The Golem95 Library

The Thomas Binoth Symposium

Edinburgh, 12.03.10

Tobias Kleinschmidt

The Golem Collaboration:

T. Binoth, G. Cullen, A. Guffanti, J.P. Guillet, G. Heinrich, S. Karg, N. Kauer, TK, E. Pilon, T. Reiter, M. Rodgers, I. Wigmore

arXiv: 0810.0992 lappweb.in2p3.fr/lapth/Golem/golem95.html

In Memory of Thomas

Outline

- Introduction
- The Golem95 Library
  - Reduction Formalism for Scalar and Tensor Integrals
  - Overview of Form Factors implemented in public version
  - Going from massless to massive integrals
  - Going from real masses to complex masses
- Summary

Computations at Loop Level

Why NLO?

Computations at Loop Level

Why NLO? Precision!

Computations at Loop Level

Why NLO? Precision!

- Reduce dependence on renormalization scale for observables including $\alpha(\mu)$
- Match experimental precision to extract important parameters used as input in further experiments/analyses (Luminosity, SM-benchmark processes, ...)
- Match experimental precision to obtain more accurate values for masses and couplings
- Better simulations for background to disentangle signals of new physics from standard model processes (inclusion of off-shell effects)
- ...

Computations at Loop Level

Why NLO? Precision!

- Reduce dependence on renormalization scale for observables including $\alpha(\mu)$
- Match experimental precision to extract important parameters used as input in further experiments/analyses (Luminosity, SM-benchmark processes, ...)
- Match experimental precision to obtain more accurate values for masses and couplings
- Better simulations for background to disentangle signals of new physics from standard model processes (inclusion of off-shell effects)
- ...

○ Equally Valid for LHC and future linear collider:

The experimental accuracy should be matched by the theoretical precision. This involves the computation of multi-leg scattering amplitudes at NLO.

process	relevant for
$pp \to VV$ jet	$t\bar{t}H$, new physics
$pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
$pp \to t\bar{t}+$ 2 jets	$t\bar{t}H$
$pp \to VVb\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
$pp \to VV+$ 2 jets	VBF$\to H \to VV$
$pp \to V+$ 3 jet	various new physics signatures
$pp \to VVV$	SUSY trilepton

The LHC *priority* wishlist, Les Houches '05. [hep-ph/0604120]

process	relevant for
$pp \to VV$ jet	$t\bar{t}H$, new physics
$pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
$pp \to t\bar{t}+$ 2 jets	$t\bar{t}H$
$pp \to VVb\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
$pp \to VV+$ 2 jets	VBF$\to H \to VV$
$pp \to V+$ 3 jet	various new physics signatures
$pp \to VVV$	SUSY trilepton

The LHC *priority* wishlist, Les Houches '05. [hep-ph/0604120]

- Lot of progress for 2 → 3 processes at NLO in past years
- Few 2 → 4 processes at NLO coming in now
  - $q\bar{q} \to b\bar{b}t\bar{t}$ Bredenstein, Denner, Dittmaier, Pozzorini
  - $pp \to b\bar{b}b\bar{b}$ GOLEM
  - $pp \to W + 3j$ BlackHat (Dixon et. al.); Rocket (Ellis et. al.)
  - $pp \to t\bar{t} + 2j$ Helac (Bevilacqua et. al.)

process	relevant for
$pp \to VV$ jet	$t\bar{t}H$, new physics
$pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
$pp \to t\bar{t}+$ 2 jets	$t\bar{t}H$
$pp \to VVb\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
$pp \to VV+$ 2 jets	VBF$\to H \to VV$
$pp \to V+$ 3 jet	various new physics signatures
$pp \to VVV$	SUSY trilepton

The LHC *priority* wishlist, Les Houches '05. [hep-ph/0604120]

- Lot of progress for 2 → 3 processes at NLO in past years
- Few 2 → 4 processes at NLO coming in now
  - $q\bar{q} \to b\bar{b}t\bar{t}$ Bredenstein, Denner, Dittmaier, Pozzorini
  - $pp \to b\bar{b}b\bar{b}$ GOLEM
  - $pp \to W + 3j$ BlackHat (Dixon et. al.); Rocket (Ellis et. al.)
  - $pp \to t\bar{t} + 2j$ Helac (Bevilacqua et. al.)

Contribution by many people!

Andersen, Berger, Bern, Bevilacqua, Binoth, Bozzi, Bredenstein, Britto, Campaniero, Campbell, Dawson, del Duca, Denner, Dittmaier, Dixon, Ellis, Englert, Febres Cordero, Feng, Figy, Forde, Giele, Gleisberg, Glover, Guffanti, Guillet, van Hameren, Hankele, Ita, Jäger, Kallweit, Karg, Kauer, Kosower, Kunszt, Lazopoulos, Mahmoudi, Maitre, Mastrolia, McElmurry, Melnikov, Miller, Nagy, Oleari, Orr, Ossola, Papadopoulos, Petriello, Pittau, Pozzorini, Reina, Reiter, Reuter, Sanguinetti, Smillie, Soper, Spannowsky, Uwer, Wackeroth, Weinzierl, Zanderighi, Zeppenfeld,...and many others

Ingredients of NLO Calculations

Ingredients of NLO Calculations

- Real emissions (Tree graphs) with n+1 final state partons

  Several generators for creation of efficient matrix elements:

  e.g. Alpha [Caravaglios et.al. '95], Comix [Gleisberg, Höche '08],

  MadGraph [Stelzer et.al. '03], O'Mega [Ohl et.al. '01].

  ↬ Contain infrared soft and collinear divergences

- Subtraction Terms

  Cancel Divergences in real corrections locally

  Mainly used: [Catani, Seymour `96; Catani, Seymour, Dittmaier, Troscanyi `02]

- Virtual Corrections

  ↬ Contain Problems! '<u>Bottleneck part of NLO computations</u>'

Ingredients of NLO Calculations

- Real emissions (Tree graphs) with n+1 final state partons

  Several generators for creation of efficient matrix elements:

  e.g. Alpha [Caravaglios et.al. '95], Comix [Gleisberg, Höche '08],

  MadGraph [Stelzer et.al. '03], O'Mega [Ohl et.al. '01].

