Conformal vs confining scenario in SU(2) with adjoint fermions

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Outline

Outline

Technicolor

Bosonic sector

$$\begin{array}{ccccc} SU(3)_c & \times & SU(2)_L & \times & U(1)_Y \\ G_{\mu\nu} & & W_{\mu\nu} & & B_{\mu\nu} \end{array}$$

Fermionic sector

$$\begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} \quad , \quad u_R^i \quad , \quad d_R^i$$

$$\begin{pmatrix} e_L^i \\ \nu_L^i \end{pmatrix} \quad , \quad e_R^i \quad , \quad \nu_R^i(?) \qquad i = 1, 2, 3$$

Higgs sector

Higgs field
– complex scalar field in the $\frac{1}{2}$ repr. of $SU(2)_L$
Mexican-hat potential
– $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ and Higgs mechanism
Yukawa coupling
– fermion masses

Technicolor

Bosonic sector

$$\begin{array}{ccccc} SU(3)_c & \times & SU(2)_L & \times & U(1)_Y \\ G_{\mu\nu} & & W_{\mu\nu} & & B_{\mu\nu} \end{array}$$

Fermionic sector

$$\begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} \quad , \quad u^i_R \quad , \quad d^i_R$$

$$\begin{pmatrix} e^i_L \\ \nu^i_L \end{pmatrix} \quad , \quad e^i_R \quad , \quad \nu^i_R(?) \qquad i = 1, 2, 3$$

Chiral symmetry breaking

$$\langle \bar{u}_R u_L + \bar{d}_R d_L \rangle = \frac{\Sigma}{2}$$

– $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
– small mass for $W^{\pm}$, Z bosons
– massless fermions

Technicolor

Bosonic sector

$$\begin{array}{ccccc} SU(3)_c & \times & SU(2)_L & \times & U(1)_Y \\ G_{\mu\nu} & & W_{\mu\nu} & & B_{\mu\nu} \end{array}$$

Fermionic sector

$$\begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} \quad , \quad u^i_R \quad , \quad d^i_R$$

$$\begin{pmatrix} e^i_L \\ \nu^i_L \end{pmatrix} \quad , \quad e^i_R \quad , \quad \nu^i_R(?) \qquad i = 1, 2, 3$$

Technicolor sector

$$SU(N_{TC}) \text{ gauge theory}$$

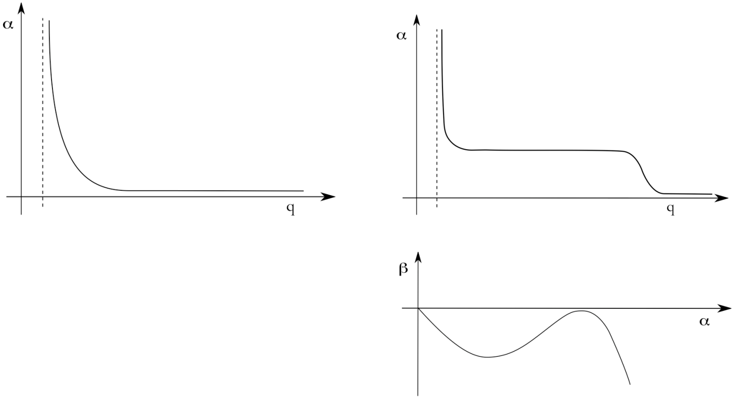
$$\begin{pmatrix} U^i_L \\ D^i_L \end{pmatrix} \quad , \quad U^i_R \quad , \quad D^i_R \qquad i = 1, \ldots, N_f$$

$$\langle \bar{U}_R U_L + \bar{D}_R D_L \rangle = \frac{\Sigma_{TC}}{2}$$

(Walking) technicolor

- $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- Λ_{TC} is tuned to give the right mass to the $W^{\pm}$, Z bosons
- 4-operator coupling $\bar{Q}Q\bar{q}q$ to give mass to the SM fermions; effectively generated by some more fundamental theory (*extended technicolor*, ETC) at higher energy Λ_{ETC}
- in general too many technipions exists
- ETC generates also masses for the extra technipions (good!) and flavor changing neutral currents (FCNC, bad!)
- we can require Λ_{ETC} to be high enough in order to suppress FCNC, but then we need an enhancement mechanism to get reasonable masses for the SM fermions and high masses for the extra technipions...
- ...walking technicolor

(Walking) technicolor

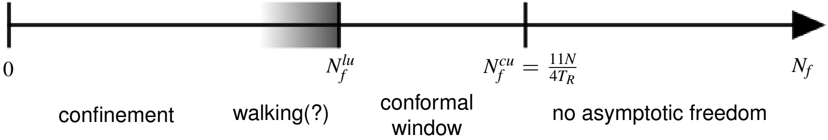


Conformal window

$$\beta(\mu) = \mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \ldots$$

$$b_0 = \frac{1}{(4\pi)^2} \left(\frac{11}{3} N - \frac{4}{3} T_R N_f \right)$$

$$b_1 = \frac{1}{(4\pi)^4} \left(\frac{34}{3} N^2 - \frac{20}{3} N T_R N_f - 4 \frac{N^2 - 1}{d_R} T_R^2 N_f \right)$$



Banks-Zaks (perturbative) fixed point:

$$g_*^2 \simeq -\frac{b_0}{b_1}$$

Hunting for the conformal window

$SU(3)$ with $N_f = 8, 12, 16$ staggered fundamental fermions ($N_f^{cu} = 16.5$)

A. Deuzeman, M.P. Lombardo, E. Pallante.

- Phase structure of the bare-parameter space.

