Why (your) mass isn't gauge dependent

Des Johnston

Nielsen Identities

Edinburgh, May 2010

Apologies and some justifications

- It's been a *VERY* long time since I've done PPT related stuff..
- The topic is something that has been picked over for many years
- The Higgs (particle) is back on the agenda

Plan of Talk

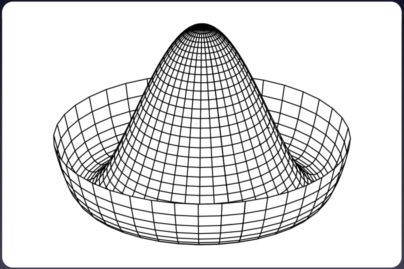
- Symmetry breaking
- Higgs mechanism
- Quantum corrections, gauge fixing, effective potential
- The problem (and the solution)
- Other models - Standard, gravity...

Form of Potential

Spontaneously broken symmetry

$$V(|\phi|^2) = -\frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4!}|\phi|^4$$

$$\phi = (\phi_1 + i\phi_2)$$



Goldstone bosons and all that

- Complex scalar field theory

$$L(x) = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi^* - V(|\phi|^2)$$

- Minimum: $\phi_0 = \left(\frac{6\mu^2}{\lambda}\right)^{1/2}$

- Take: $\phi(x) = \phi_0 + \phi_1(x) + i\phi_2(x)$

- Lagrangian: $\frac{1}{2}(\partial_\mu \phi_1)^2 - \mu^2 \phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \dots$

Abelian Higgs

- Scalar electrodynamics - Lagrangian

$$L(x) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi_i D^\mu\phi_i - V(\phi^2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad \phi^2 = \phi_1^2 + \phi_2^2$$

$$D_\mu = \partial_\mu + ieA_\mu$$

- Same symmetry breaking as before

$$\frac{1}{2}(\partial_\mu\phi_1)^2 - \mu^2\phi_1^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{1}{2}e^2\phi_0^2 A_\mu A^\mu + e\phi_0 A^\mu \partial_\mu \phi_2 + \cdots$$

Abelian Higgs II

- ϕ_2 is surplus to requirements
- Gauge invariance to the rescue

$$\begin{aligned}\phi(x) &= \phi(x)\exp[i\theta(x)] \\ A_\mu(x) &= A_\mu(x) + \partial_\mu\theta(x)\end{aligned}$$

- Unitary gauge

$$\phi(x) = v + \phi_1(x)$$

- ϕ_2 is "gauged away"

Quantum Abelian Higgs

- Scalar electrodynamics - Lagrangian in gory detail

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i-e\epsilon_{ij}(\partial_\mu\phi_i)\phi_jA^\mu+\frac{1}{2}e^2A_\mu A^\mu\phi^2+V(\phi^2)$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \;\;,\;\; \phi^2=\phi_1^2+\phi_2^2$$

$$\epsilon_{ij}=\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)$$

Gauge fixing/regularization

- Add (R_ξ) gauge fixing terms

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i-e\epsilon_{ij}(\partial_\mu\phi_i)\phi_j A^\mu+\frac{1}{2}e^2A_\mu A^\mu\phi^2+V(\phi^2)$$

$$+\frac{1}{2}\xi B^2+B(\partial_\mu A^\mu+e\xi v_i\phi_i)+\partial_\mu\psi^*\partial^\mu\psi-e^2\xi\psi^*\psi\epsilon_{ij}v_i\phi_j$$

- Use dimensional regularization/$\overline{\text{MS}}$ subtraction
- $v_i=\epsilon_{ij}\phi_{i0}$ is 't Hooft gauge

The usual setup

- Functional integral

$$Z[J] = \int [D\Phi] \exp \left(i \int d^4x \, [L(x) + J\Phi] \right)$$

  Connected generating functional

$$W[J] = -i\hbar \ln Z[J]$$

- One particle irreducible generating functional

$$\Gamma[\bar{\Phi}] = W[J] - J\bar{\Phi}$$

Feynman diagrams 0

Some useful definitions before we start...

