QCD flavour-blindness on the lattice and in the real world

Paul Rakow for QCDSF

UNIVERSITY OF LIVERPOOL

QCDSF

W Bietenholz, V Bornyakov, N Cundy, M Göckeler, T Hemmert,
R Horsley, WG Lockhart, Y Nakamura, H Perlt, D Pleiter, PR,
A Schäfer, G Schierholz, A Schiller, H Stüben, F Winter,
JM Zanotti, $\cdots$

Bietenholz et al; Phys. Rev. D 84, 054509 (2011);
arXiv:1102.5300 [hep-lat]
arXiv:1012.4371 [hep-lat] (Lat10)
Phys.Lett.B690:436-441,2010; arXiv:1003.1114 [hep-lat]

Abstract

The QCD interaction is flavour-blind. Neglecting electromagnetic and weak interactions, the only difference between flavours comes from the mass matrix. We investigate how flavour-blindness constrains hadron masses after flavour SU(3) is broken by the mass difference between the strange and light quarks, to help us extrapolate 2+1 flavour lattice data to the physical point.

We have our best theoretical understanding when all 3 quark flavours have the same masses (because we can use the full power of flavour $SU(3)$); nature presents us with just one instance of the theory, with $m_s/m_l \approx 25$. We are interested in interpolating between these two cases.

Plan

A talk about flavour symmetry breaking, not about lattice.
All the lattice knowledge you need to follow this talk will fit on one slide.

- Consider symmetry of QCD action with unequal quark masses
- Construct basis functions, which must have well-defined symmetry.
- Use these basis functions to describe the quark mass dependence of any physical quantity (masses, matrix elements, etc.)
- Compare with non-perturbative lattice results, fix coefficients, extrapolate to real world

Lattice

- On the lattice we can choose our quark masses, so we can investigate fictional universes where $m_s/m_l \neq 25$, and so gain a clearer understanding of symmetry breaking

Lattice

- On the lattice we can choose our quark masses, so we can investigate fictional universes where $m_s/m_l \neq 25$, and so gain a clearer understanding of symmetry breaking
- Computational expense gets too high if the u and d quarks are too light. We still need to extrapolate a little from the lattice results to the physical point.

Introduction

Standard Theorist's Approach:

$$\text{Action} = \text{Large Piece} + \text{Small Piece}$$

Treat the Small Piece as a perturbation.
Apply this to QCD.

Introduction

Perturbative QCD

$$\begin{aligned} \text{Large Piece} &= \text{Kinetic Terms} \\ &\quad + \text{Quark Mass Terms} \end{aligned}$$

$$\begin{aligned} \text{Small Piece} &= \text{Gluon-Gluon Vertices} \\ &\quad + \text{Quark-Gluon Vertices} \end{aligned}$$

Perturb about non-interacting QCD.
Works best at high energies, short distances.

Introduction

Chiral Perturbation Theory

$$\text{Large Piece} = \text{Kinetic Terms} + \text{Gluon-Gluon Vertices} + \text{Quark-Gluon Vertices}$$

$$\text{Small Piece} = \text{Quark Mass Terms}$$

Perturb about massless QCD

Introduction

This Talk

$$\begin{aligned}
\text{Large Piece} &= \text{Kinetic Terms} \\
&\quad + \text{Gluon-Gluon Vertices} \\
&\quad + \text{Quark-Gluon Vertices} \\
&\quad + \text{Singlet Quark Mass Term}
\end{aligned}$$

$$\text{Small Piece} = \text{Non-Singlet Quark Mass Terms}$$

Perturb about $SU(3)$ symmetric QCD.

Introduction

This Talk

$$
\begin{aligned}
\text{Large Piece} \quad &= \quad \text{Kinetic Terms} \\
&\quad + \text{Gluon-Gluon Vertices} \\
&\quad + \text{Quark-Gluon Vertices} \\
&\quad + \text{Singlet Quark Mass Term}
\end{aligned}
$$

$$
\text{Small Piece} \quad = \quad \text{Non-Singlet Quark Mass Terms}
$$

Long history: M. Gell Man, Phys Rev 125 (1962) 1067.
S. Okubo, Prog Theor Phys 27 (1962) 949.