$N_f = 8$ confining, $N_f = 12$ conformal.

T. Appelquist, G. T. Fleming, E. T. Neil. LSD collaboration.

- SF running coupling.

$N_f = 8$ confining, $N_f = 12$ conformal.

Z. Fodor, K. Holland, J. Kuti, D. Nogradi, C. Schroeder.

- Mesonic spectrum.
- Finite volume effects and ϵ-regime.
- WL running coupling.

$N_f = 8$ confining, $N_f = 12$ confining?, $N_f = 16$ conformal.

X. Jin, R. D. Mawhinney.

- Mesonic spectrum.
- Quark potential.
- Phase structure.

$N_f = 8$ confining, $N_f = 12$ controversial.

A. Hasenfratz.

- Montecarlo Renormalization Group.

No results yet.

Hunting for the conformal window

$SU(3)$ with $N_f = 2$ Wilson 2S fermions ($N_f^{cu} = 3.3$)

T. DeGrand, Y. Shamir, B. Svetitsky.

- Finite-temperature phases.
- SF descrete beta function.

Controversial.

D.K. Sinclair, J.B. Kogut.

- Finite-temperature phases.

Confining.

Hunting for the conformal window

$SU(2)$ with $N_f = 2$ Wilson adjoint fermions ($N_f^{cu} = 2.75$)

A. Hietanen , T. Karavirta, A. Mykkanen, J. Rantaharju, K. Rummukainen, K. Tuominen.

- Spectrum.
- SF running coupling.

Conformal.

S. Catterall, J. Giedt, F. Sannino, J. Schneible.

- Mesonic spectrum.

Conformal.

L. Del Debbio, B. Lucini, AP, C. Pica, A. Rago.

- Mesonic and gluonic spectrum.
- SF running coupling.
- Finite-size scaling.

Outline

Running coupling

Confinement and χSB.
- Conformal anomaly + asymptotic freedom.
- The RG flow has an UV gaussian fixed point.
- Λ separates the asymptotically free and non-perturbative regions.

$$g(\mu) = \begin{cases} \dfrac{1}{2b_0 \log\left(\frac{\mu}{\Lambda}\right)} & \mu \to \infty \\ +\infty & \mu \to 0 \end{cases}$$

IR conformality.
- Conformal anomaly + asymptotic freedom.
- The RG flow has an UV gaussian and an IR fixed point. The theory flows from the UV to the IR fixed point.
- Λ separates the asymptotically free and scale-invariant regions.

$$g(\mu) = \begin{cases} \dfrac{1}{2b_0 \log\left(\frac{\mu}{\Lambda}\right)} & \mu \to \infty \\ g_* & \mu \to 0 \end{cases}$$

Conformality.
- Conformal symmetry.
- The RG flow has an UV gaussian and an IR fixed point. The theory sits in the IR fixed point.

$$g(\mu) = g_*$$

Physical definition of Λ

Mass of particles

Confinement and χSB.
The pions are the Goldstone bosons for χSB. All the other particles are massive.

$$M_\rho = a_\rho \Lambda$$

IR conformality (or conformality).
Unparticles. The correlators decay with power law.

$$C(x,g,\mu) = \langle \Phi_R(x) \Phi_R(0) \rangle (g,\mu)$$

$$\left\{ x \frac{\partial}{\partial x} + \beta(g) \frac{\partial}{\partial g} + 2 \left[d_\Phi - \gamma_\Phi(g) \right] \right\} C = 0$$

$$C(x,g,\mu) \simeq A \mu^{2d_\Phi} \left(\frac{\mu}{x} \right)^{2(d_\Phi - \gamma_\Phi)} \qquad x \gg \Lambda$$

Physical definition of Λ

$e^+e^- \to \bar{q}q$ cross section

Confinement and χSB. Λ can be defined as the energy at which the cross section deviates from free-theory behaviour by a certain relative amount.

$$\sigma(s) = \frac{A_0}{s} \left\{ 1 + \sum_{k=1}^{\infty} a_k g^{2k}(s^{1/2}) \right\}$$

$$s\sigma(s)\big|_{s=\Lambda^2} = A_0(1+1\%)$$

Conformality. The behaviour of the cross section is completely determined by scale invariace.

$$\sigma(s) = \frac{A_*}{s}$$

IR conformality. The cross section interpolates between the free and the conformal theories.

$$\sigma(s) = \begin{cases} \frac{A_0}{s} & s \to \infty \\ \frac{A_*}{s} & s \to 0 \end{cases}$$

$$s\sigma(s)\big|_{s=\Lambda^2} = \frac{A_0 + A_*}{2}$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \, \langle \Phi_R(t,\mathbf{x}) \Phi_R(0) \rangle (g,m,\mu)$$

Weinberg-Callan-Symanzik equation.

