Re}\sum_{\substack{x\\ i}} \mathrm{Tr}\left(U_{x,i}\Omega_{x+i}U^{\dagger}_{x,i}\Omega^{\dagger}_{x}\right)$$

$$\xrightarrow{\text{“}a_s \to 0\text{”}}$$

$$S^{\text{one-site}} = \int d^3x \left(\frac{1}{g^2}\mathrm{Tr}\sum_{i<j\in[1,3]} F_{ij}^2 + f^2\,\mathrm{Tr}\sum_i |D_i\,\Omega|^2\right)$$

$$D_i\Omega(x) = \partial_i\Omega(x) + i[A_i,\Omega]$$

$$\Omega \in SU(N)$$

$$V(\theta) = \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3 \log\left[a_t^2p^2 + \sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right] = \Lambda^3 + \sum_{a\neq b}\int\left(\frac{dp}{2\pi}\right)^3 \log\left[1 + \frac{1}{a_t^2p^2}\sin^2\left(\frac{\theta^a-\theta^b}{2}\right)\right]$$

$$= \Lambda^3 + \Lambda\sum_{a\neq b}\sin^2\left(\frac{\theta^a-\theta^b}{2}\right) + \dots$$

$$= \Lambda^3 + \Lambda\,|\mathrm{tr}\,\Omega_{\text{classical}}|^2 + \dots,\ \Omega^{ab}_{\text{classical}} = e^{i\theta^a}$$

$\longrightarrow$ $S^{\text{one-site}}$ is non-renormalizable ... need counter-terms...

Bernard&Appelquist `80, Longhitano `80, Banks&Ukawa `84, Gasser-Leutwyler '84, Arkani-Hamed-Cohen-Georgi `01, Pisarski `06,

II.A. Weak coupling analysis: related developments

What does this teach us?

- Continuum limit in space with **$L_1=1$** (or for any **$D > 2L_1$**). Neuberger `02
  - need counter-terms $\longrightarrow$ new **L**ow **E**nergy **C**onstants (**LEC**).
  - means treating theory as an **E**ffective **F**ield **T**heory (**EFT**), and at one-loop:

$$V(\theta) \longrightarrow V(\theta) + b_1 \left|\mathrm{tr}\, \Omega_{\mathrm{classical}}\right|^2 + b_2 \left|\mathrm{tr}\, \Omega^2_{\mathrm{classical}}\right|^2$$

Dim-reg hides this and implicitly sets $b_1=b_2=0 \longrightarrow b_{1,2} > 0$ fixes $Z_N \longrightarrow Z_2$ breaking.

II.A. Weak coupling analysis: related developments

What does this teach us?

- Continuum limit in space with **$L_1=1$** (or for any **$D > 2L_1$**). Neuberger `02
  - need counter-terms ⟶ new **L**ow **E**nergy **C**onstants (**LEC**).
  - means treating theory as an **E**ffective **F**ield **T**heory (**EFT**), and at one-loop:

$$V(\theta) \longrightarrow V(\theta) + b_1 \left|\mathrm{tr}\, \Omega_{\text{classical}}\right|^2 + b_2 \left|\mathrm{tr}\, \Omega^2_{\text{classical}}\right|^2$$

Dim-reg hides this and implicitly sets $b_1=b_2=0 \longrightarrow b_{1,2} > 0$ fixes $Z_N \longrightarrow Z_2$ breaking.

On $L_1=1$:

But Z_N-realization may depend on lattice action ...

Lattice results cannot be anticipated in advance (from Kovtun-Unsal-Yaffe `07)

II.A. Weak coupling analysis: related developments

What does this teach us?

- Continuum limit in space with **$L_1=1$** (or for any **$D > 2L_1$**). Neuberger `02
  - need counter-terms $\longrightarrow$ new **L**ow **E**nergy **C**onstants (**LEC**).
  - means treating theory as an **E**ffective **F**ield **T**heory (**EFT**), and at one-loop:

$$V(\theta) \longrightarrow V(\theta) + b_1 \left|\mathrm{tr}\,\Omega_{\mathrm{classical}}\right|^2 + b_2 \left|\mathrm{tr}\,\Omega^2_{\mathrm{classical}}\right|^2$$

Dim-reg hides this and implicitly sets $b_1=b_2=0 \longrightarrow b_{1,2} > 0$ fixes $Z_N \longrightarrow Z_2$ breaking.

On $L_1=1$:

But Z_N-realization may depend on lattice action ...

Lattice results cannot be anticipated in advance (from Kovtun-Unsal-Yaffe `07)

Update: Bedaque et al `09 (postscript): add b1,b2>0, and find $Z_N \longrightarrow Z_3$!

II.A. Weak coupling analysis: (very recent) related developments

II.A. Weak coupling analysis: (very recent) related developments

- Myers-Hollowood `09 generalized Kovtun-Unsal-Yaffe `07 to nonzero mass

  ** see also earlier Myers-Ogilvie`08

L = size of S1 For $ML > 0$ get $Z_N \to Z_K$ with $K \sim 1/ML$

II.A. Weak coupling analysis: (very recent) related developments

- Myers-Hollowood `09 generalized Kovtun-Unsal-Yaffe `07 to nonzero mass
  ** see also earlier Myers-Ogilvie`08

L = size of S1 For $ML > 0$ get $Z_N \to Z_K$ with $K \sim 1/ML$

- BB, in progress: Similar results seem to be obtained (preliminary)

$$V(\theta) = \sum_{a\neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a - \theta^b}{2}\right)\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a - \theta^b\right) + m_W^2(\theta, p)\right]\right\}$$

$$\equiv \sum_{a\neq b}\sum_r V_r\, e^{ir(\theta_a - \theta_b)} \underset{\Omega_{ab} = e^{i\theta_a}\delta_{ab}}{\uparrow}{=} \sum_r V_r\, |\mathrm{tr}\,\Omega^r|^2 + \text{const.}$$

$$V_r = \int \frac{d\omega}{2\pi} e^{-ir\omega} \int \left(\frac{dp}{2\pi}\right)^3 \left\{\log\left[\hat{p}^2 + \hat{\omega}^2\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \hat{\hat{\omega}}^2 + m_W^2(\omega, p)\right]\right\}$$

II.A. Weak coupling analysis: (very recent) related developments

- Myers-Hollowood `09 generalized Kovtun-Unsal-Yaffe `07 to nonzero mass

  ** see also earlier Myers-Ogilvie`08

L = size of S1 For $ML > 0$ get $Z_N \to Z_K$ with $K \sim 1/ML$

- BB, in progress: Similar results seem to be obtained (preliminary)

$$V(\theta) = \sum_{a \neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a - \theta^b}{2}\right)\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a - \theta^b\right) + m_W^2(\theta, p)\right] \right\}$$

$$\equiv \sum_{a \neq b} \sum_r V_r \, e^{ir(\theta_a - \theta_b)} \underset{\uparrow}{=} \sum_r V_r \, |\mathrm{tr}\, \Omega^r|^2 + \text{const.}$$

$$\Omega_{ab} = e^{i\theta_a}\, \delta_{ab}$$

dispersion relations of:

fermions gluons

$$V_r = \int \frac{d\omega}{2\pi} e^{-ir\omega} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + \hat{\omega}^2\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \hat{\hat{\omega}}^2 + m_W^2(\omega, p)\right] \right\} = \int \left(\frac{dp}{2\pi}\right)^3 \left[2N_f e^{-rE_f(p)} - e^{-rE_g(p)}\right]$$

II.A. Weak coupling analysis: (very recent) related developments

- Myers-Hollowood `09 generalized Kovtun-Unsal-Yaffe `07 to nonzero mass

  ** see also earlier Myers-Ogilvie`08

L = size of S1 For $ML > 0$ get $Z_N \to Z_K$ with $K \sim 1/ML$

- BB, in progress: Similar results seem to be obtained (preliminary)

$$V(\theta) = \sum_{a \neq b} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + 4\sin^2\left(\frac{\theta^a - \theta^b}{2}\right)\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \sin^2\left(\theta^a - \theta^b\right) + m_W^2(\theta, p)\right]\right\}$$

$$\equiv \sum_{a \neq b} \sum_r V_r\, e^{ir(\theta_a - \theta_b)} = \sum_r V_r\, |\mathrm{tr}\,\Omega^r|^2 + \mathrm{const.}$$

$$\Omega_{ab} = e^{i\theta_a}\,\delta_{ab}$$

dispersion relations of:

fermions gluons

$$V_r = \int \frac{d\omega}{2\pi} e^{-ir\omega} \int \left(\frac{dp}{2\pi}\right)^3 \left\{ \log\left[\hat{p}^2 + \hat{\omega}^2\right] - 2N_f \log\left[\hat{\hat{p}}^2 + \hat{\hat{\omega}}^2 + m_W^2(\omega, p)\right]\right\} = \int \left(\frac{dp}{2\pi}\right)^3 \left[2N_f e^{-rE_f(p)} - e^{-rE_g(p)}\right]$$

Given mass **m**, there will be **r**'s for which fermions unimportant and **Vr < 0.**

$$\boxed{\begin{array}{c} \mathbf{Z_N \longrightarrow Z_r} \\ \textbf{with } \mathbf{r = f(m)} \end{array}}$$

But really need a non-perturbative lattice study

- Really interested in $L_{1,2,3,4}=1$, but IR div's.
- What happens at $g^2 N \simeq 1-3$?
- Non-perturbative effect (e.g. QEK and TEK model).

$\Downarrow$

Simulate $N_f=1$ Wilson adjoint fermions BB+S.Sharpe, 0906.3538

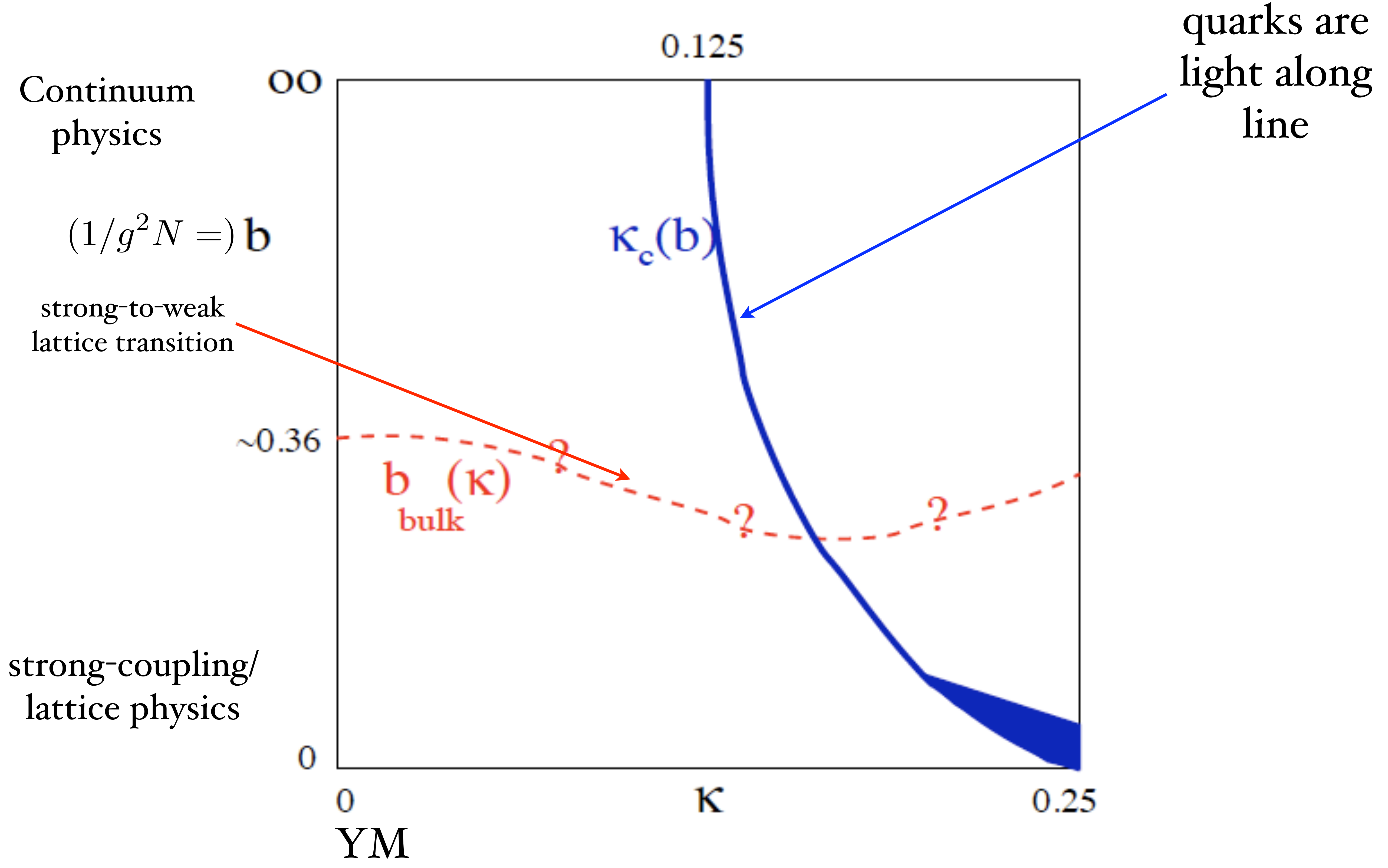
Goal : Map single-site theory in κ and $g^2 N$

Look for intact ZN.

** Lattice`09: contrasting preliminary results Hietanen+Narayanan

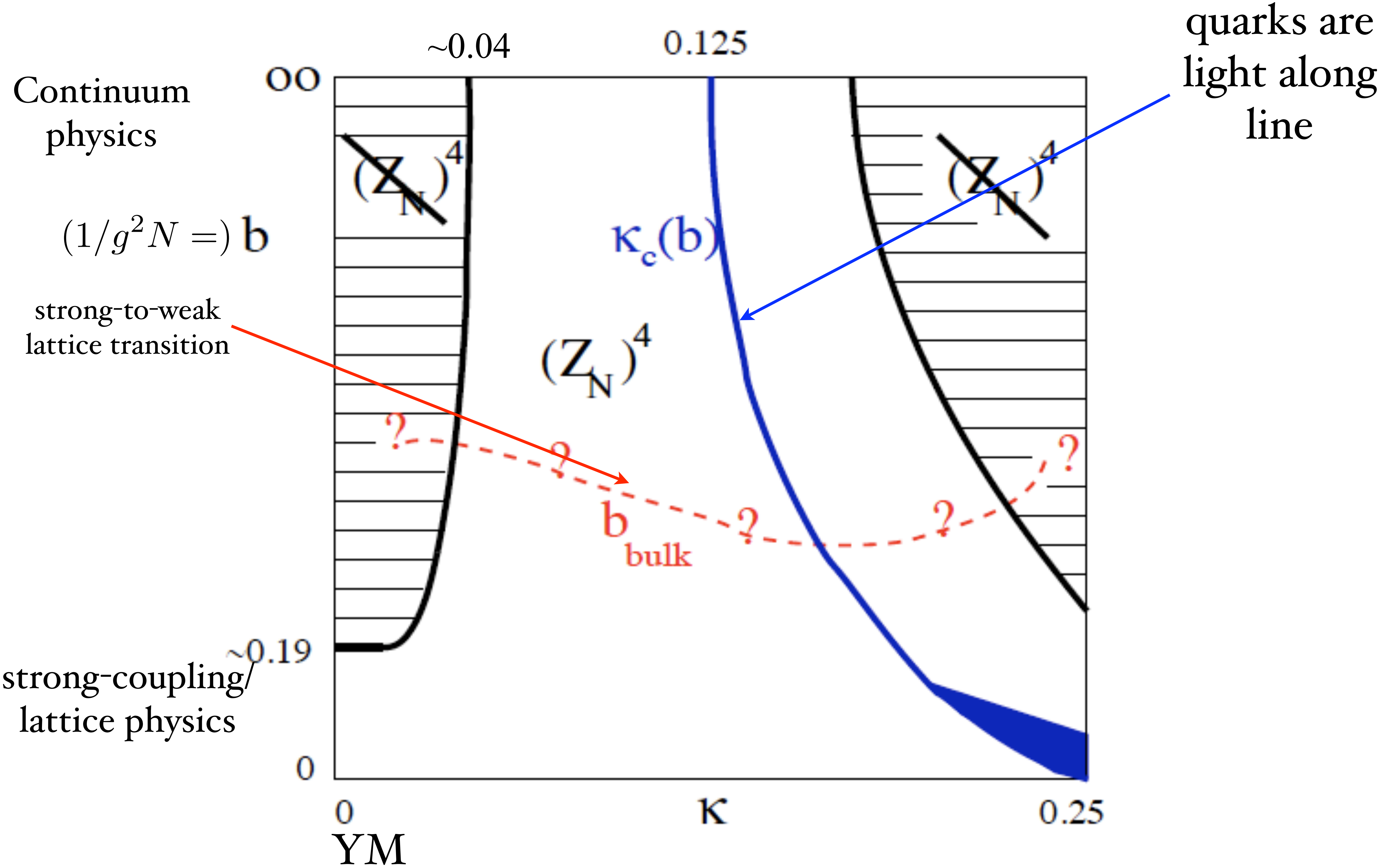
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? L1,2,3,4=oo :



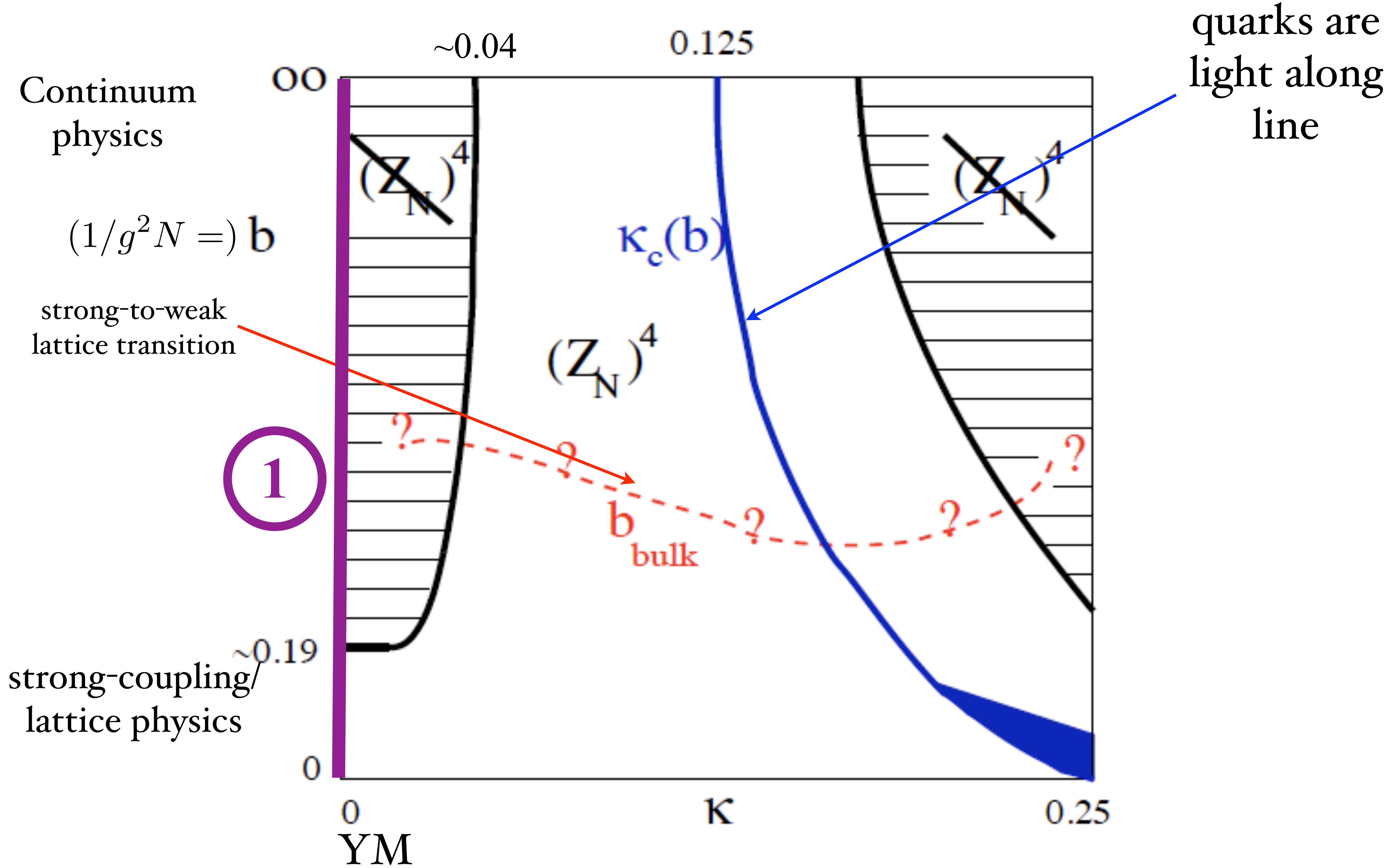
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:



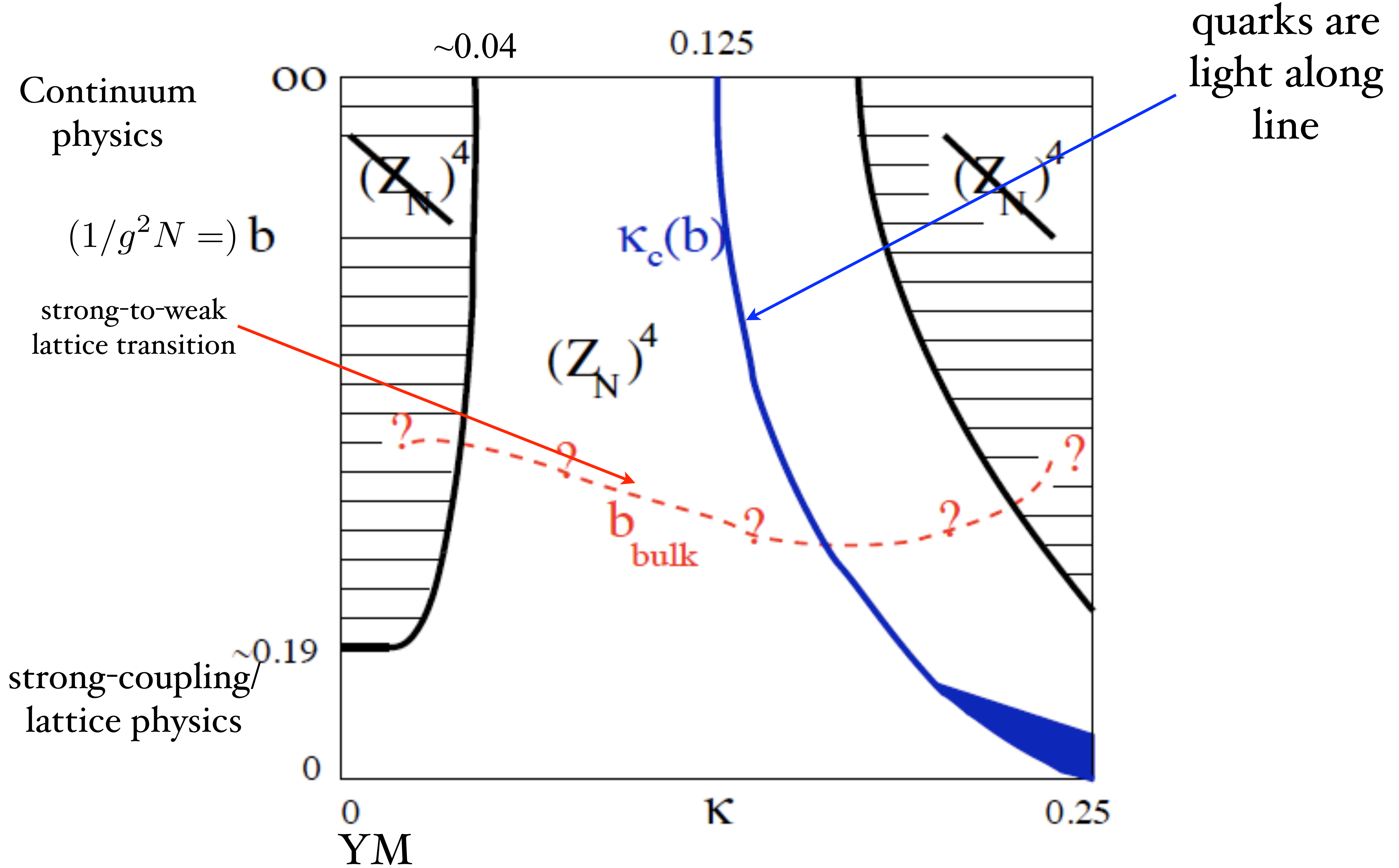
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:



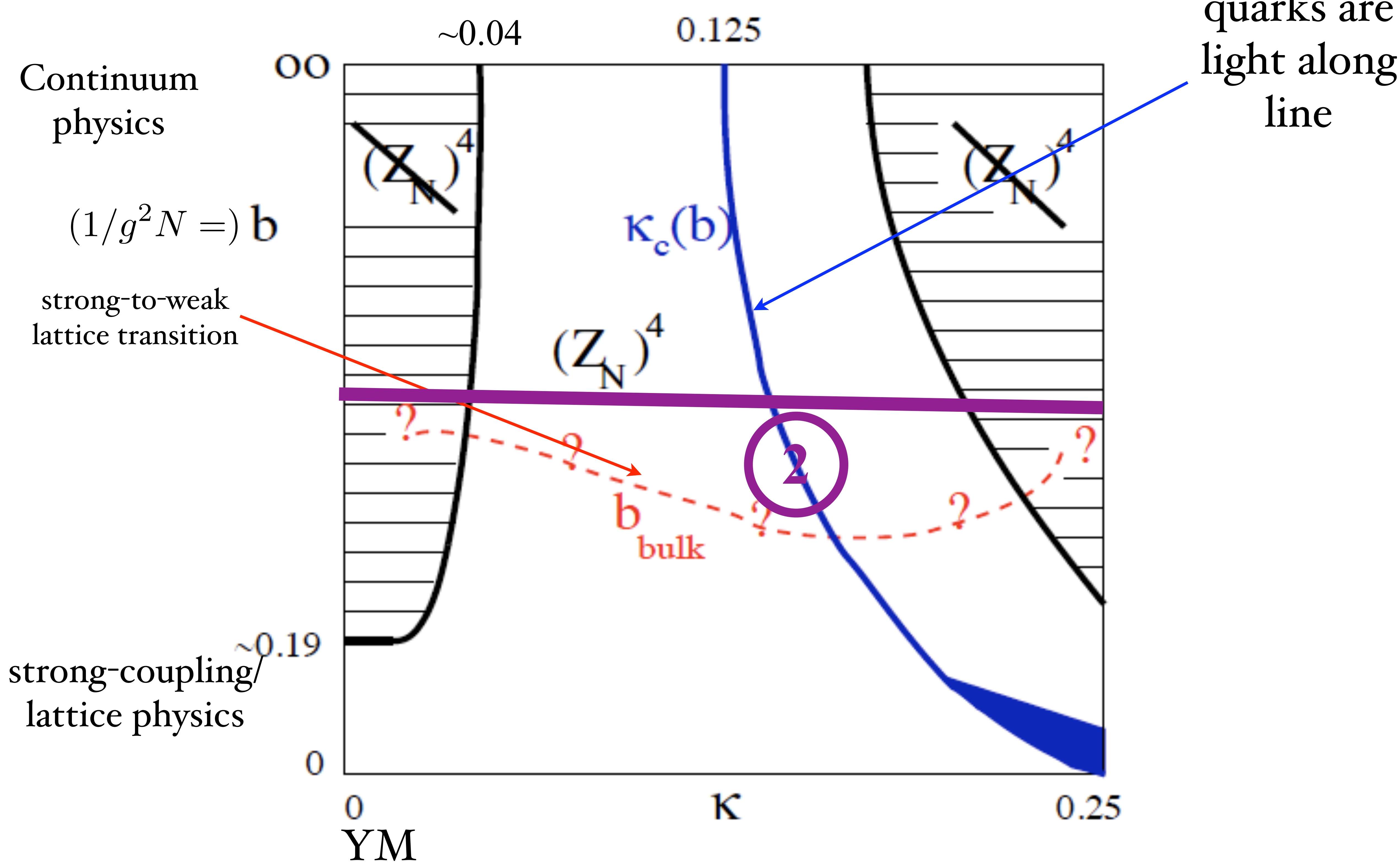
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:



II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:



II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:

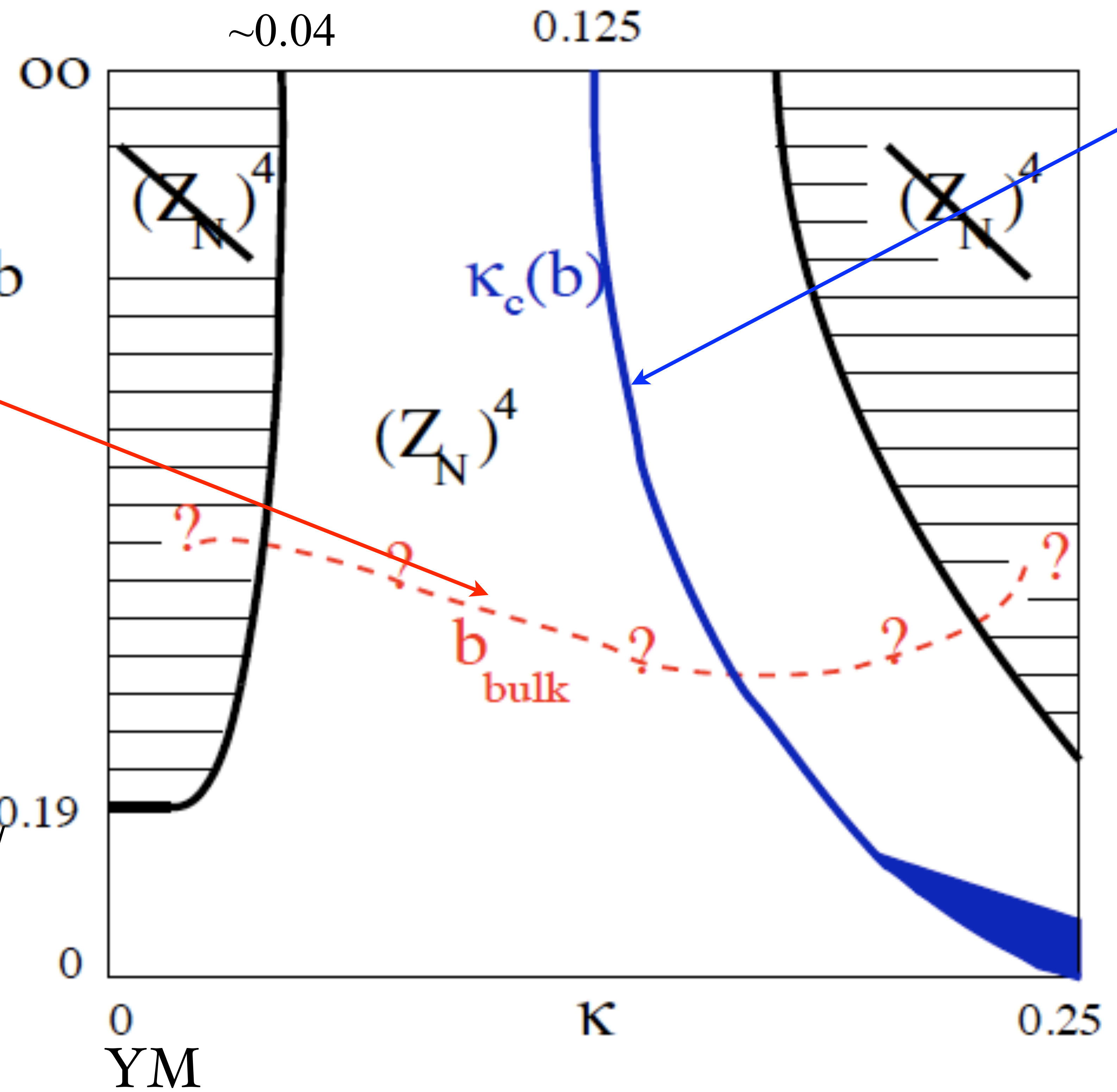
Continuum physics

$(1/g^2N =)$ b

strong-to-weak lattice transition

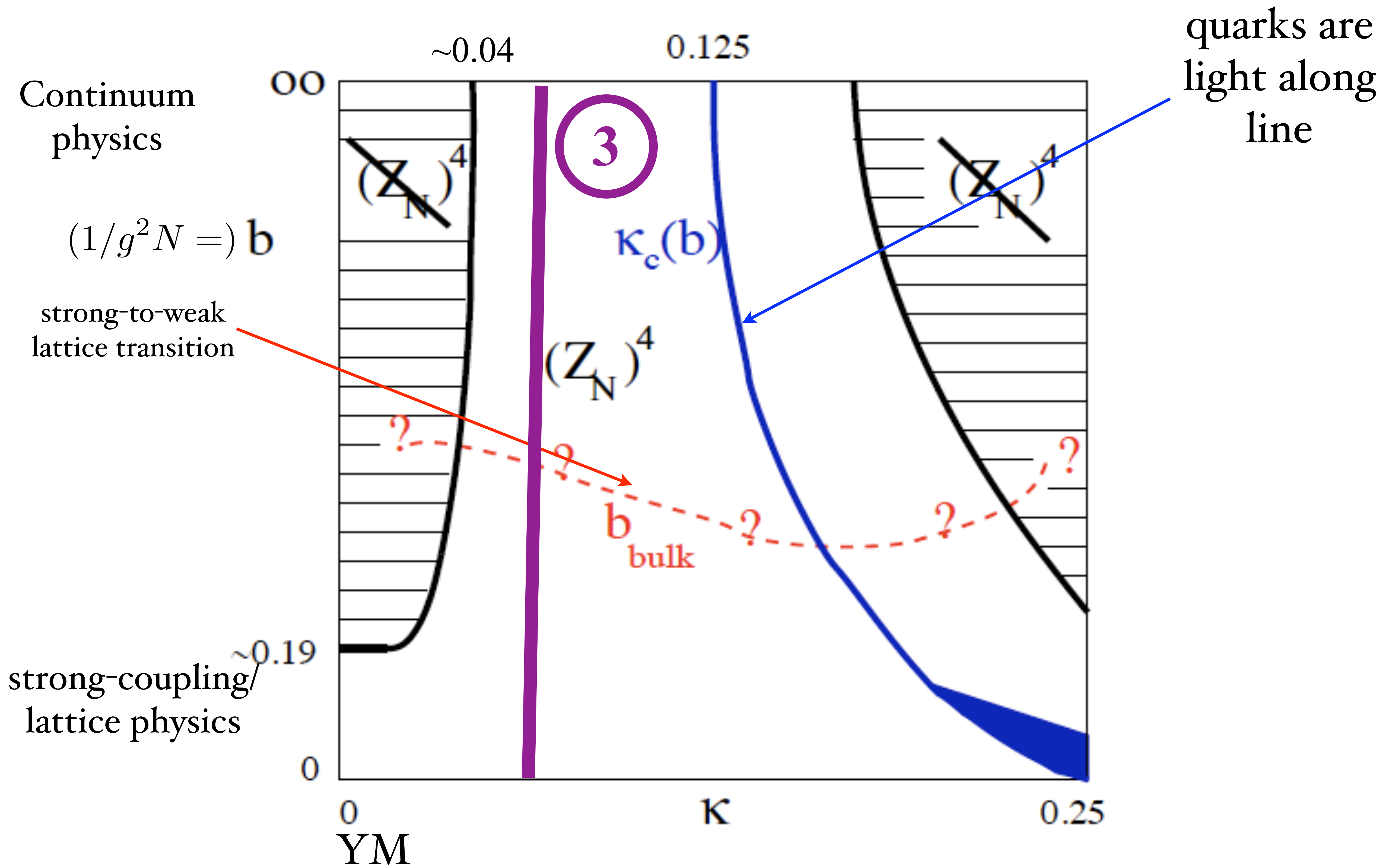
strong-coupling/ lattice physics

quarks are light along line



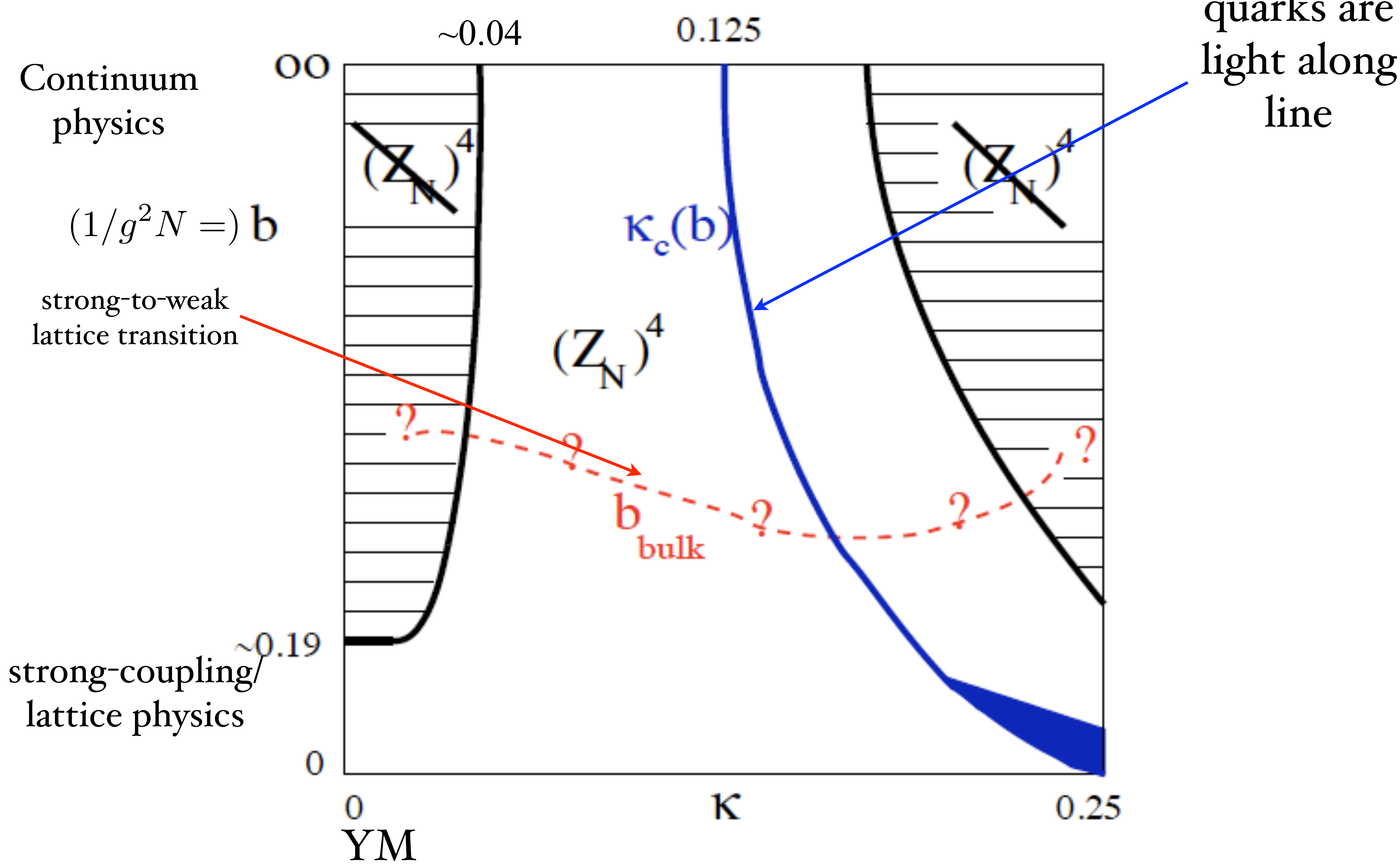
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:



II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What should we expect? $L_{1,2,3,4}=1$:



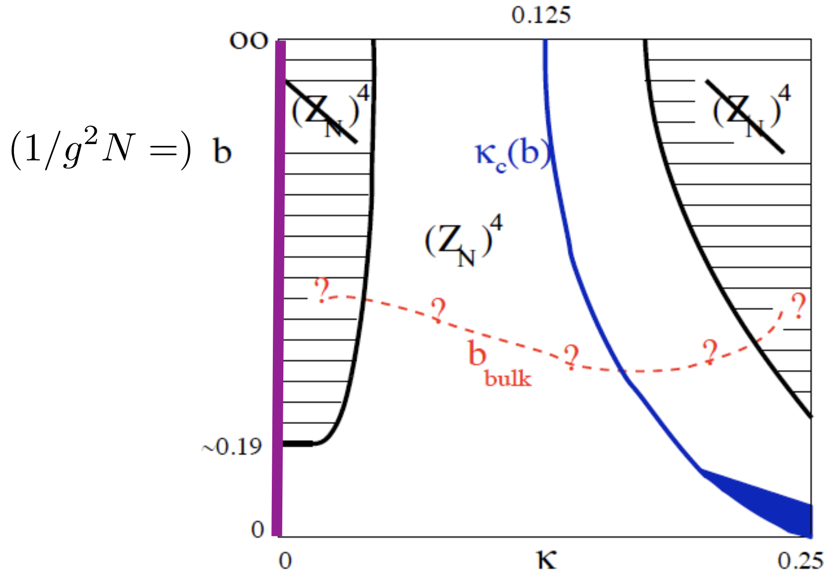
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

- Use Metropolis algorithm with weight

$$P(U) = \exp\left\{\frac{2}{g^2}\sum_{\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\,\left[U_\mu\, U_\nu\, U_\mu^\dagger\, U_\nu^\dagger\right]\right\} \times \det\left\{1-\kappa\sum_\mu\left[(1+\gamma_\mu)\,U_\mu^G + (1-\gamma_\mu)\,U_\mu^{\dagger\,G}\right]\right\}$$

- Determinant is real & positive
- Update N(N-1)/2 SU(2) subgroups in turn on each of the 4 links
- Evaluate determinant explicitly: 50-60% accept.
- Scaling is $(N^2)^3 x N^2 \longrightarrow$ can reach N=15 on PCs
- Measure every 5 sweeps after ~50 sweeps thermalizations
  (1 sweep = 5 hits of Metropolis)
- 100-3700 measurements