  ↝ Contain infrared soft and collinear divergences

- Subtraction Terms

  Cancel Divergences in real corrections locally

  Mainly used: [Catani, Seymour `96; Catani, Seymour, Dittmaier, Troscanyi `02]

- Virtual Corrections

  ↝ Contain Problems! '<u>Bottleneck part of NLO computations</u>'

Level of Complexity rises due to:

- Length of Expressions
- Complexity of Integrals
- Infrared divergences, internal singularities

Ingredients of NLO Calculations

- Real emissions (Tree graphs) with n+1 final state partons

  Several generators for creation of efficient matrix elements:

  e.g. Alpha [Caravaglios et.al. '95], Comix [Gleisberg, Höche '08],

  MadGraph [Stelzer et.al. '03], O'Mega [Ohl et.al. '01].

  - Contain infrared soft and collinear divergences

- Subtraction Terms

  Cancel Divergences in real corrections locally

  Mainly used: [Catani, Seymour `96; Catani, Seymour, Dittmaier, Troscanyi `02]

- Virtual Corrections

  - Contain Problems! '<u>Bottleneck part of NLO computations</u>'

Level of Complexity rises due to:

- Length of Expressions
- Complexity of Integrals
- Infrared divergences, internal singularities

Very time consuming!

Not only computing time!

Ingredients of NLO Calculations

- Real emissions (Tree graphs) with n+1 final state partons

  Several generators for creation of efficient matrix elements:

  e.g. Alpha [Caravaglios et.al. '95], Comix [Gleisberg, Höche '08], MadGraph [Stelzer et.al. '03], O'Mega [Ohl et.al. '01].

  ↬ Contain infrared soft and collinear divergences

- Subtraction Terms

  Cancel Divergences in real corrections locally

  Mainly used: [Catani, Seymour `96; Catani, Seymour, Dittmaier, Troscanyi `02]

- Virtual Corrections

  ↬ Contain Problems! '<u>Bottleneck part of NLO computations</u>'

Level of Complexity rises due to:

- Length of Expressions
- Complexity of Integrals
- Infrared divergences, internal singularities

Very time consuming!

Not only computing time!

Aim for Automation!

The Two Major Approaches for Loop Calculations

Feynman Diagram based

- ☼Start with individual Feynman graphs
- Tensor Reduction (PV-Style) ➪ Set of basis integrals
- ✓ Scalar integrals known analytically
- ✗ Yields large expressions for coefficients
- ✗ can have delicate numerical stability
- ➪ use modified reduction scheme to avoid inverse Gram determinants.

GOLEM

Denner, Dittmaier,...

Unitarity based

- ☀Start with Amplitude
- Decompose Amplitude into scalar integrals and coefficients.
- Coefficients are product of on-shell tree amplitudes, obtained by cutting techniques.
- ✗ large expressions for coefficients...
- ✓ ...but simpler than from PV-style reductions?
- ✓ P-algorithm ($\tau \propto N^9$ for N-gluon ampl.)

BlackHat: Bern, Dixon, Forde, Gleisberg, Kosower, Maitre,

Rocket: Ellis, Giele, Kunszt, Melnikov, Zanderighi

Helac-1Loop: Bevilaqua, Czakon, van Hameren, Papadopoulus, Pittau, Worek

The GOLEM Project

General One Loop Evaluator for Matrix Elements

- Aim at automated evaluation of numerically stable one-loop amplitudes for multi-leg processes

- Represent one-loop amplitudes as sum of diagrams, sorted by color:

$$\mathcal{A}^{\{c,\lambda\}}(\{p_j, m_j\}) = \sum_{\{c_i\},\alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^D k}{i\pi^{D/2}} \frac{\mathcal{N}^{\{\lambda\}}}{D_1 \ldots D_N} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\ldots,\mu_R} \, I_N^{\mu_1 \ldots \mu_R}(\{p_j, m_j\})$$

- GOLEM95: Library providing form factors entering the calculation of one-loop amplitudes.

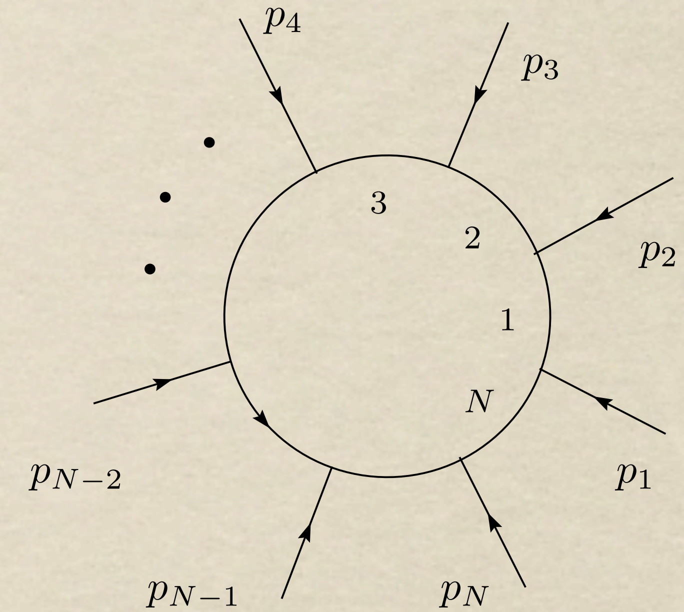
  Reduction to a certain set of basis integrals by formalism avoiding inverse Gram determinants.