Introduction

This Talk

$$\begin{aligned}\text{Large Piece} &= \text{Kinetic Terms}\\ &+ \text{Gluon-Gluon Vertices}\\ &+ \text{Quark-Gluon Vertices}\\ &+ \text{Singlet Quark Mass Term}\end{aligned}$$

$$\text{Small Piece} = \text{Non-Singlet Quark Mass Terms}$$

Not as familiar as chiral perturbation theory,
but useful for organising and analysing the data.

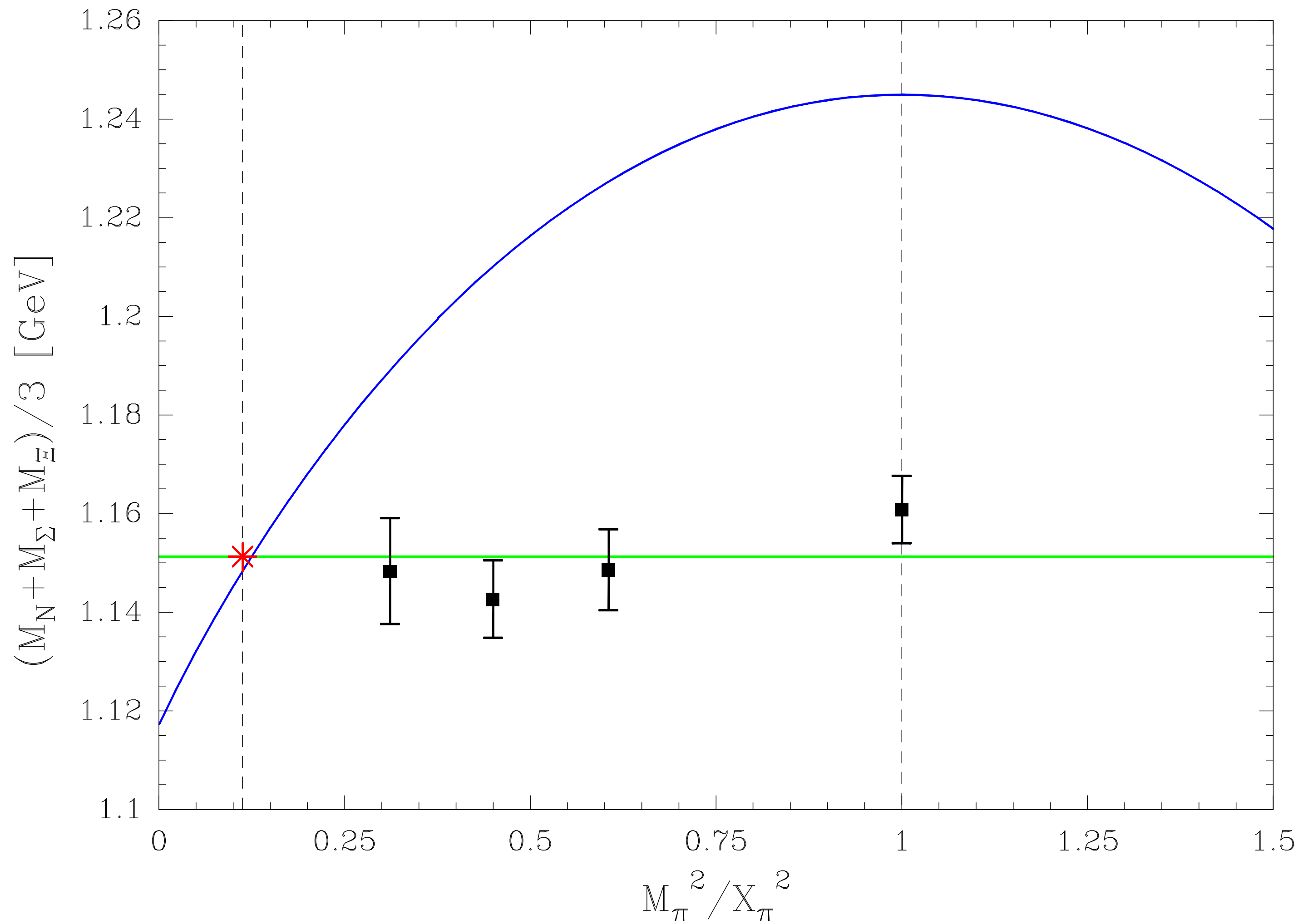
Do we need a new approach?