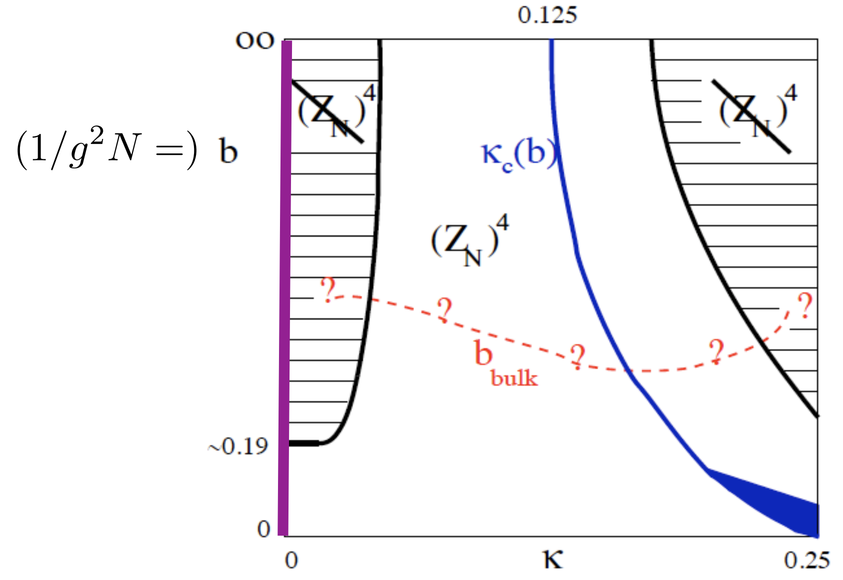
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



Scan no. 1 : infinitely massive quarks

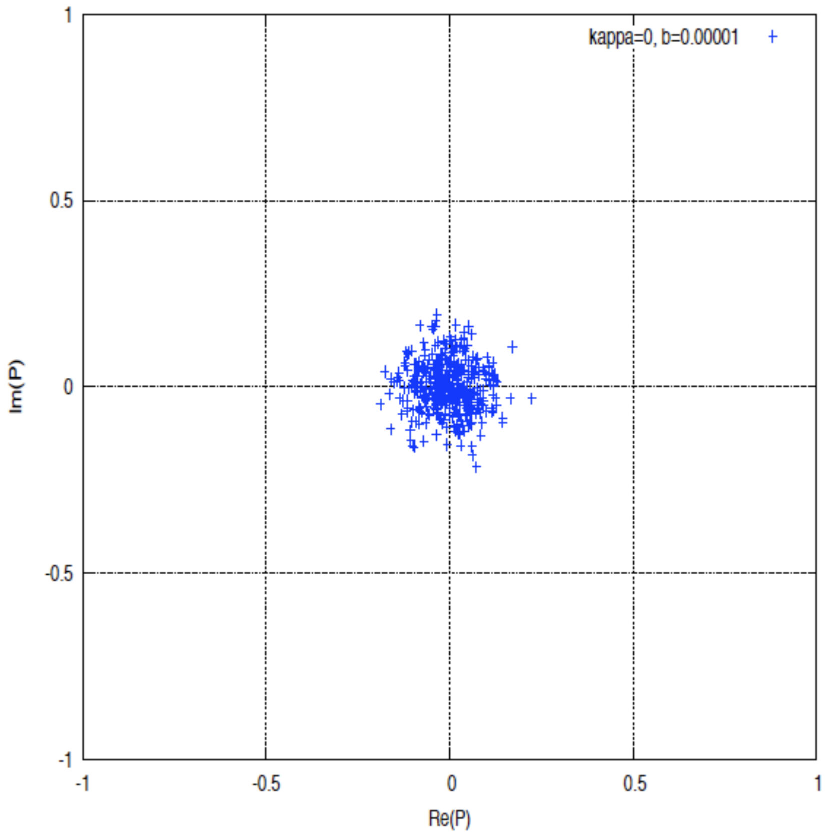
X-axis: Real(Polyakov)
Y-axis: Imag(Polyakov)

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



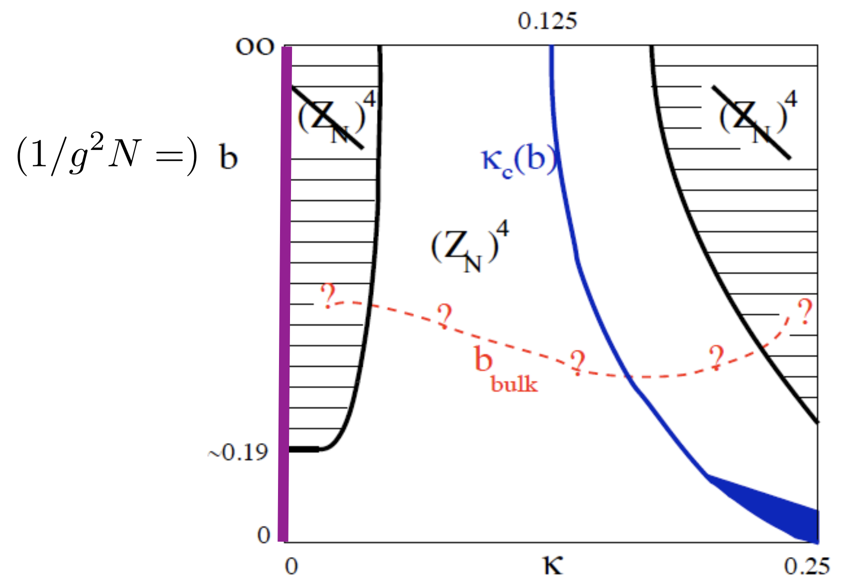
Scan no. 1 : infinitely massive quarks

X-axis: Real(Polyakov)
Y-axis: Imag(Polyakov)



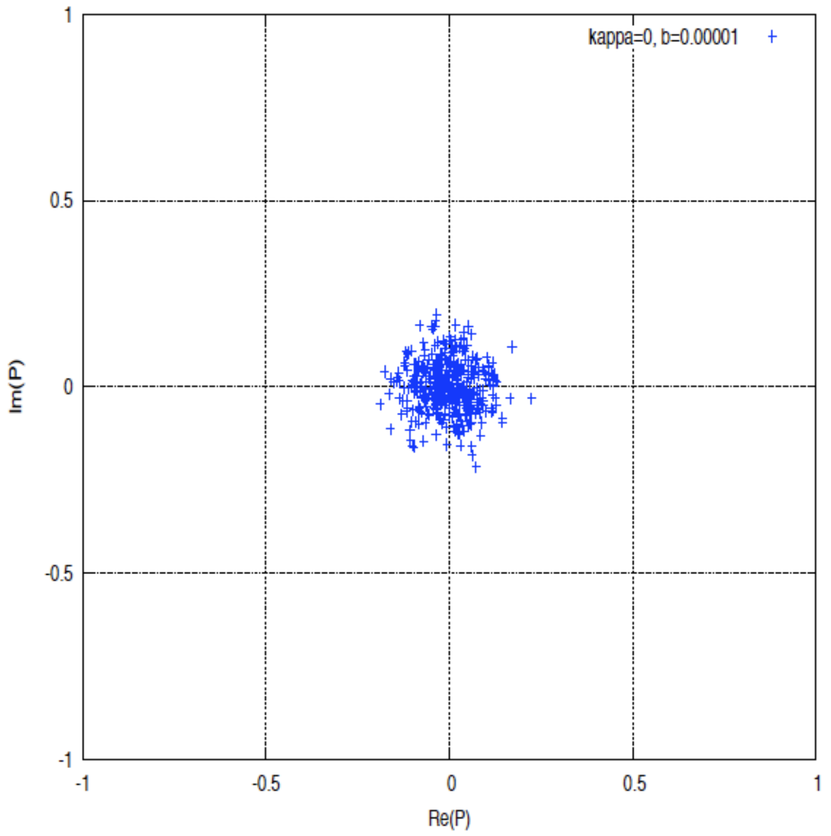
$$b = 0$$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

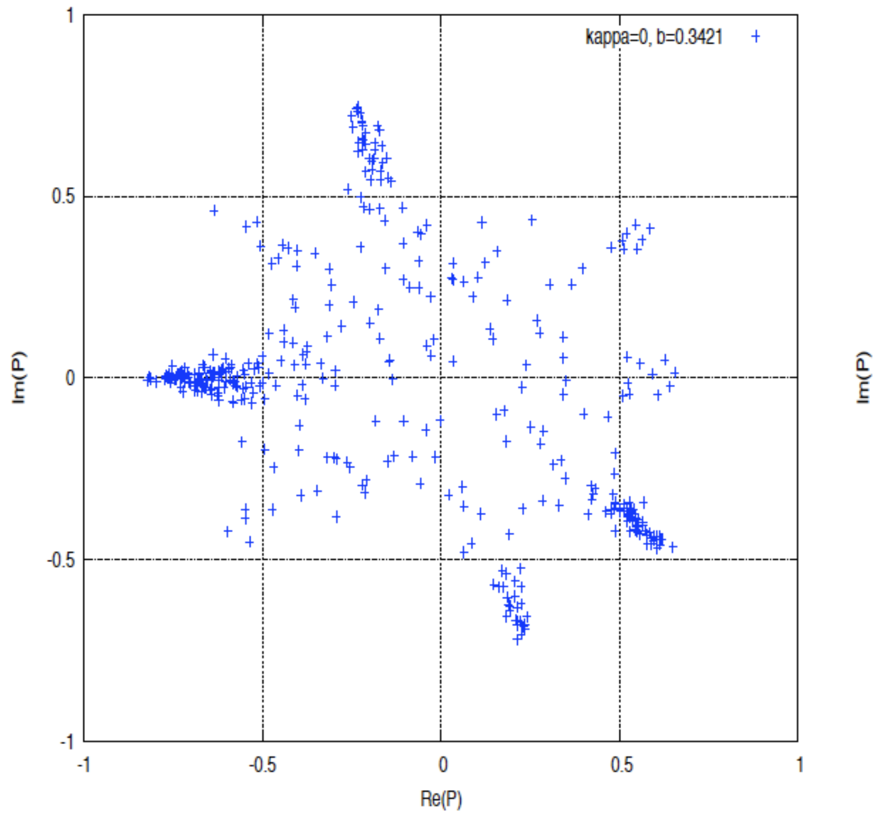


Scan no. 1 : infinitely massive quarks

X-axis: Real(Polyakov)
Y-axis: Imag(Polyakov)

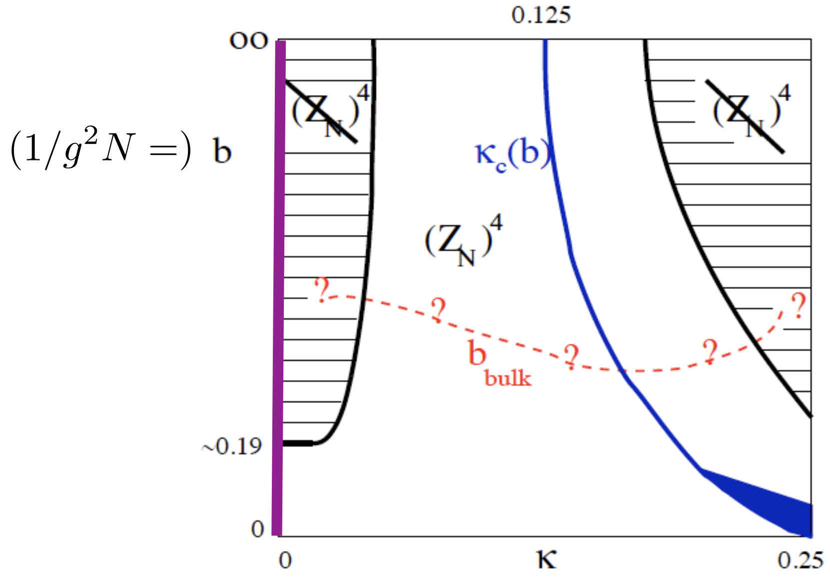


$b = 0$



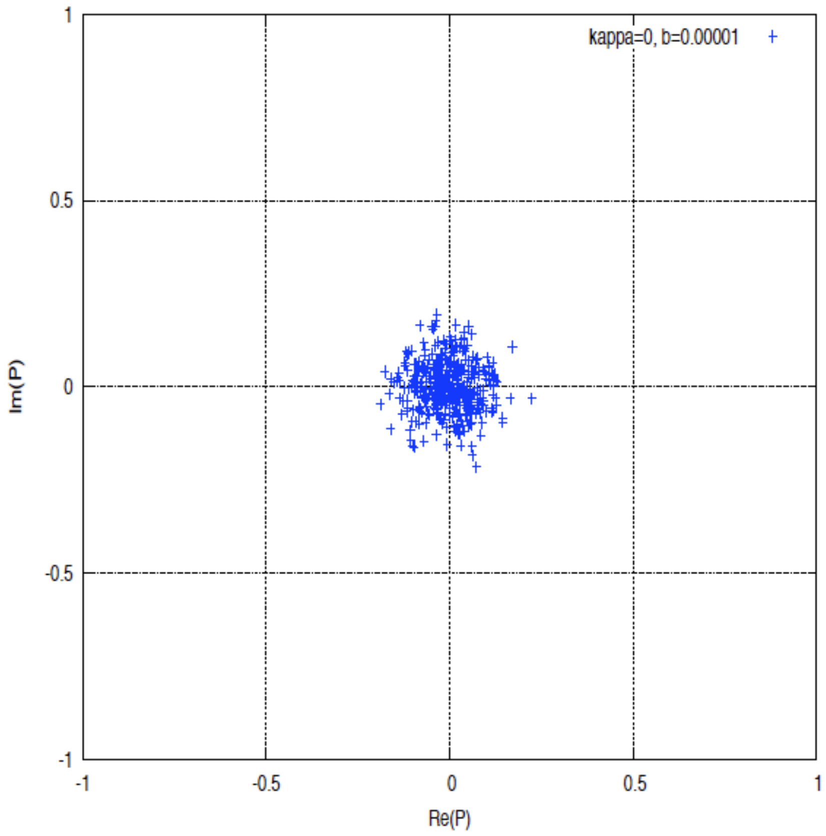
$b \approx 0.3$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

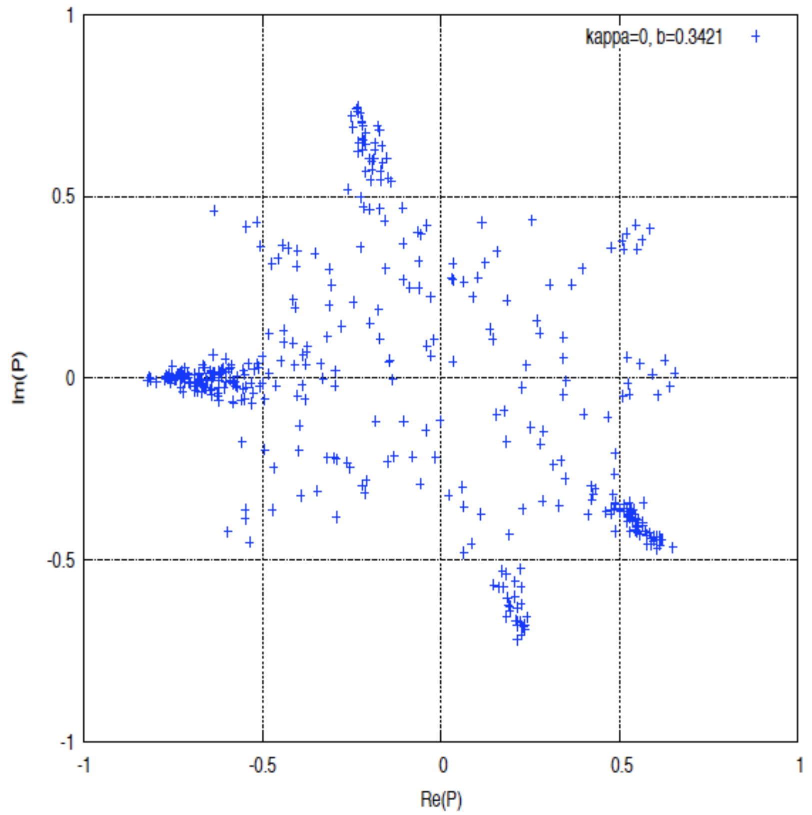


Scan no. 1 : infinitely massive quarks

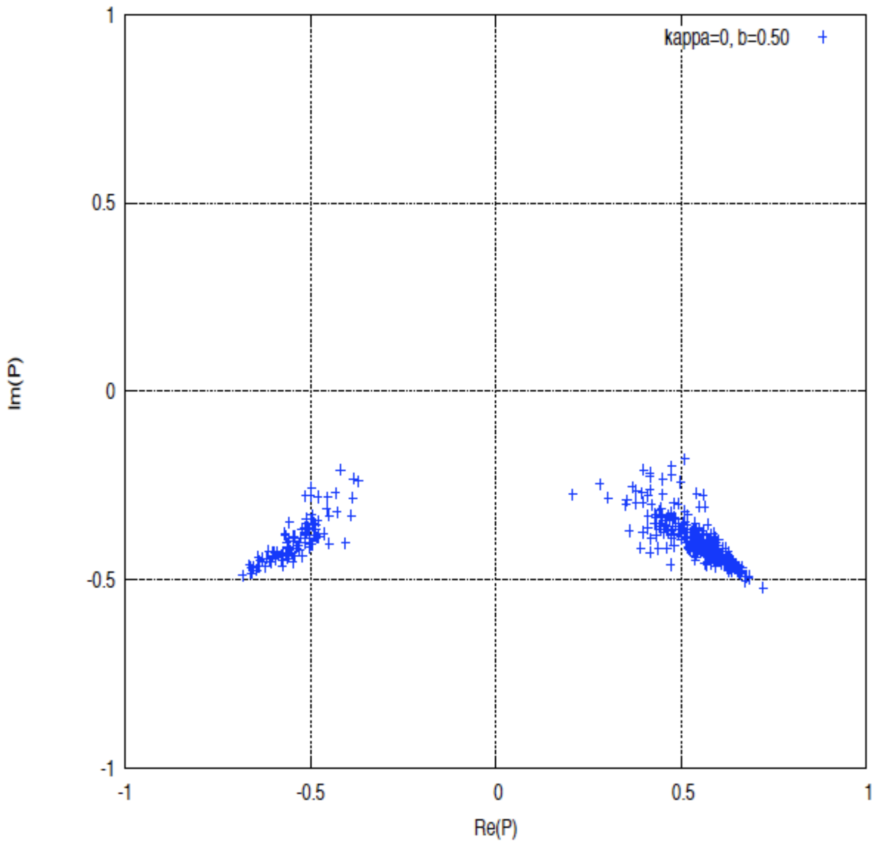
X-axis: Real(Polyakov)
Y-axis: Imag(Polyakov)



$b = 0$

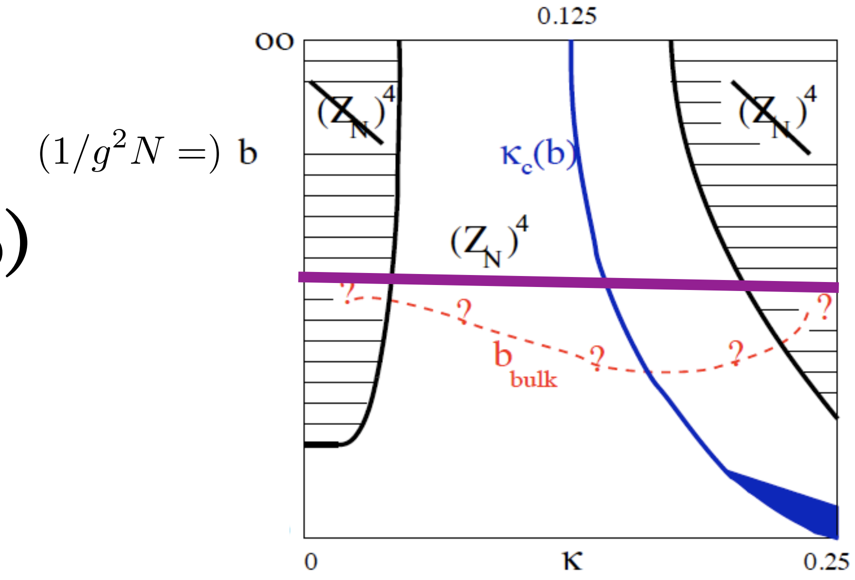


$b \approx 0.3$



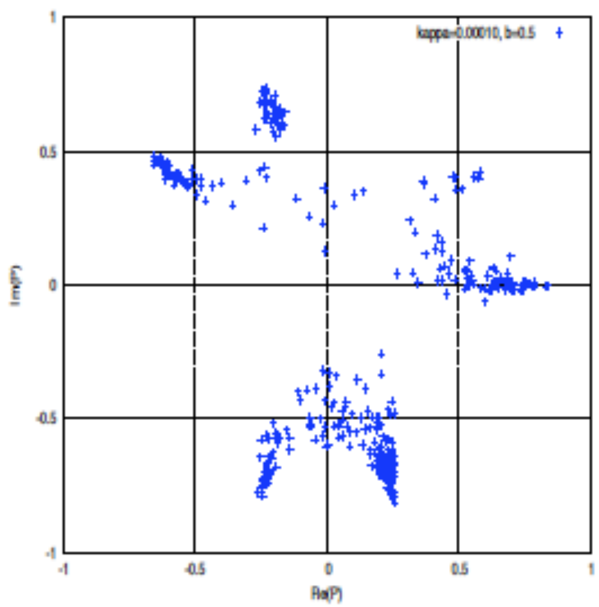
$b = 0.5$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