An algebraic/numerical formalism for one-loop multi-leg amplitudes [hep-ph/0504267]

Notation and Conventions

$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\, \frac{q_{a_1}^{\mu_1}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}$$

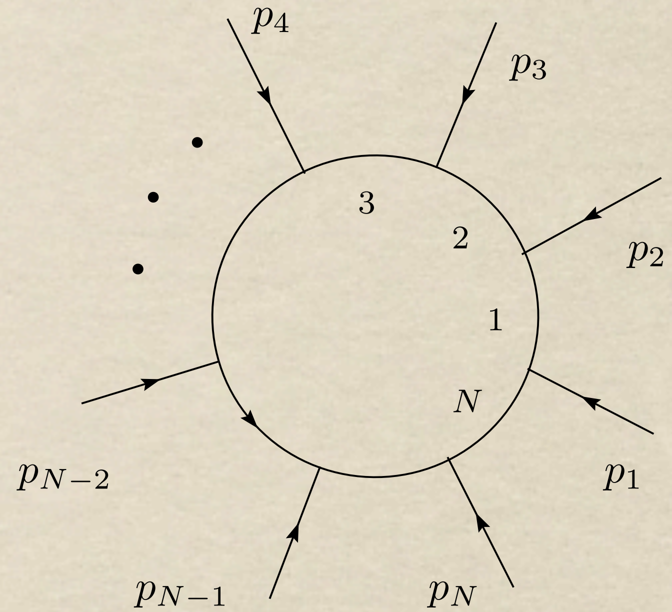
$$q_i = k + r_i$$



Invariant under translations: $r_i^\mu \to r_i^\mu + r_a^\mu$

An algebraic/numerical formalism for one-loop multi-leg amplitudes [hep-ph/0504267]

Notation and Conventions



$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\, \frac{q_{a_1}^{\mu_1}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}$$

$$q_i = k + r_i$$

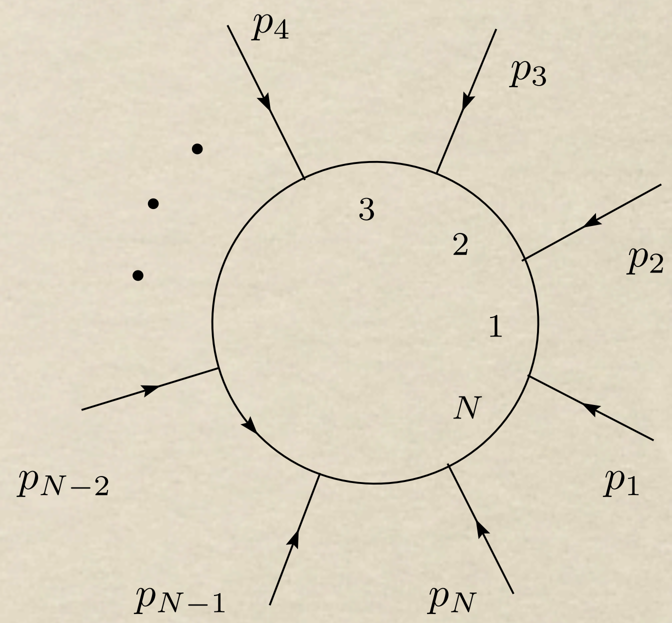
Invariant under translations: $r_i^\mu \longrightarrow r_i^\mu + r_a^\mu$

Shift invariant 4-vectors: $\Delta_{ij}^\mu = r_i^\mu - r_j^\mu = q_i^\mu - q_j^\mu$

Kinematic matrix: $\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$

An algebraic/numerical formalism for one-loop multi-leg amplitudes [hep-ph/0504267]

Notation and Conventions



$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\, \frac{q_{a_1}^{\mu_1}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}$$

$$q_i = k + r_i$$

Invariant under translations: $r_i^\mu \longrightarrow r_i^\mu + r_a^\mu$

Shift invariant 4-vectors: $\Delta_{ij}^\mu = r_i^\mu - r_j^\mu = q_i^\mu - q_j^\mu$

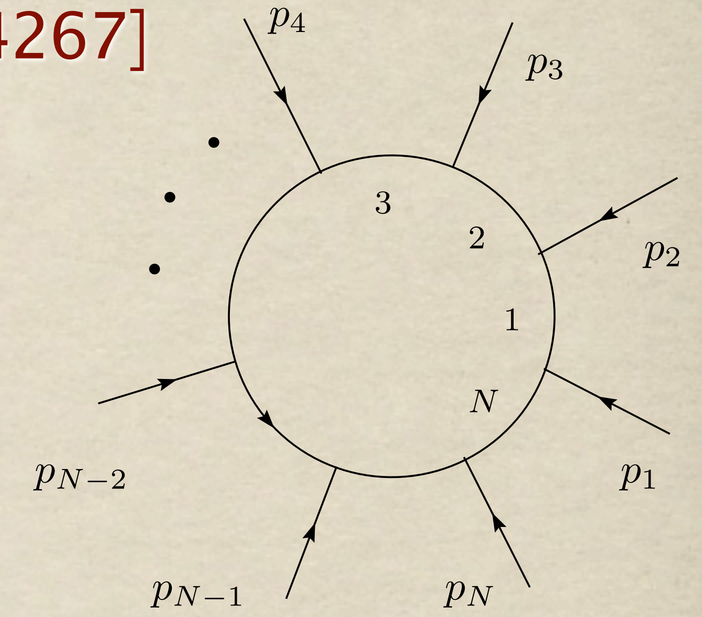
Kinematic matrix: $\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$