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\,[d_\Phi - \gamma_\Phi(g)] \right\} C(t,g,m,\mu) = 0$$

$$\mu\frac{dg}{d\mu} = \beta(g)$$

$$\frac{\mu}{m}\frac{dm}{d\mu} = -\gamma(g)$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \, \langle \Phi_R(t,\mathbf{x}) \Phi_R(0) \rangle (g,m,\mu)$$

Weinberg-Callan-Symanzik equation. Close to the fixed point...

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\,[d_\Phi - \gamma_\Phi(g)] \right\} C(t,g,m,\mu) = 0 + \text{corrections}$$

$$\mu\frac{dg}{d\mu} = \beta(g)$$

$$\frac{\mu}{m}\frac{dm}{d\mu} = -\gamma(g)$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \, \langle \Phi_R(t,\mathbf{x}) \Phi_R(0) \rangle (g,m,\mu)$$

Weinberg-Callan-Symanzik equation.

$$\left\{ t\frac{\partial}{\partial t} - [1+\gamma]\, m\frac{\partial}{\partial m} + 2\,[d_\Phi - \gamma_\Phi] \right\} C(t,g,m,\mu) = 0$$

Solution of the Weinberg-Callan-Symanzik equation.

$$C(t,g,m,\mu) \simeq b^{2(d_\Phi - \gamma_\Phi)} C(bt, g_*, b^{-(1+\gamma)} m, \mu) =$$

$$\simeq \mu^{2d_\Phi} \left(\frac{m}{\mu} \right)^{2\frac{d_\Phi - \gamma_\Phi}{1+\gamma}} F\left(tm^{\frac{1}{1+\gamma}}, \mu \right)$$

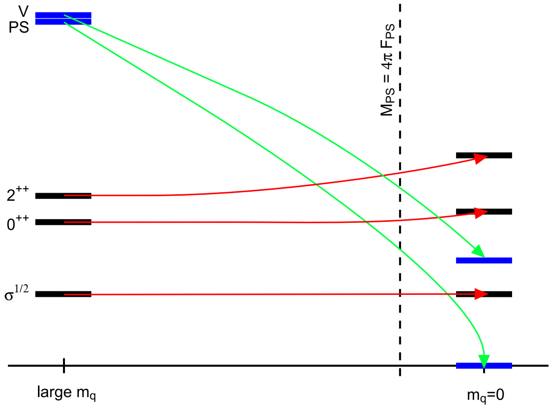
The mass term breaks the asymptotic scale invariance. A mass gap is expected to be generated.

$$C(t,g,m,\mu) \simeq A \exp - M_\Phi t$$

$$M_\Phi = a_\Phi \mu \left(\frac{m}{\mu} \right)^{\frac{1}{1+\gamma}} \qquad m \to 0$$

Some cartoons

Confinement and χSB



large fermion mass
(pure YM + NR fermions)

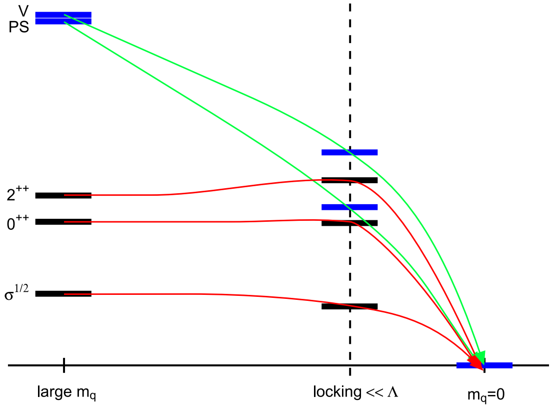
$$M_{PS} \simeq M_V \simeq 2m_q$$
$$M_V/M_{PS} \simeq 1$$
$$M_G \simeq M_G^{(YM)}$$

chiral limit (χPT)

$$M_{PS}^2 \simeq -\frac{\langle\bar{\psi}\psi\rangle}{F_{PS}^2} m_q$$
$$M_V \simeq M_V^{(0)}$$
$$M_G \simeq M_G^{(0)}$$

Some cartoons

IR conformality



large fermion mass
(pure YM + NR fermions)

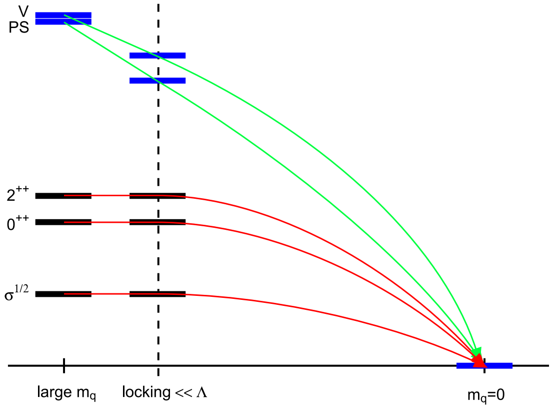
$$M_{PS} \simeq M_V \simeq 2m_q$$
$$M_V/M_{PS} \simeq 1$$
$$M_G \simeq M_G^{(YM)}$$
$$\sigma \simeq \sigma^{(YM)}$$

chiral limit (scaling region)

$$M_X \simeq a_X \mu^{1-\alpha} m_q^{\alpha}$$
$$\sigma^{1/2} \simeq a_\sigma \mu^{1-\alpha} m_q^{\alpha}$$
$$M_X/M_Y \simeq a_X/a_Y$$
$$M_X/\sigma^{1/2} \simeq a_X/a_\sigma$$