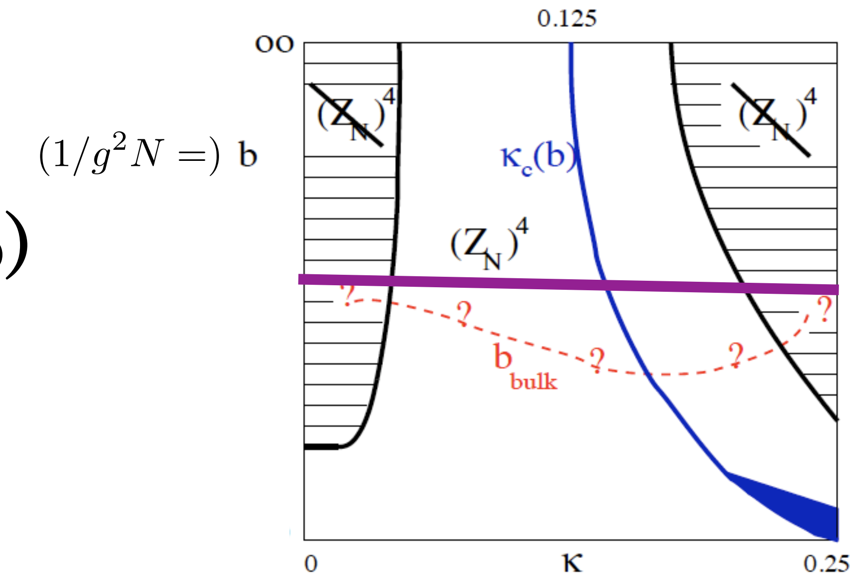


Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

$\kappa \approx 0$

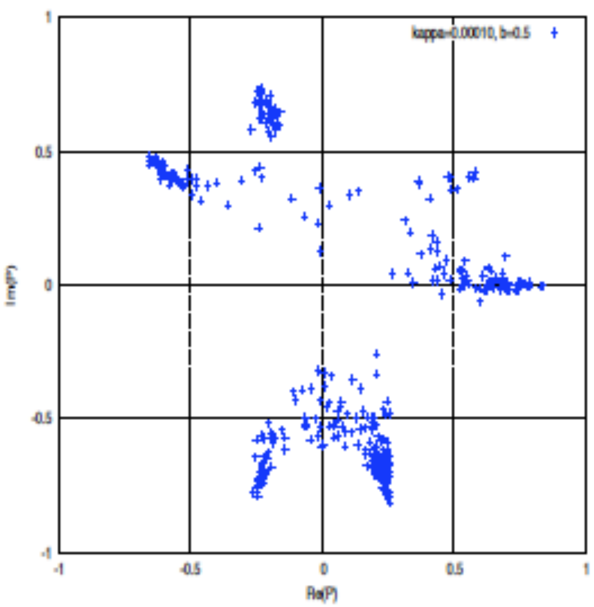


II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

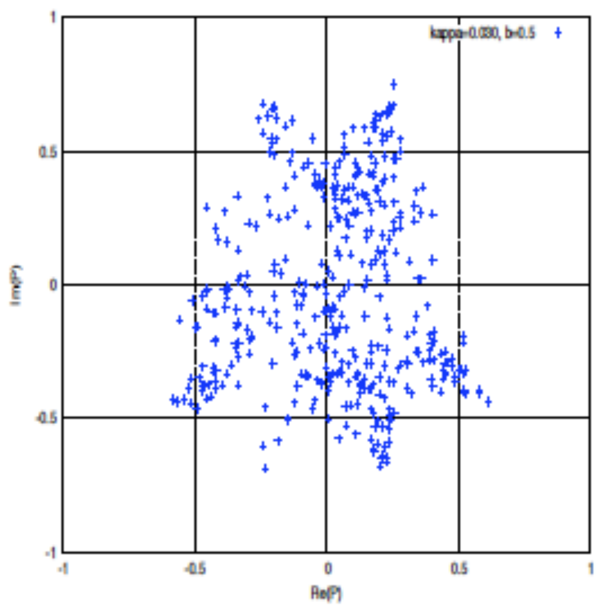


Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

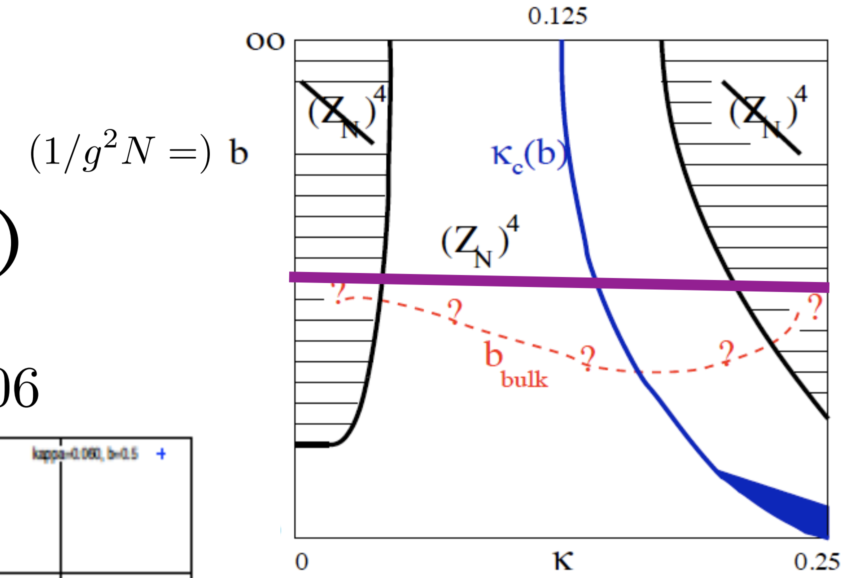
$\kappa \approx 0$



$\kappa = 0.03$

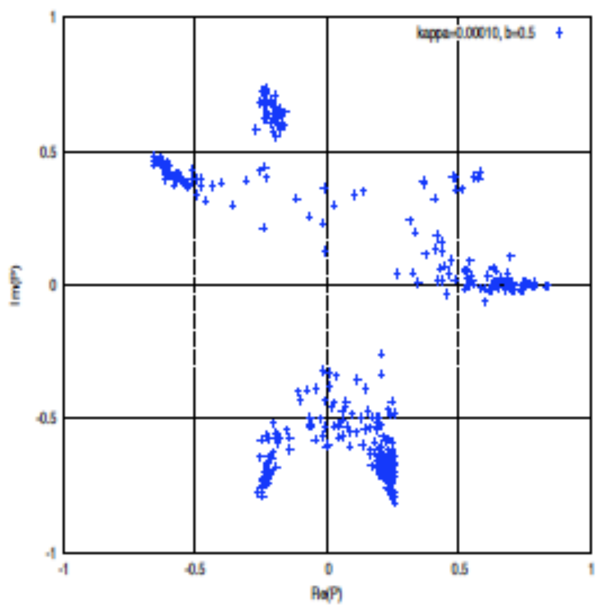


II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

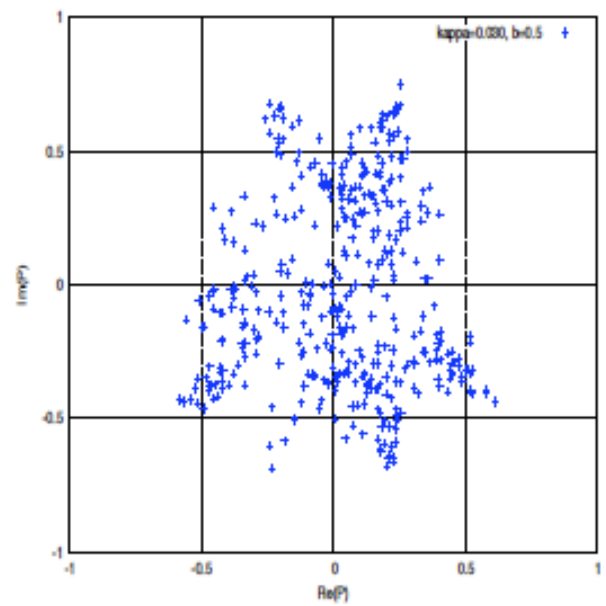


Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

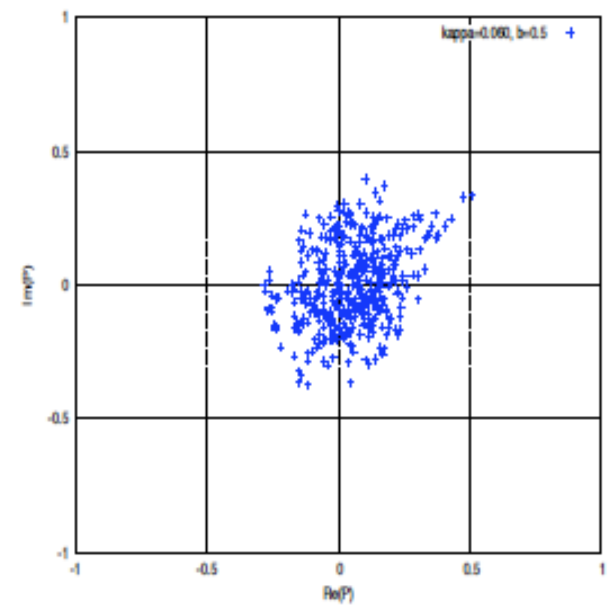
$\kappa \approx 0$



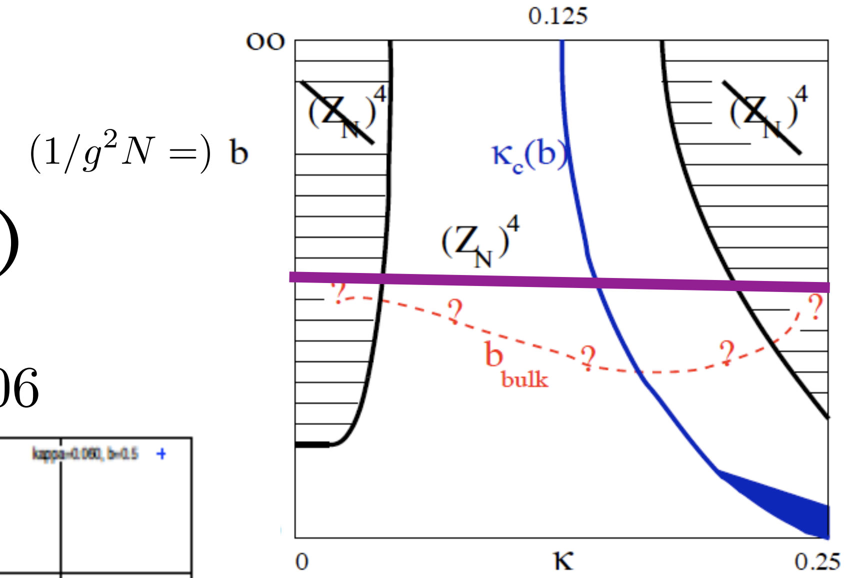
$\kappa = 0.03$



$\kappa = 0.06$

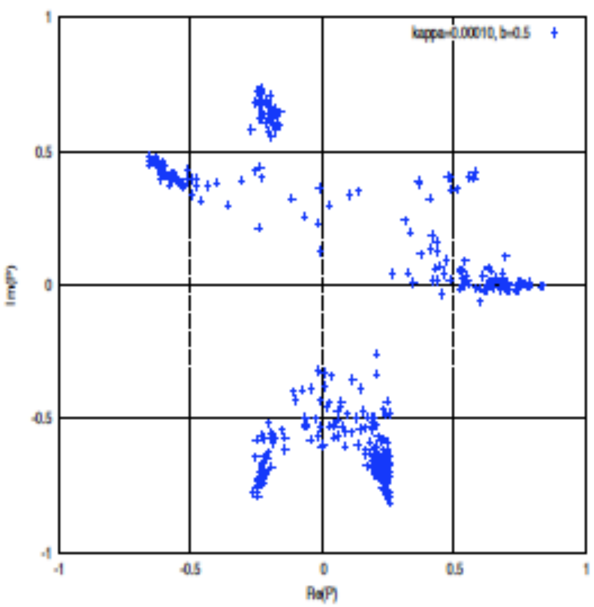


II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

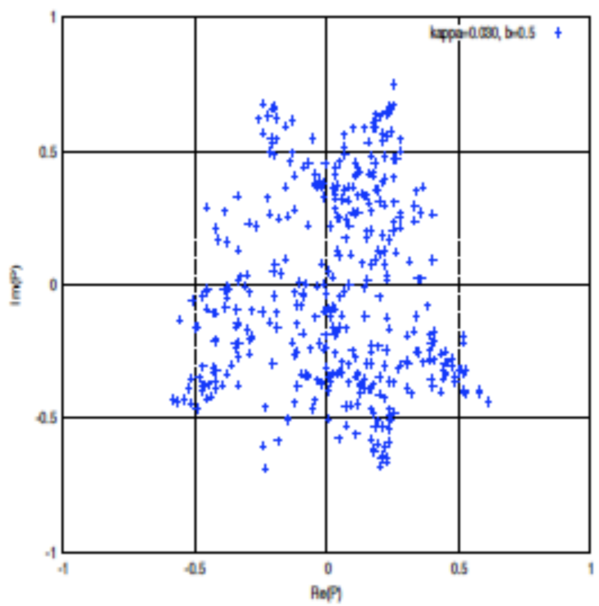


Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

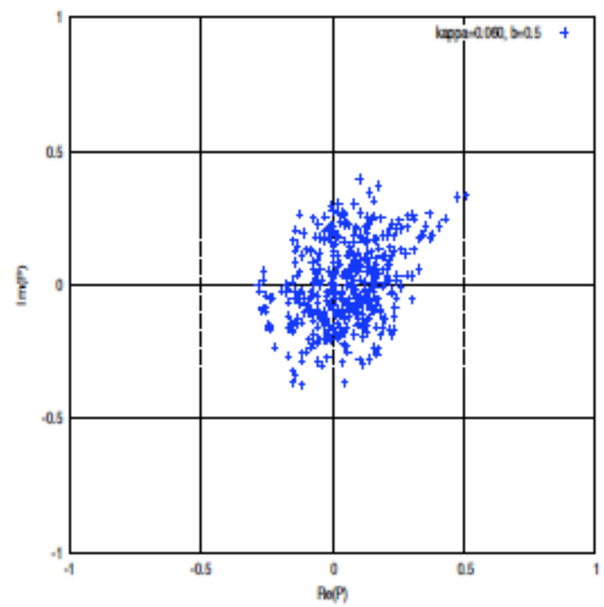
$\kappa \approx 0$

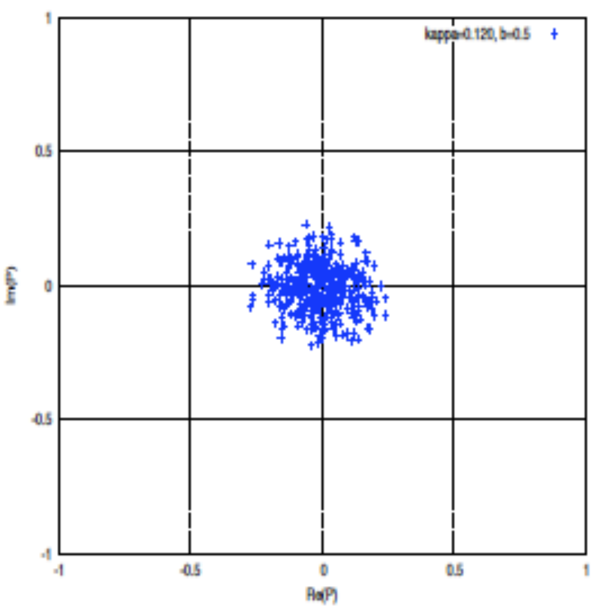


$\kappa = 0.03$



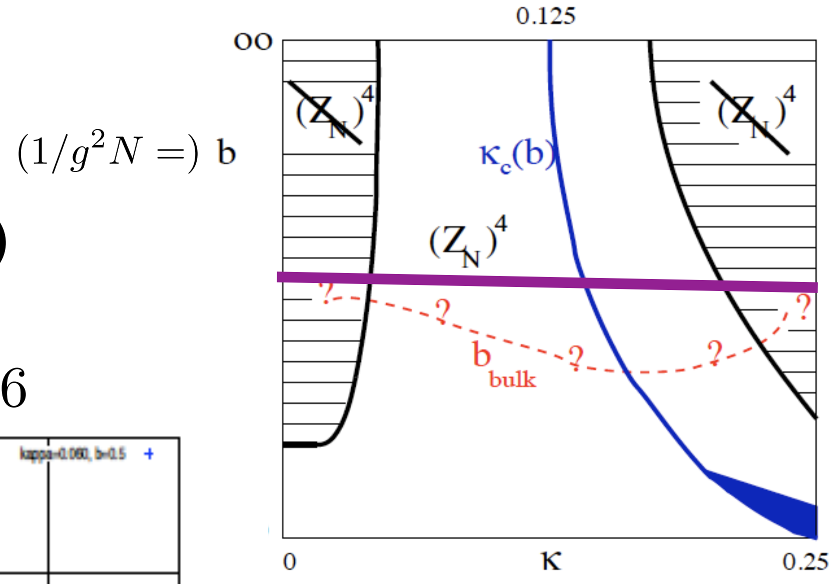
$\kappa = 0.06$





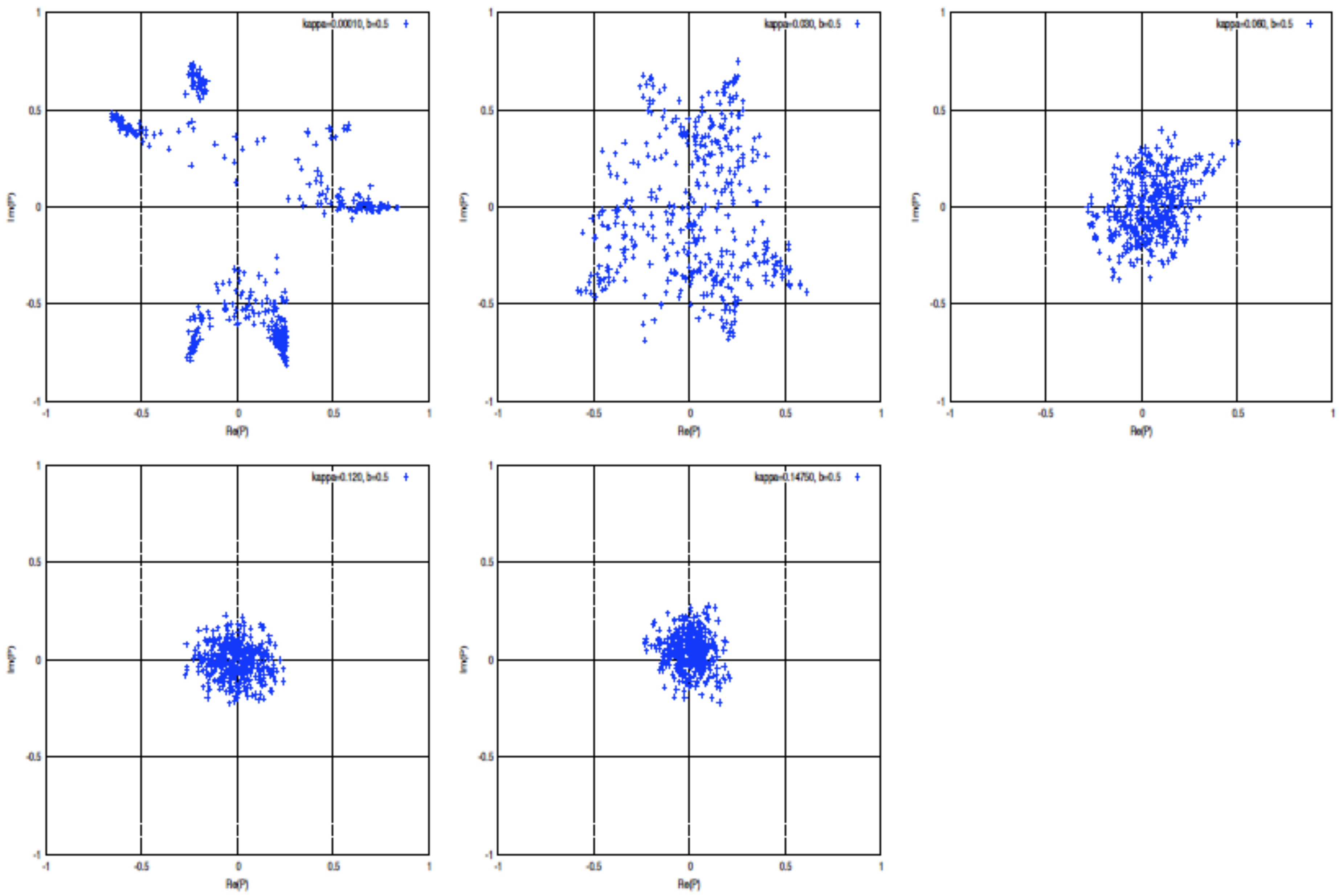
$\kappa = 0.12$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