Maybe chiral perturbation theory is all we need?

To get symmetrical results for the hyperons, we need to treat the K and η on the same footing as the π . We may be getting to the point where the s quark is too heavy for this to work well, without using complicated corrections and resummations.

Do we need a new approach?

Simplest possible chiral PT calculation of $(M_N + M_\Sigma + M_\Xi)/3$
Terms up to M_π^3, M_K^3, M_η^3.



Quark Masses

Notation

$$
\begin{aligned}
\text{Fix} \qquad \overline{m} &\equiv \frac{1}{3}(m_u + m_d + m_s) \\
\delta m_u &\equiv m_u - \overline{m} \\
\delta m_d &\equiv m_d - \overline{m} \\
\delta m_s &\equiv m_s - \overline{m}
\end{aligned}
$$

$$
\delta m_u + \delta m_d + \delta m_s = 0
$$

$$
\begin{aligned}
m_l &\equiv \frac{1}{2}(m_u + m_d) \\
\delta m_l &\equiv m_l - \overline{m}
\end{aligned}
$$

Quark Masses

The quark mass matrix is

$$
\begin{aligned}
\mathcal{M} &= \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \\
&= \overline{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \tfrac{1}{2}(\delta m_u - \delta m_d) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \tfrac{1}{2}\delta m_s \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}
\end{aligned}
$$

$\mathcal{M}$ has a flavour singlet part (proportional to I) and a flavour octet part, proportional to λ_3, λ_8.
In clover case, the singlet and non-singlet parts of the mass matrix renormalise differently.

Flavour Hierarchy

$$\begin{aligned} \text{Large Piece} \quad &= \quad \text{Kinetic Terms} \\ &\quad + \text{Gluon-Gluon Vertices} \\ &\quad + \text{Quark-Gluon Vertices} \\ &\quad + \text{Singlet Quark Mass Term} \end{aligned}$$

$$\text{Small Piece} \quad = \quad \text{Non-Singlet Quark Mass Terms}$$

All terms in Large Piece are flavour singlets, leave $SU(3)$ unbroken.
Small Piece is pure flavour octet.
Higher $SU(3)$ representations completely absent from QCD action.

Flavour Hierarchy

Higher representations of $SU(3)$ are absent from the QCD action, but they appear at higher orders in the perturbation. Square an octet — generates 27-plet.