Gram matrix: $G_{ij} = 2\, r_i \cdot r_j$

Passarino-Veltman reduction induces numerical problems → $det(G)^{-r}$

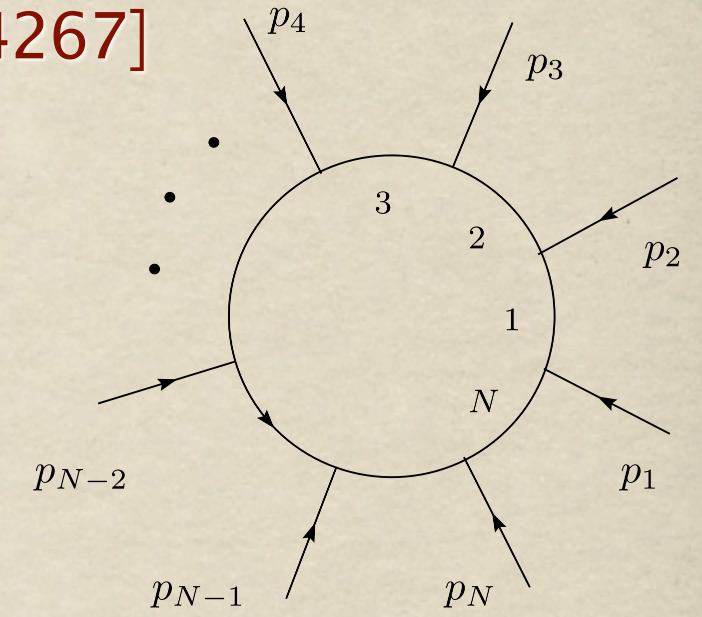
⇨ find better suited scheme for tensor reduction!

An algebraic/numerical formalism for one-loop multi-leg amplitudes [hep-ph/0504267]



$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\, \frac{q_{a_1}^{\mu_1}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}$$

An algebraic/numerical formalism for one-loop multi-leg amplitudes [hep-ph/0504267]



$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\,\frac{q_{a_1}^{\mu_1}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}$$

Tensor Integrals → Form Factor Representation [Phys. Lett. B 263 (1991) 107]

$$I_N^{n,\,\mu_1\ldots\mu_r}(a_1,\ldots,a_r\,;S) = \sum_{j,m} \tau_m \left[(g^{\cdot\cdot})^{\otimes m}\,\Delta^{\cdot}_{j_1\cdot}\cdots\Delta^{\cdot}_{j_r\cdot}\right]^{\{\mu_1\cdots\mu_r\}}_{\{a_1\cdots a_r\}}\; I_N^{n+2m}(j_1\ldots,j_{r-2m}\,;S)$$

$$I_N^n(l_1,\ldots,l_r;S) = (-1)^N\Gamma(N-\frac{n}{2})\int\prod_{i=1}^N dz_i\,\delta(1-\sum_{l=1}^N z_l)z_{l_1}\ldots z_{l_r}\left(R^2\right)^{n/2-N}$$

$$R^2 = -\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta$$

- Use similar representation for N={3,4}.
- Reduction and numerical evaluation possible

Reduction of Scalar Integrals

- Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$
\begin{aligned}
I_N^n(S) &= I_{div}(S) + I_{fin}(S) \\
&= \sum_{i\in S} b_i(S) \int d\bar{k} \, \frac{(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)} + \int d\bar{k} \, \frac{1 - \sum_{i\in S} b_i(S)\,(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)}
\end{aligned}
$$

Reduction of Scalar Integrals

- Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$\begin{aligned} I_N^n(S) &= I_{div}(S) + I_{fin}(S) \\ &= \sum_{i\in S} b_i(S) \int d\bar{k}\, \frac{(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)} + \int d\bar{k}\, \frac{1 - \sum_{i\in S} b_i(S)\,(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)} \end{aligned}$$

- Introduce Feynman parameters, momentum shift to quadratic form of loop momentum:

$$1 - \sum_{i\in S} b_i(S)\,(q_i^2 - m_i^2) = -(l^2 + R^2) \sum_{i\in S} b_i(S) + \sum_{j\in S} z_j \Big[1 - \sum_{i\in S} b_i(S)\, \{\mathcal{S}_{ij}\} \Big]$$

Reduction of Scalar Integrals

- Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$\begin{aligned} I_N^n(S) &= I_{div}(S) + I_{fin}(S) \\ &= \sum_{i\in S} b_i(S) \int d\bar{k}\, \frac{(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)} + \int d\bar{k}\, \frac{1 - \sum_{i\in S} b_i(S)\,(q_i^2 - m_i^2)}{\prod_{j\in S}(q_j^2 - m_j^2 + i\,\delta)} \end{aligned}$$

- Introduce Feynman parameters, momentum shift to quadratic form of loop momentum:

$$1 - \sum_{i\in S} b_i(S)\,(q_i^2 - m_i^2) \quad = \quad -(l^2 + R^2) \sum_{i\in S} b_i(S) + \sum_{j\in S} z_j \left[1 - \sum_{i\in S} b_i(S)\, \{\mathcal{S}_{ij}\} \right]$$

$$\begin{aligned} I_{fin}(S) &= -B(S)\,\Gamma(N) \int_0^1 \prod_{i\in S} dz_i\, \delta(1 - \sum_{l\in S} z_l) \int \frac{d^n l}{i\pi^{n/2}}\, \frac{l^2 + R^2}{(l^2 - R^2)^N} \\ &= -B(S)\,(N - n - 1)\, I_N^{n+2}(S) \end{aligned}$$

$$B(S) = \sum_{i\in S} b_i(S)$$

Reduction of Scalar Integrals

- Reduction: $I_N^n(S) = I_{N-1}^n(S) + I_N^{n+2}(S)$
- Condition: $\sum_{i \in S} b_i(S)\, \mathcal{S}_{ij} = 1 \;\; , \;\; j = 1, \ldots, N$

Reduction of Scalar Integrals

- Reduction: $I_N^n(S) = I_{N-1}^n(S) + I_N^{n+2}(S)$
- Condition: $\sum_{i\in S} b_i(S)\,\mathcal{S}_{ij} = 1 \;\;,\;\; j = 1,\ldots,N$
- S invertible, N≤6 → Unique solution: $b_i = \sum_{k\in S} \mathcal{S}_{ki}^{-1}$