Some cartoons

IR conformality



at all scales

$$M_V/M_{PS} \simeq 1+\epsilon$$
$$M_{PS} \gg \sigma^{1/2}$$
$$M_G/\sigma^{1/2} \simeq \left[M_G/\sigma^{1/2}\right]^{(YM)}$$

Miransky's scenario for Banks-Zaks fixed point

V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

- Banks-Zaks fixed point $g_*^2 = -\frac{b_0}{b_1} \ll 1$.
- The fermionic mass destroys the IR fixed point. Define the fermionic pole mass M_q.

$$M_q = a_q \mu \left(\frac{m_q}{\mu} \right)^{\frac{1}{1+\gamma}} \qquad M_q \ll \Lambda$$

- In the limit $M_q \gg \Lambda$, the fermions decouple and theory is pure YM. Consider the regime $M_q \ll \Lambda$.
- The running coupling constant:

$$g(m,\mu) = \begin{cases} g(0,\mu) & \mu > M_q \\ g_* & \mu = M_q \\ \frac{1}{2b_0^{YM} \ln \frac{\mu}{\Lambda_{YM}}} & \mu \ll M_q \end{cases}$$

- Λ_{YM} is not a new scale in the theory, but is computed by requiring continuity about the energy scale $\mu \simeq M_q$.

$$\Lambda_{YM} = M_q \, e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M_q$$

Miransky's scenario for Banks-Zaks fixed point

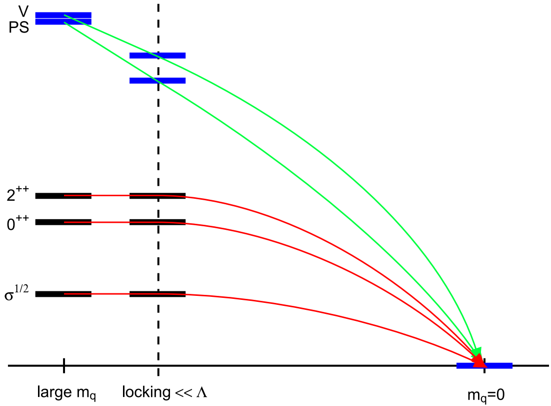
V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

$$\Lambda_{YM} = M_q \, e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M_q \ll \Lambda$$

- At energies much lower than M_q, the original theory is effectively described by a pure Yang-Mills theory with scale Λ_{YM}.
- Glueballs are lighter than mesons.
- A deconfinement transition occurs at a temperature $T_c \simeq \Lambda_{YM}$.
- Mesons are effectively quenched. The mesons are bound states of the quark-antiquark pair interacting via the YM static potential, the bound energy is small with respect to the mass of the constituents, and the correction to the potential due to quark-antiquark pair creation are negligible.
- As the mass M_q is reduced, the IR physics is always the same, provided that all the masses are rescaled with M_q.

Some cartoons again

IR conformality



at all scales

$$M_V/M_{PS} \simeq 1+\epsilon$$

$$M_{PS} \gg \sigma^{1/2}$$

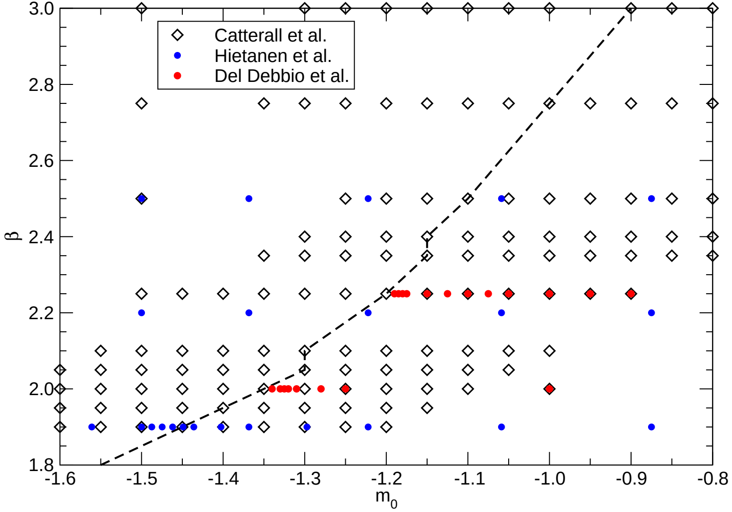
$$M_G/\sigma^{1/2} \simeq \left[M_G/\sigma^{1/2}\right]^{(YM)}$$