δm_q^0	1					
δm_q^1		8				
δm_q^2	1	8			27	
δm_q^3	1	8	10	$\overline{10}$	27	64

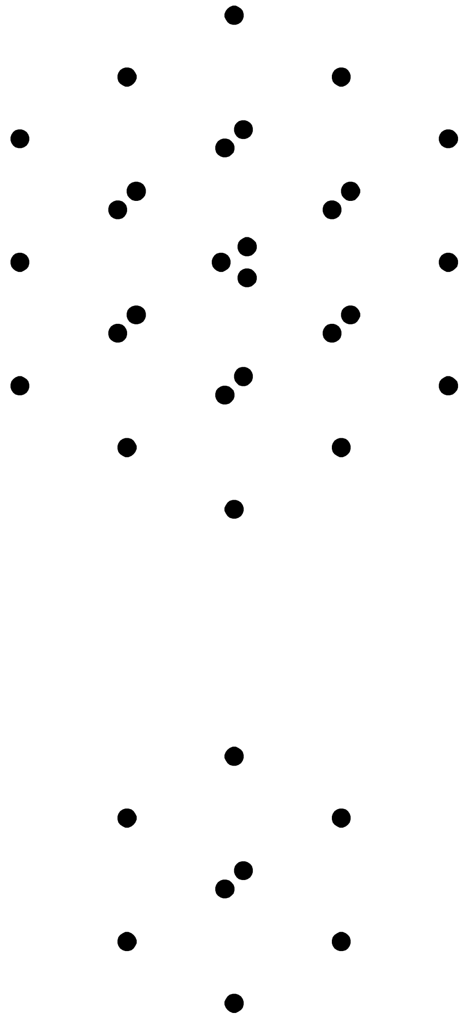
Flavour Hierarchy

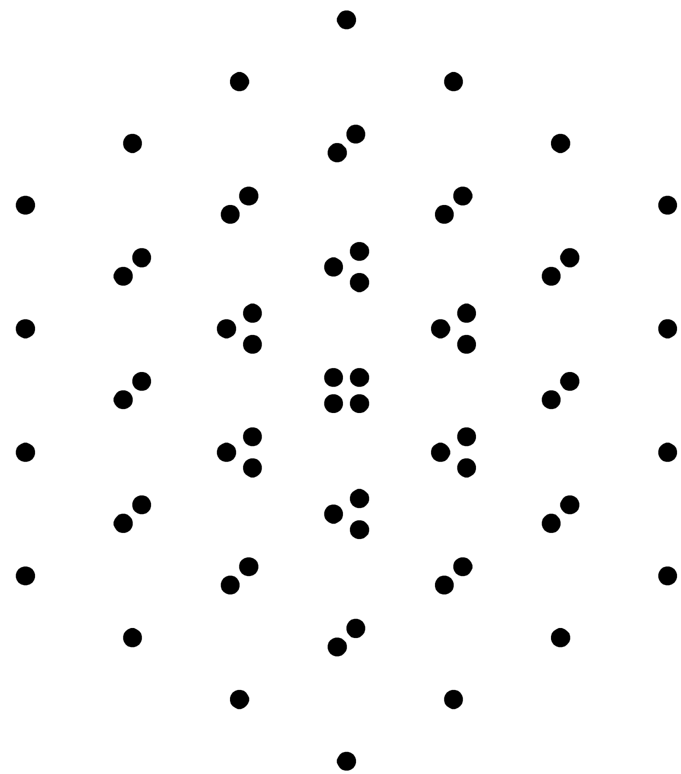
Decuplet mass matrix

$$10 \otimes \overline{10} = 1 \oplus 8 \oplus 27 \oplus 64$$

Δ^-	Δ^0	Δ^+	Δ^{++}	Σ^{*-}	Σ^{*0}	Σ^{*+}	Ξ^{*-}	Ξ^{*0}	Ω^-	$SU(3)$
1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	0	0	0	1	1	2	8
3	3	3	3	-5	-5	-5	-3	-3	9	27
-1	-1	-1	-1	4	4	4	-6	-6	4	64

Flavour Hierarchy





Flavour Hierarchy

$$\begin{aligned}
4M_\Delta + 3M_{\Sigma^*} + 2M_{\Xi^*} + M_\Omega &= 13.82\ \text{GeV} && \text{singlet}\\
-2M_\Delta \qquad\qquad + M_{\Xi^*} + M_\Omega &= 0.742\ \text{GeV} && \text{octet}\\
4M_\Delta - 5M_{\Sigma^*} - 2M_{\Xi^*} + 3M_\Omega &= -0.044\ \text{GeV} && 27-\text{plet}\\
-M_\Delta + 3M_{\Sigma^*} - 3M_{\Xi^*} + M_\Omega &= -0.006\ \text{GeV} && 64-\text{plet} \quad ,
\end{aligned}$$

[PDG masses]

Strong Hierarchy:

1	8	27	64
$(m_s - m_l)^0$	$(m_s - m_l)^1$	$(m_s - m_l)^2$	$(m_s - m_l)^3$

Strategy

Keep Large Piece constant,
Vary Small Piece until we reach the physical point.