Reduction of Scalar Integrals

- Reduction: $I_N^n(S) = I_{N-1}^n(S) + I_N^{n+2}(S)$
- Condition: $\sum_{i \in S} b_i(S)\, \mathcal{S}_{ij} = 1 \;\;,\;\; j = 1, \ldots, N$
- S invertible, N≤6 → Unique solution: $b_i = \sum_{k \in S} \mathcal{S}_{ki}^{-1}$
- For N>6→ solve equivalent set of equations:

$$\sum_{i=1}^{N-1} G_{ki} b_i = B v_k \quad , \quad \sum_{l=1}^{N-1} v_i b_i = 1 \quad , \quad B = \sum_{i=1}^{N} b_i \quad , \quad v_i = \Delta_{iN}^2 - m_i^2$$

- Solutions b_i span Ker(G). $B = 0$

Reduction of Scalar Integrals

- Reduction: $I_N^n(S) = I_{N-1}^n(S) + I_N^{n+2}(S)$
- Condition: $\sum_{i\in S} b_i(S)\,\mathcal{S}_{ij} = 1 \;,\; j = 1,\ldots,N$
- S invertible, N≤6 → Unique solution: $b_i = \sum_{k\in S} \mathcal{S}_{ki}^{-1}$
- For N>6→ solve equivalent set of equations:

$$\sum_{i=1}^{N-1} G_{ki} b_i = B v_k \quad , \quad \sum_{l=1}^{N-1} v_i b_i = 1 \quad , \quad B = \sum_{i=1}^{N} b_i \quad , \quad v_i = \Delta_{iN}^2 - m_i^2$$

- Solutions b_i span Ker(G). $B = 0$

⇨ Any N-point scalar integral (N>4) reduces to sum of (n+2) dimensional Box integrals and IR divergent Triangles!

$$I_6^n(S) \;=\; \sum_{j\in S} b_j \sum_{k\in S\setminus\{j\}} b_k^{\{j\}} \left[B^{\{j,k\}}\, I_4^{n+2}(S\setminus\{j,k\}) + \sum_{l\in S\setminus\{j,k\}} b_l^{\{j,k\}}\, I_3^n(S\setminus\{j,k,l\}) \right]$$

Reduction of Tensor Integrals

- Completely analogue to scalar case!

  Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$
\begin{aligned}
I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) \quad &= \int d\bar{k}\, \frac{\left[q_{a_1}^{\mu_1} + \sum_{j\in S} \mathcal{C}_{ja_1}^{\mu_1}\,(q_j^2 - m_j^2)\right]\; q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)} \\
&- \sum_{j\in S} \mathcal{C}_{ja_1}^{\mu_1} \int d\bar{k}\, \frac{(q_j^2 - m_j^2)\; q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_{a_r}}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}
\end{aligned}
$$

Reduction of Tensor Integrals

- Completely analogue to scalar case!

  Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\,\frac{\left[q_{a_1}^{\mu_1}+\sum_{j\in S}\mathcal{C}_{ja_1}^{\mu_1}\,(q_j^2-m_j^2)\right]\,q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2-m_i^2+i\delta)} - \sum_{j\in S}\mathcal{C}_{ja_1}^{\mu_1}\int d\bar{k}\,\frac{(q_j^2-m_j^2)\,q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_{a_r}}}{\prod_{i\in S}(q_i^2-m_i^2+i\delta)}$$

- Condition: $\displaystyle\sum_{j\in S}\mathcal{S}_{ij}\,\mathcal{C}_{j\,a}^{\mu} = \Delta_{i\,a}^{\mu}$

Reduction of Tensor Integrals

- Completely analogue to scalar case!

  Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\, \frac{\left[q_{a_1}^{\mu_1} + \sum_{j\in S} \mathcal{C}_{ja_1}^{\mu_1}\,(q_j^2 - m_j^2)\right]\; q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)} - \sum_{j\in S} \mathcal{C}_{ja_1}^{\mu_1} \int d\bar{k}\, \frac{(q_j^2 - m_j^2)\; q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_{a_r}}}{\prod_{i\in S}(q_i^2 - m_i^2 + i\delta)}$$

- Condition: $\sum_{j\in S} \mathcal{S}_{ij}\, \mathcal{C}_{j\,a}^{\mu} = \Delta_{i\,a}^{\mu}$

- Solutions:

N≤6

$$\mathcal{C}_{ja}^{\mu} = \sum_{k\in S} \left(\mathcal{S}^{-1}\right)_{jk} \Delta_{ka}^{\mu}$$

N>7

$$\mathcal{C}_{ja}^{\mu} = \ldots \qquad \sum_{j\in S} \mathcal{C}_{ja}^{\mu} = 0$$

Reduction of Tensor Integrals

- Completely analogue to scalar case!

  Split scalar N-point integral into IR-finite part and a simpler, possibly divergent part.

$$I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = \int d\bar{k}\,\frac{\left[q_{a_1}^{\mu_1}+\sum_{j\in S}\mathcal{C}_{ja_1}^{\mu_1}\,(q_j^2-m_j^2)\right]\,q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_r}}{\prod_{i\in S}(q_i^2-m_i^2+i\delta)} - \sum_{j\in S}\mathcal{C}_{ja_1}^{\mu_1}\int d\bar{k}\,\frac{(q_j^2-m_j^2)\,q_{a_2}^{\mu_2}\ldots q_{a_r}^{\mu_{a_r}}}{\prod_{i\in S}(q_i^2-m_i^2+i\delta)}$$

- Condition: $\sum_{j\in S}\mathcal{S}_{ij}\,\mathcal{C}_{j\,a}^{\mu}=\Delta_{i\,a}^{\mu}$

- Solutions:

N≤6

$$\mathcal{C}_{ja}^{\mu}=\sum_{k\in S}\left(\mathcal{S}^{-1}\right)_{jk}\Delta_{ka}^{\mu}$$

N>7

$$\mathcal{C}_{ja}^{\mu}=\ldots \qquad \sum_{j\in S}\mathcal{C}_{ja}^{\mu}=0$$

$$I_N^{n,\mu_1\ldots\mu_r}(a_1,\ldots,a_r;S) = -\sum_{j\in S}\mathcal{C}_{ja_1}^{\mu_1}\,I_{N-1}^{n,\mu_2\ldots\mu_r}(a_2,\ldots,a_r;S\setminus\{j\}) \quad (N\geq 6)$$

Set of Basis Integrals

- After reduction of higher N-point scalar/tensor integrals
- End point of (our) reduction: Basis functions:
  - Boxes (N=4): (4,6,8)-dimensional with up to (0,3,1) Feynman parameters in numerator
  - Triangles (N=3): (4,6)-dimensional with up to (3,1) Feynman parameters in numerator (possibly IR-divergent!)
  - Bubbles (N=2): Further reduction to 4-dimensional scalar integrals
  - Tadpoles (N=1): 4-dimensional scalar integral
- Numerically stable functions! No inverse Gram determinants involved
- Further reduction can induces instabilities!