Outline

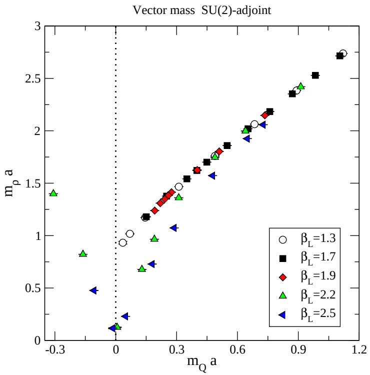
General strategy

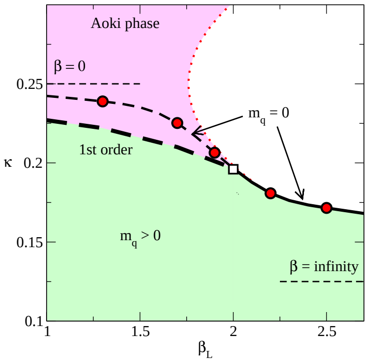
- In order to distinguish between confinement and IR-conformality, the study of the chiral limit is essential.
- The IR-conformality is characterised by the presence of a scaling region (all the masses go to zero with the same power law).
- In principle one could investigate the existence of a power-law behaviour, but unfortunately the quality of the fits is poor. A stabler numerical strategy is to look for plateaux in ratios of masses.
- In the case of a perturbative-like IR fixed point, the quasi-degeneracy of PS and V mesons is a common feature to the high-mass and chiral regimes. Instead the glueballs and the string tension have drammatically different behaviours in the two opposite regimes.

What has been simulated so far



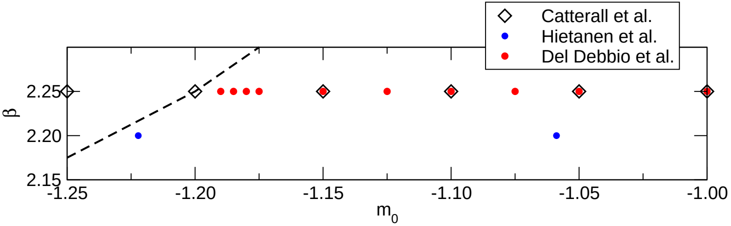
Large masses or conformality remnants?

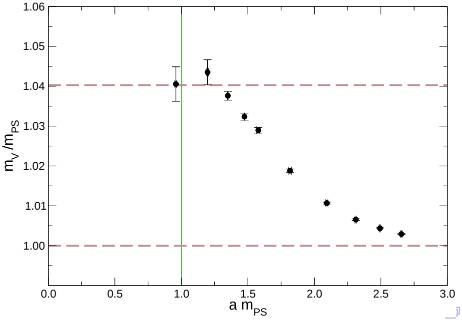




A. J. Hietanen, K. Rummukainen, K. Tuominen. Spectrum of SU(2) lattice gauge theory with two adjoint Dirac flavours. arXiv:0812.1467 [hep-lat].

Large masses or conformality remnants?





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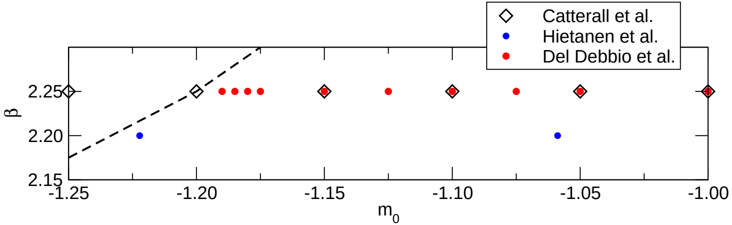
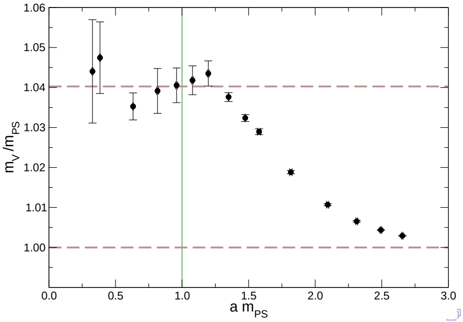


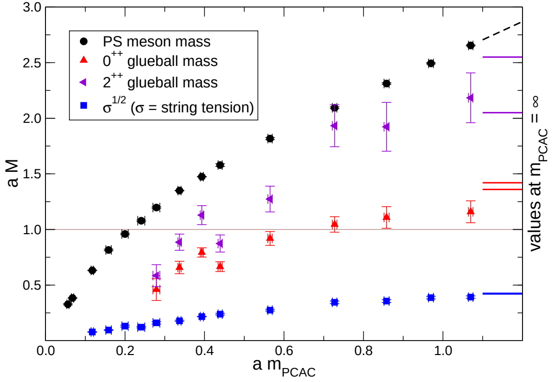


Outline

Some simulation details

- SU(2) 1x1 plaquette action in the fundamental representation
- + 2 Dirac Wilson fermions in the adjoint representation
- fixed lattice spacing: $\beta = 2.25$
- 4 volumes: $16\text{x}8^3$, $24\text{x}12^3$, $32\text{x}16^3$, $64\text{x}24^3$