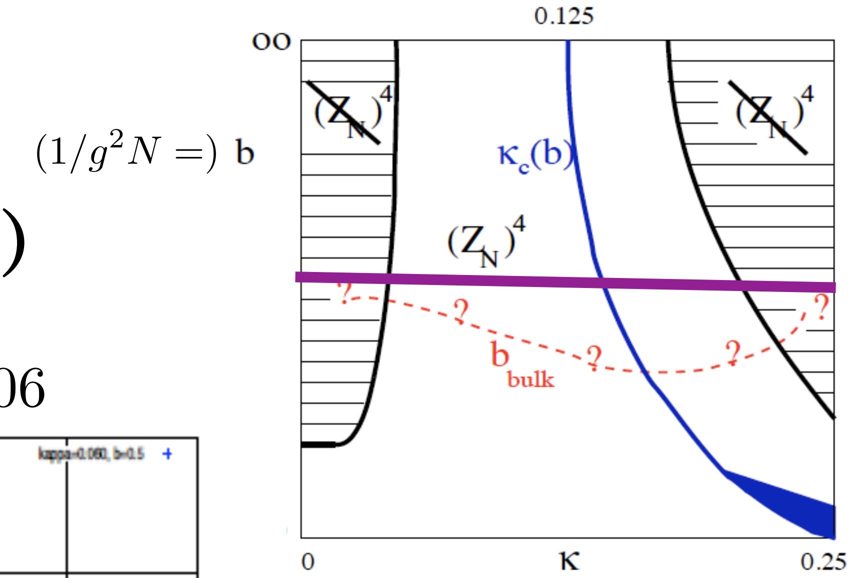
Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

$\kappa \approx 0$ $\kappa = 0.03$ $\kappa = 0.06$



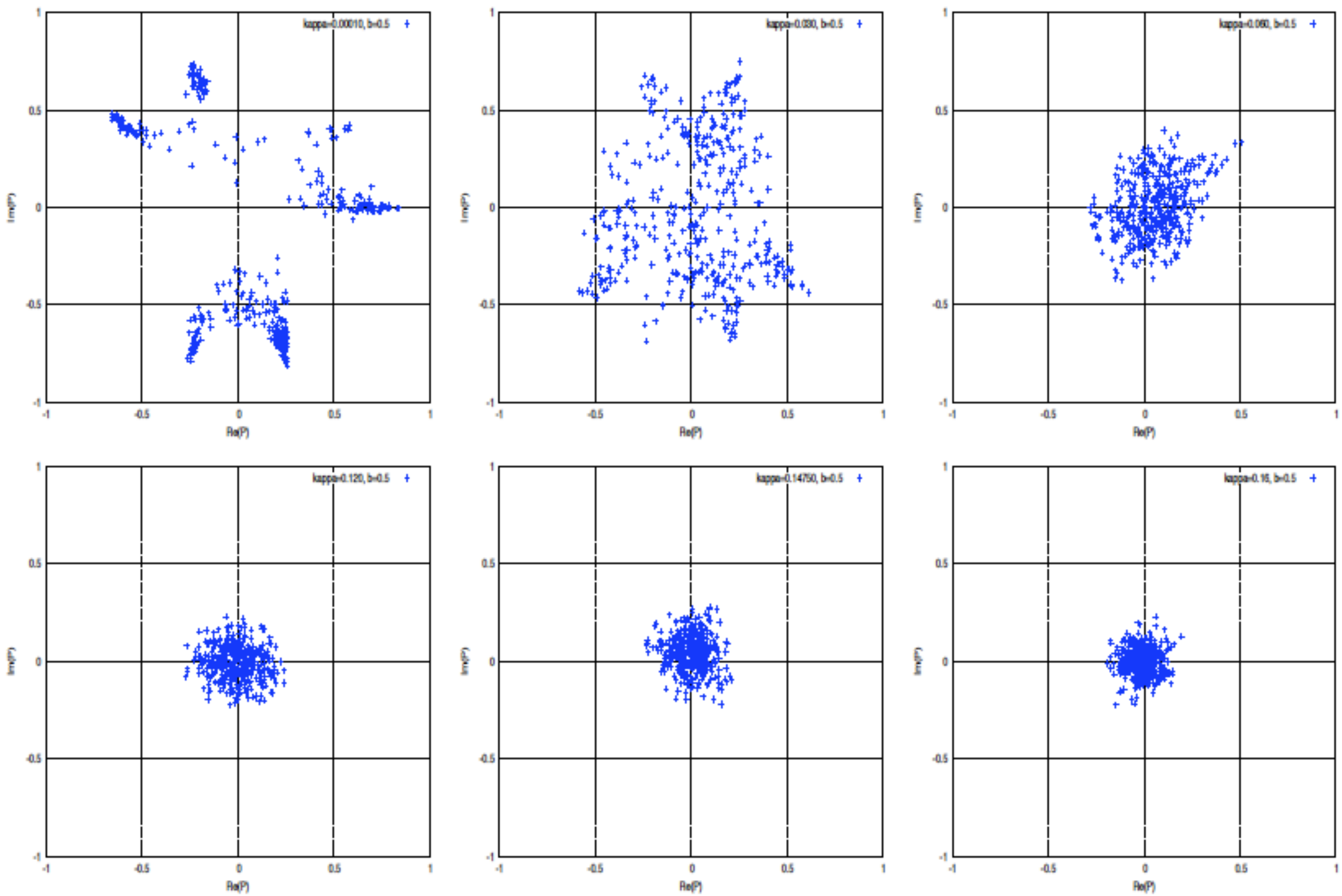
$\kappa = 0.12$ $\kappa = 0.1475$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



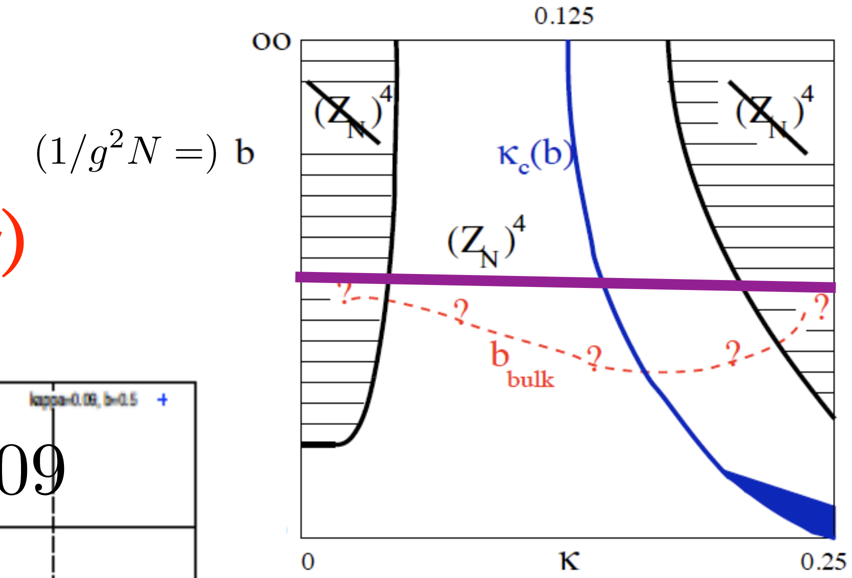
Scan no. 2 : decreasing the quark mass b=0.5 ,SU(10)

$\kappa \approx 0$ $\kappa = 0.03$ $\kappa = 0.06$

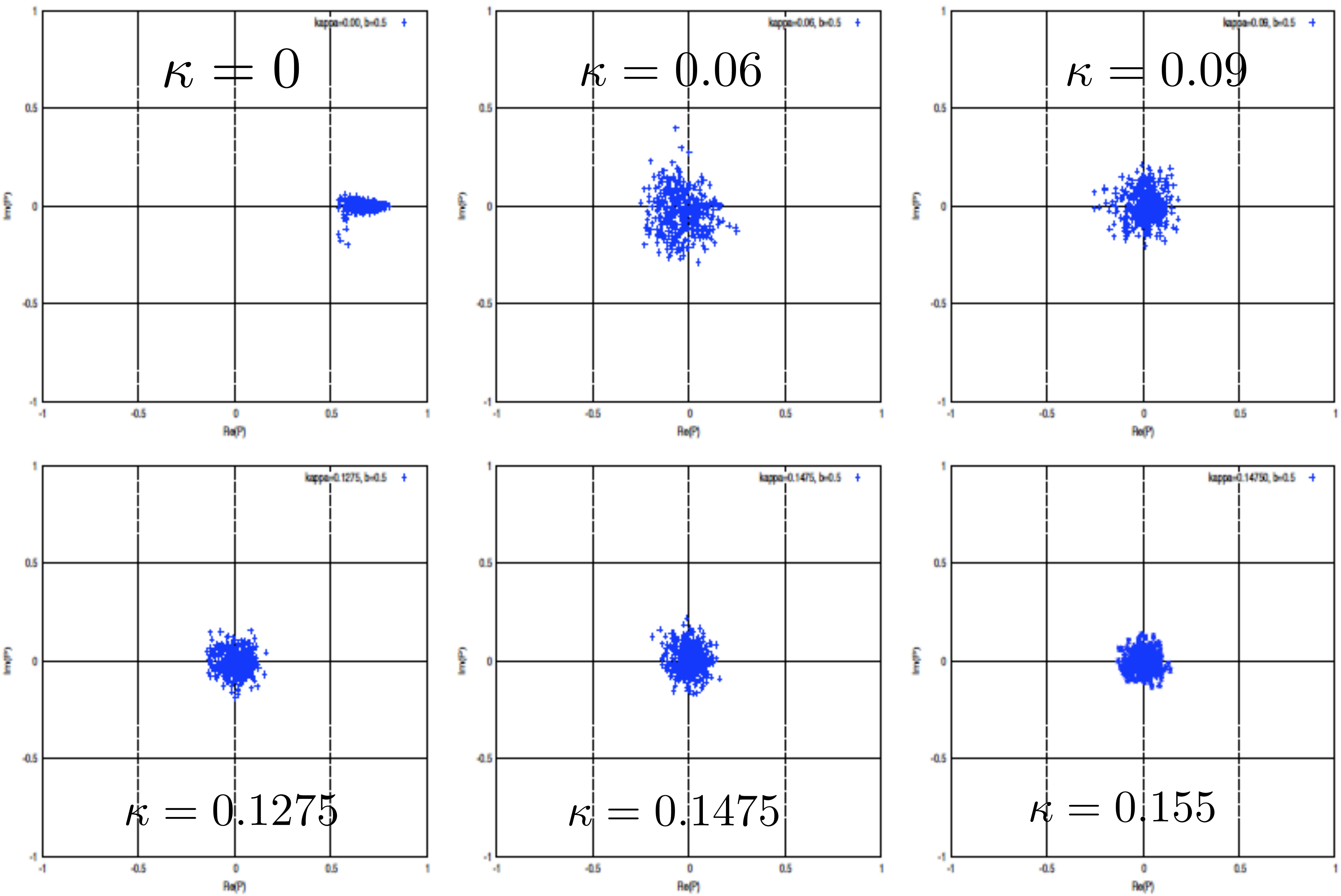


$\kappa = 0.12$ $\kappa = 0.1475$ $\kappa = 0.16$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

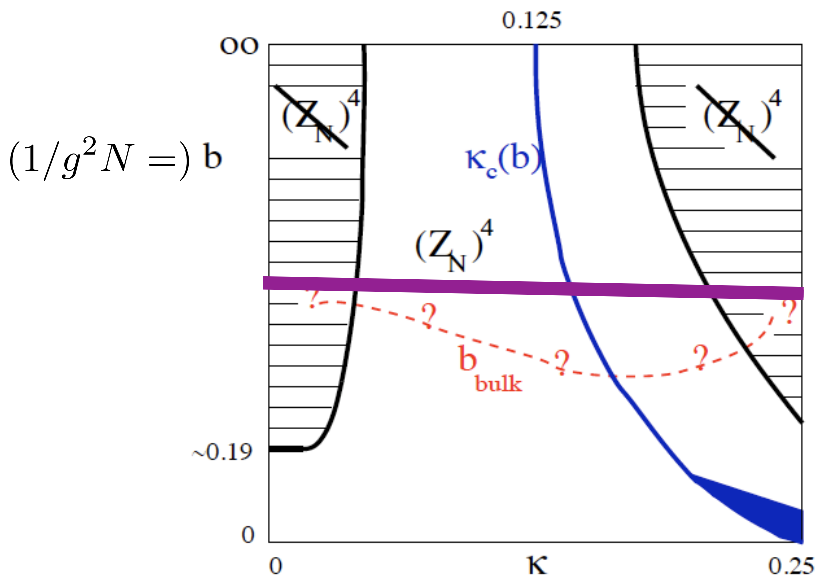


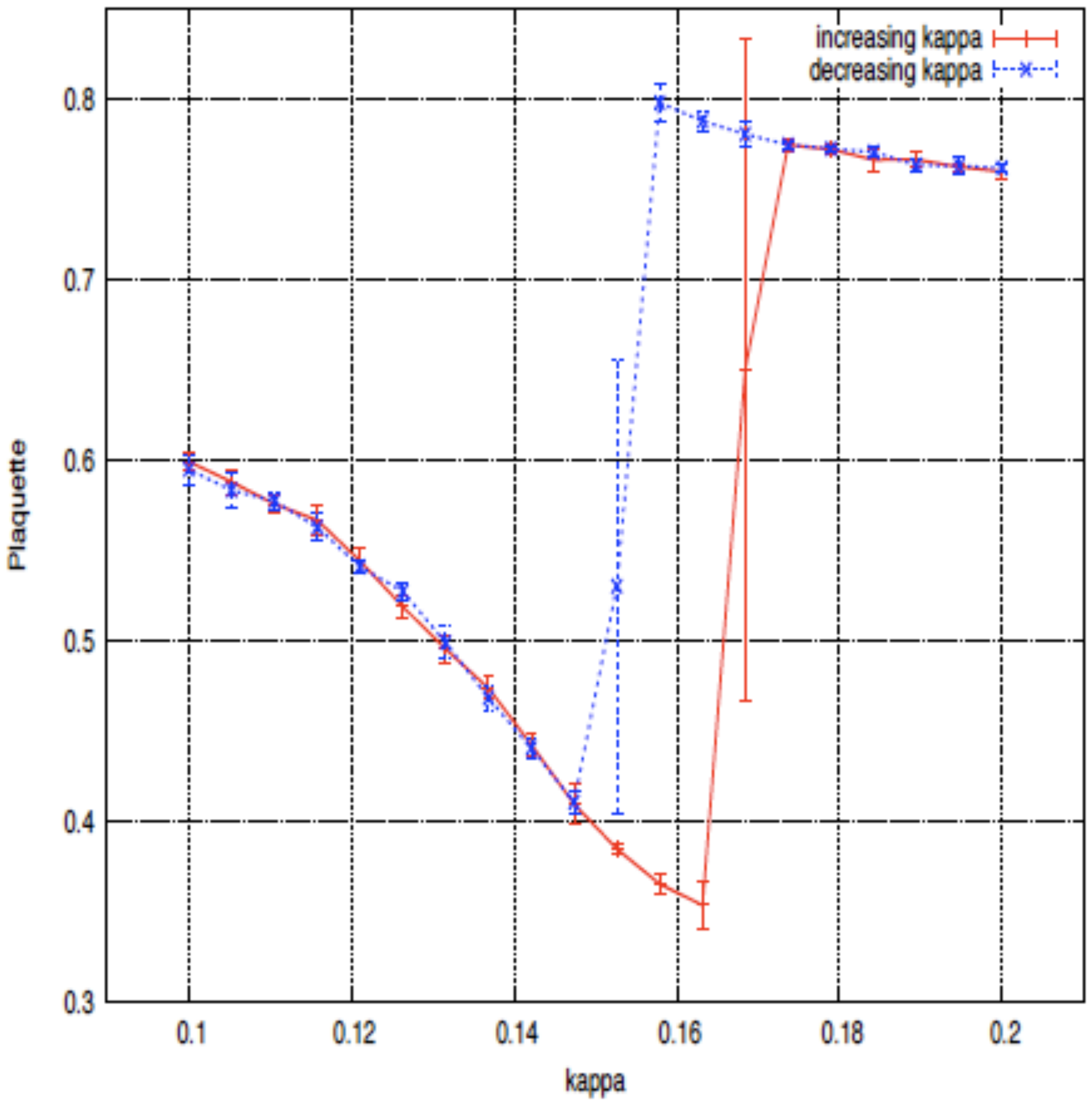
Scan no. 2 : decreasing the quark mass b=0.5 ,SU(15)



II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

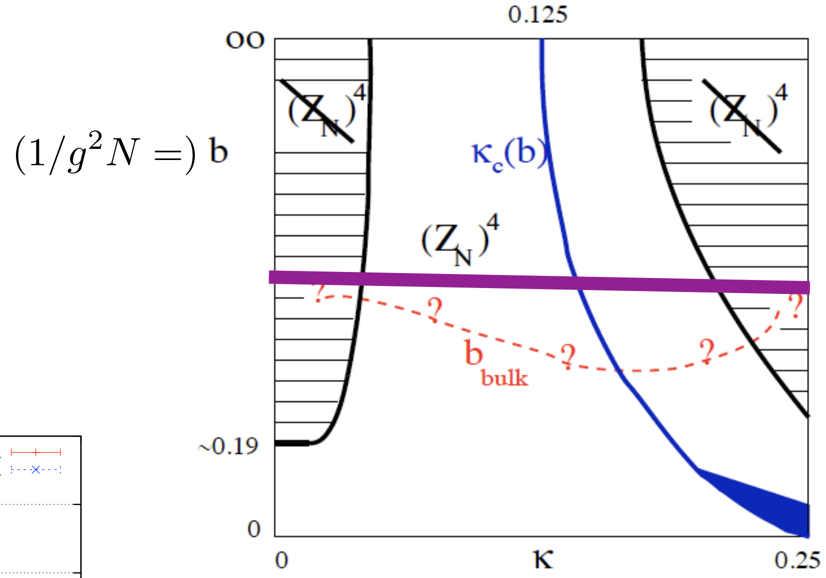
Scan no. 2 : looking for the "critical" line