Set of Basis Integrals

$$I_4^{n+2}(l;S) \quad = \quad \frac{1}{B}\left\{ b_l\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j\in S} \mathcal{S}^{-1}_{j\,l}\, I_3^n(S\setminus\{j\}) - \frac{1}{2}\sum_{j\in S} b_j\, I_3^n(l;S\setminus\{j\}) \right\}$$

- B proportional to Gram determinant!

Set of Basis Integrals

$$I_4^{n+2}(l;S) \quad = \quad \frac{1}{B}\left\{ b_l\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j\in S} \mathcal{S}^{-1}_{j\,l}\, I_3^n(S\setminus\{j\}) - \frac{1}{2}\sum_{j\in S} b_j\, I_3^n(l;S\setminus\{j\}) \right\}$$

- B proportional to Gram determinant!

⇨ Mixed algebraic/numerical approach

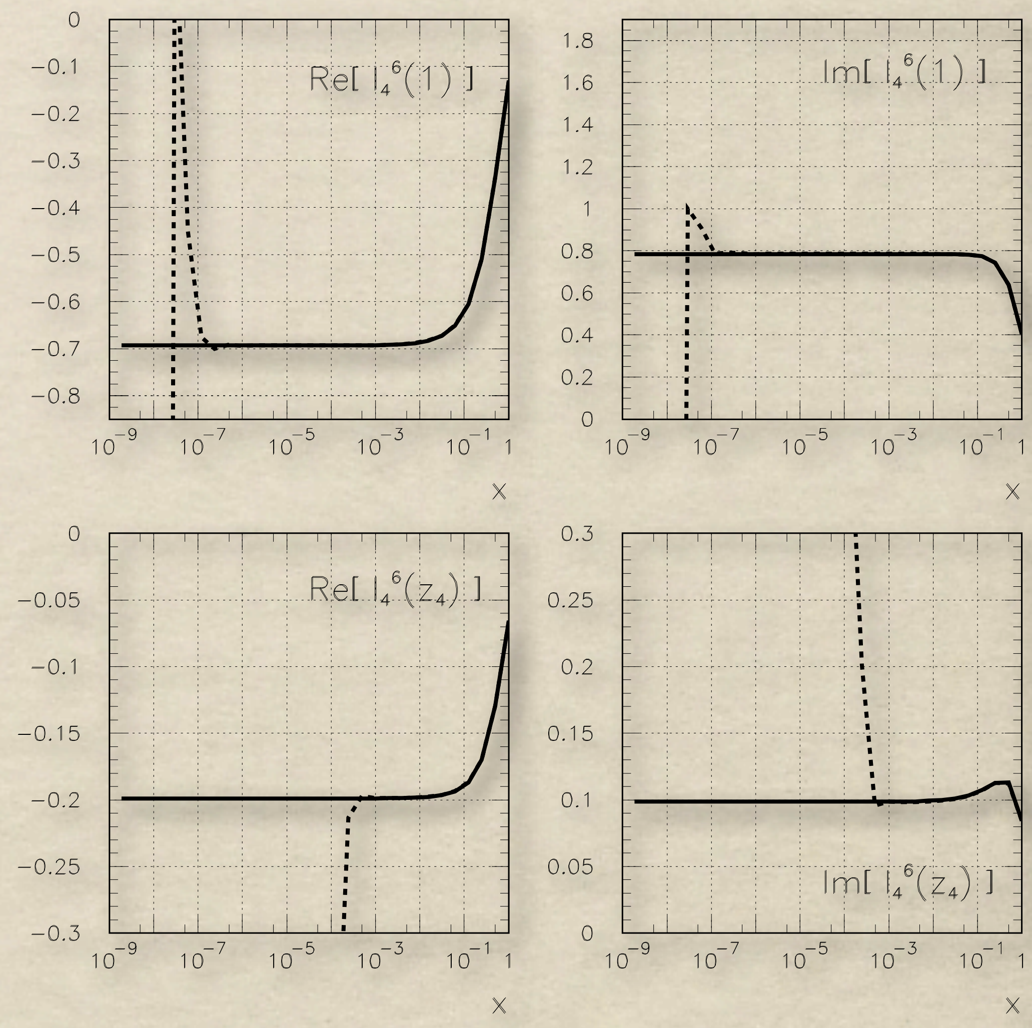
Set of Basis Integrals

$$I_4^{n+2}(l;S) \quad = \quad \frac{1}{B}\left\{ b_l\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j\in S} \mathcal{S}^{-1}_{j\,l} I_3^n(S\setminus\{j\}) - \frac{1}{2}\sum_{j\in S} b_j\, I_3^n(l;S\setminus\{j\}) \right\}$$

- B proportional to Gram determinant!