$$v_i = v\, e_i \quad , \quad e_i = (0,1)$$
$$\phi_{i0} = \phi_0\, \eta_i \quad , \quad \eta_i = (1,0)$$

$$m_1^2 = \frac{1}{2}\lambda\phi^2 - \mu^2$$
$$m_2^2 = \frac{1}{6}\lambda\phi^2 - \mu^2$$

$$D_N = k^4 - k^2(m_2^2 - 2e^2\xi v\phi) + e^2\phi^2(e^2\xi^2 v^2 + \xi m_2^2)$$

Feynman diagrams I

Propagators for the theory

Ghost:

$$\frac{i}{k^2 - e^2 \epsilon_{ij} \xi v_i \phi_j}$$

Scalar:

$$\frac{i(k^2 - \xi e^2 \phi^2)}{D_N}(\delta_{ij} - \eta_i \eta_j) + \frac{i \eta_i \eta_j}{k^2 - m_1^2}$$

Gauge:

$$-iC \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - iD \frac{k_\mu k_\nu}{k^2}$$

$$C = \frac{1}{k^2 - e^2 \phi^2} \, , \; D = \frac{\xi (k^2 - m_2^2 - e^2 \xi v^2)}{D_N}$$

Feynman diagrams II

Propagagators in the 't Hooft gauge

Ghost:

$$\frac{i}{k^2 - e^2\xi\phi^2}$$

Scalar:

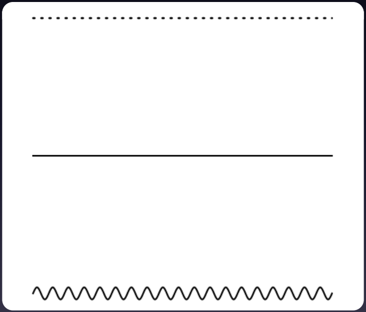
$$\frac{i}{(k^2 - m_2^2 - \xi e^2\phi^2)}(\delta_{ij} - \eta_i\eta_j) + \frac{i\eta_i\eta_j}{k^2 - m_1^2}$$

Gauge:

$$-i\frac{1}{k^2 - e^2\phi^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - i\frac{\xi}{(k^2 - e^2\xi\phi^2)}\frac{k_\mu k_\nu}{k^2}$$

Feynman diagrams III

Graphically:



Feynman diagrams IV

Vertices for the theory

Scalar/Scalar/Gauge:

$$-e\epsilon_{ij}(k_1 + k_2)^{\mu}$$

Scalar/Ghost/Ghost:

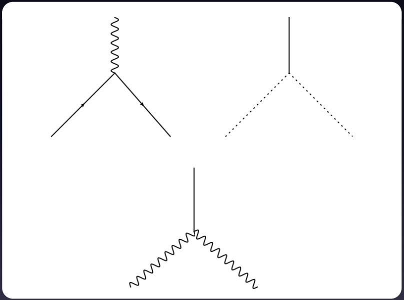
$$-ie^2\xi v_i \epsilon_{ij}$$

Scalar/Gauge/Gauge:

$$2ie^2\phi_i g_{\mu\nu}$$

Feynman diagrams V

Vertices for the theory



Effective Potential I

- Quantum equivalent of classical potential

- Should use this to discuss corrections to classical symmetry breaking picture

- Done perturbatively using loop (i.e. $\hbar$) expansion

$$V = i \sum_n \frac{1}{n!} \Gamma_{i_1 \dots i_n}(0, \dots, 0) \phi_{i_1} \cdots \phi_{i_n}$$

Effective Potential II

$$V^{(1)} = i \int d^4k \left[\ln(k^2 + e^2 \xi v \phi) - \frac{3}{2} \ln(-k^2 + e^2 \phi^2) - \frac{1}{2} \ln(k^2 - m_1^2) - \frac{1}{2} \ln D_N \right]$$

$$\xi \frac{\partial V^{(1)}}{\partial \xi} = -\frac{1}{2} i e \xi \phi m_2^2 \int d^4k \frac{(2v + \phi) k^2 - v \xi e^2 \phi^2}{(k^2 + e^2 \xi v \phi) D_N}$$

Ugly, gauge parameter dependent

And the mass too...