Strategy

Start from a point with all 3 sea quark masses equal,

$$m_u = m_d = m_s \equiv m_0$$

and extrapolate towards the physical point, keeping the average sea quark mass

$$\overline{m} \equiv \frac{1}{3}(m_u + m_d + m_s)$$

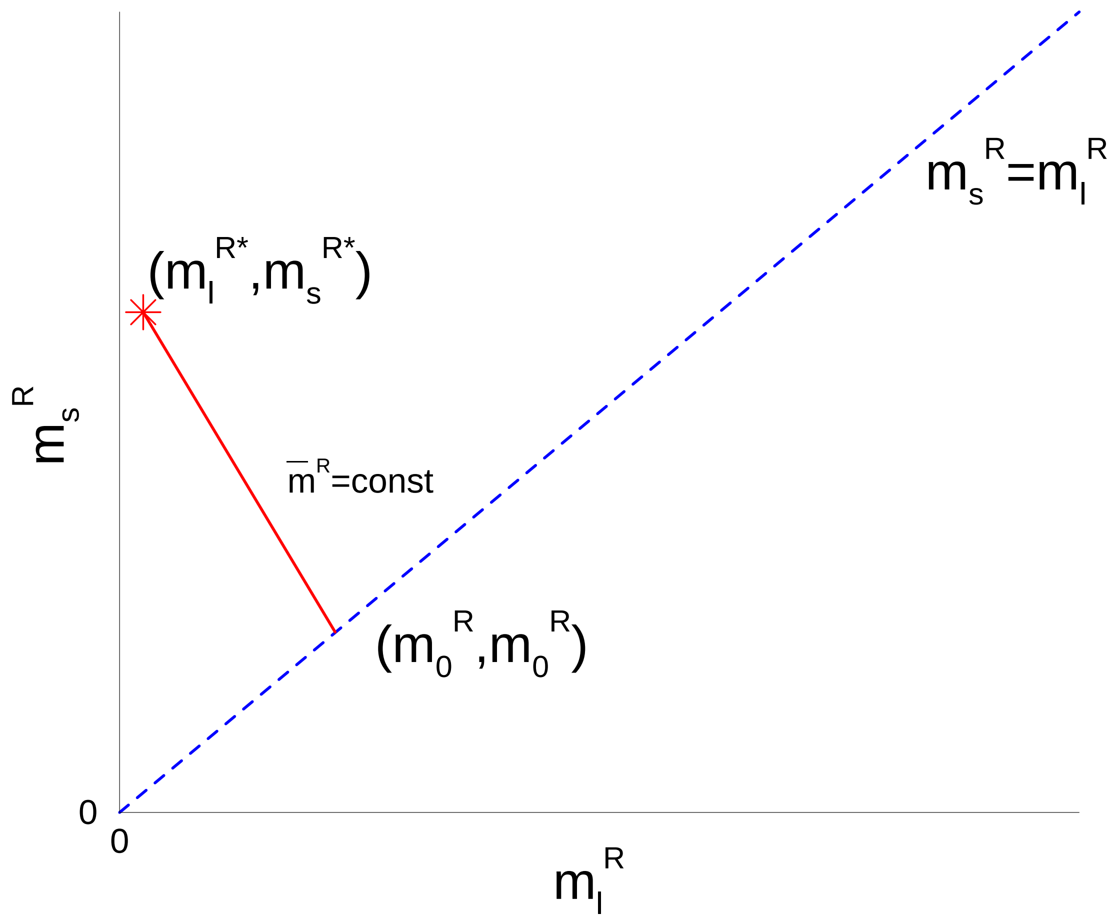
constant.
Starting point has

$$m_0 \approx \frac{1}{3} m_s^{phys}$$

As we approach the physical point, the u and d become lighter, but the s becomes heavier. Pions are decreasing in mass, but K and η increase in mass as we approach the physical point.