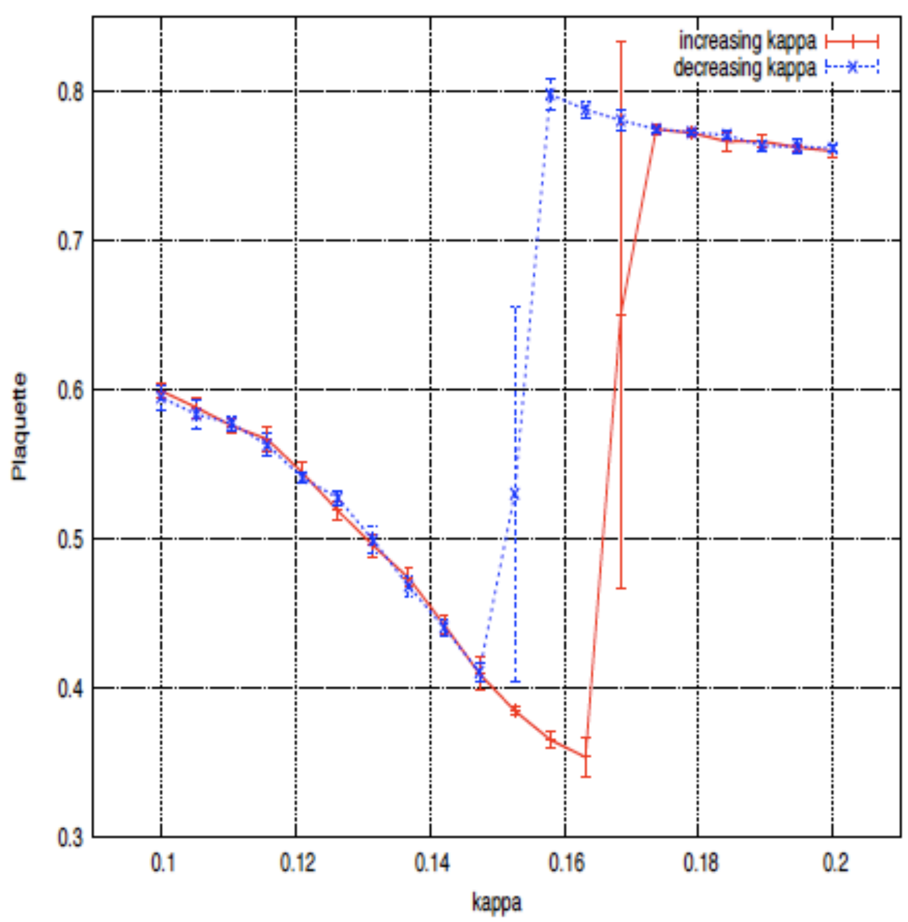


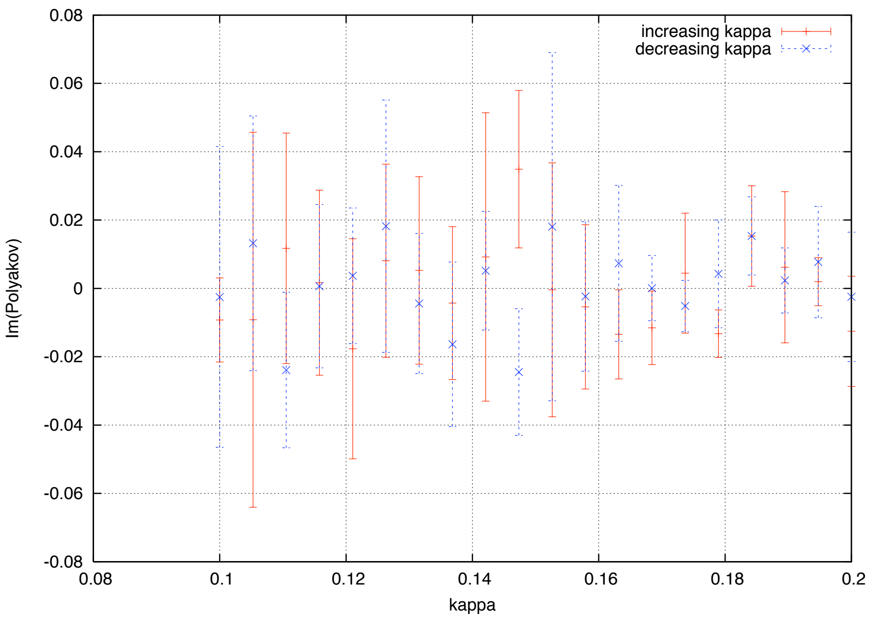
b=0.35:
1st transition
at kappa~0.15

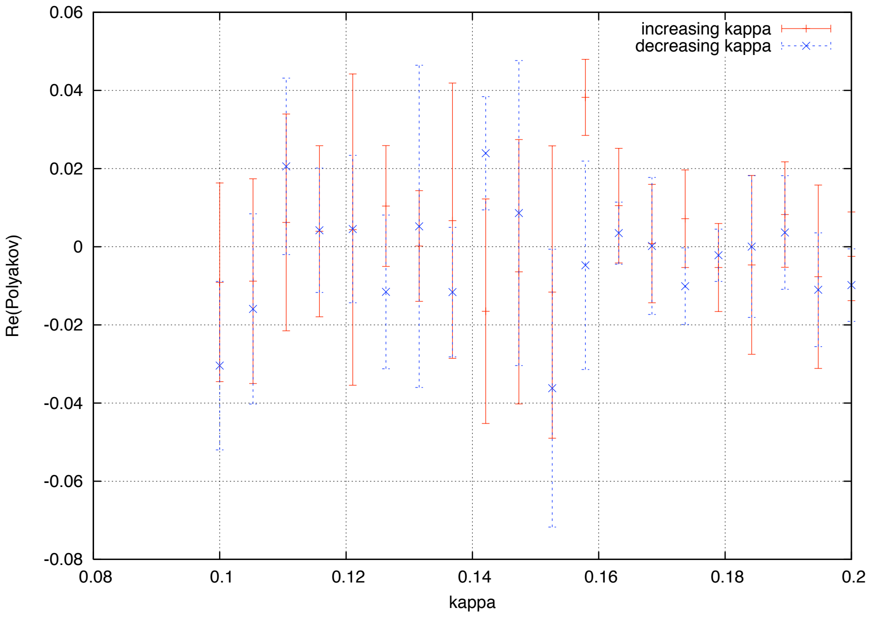
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



Scan no. 2 : looking for the "critical" line

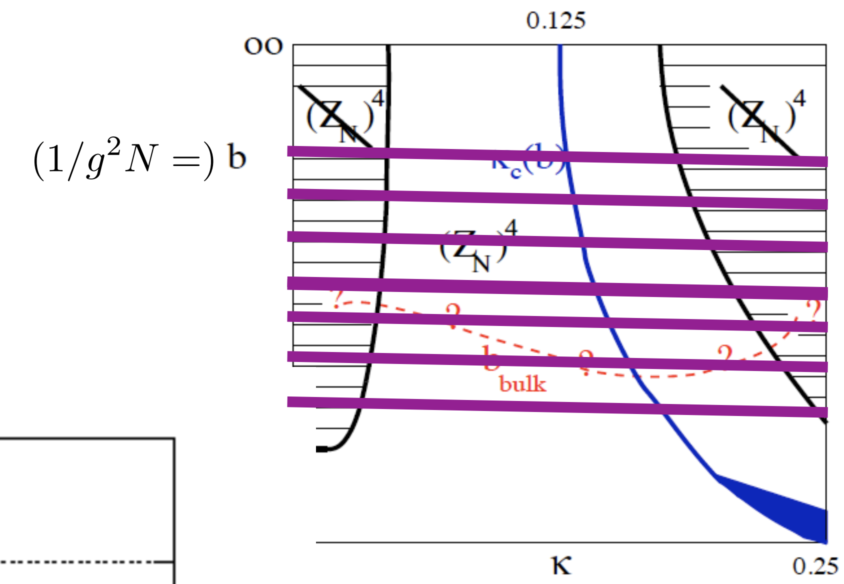




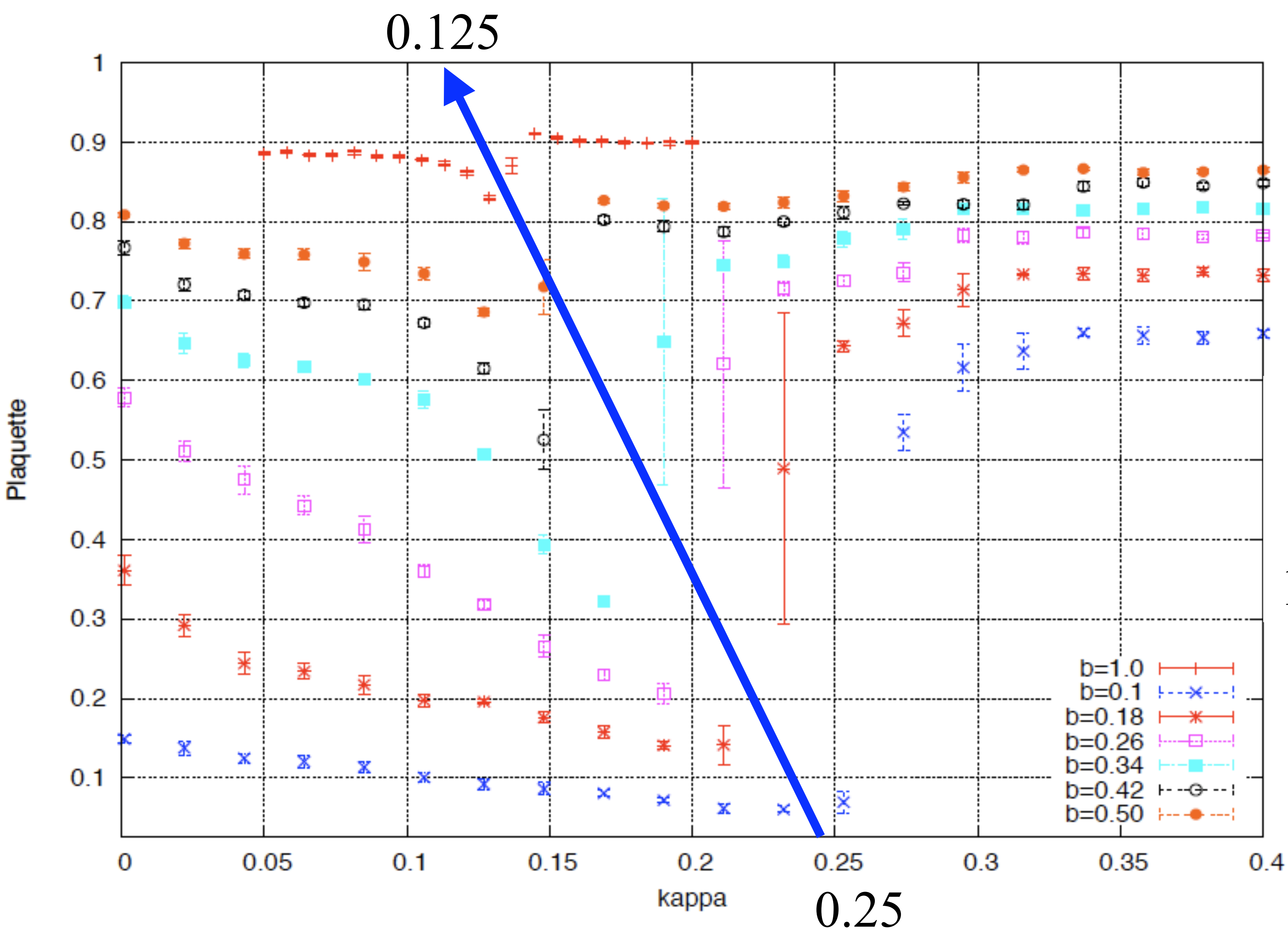


b=0.35:
1st transition
at kappa~0.15

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

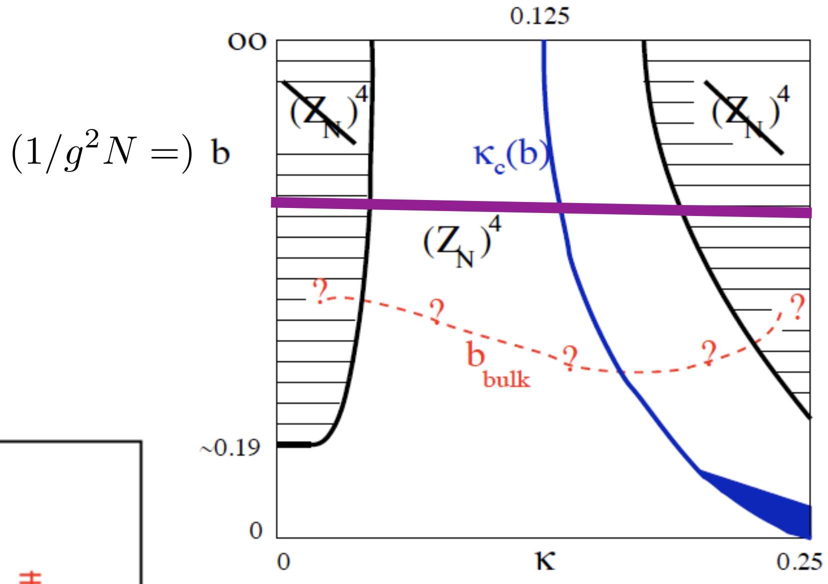


Scan no. 2 : looking for the “critical” line

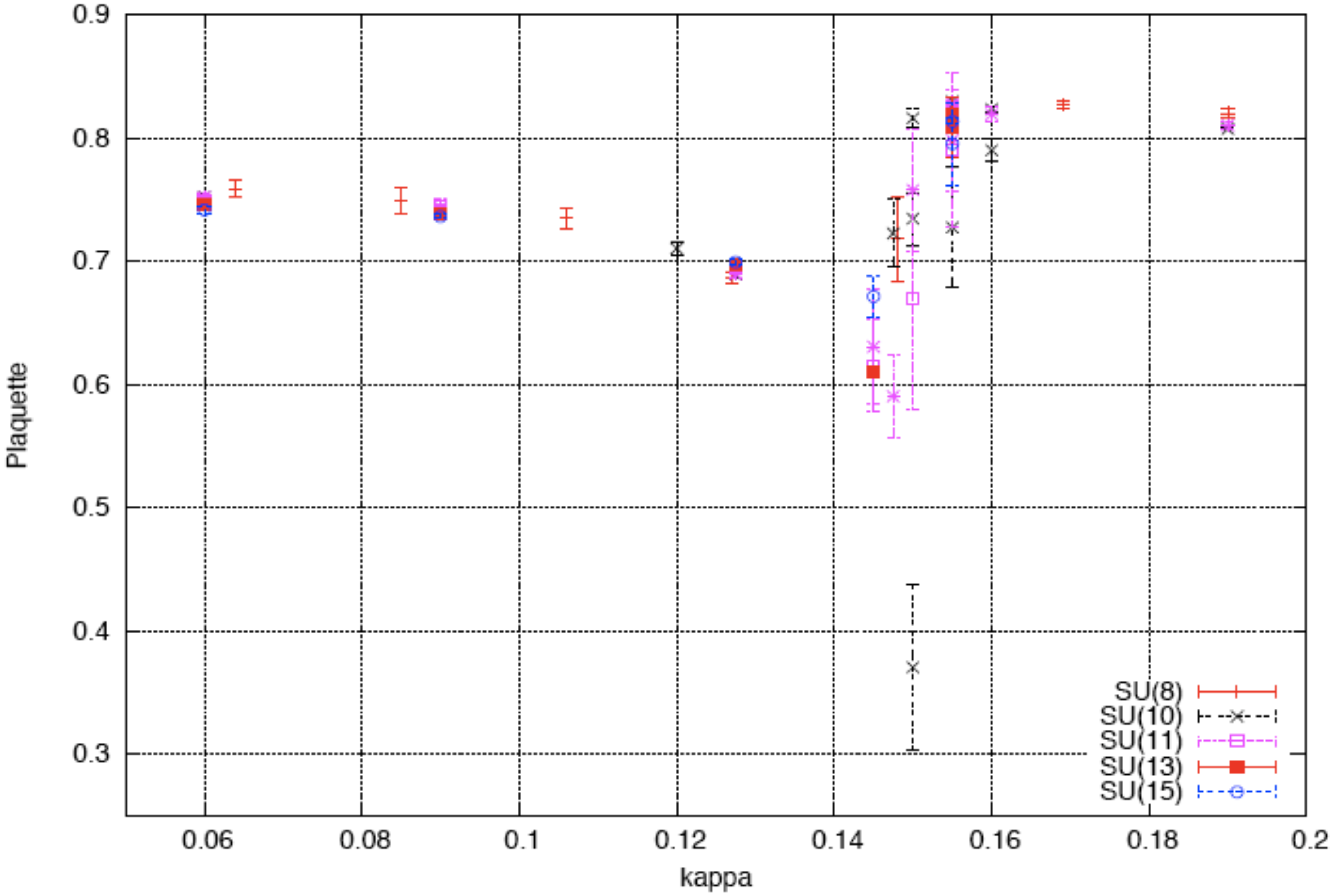


1st transition structure at all b.
Extending from kappa=0.25 to 0.125

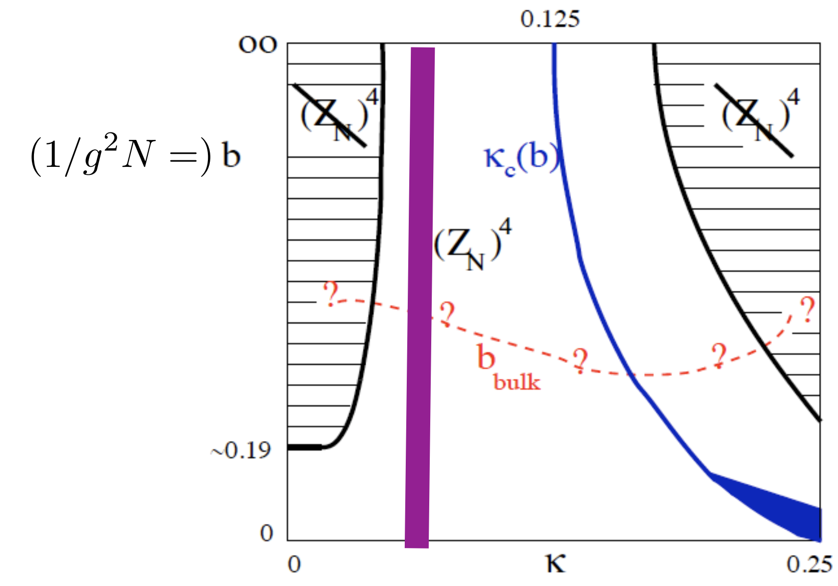
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



"Transition" present for all N studied
e.g. b=0.5, N=8, 10, 11, 13, 15:



II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538



All results consistent with phase diagram and validity of reduction.

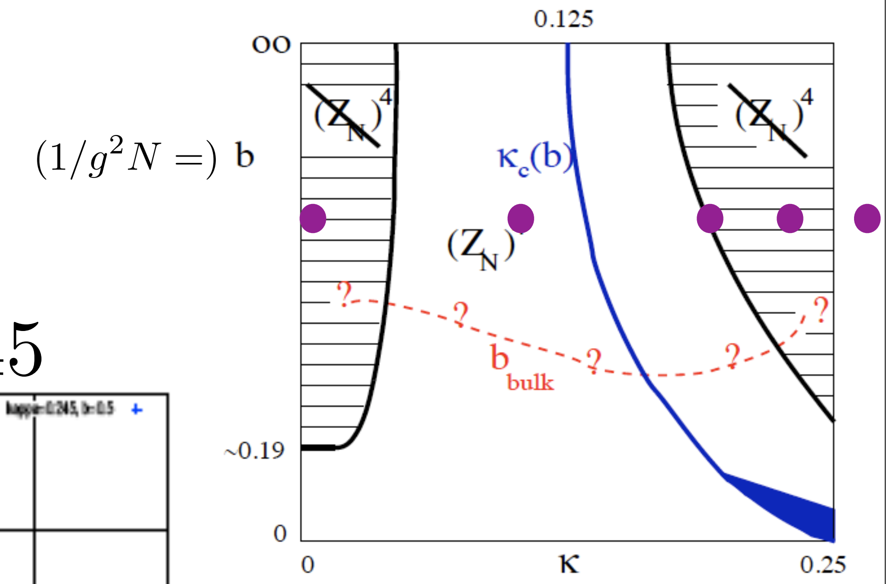
But:

More order parameters for nontrivial breaking of Z_N.

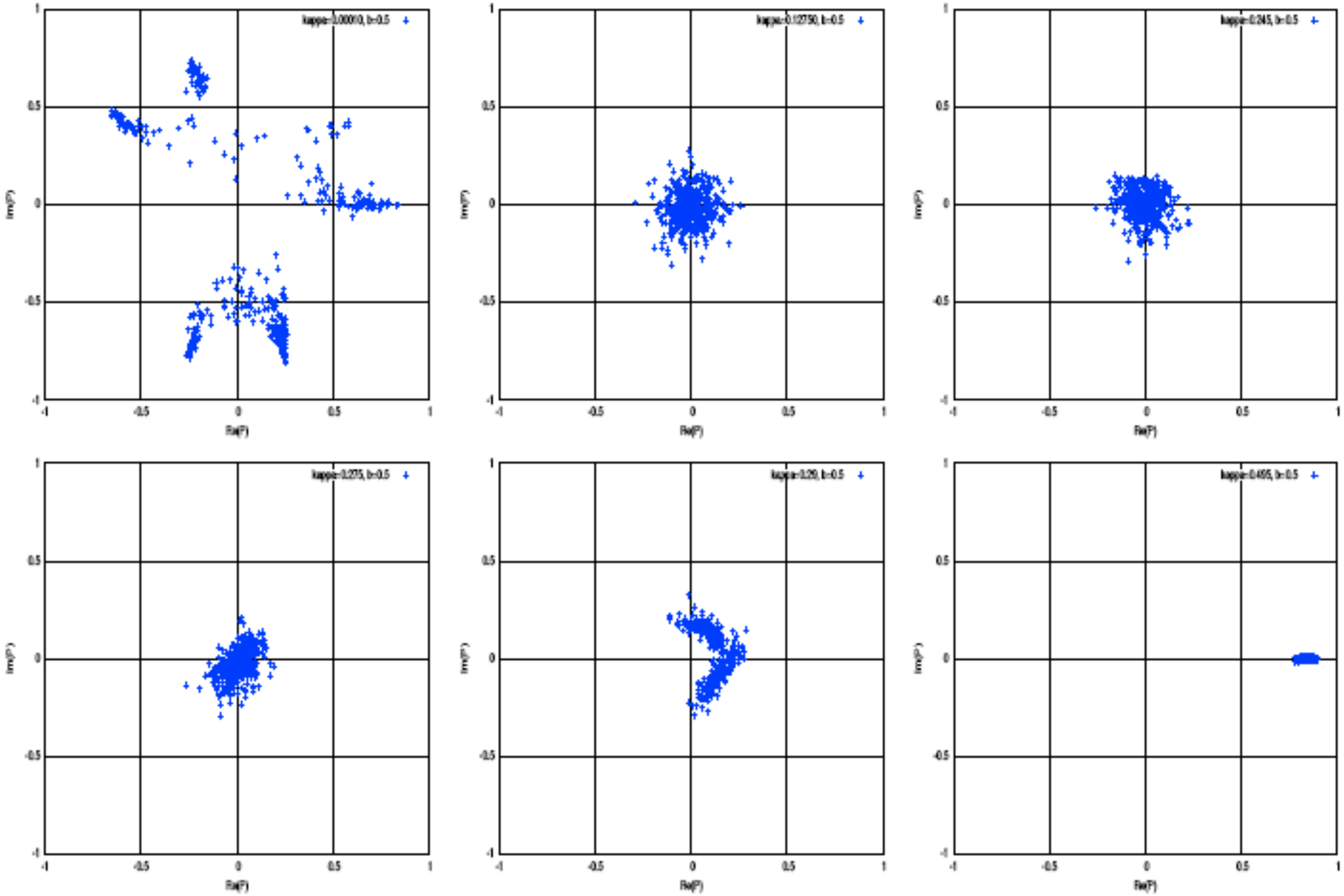
For example, $\mathrm{tr}\, U_\mu U_\nu \neq 0$ in QEK