Spectrum hierarchy



PS and V almost degenerate on the scale of the plot

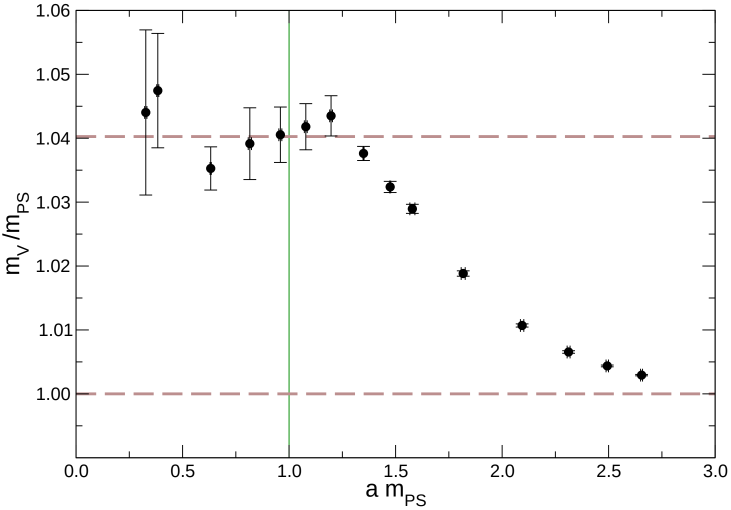
PS heavier than the two lightest glueballs

$\sigma^{1/2}$ always smaller than a^{-1}, but comparable with the spatial volumes for the smallest masses

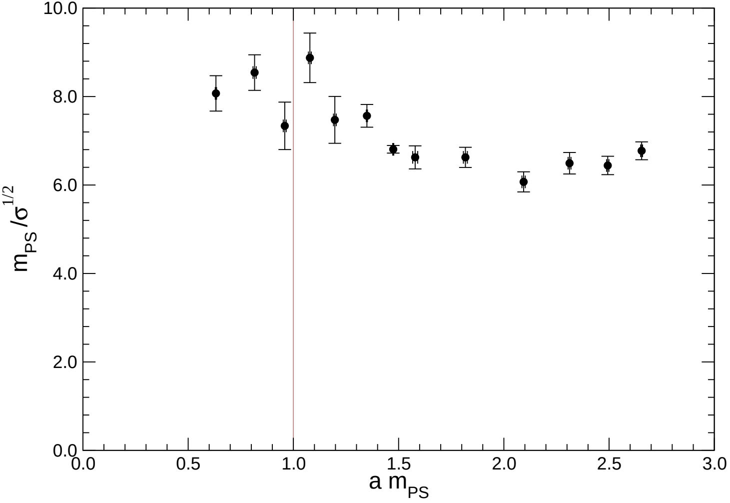
few glueballs with masses below a^{-1}

at masses with the pseudoscalar lighter than a^{-1}, there are no measured glueballs

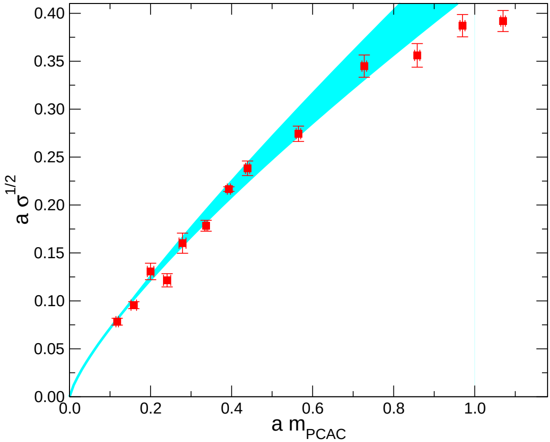
Scaling region



Scaling region



Scaling region



$$a\sigma^{1/2} = A_\sigma (am_q)^{\frac{1}{1+\gamma}}$$

$$\gamma \simeq 0.19 \ \ldots \ 0.30$$

IR dynamics

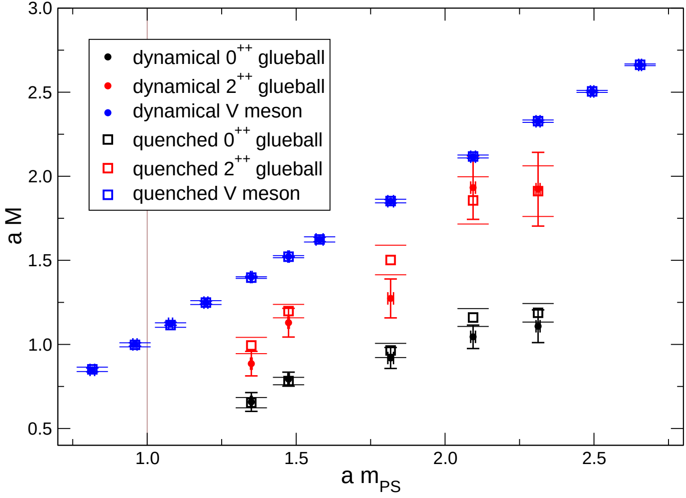
The spectrum shows some qualitative features of the perturbative IR fixed point. This would suggest that the theory is effectively decribed at low energy by pure Yang-Mills. How can we check this?

$$\int \left[dU_\mu \, d\psi \, d\bar{\psi}\right]_{k>\mu} e^{-\beta S_{YM}(U)+\bar{\psi}D_m^{-1}(U)\psi} = \int \left[dU_\mu \, d\psi \, d\bar{\psi}\right]_{k>\mu} e^{-\beta_q(\beta,m)S_{YM}(U)}$$

We need to find $\beta_q(\beta, m)$ and to check the equality above. Practically:
1. choose β, m and measure the string tension $a\sigma^{1/2}$ in the dynamical theory;
2. define $\beta_q(\beta, m)$ as the coupling of the YM theory that reproduces the same value $a\sigma^{1/2}$ for the string tension;
3. measure other purely-gluonic observables (like glueball masses) in the two theories, and check that they match.