Strategy

Keep Large Piece constant,
Vary Small Piece until we reach the physical point.



Singlet Quantities

Consider a flavour singlet quantity (eg string tension σ) at the symmetric point (m_0, m_0, m_0).

$$\frac{\partial \sigma}{\partial m_u} = \frac{\partial \sigma}{\partial m_d} = \frac{\partial \sigma}{\partial m_s} \; .$$

If we keep $m_u + m_d + m_s$ constant, $dm_s = -dm_u - dm_d$ so

$$d\sigma = dm_u \frac{\partial \sigma}{\partial m_u} + dm_d \frac{\partial \sigma}{\partial m_d} + dm_s \frac{\partial \sigma}{\partial m_s} = 0$$

The effect of making the strange quark heavier exactly cancels the effect of making the light quarks lighter, so we know that σ must have a stationary point at the symmetrical point.

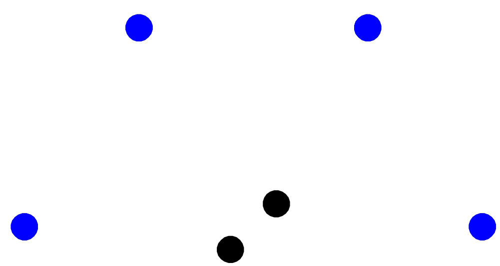
Singlet Quantities

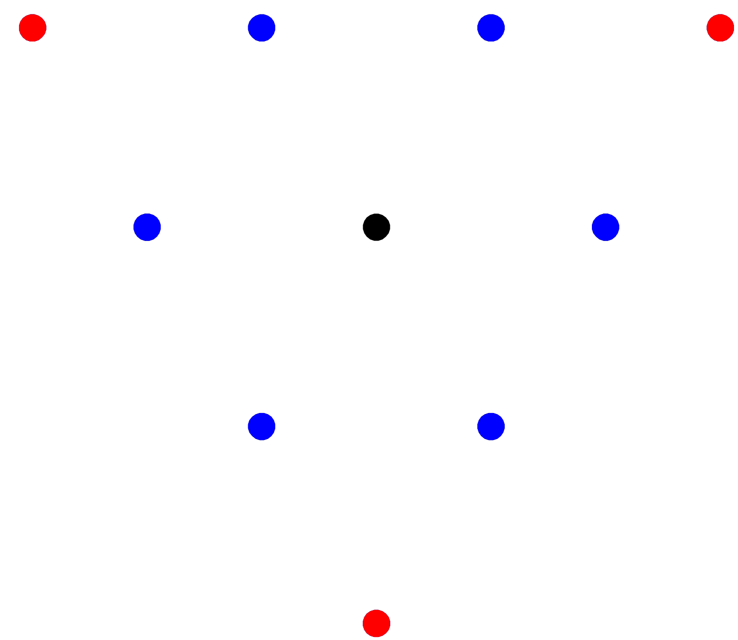
Any permutation of the quarks, eg

$$u \leftrightarrow s, \qquad u \to d \to s \to u$$

doesn't really change physics, it just renames the quarks.
Group S_3 , permutations of three objects, symmetry group of the equilateral triangle.
Any quantity unchanged by all permutations will also be flat at the symmetric point.