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

b=0.5, SU(10): $\mathrm{tr}\, U_\mu$



$\kappa = 0.0001$ $\kappa = 0.1275$ $\kappa = 0.245$

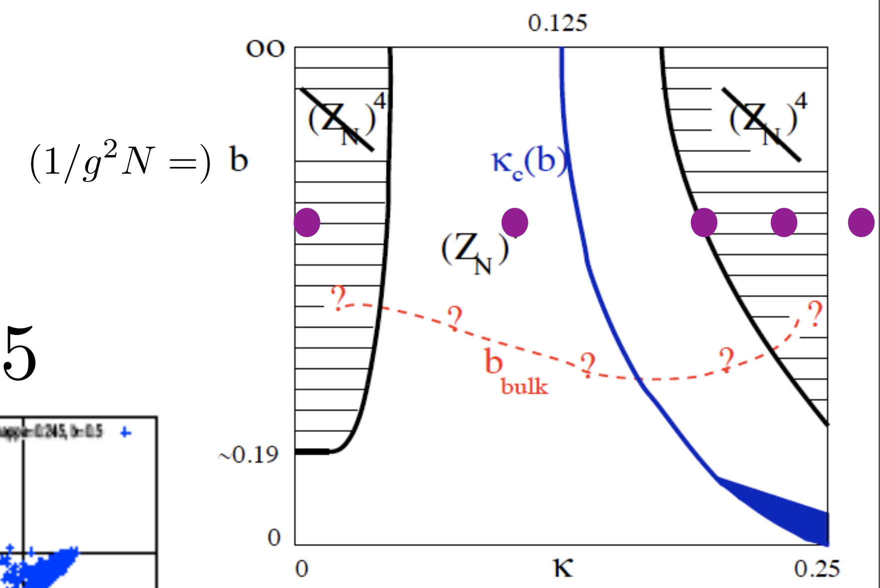


$\kappa = 0.275$ $\kappa = 0.29$ $\kappa = 0.495$

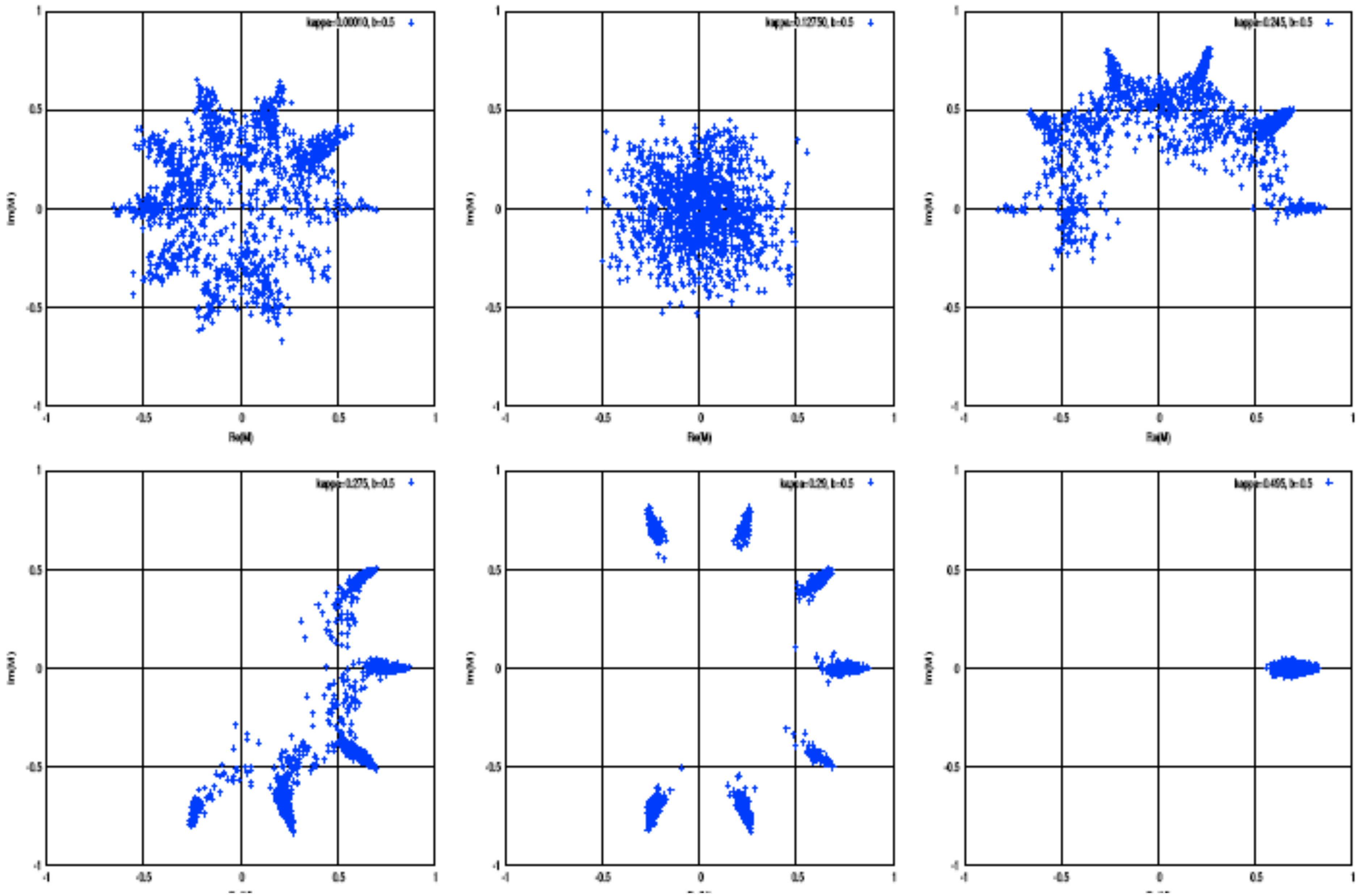
Indicate Z_N breaking for $\kappa \gtrsim 0.28$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

b=0.5, SU(10): $\mathrm{tr}\, U_\mu U_\nu$



$\kappa = 0.0001$ $\kappa = 0.1275$ $\kappa = 0.245$



$\kappa = 0.275$ $\kappa = 0.29$ $\kappa = 0.495$

Indicate Z_N breaking for $\kappa \gtrsim 0.24$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

Perform long runs and measure order parameters of the form

$\mathrm{tr}\,[P_1^{n_1}\,P_2^{n_2}\,P_3^{n_3}\,P_4^{n_4}\,]$ with $n_i \in [-5, 5]$ $\Rightarrow$ 14641 order parameters !

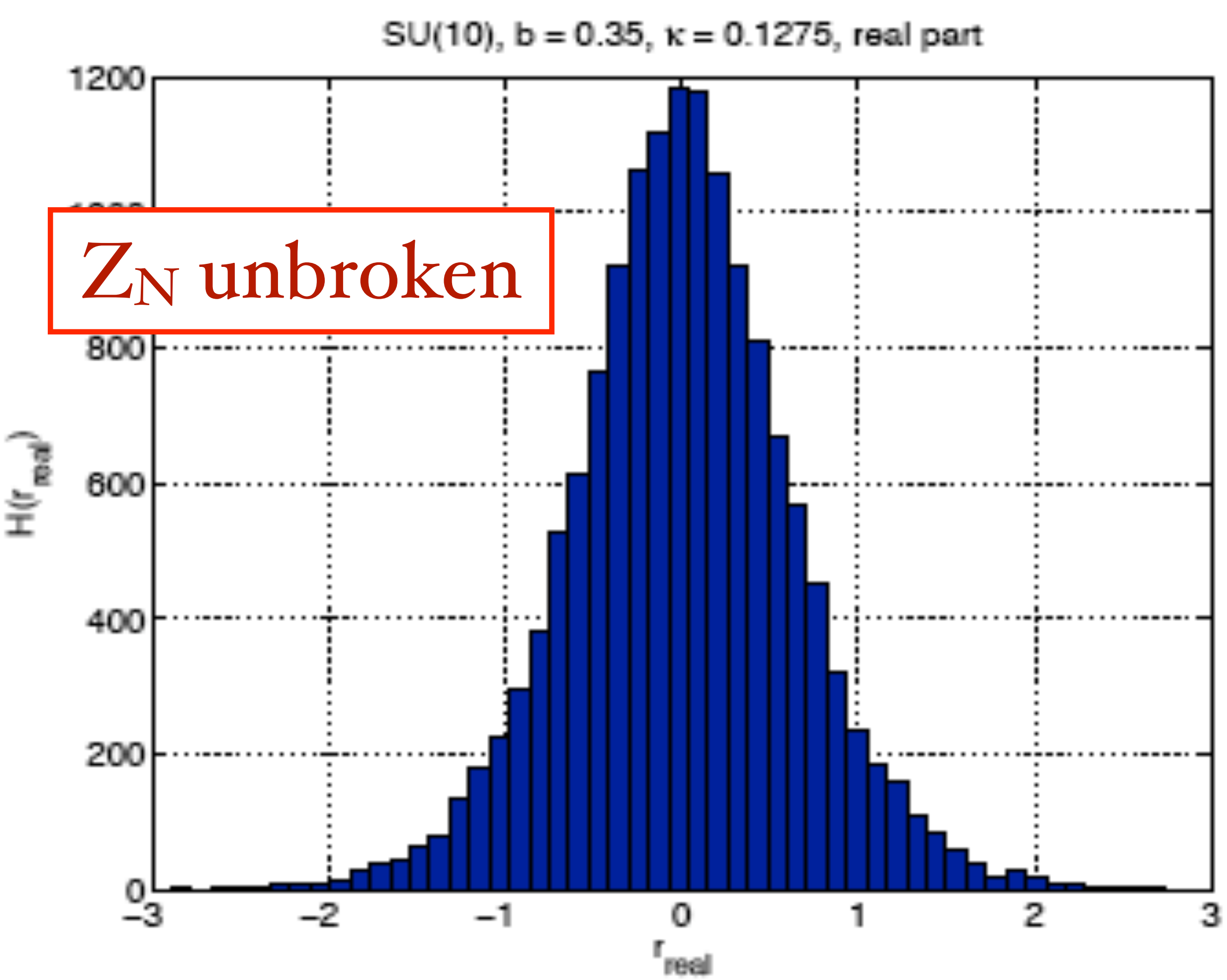
- For each histogram signal-to-noise for real and imag. part

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

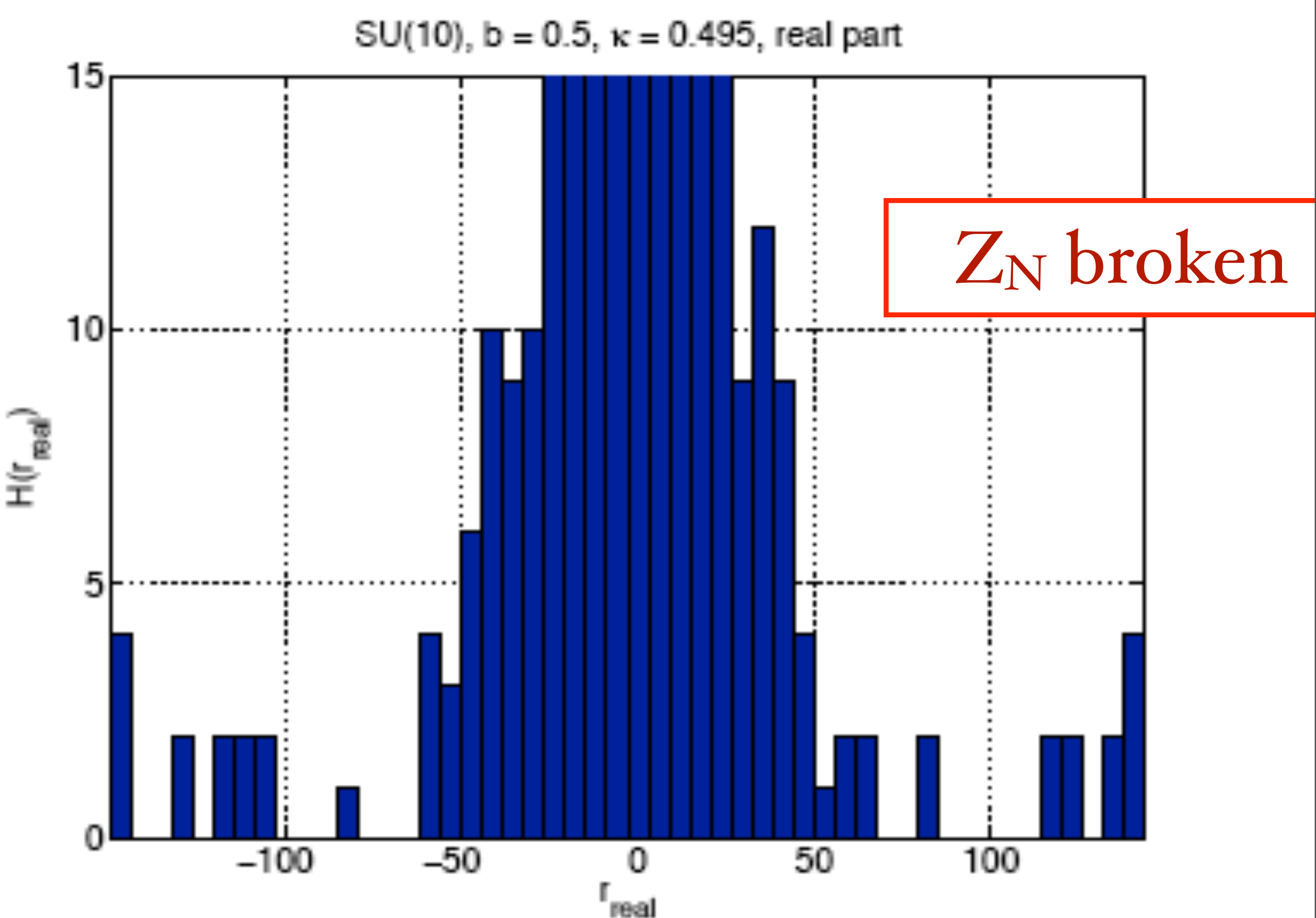
Perform long runs and measure order parameters of the form

$$\mathrm{tr}\,[P_1^{n_1}\,P_2^{n_2}\,P_3^{n_3}\,P_4^{n_4}\,] \quad \text{with} \;\; n_i \in [-5,5] \;\; \longrightarrow \;\; 14641 \text{ order parameters !}$$

- For each histogram signal-to-noise for real and imag. part



$N=10,\;\; b=0.35,\;\; \kappa=0.1275$

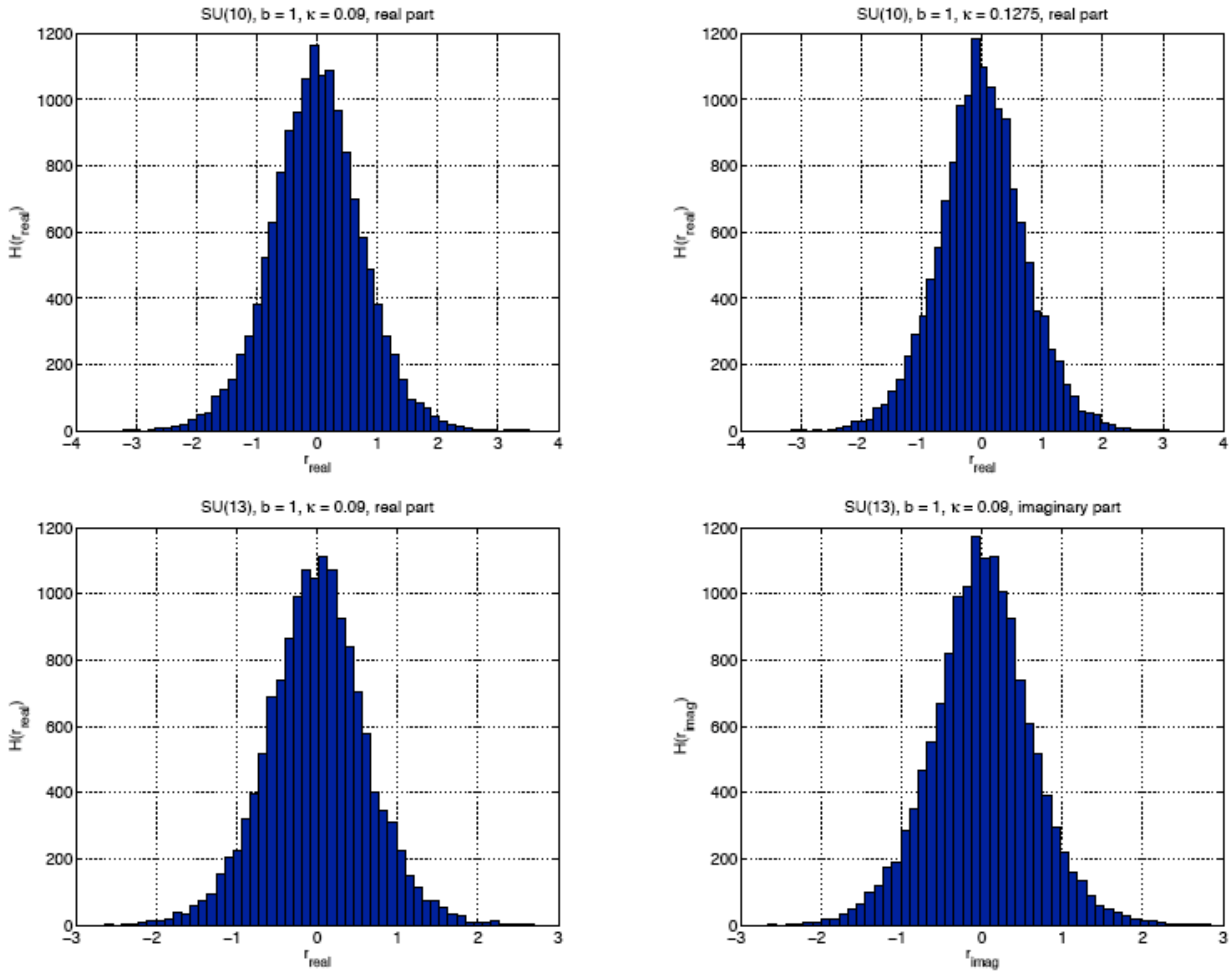


$N=10,\;\; b=0.5,\;\; \kappa=0.495$

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

Results for K_n at b=1:

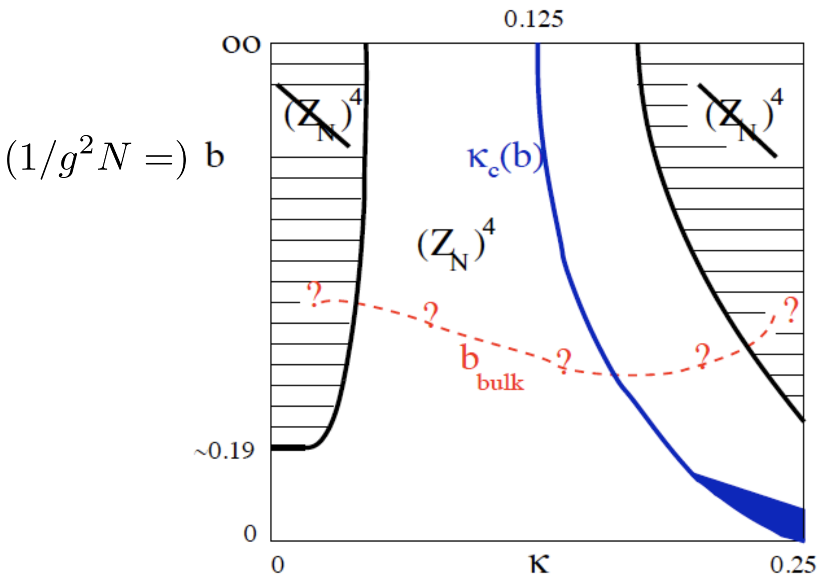
$N = 10,\ b = 1.0,\ \kappa = 0.09$ $N = 10,\ b = 1.0,\ \kappa = 0.1275$



$N = 13,\ b = 1.0,\ \kappa = 0.09$

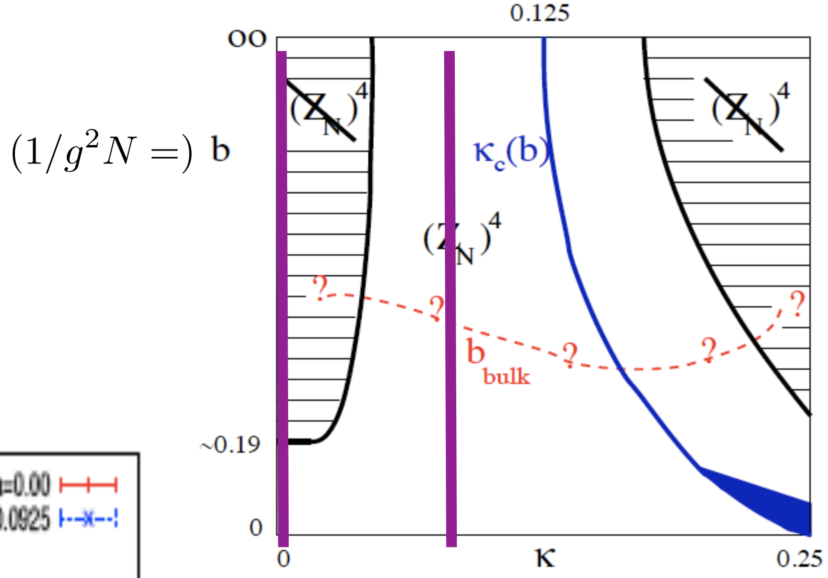
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

What about the bulk transition?