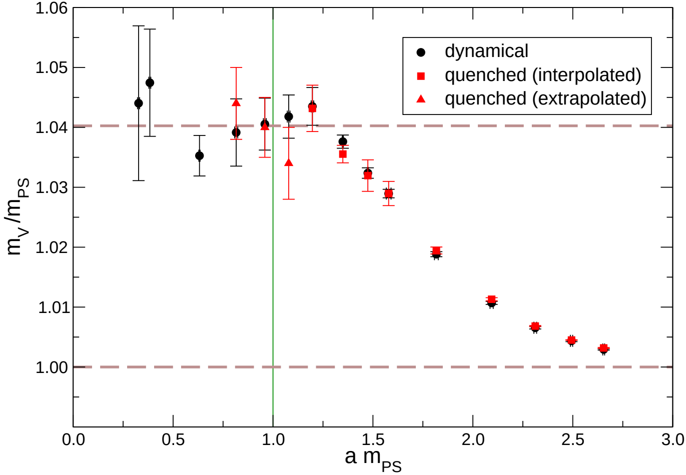
For mesonic observables, we need to compute the mass $m_q(\beta, m)$ to be used in the quenched theory. Practically:
4. measure the PS meson mass aM_{PS} in the dynamical theory;
5. define $m_q(\beta, m)$ as the mass for the quenched theory that reproduces the same value aM_{PS} for the PS meson mass;
6. measure other mesonic observables (like the V meson mass) in the two theories, and check that they match.

IR dynamics



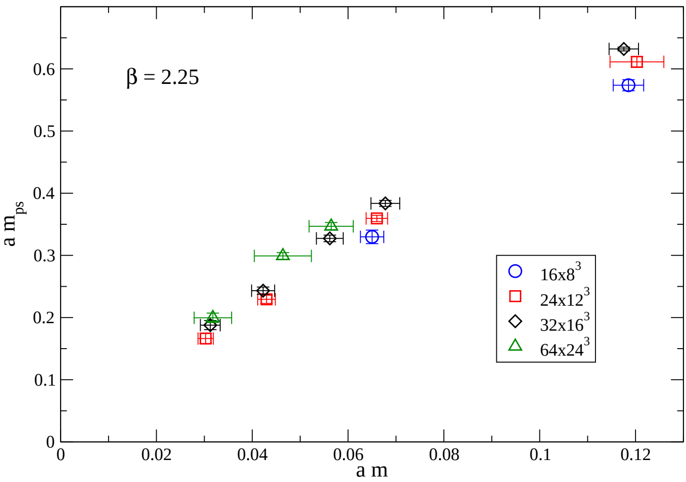
dynamical 0^{++} glueball
dynamical 2^{++} glueball
dynamical V meson
quenched 0^{++} glueball
quenched 2^{++} glueball
quenched V meson
a M
a m_{PS}
3.0
2.5
2.0
1.5
1.0
0.5
1.0
1.5
2.0
2.5

IR dynamics

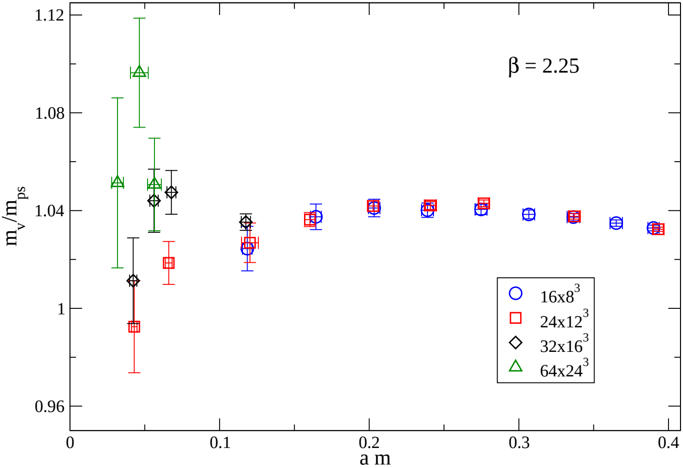

dynamical
quenched (interpolated)
quenched (extrapolated)
1.06
1.05
1.04
1.03
1.02
1.01
1.00
m_V/m_{PS}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$a\, m_{PS}$