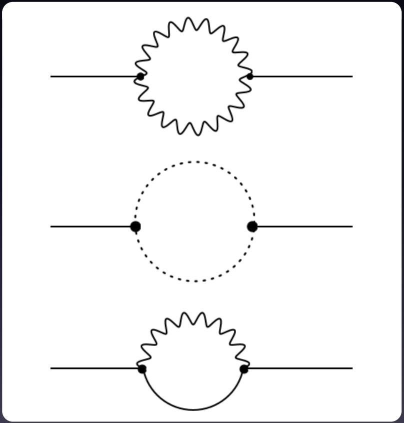
- Masses, 2nd derivatives of (effective) potentials
- Choose $\lambda \sim O(e^4)$ (Coleman-Weinberg)

$$m^{2(1)} = m_1^2 + \Sigma^{(1)}(0) + m_1^2 \frac{\partial \Sigma^{(1)}(p^2)}{\partial p^2}$$

$$m_1^2 = \frac{1}{2}\lambda\phi^2 - \mu^2$$

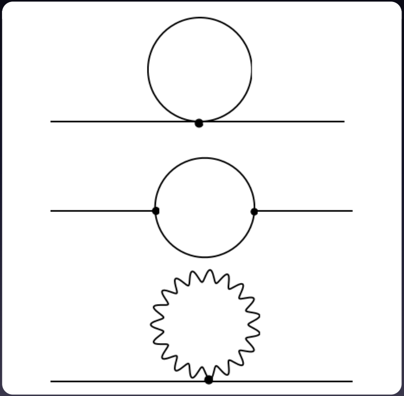
Included Diagrams

Work that can't be avoided



Excluded Diagrams

Work that *can* be avoided



And finally...

Maybe not so ugly, but still gauge parameter dependent:

$$\xi \frac{\partial m^{2(1)}}{\partial \xi} = \frac{e^2 \xi \lambda \phi_0^2}{32\pi^2} \ln \left[\frac{e^2 \xi \phi_0^2}{M^2} \right]$$

One loop correction to effective potential and Higgs mass look like they depend on ξ

What's going on here?

- Effective potential and mass shouldn't be gauge parameter dependent
- Nielsen (1975) - don't forget about *implicit* dependence
- All would be well if

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0$$
$$\xi \frac{\partial m^2}{\partial \xi} + C(\phi, \xi) \frac{\partial m^2}{\partial \phi} = 0$$

What's going on here?

- Why is this a fix?
- Minimum of effective potential at some $\bar{\phi}$

$$\frac{\partial V(\phi, \xi)}{\partial \phi} = 0$$

- V_{min} stays the same under $\xi \to \xi + d\xi$

$$V(\bar{\phi} + \delta\bar{\phi}, \xi + d\xi) = V(\bar{\phi}, \xi) = V_{min}$$

$$\frac{\partial V}{\partial \xi} + \frac{d\bar{\phi}}{d\xi}\frac{\partial V}{\partial \bar{\phi}} = 0$$

Nielsen Identities

- The man himself N. K. Nielsen - not H. B. Nielsen

- Proved requisite identities exist using BRST symmetry

(Standard) BRST symmetry I

- "Modern" way of dealing with gauge invariance
- Valid for the gauge-fixed Lagrangian

$$\delta^2 = 0$$

$$\delta \psi^* = \epsilon B \ , \ \delta \psi = 0 \ , \ \delta B = 0 \ , \ \delta A_\mu = \epsilon \partial_\mu \psi$$

$$\delta \phi_i = \epsilon e \epsilon_{ij} \psi \phi_j$$

(Standard) BRST symmetry II

- The BRST identities are encapsulated in:

$$S(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta\rho^\mu}\frac{\delta\Gamma}{\delta A_\mu} + \frac{\delta\Gamma}{\delta K_i}\frac{\delta\Gamma}{\delta\phi_i} + \frac{\delta\Gamma}{\delta\sigma}\frac{\delta\Gamma}{\delta\psi} + B\frac{\delta\Gamma}{\delta\psi^*}\right) = 0$$

- ρ^μ, K_i, σ sources for δA_μ, $\delta\phi_i$ and $\delta\psi$
- Nothing that looks like $\frac{\partial\Gamma}{\partial\xi} \to \frac{\partial V}{\partial\xi}$
- Trick, Piguet and Sibold