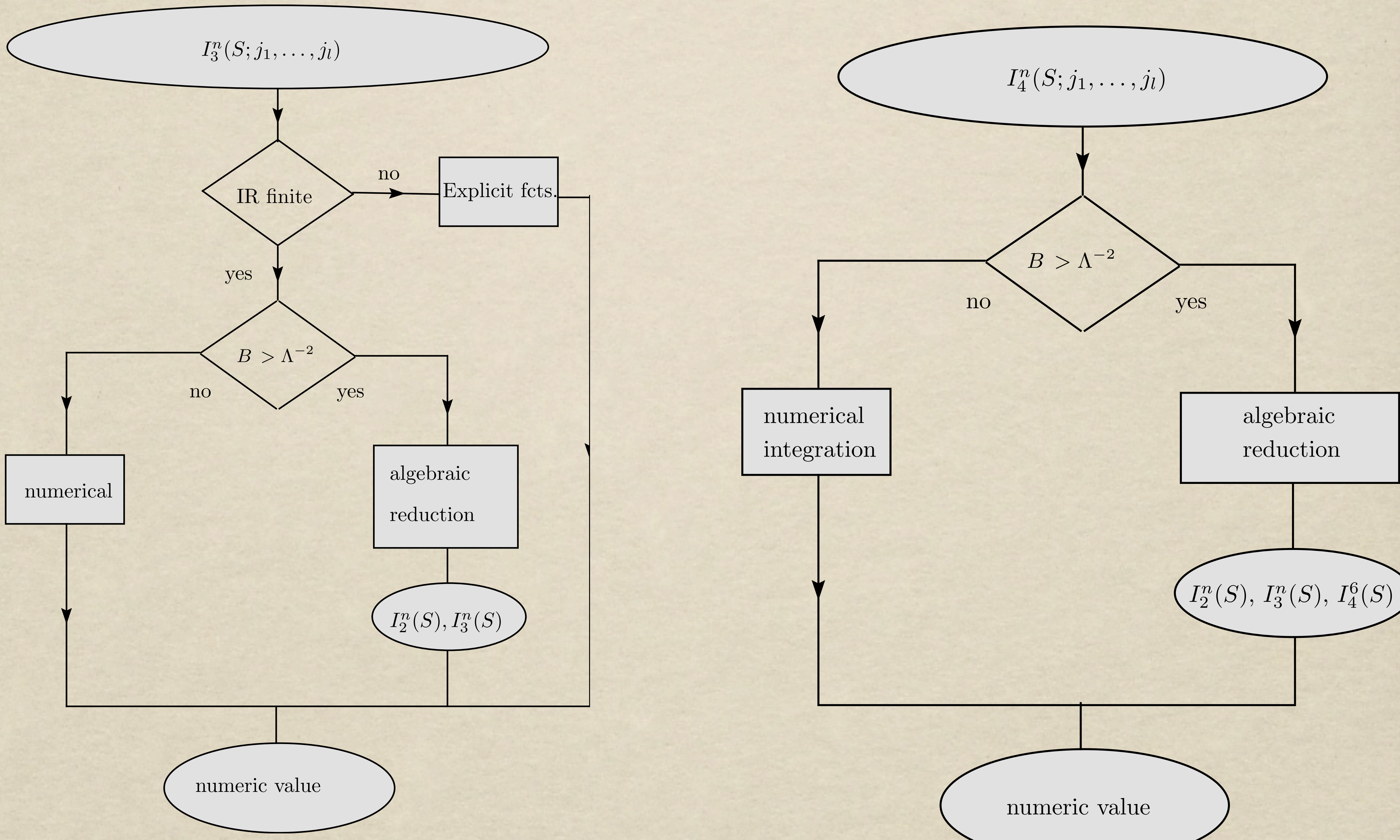
⇨ Mixed algebraic/numerical approach

- Find (one-dimensional) representation of form factors
- Switch to numerical evaluation if B is small



$$det(G) \propto x^2$$

Set of Basis Integrals



Public Version

lappweb.in2p3.fr/lapth/Golem/golem95.html

- Code, Instructions, Demos
- algebraic separation of IR poles
- caches avoid multiple evaluations
- All Form Factors coded ($N \leq 6$, massless)

Public Version

lappweb.in2p3.fr/lapth/Golem/golem95.html

- Code, Instructions, Demos
- algebraic separation of IR poles
- caches avoid multiple evaluations
- All Form Factors coded ($N \leq 6$, massless)
- some applications...
  - $gg \to WW \to l\nu l\nu$ Binoth, Ciccolini, Krämer, Kauer
  - $gg \to HH, HHH$ Binoth, Karg, Kauer, Rückl
  - $pp \to Hjj$ in WBF/GF Andersen, Binoth, Heinrich, Smillie
  - $pp \to b\bar{b}b\bar{b}$ Binoth, Greiner, Guffanti, Guillet, Reiter, Reuter
  - ...

Going Massive

- Massless form factors for QCD computations
- Full SM and beyond requires inclusion of massive loops!

Going Massive

- Massless form factors for QCD computations
- Full SM and beyond requires inclusion of massive loops!
- Reduction of scalar/tensor integrals not affected, but...
- Form factors much more complex
  - Find suitable (numerically stable) representations
  - Code (G. Heinrich, JP. Guillet) and Checks (M. Rodgers)

Going Massive

- Massless form factors for QCD computations

○ Full SM and beyond requires inclusion of massive loops!

- Reduction of scalar/tensor integrals not affected, but...
- Form factors much more complex
  - Find suitable (numerically stable) representations
  - Code (G. Heinrich, JP. Guillet) and Checks (M. Rodgers)

Current Implementation

- Boxes: Now distinction between massless and massive case
  - Massless case as before
  - Massive case: analytic reduction
    - divergent: implemented in GOLEM
    - finite: Scalar Box D_0 call to LoopTools (T.Hahn)

- Triangles: Similar to Boxes, call to LoopTools for finite Triangles

Going Massive

- Scalar Box D_0:
  - Analytic result known since 30 years
  - Contains approx. 50 DiLogs
  - Problem: find numerically stable representation depending on kinematics
  - Implemented by van Oldenborgh in FF/LoopTools

Going Massive

- Scalar Box D_0:
  - Analytic result known since 30 years
  - Contains approx. 50 DiLogs
  - Problem: find numerically stable representation depending on kinematics
  - Implemented by van Oldenborgh in FF/LoopTools
- Possible Solutions for GOLEM
  - Call to LoopTools/FF, provide standalone package containing FF
  - Find own stable analytical representations
  - Find one-dimensional representation for numerical integration

⇨ For now: stick to point 1 and postpone discussion for later!