Outline

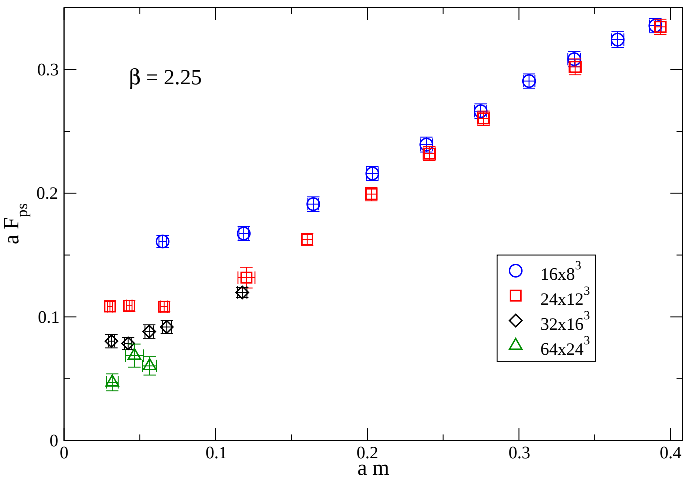
Mesonic quantities



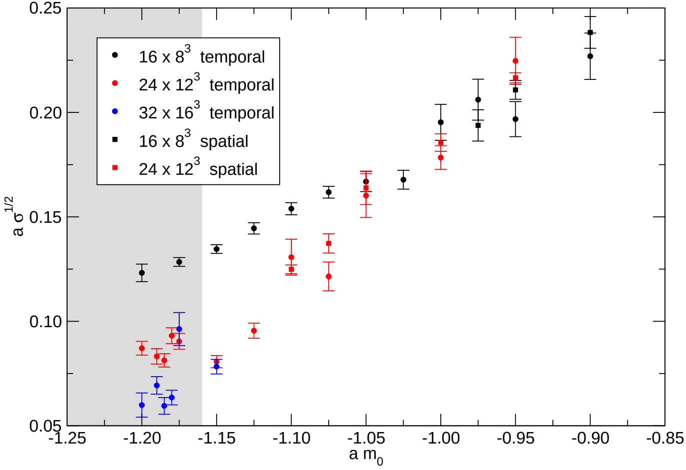
Mesonic quantities



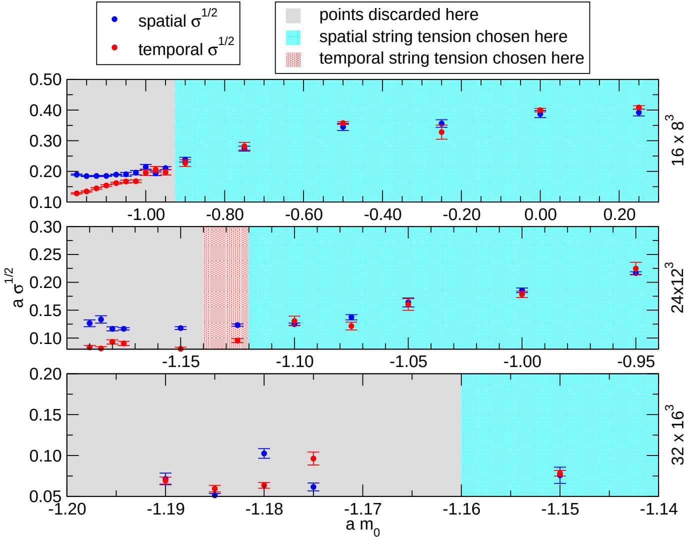
Mesonic quantities



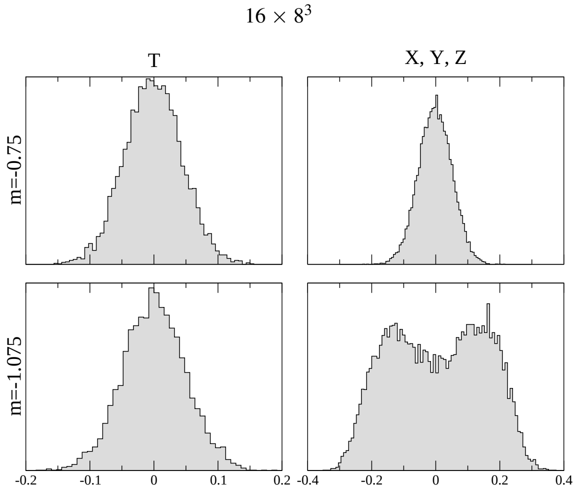
String tension



String tension



Polyakov loops



$$16 \times 8^3 : \quad am_q^* \simeq 0.35$$

$$24 \times 12^3 : \quad am_q^* \simeq 0.26$$

Outline

Conclusions

- It is important to simulate at small enough masses, and look for the scaling region in both the mesonic and gluonic sectors (how chiral can we safely simulate at fixed β?).
- If we are observing the physics of an IR fixed point, it shows the qualitative features of the Banks-Zaks fixed point.
  - The spectrum shows a well-defined hierarchy: the PS meson is heavier than the two lightest glueballs.
  - The PS and V mesons are almost (but not exactly) degenerate.
  - The IR spectrum is reproduced by the quenched theory.
- Possible scenarios (assuming that we are actually observing a scaling region):
  - **QCD-like scenario.** The scaling region is due to the vicinity of an IR fixed point which is not in the basin of the UV fixed point defining the continuum limit. Going to the continuum limit, it could simply disappear.
  - **Walking scenario.** The scaling region survives in the continuum limit, but it is bounded from below by some scale. When the PS meson becomes lighter than this scale, the theory enters into a chiral symmetry breaking regime.
  - **Conformal scenario.** The scaling region survives in the continuum limit and extends to the chiral limit.
- Ideally we would like to move towards the continuum limit.
- The presented ideas can be used for different theories.