Extended BRST symmetry

- Introduce a BRST transform on the gauge *parameter*

$$\delta\xi = \epsilon\chi$$

- This requires extra terms in the Lagrangian too

$$+\frac{1}{2}\chi\psi^* B + e\chi\psi^* v_i\phi_i$$

- The BRST identity now reads

$$S(\Gamma) + \chi\frac{\partial\Gamma}{\partial\xi} = 0$$

Extended BRST symmetry II

- Differentiate w.r.t. χ, substitute for B, set everything but ϕ to zero

$$\int d^4x \left(\frac{\delta\Gamma(O(x))}{\delta K_i} \frac{\delta\Gamma}{\delta\phi_i} - \frac{e\xi v_i \phi_i}{\xi} \frac{\delta\Gamma(O(x))}{\delta\psi^*} \right) - \frac{\partial\Gamma}{\partial\xi} = 0$$

- Take constant field limit

$$\xi \frac{\partial V}{\partial\xi} - \int d^4x \frac{\delta\Gamma(O(x))}{\delta K_i} \frac{\partial V}{\partial\phi} = 0$$

- Explicit expression for $C(\phi, \xi)$

$$-\int d^4x \frac{\delta\Gamma(O(x))}{\delta K_i}$$

Some observations

- We have seen that

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0$$

  is true

- Also true for other physical quantities *with the same* $C(\phi, \xi)$

- Applies in non-Abelian theories (e.g. standard model)

- Also in gravity

Matters of gravity...

- Suppose you are made of scalar, bosonic matter
- And you are happy to be described by 1-loop quantum gravity

$$\begin{aligned} L(x) &= -\frac{2}{\kappa^2}\sqrt{-g}(R+\Lambda) + L_{gf} \\ &+ \sqrt{-g}[(\nabla_\mu \phi)(\nabla_\nu \phi) g^{\mu\nu} - \frac{1}{2} m^2 \phi^2] \end{aligned}$$

- Then there is a Nielsen identity

$$\xi \frac{\partial m^2}{\partial \xi} + \eta^{\mu\nu} \tau(\phi, \xi) \frac{\partial m^2}{\partial h_{\mu\nu}} = 0$$

- Interpreted as

$$\begin{aligned} \xi &\rightarrow \xi + d\xi \\ \eta^{\mu\nu} &\rightarrow (1 + \tau d\xi)\eta^{\mu\nu} \end{aligned}$$

Matters of higher derivative gravity I

- A renormalizable, asymptotically free theory of gravity exists (Stelle 1977)
- Unfortunately, it is also non-unitary

$$L(x) = \sqrt{-g}\left[\frac{\gamma}{\kappa^2}R + \frac{\Lambda}{\kappa^4} + \beta R^2 - \frac{1}{\alpha^2}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)\right]$$

- Spin two propagator

$$D(q^2) = \frac{2}{\alpha^2\gamma}\left[\frac{1}{q^2} - \frac{1}{q^2 - (\alpha^2\gamma/\kappa^2)}\right]$$

Matters of higher derivative gravity II

- Massive ghost decays
- General recipe

$$D(q^2) = (D_0(q^2)^{-1} - \Sigma(q^2))^{-1}$$

Large N

$$D(q^2) = \frac{2}{\alpha^2\gamma}\left[\frac{1}{q^2[1 - ANq^2\ln(q^2/L^2)]}\right]$$

- Poles at $q^2 = 0$, $q = r\exp(i\theta)$ with $r \sim \gamma/N\kappa^2$
- Loop corrections $\partial M^2/\partial\xi \neq 0$

Summary

- At first sight it looks as if physical quantities depend on gauge parameters in a loop expansion
- They don't really, you are saved by:

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0$$
$$\xi \frac{\partial m^2}{\partial \xi} + C(\phi, \xi) \frac{\partial m^2}{\partial \phi} = 0$$

- $C(\phi, \xi)$ explicitly calculable from (extended) BRST identities

A reference or ten

N.K. Nielsen, *Nucl. Phys. B* 101 (1975) 173.
R. Fukuda and T. Kugo, *Phys. Rev. D* 13 (1976) 3469.
I.J.R. Aitchison and C.M. Fraser, *Ann. Phys.* 156 (1984) 1.
D. Johnston, *Nucl. Phys. B* 253 (1985) 687; 283 (1987) 317; 297 (1988) 721.
S. Ramaswamy, *Nucl. Phys. B* 453 (1995) 240.
O. M. Del Cima, D. H.T. Franco and O. Piguet, *Nucl.Phys.B* 551 (1999) 813.
P. Gambino and P. A. Grassi, *Phys. Rev. D* 62 (2000) 076002.
L. P. Alexander and A. Pilaftsis, *J.Phys.G* 36 (2009) 045006.

THE END :)