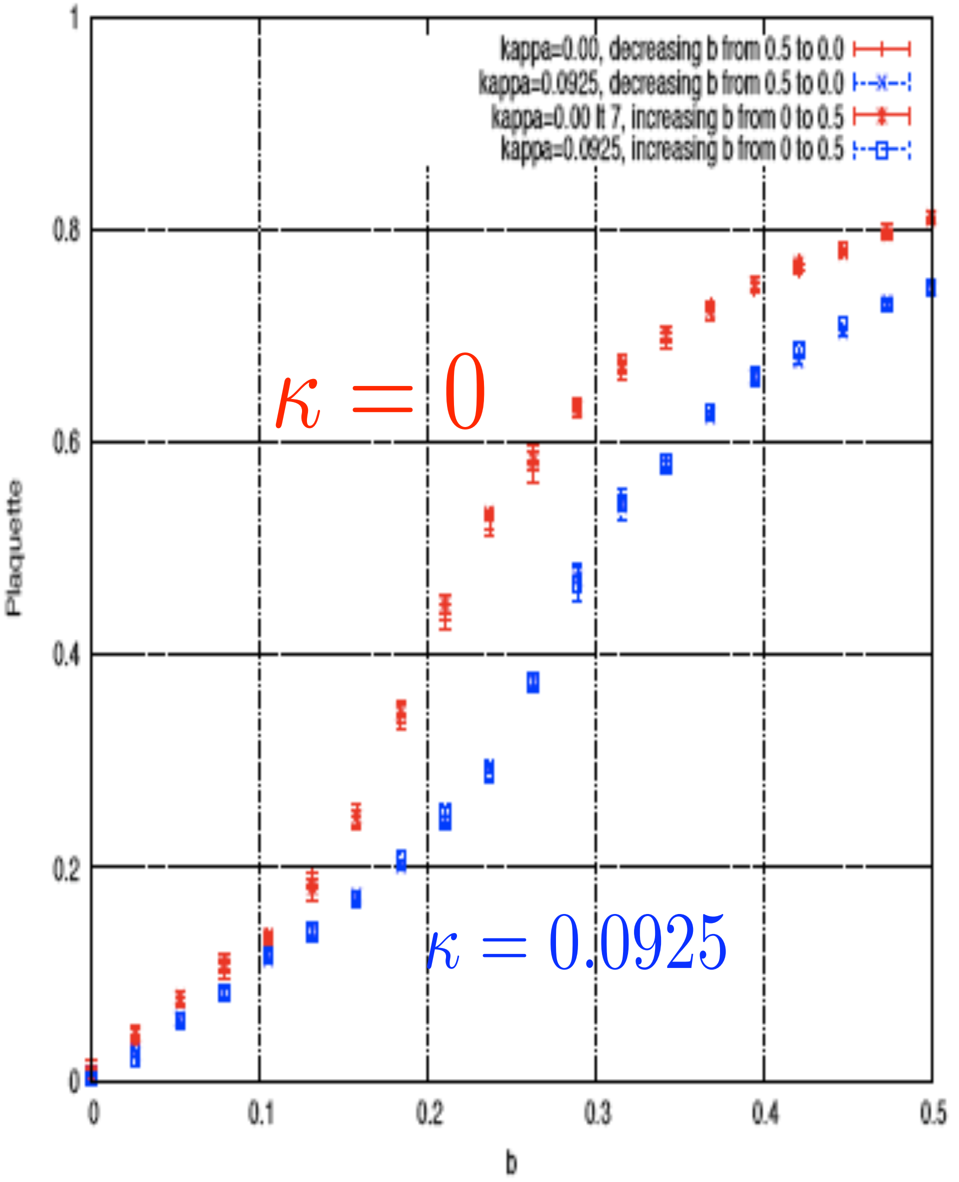


II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

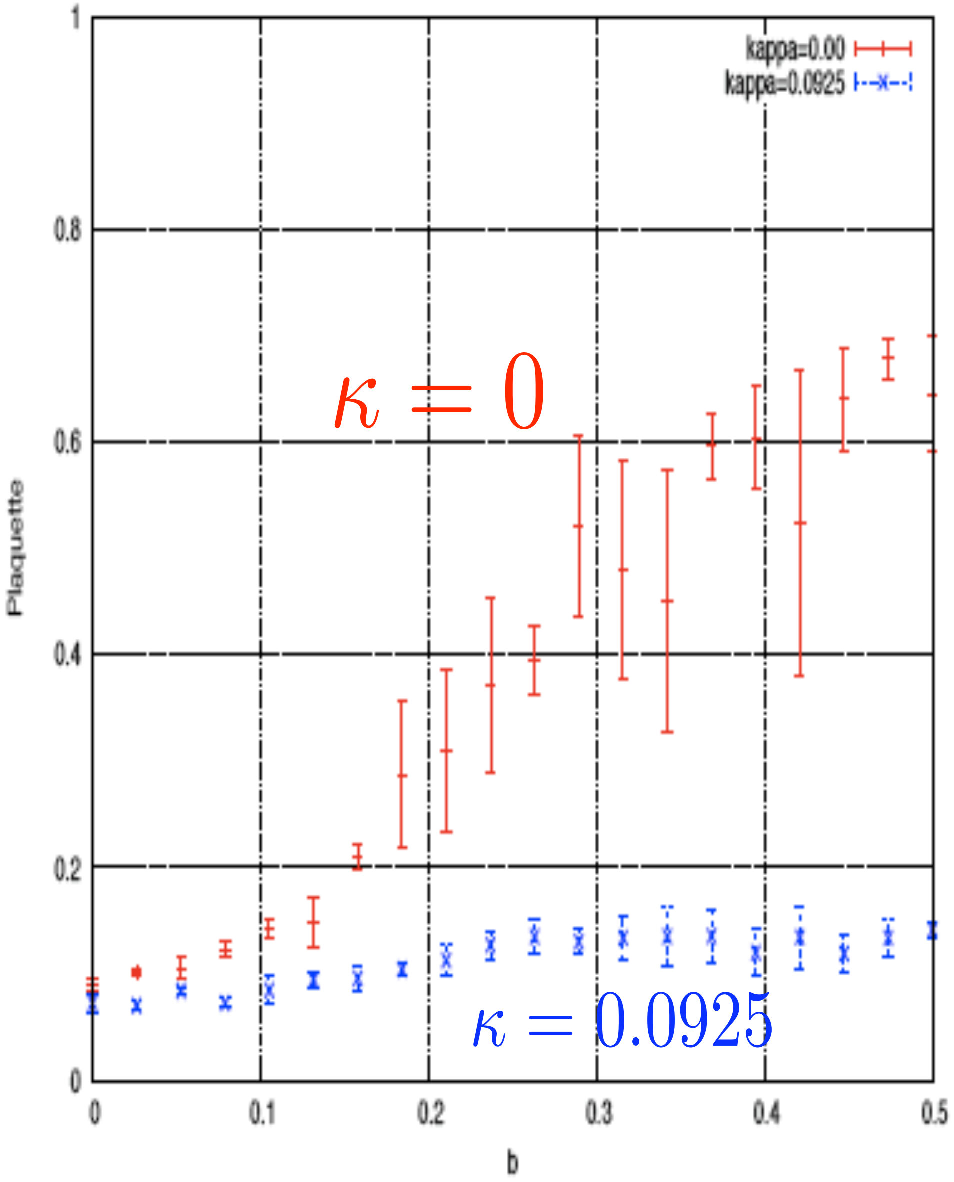
What about the bulk transition?



Plaquette



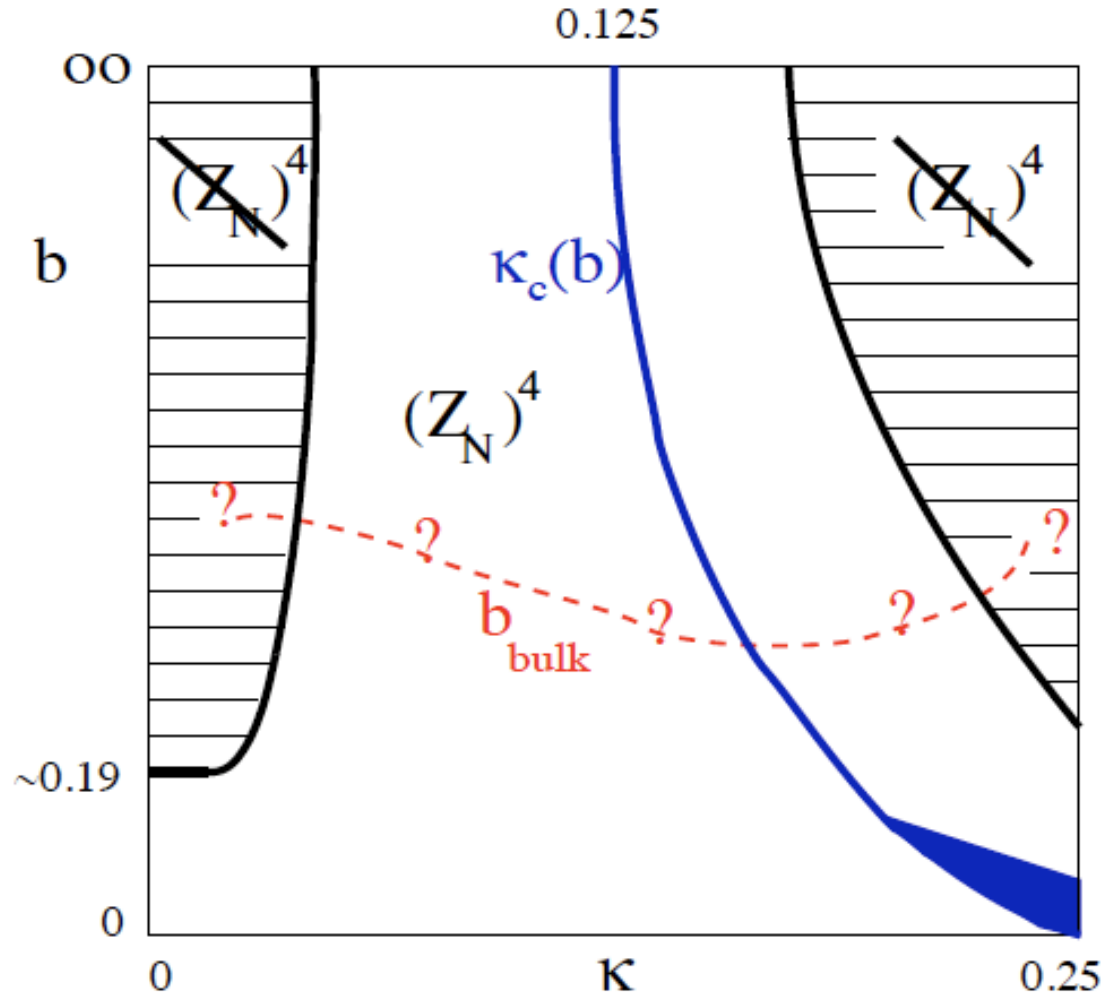
|Polyakov|



- Signs of bulk.
- Does not involve $(Z_N)^4$ breaking

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

- All our results consistent with conjecture



- Away from critical line, long distance theory is pure-gauge ⇒ original EK ?!?

- Near critical line, obtain adjoint QCD, within 1/N of physical QCD.

Results of “physical” interest: first pass

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

plaquette (action density):

- Not really physical, but an indication of $1/N^2$ corrections' size.
- Different from YM even if quarks are heavy

Integrating quarks

$\downarrow$

different effective action

- But nevertheless....

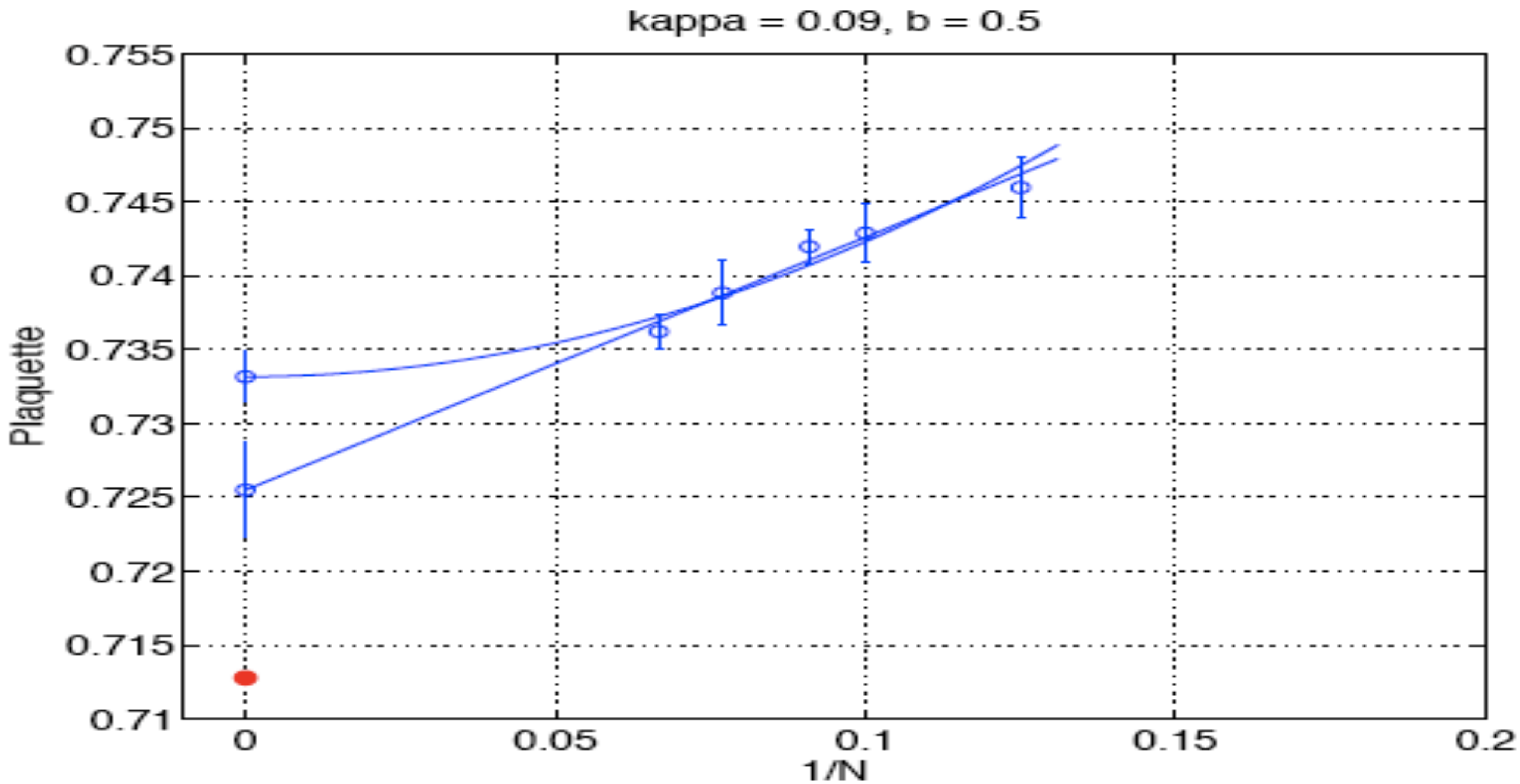
II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

plaquette (action density):

- Not really physical, but an indication of $1/N^2$ corrections' size.
- Different from YM even if quarks are heavy
- But nevertheless....

Integrating quarks

different effective action



II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

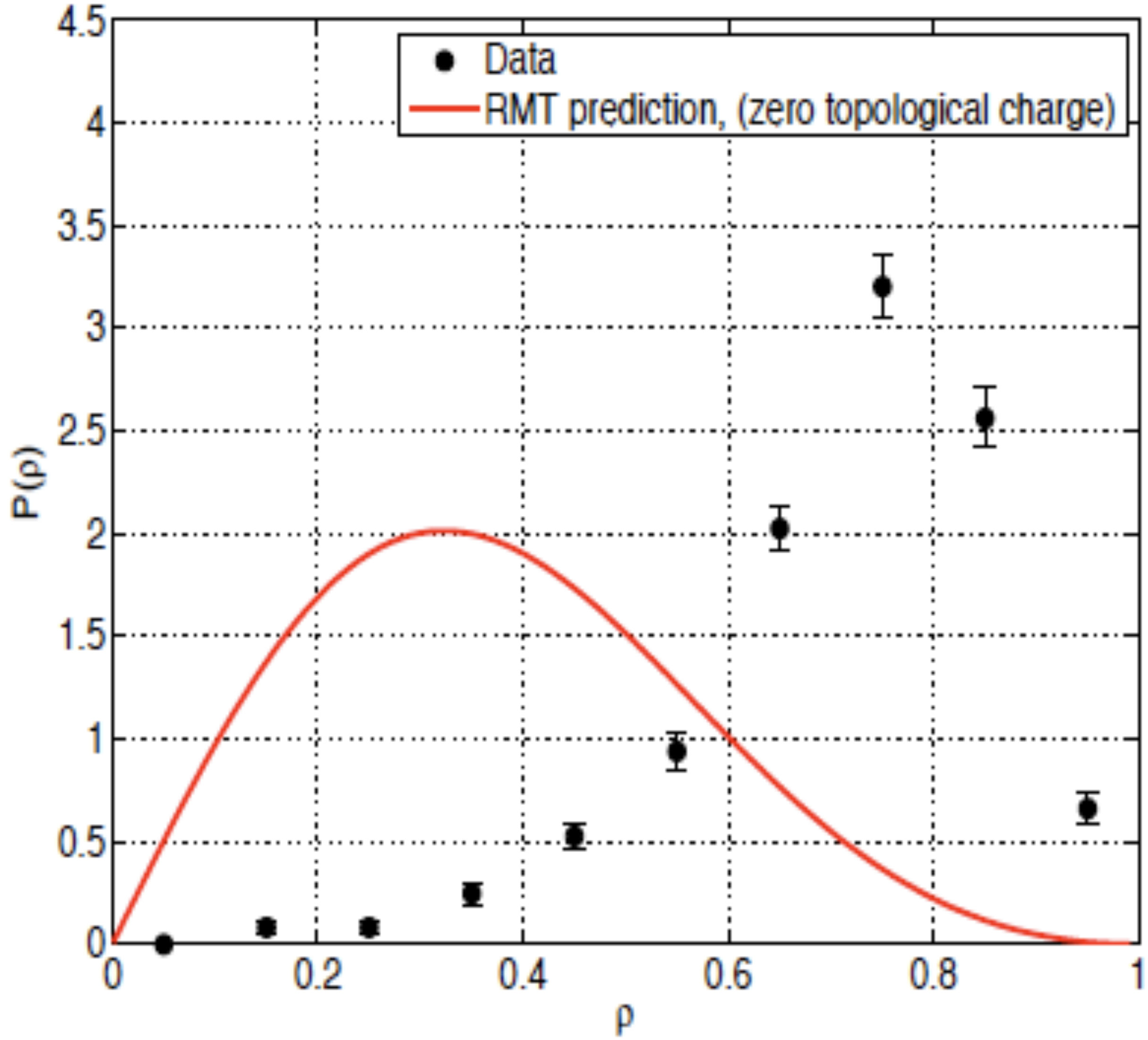
Dirac spectrum:

Eigenvalue distribution of valence Dirac operator and **quenched** **R**andom **M**atrix **T**heory

II.B. Results of non-perturbative MC lattice simulations BB+S.Sharpe, 0906.3538

Dirac spectrum:

Eigenvalue distribution of valence Dirac operator and **quenched** **R**andom **M**atrix **T**heory



Non-agreement seems typical to having too small N or volume
E.g. Narayanan & Neuberger `04

Try this with adjoints valence next ...

In any case :

Values of N that we used seem sufficient to map the phase-diagram.

III. Conclusions

Weak coupling with $L_{2,3,4}$=oo, L_1=1

- Space-time reduction seems to work for massless Wilson adjoints fermions.

  In progress: analyzing $Z_N \longrightarrow Z_K$ for massive fermions.

- Perturbation theory on L_1=1 leads to a UV-sensitive free energy.

  Do weak-coupling before MC with a different type of fermion !

Non-perturbative lattice Monte-Carlo of N_f=1 case.

- Space-time reduction seems to works for YM+Wilson adjoints fermions.

  At couplings where we foresee doing calculations.

  Both heavy and light masses.

III. Future directions

Can extract physics of QCD(Adj) and QCD(AS)

- Probably need to develop algorithms that would reduce the N^8 scaling of Metropolis!
- Mesons (using the “Gross-Kitazawa trick”)
- Realization of chiral symmetry (of both adjoint sea-quarks and valence fundamental).
- Comparisons with RMT.
- Static potential, string tensions.
- Other theories: is the two-flavor theory (nearly-)conformal?

Currently in progress...