Golem95

- Coding almost completed
- Finalize Checks:
  - Cross checks with LoopTools
  - Self-consistency checks
  - Few points ‘exceptional’ - understand and cure them

Golem95

- Coding almost completed
- Finalize Checks:
  - Cross checks with LoopTools
  - Self-consistency checks
  - Few points 'exceptional' - understand and cure them

Massive version available soon! (~May)
lappweb.in2p3.fr/lapth/Golem/golem95.html

Going Complex

Complex Masses needed for internal unstable particles

Going Complex

Complex Masses needed for internal unstable particles

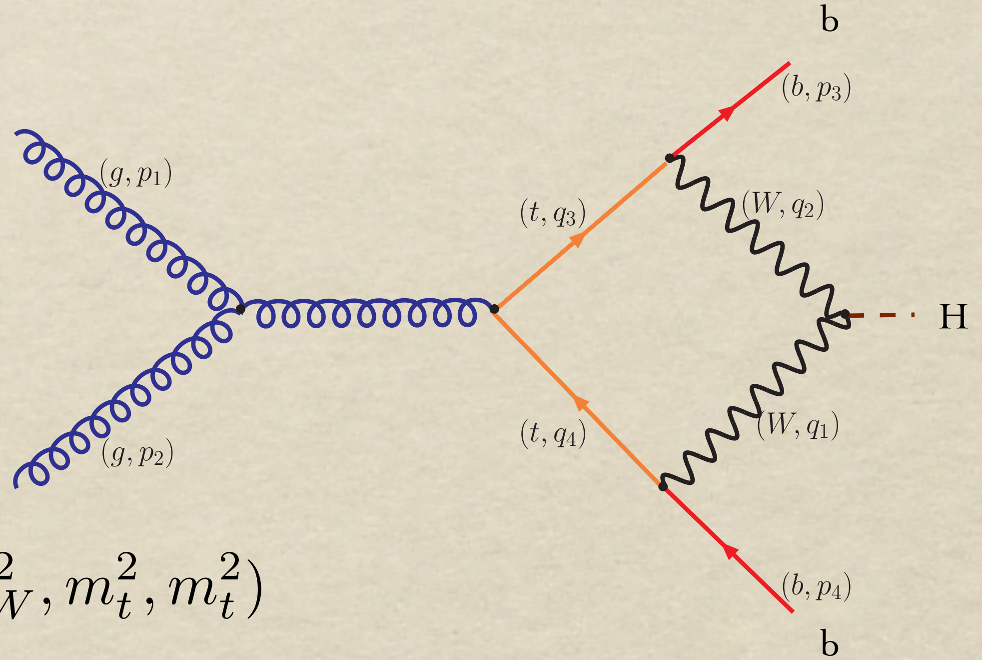
- Proper description of processes involving EW-Bosons or top quarks
- Beyond SM physics: e.g. SUSY production processes, Cascade decays

Going Complex

Complex Masses needed for internal unstable particles

- Proper description of processes involving EW-Bosons or top quarks
- Beyond SM physics: e.g. SUSY production processes, Cascade decays
- Anomalous Thresholds

e.g. $gg \to Hb\bar{b}$



$$D_0(M_H^2, 0, s, 0, (p_3+p_5)^2, (p_4+p_5)^2, M_W^2, M_W^2, m_t^2, m_t^2)$$

Le Duc Ninh

Note: det(S) = 0 !

Going Complex

Analytic Continuation: $m^2 \to m^2 - im\Gamma$

Going Complex

Analytic Continuation: $m^2 \rightarrow m^2 - im\Gamma$

Recipe

- Recalculate steps in analytic evaluation of loop integrals and check for consistency with finite width.
- For higher rank form factors: find representations for save numerical integration of tensor integrals

Going Complex

Analytic Continuation: $m^2 \to m^2 - im\Gamma$

Recipe

- Recalculate steps in analytic evaluation of loop integrals and check for consistency with finite width.
- For higher rank form factors: find representations for save numerical integration of tensor integrals

Status

- Changed infrastructure (S-matrix and related properties)
- Tadpoles (N=1): Continuation trivial
- Bubbles (N=2): Almost finished, to be checked

Going Imaginary

To Do

- Triangles (N=3)

- Scalar case: analytic expressions exist. Recheck, find (stable and efficient) representation.
- Check current representations for numerical integration and make transition to imaginary case.

Going Imaginary

To Do

Triangles (N=3)

- Scalar case: analytic expressions exist. Recheck, find (stable and efficient) representation.
- Check current representations for numerical integration and make transition to imaginary case.

Boxes (N=4)

- Higher rank: find/alter representation to include imaginary masses
- Scalar case: Same problems as discussed before
  - Analytic expression exists (Le Duc Ninh[0902.0325]). Implemented in LoopTools.
  - Can we find one-dimensional representation for numerical integration?

LoopTools

(Thomas Hahn)

- Based on FF (van Oldenborgh, Vermaseren)
- Up to N=5 scalar and tensor integrals
- Uses reduction from Denner,Dittmaier
- Form Factor representation from Denner/FF
- Complex masses implemented
- Caching system
- Switches to quadruple-precision at some internal points

- Community can only gain from having more than one Library
- Cross checks can help to find ideal method
- Which one is faster? More important: Which one is more stable?

Summary

The Golem95 Library

- Reduction Formalism, valid for massless and massive particles
- For N≥6 algebraic reduction, lowering rank at same time
- For N≤5 form factor representation, avoiding inv. Gram determinants
- IR divergences extracted analytically in terms of triangles
- Switch between analytic and numerical evaluation for stable computation, even in exceptional kinematic regions

Massive Version available soon!

Complex Masses implemented soonish!

lappweb.in2p3.fr/lapth/Golem/golem95.html