Singlet Quantities





$$2M_\Delta + M_\Omega$$

$$2(M_N + M_\Sigma + M_\Xi)$$

$$M_\Sigma + M_\Lambda$$

$$2(M_\Delta + M_{\Sigma^*} + M_{\Xi^*})$$

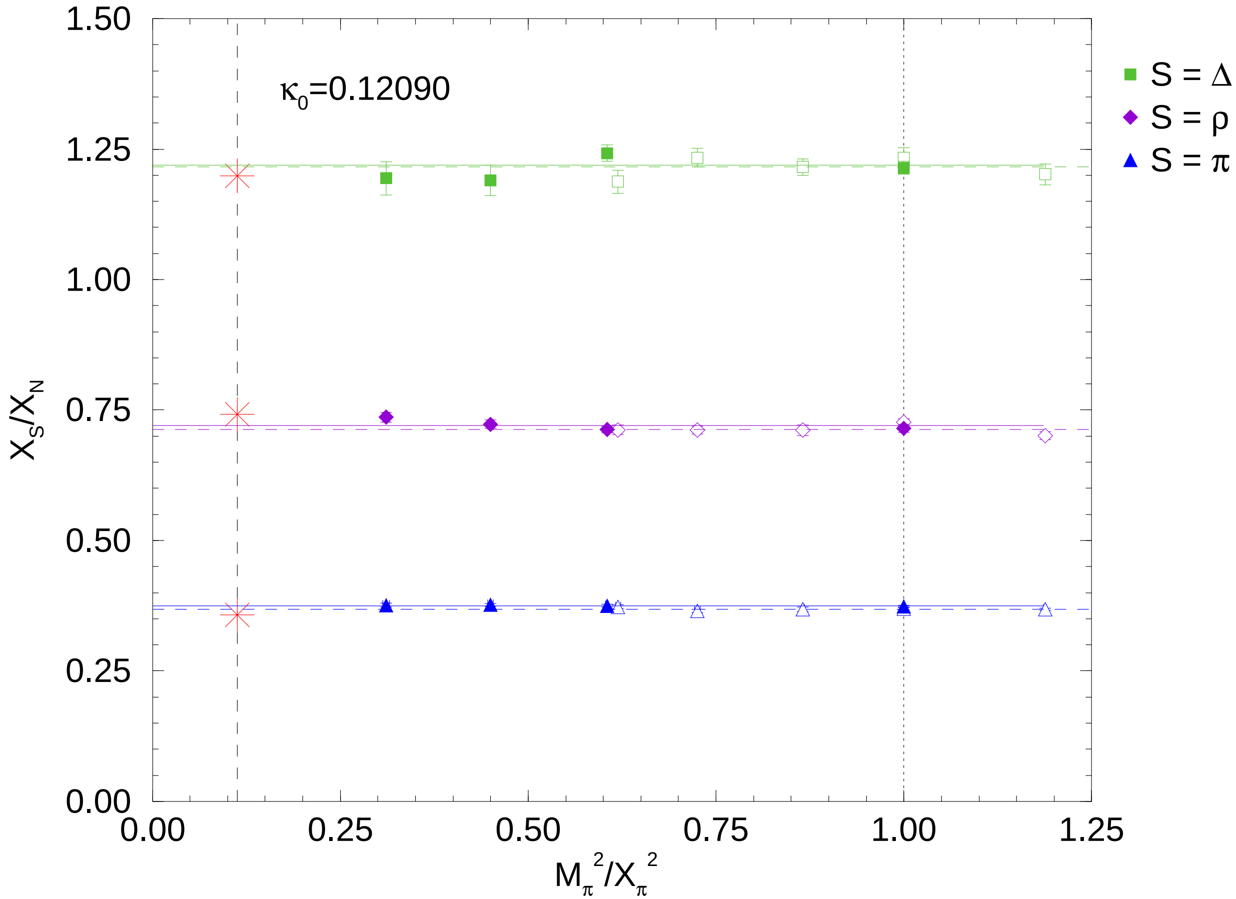
$$M_{\Sigma^*}$$

Singlet Quantities

$$
\begin{aligned}
X_\pi^2 &= (M_\pi^2 + 2M_K^2)/3 \\
X_\rho &= (M_\rho + 2M_{K^*})/3 \\
X_N &= (M_N + M_\Sigma + M_\Xi)/3 \\
X_\Delta &= (2M_\Delta + M_\Omega)/3
\end{aligned}
$$

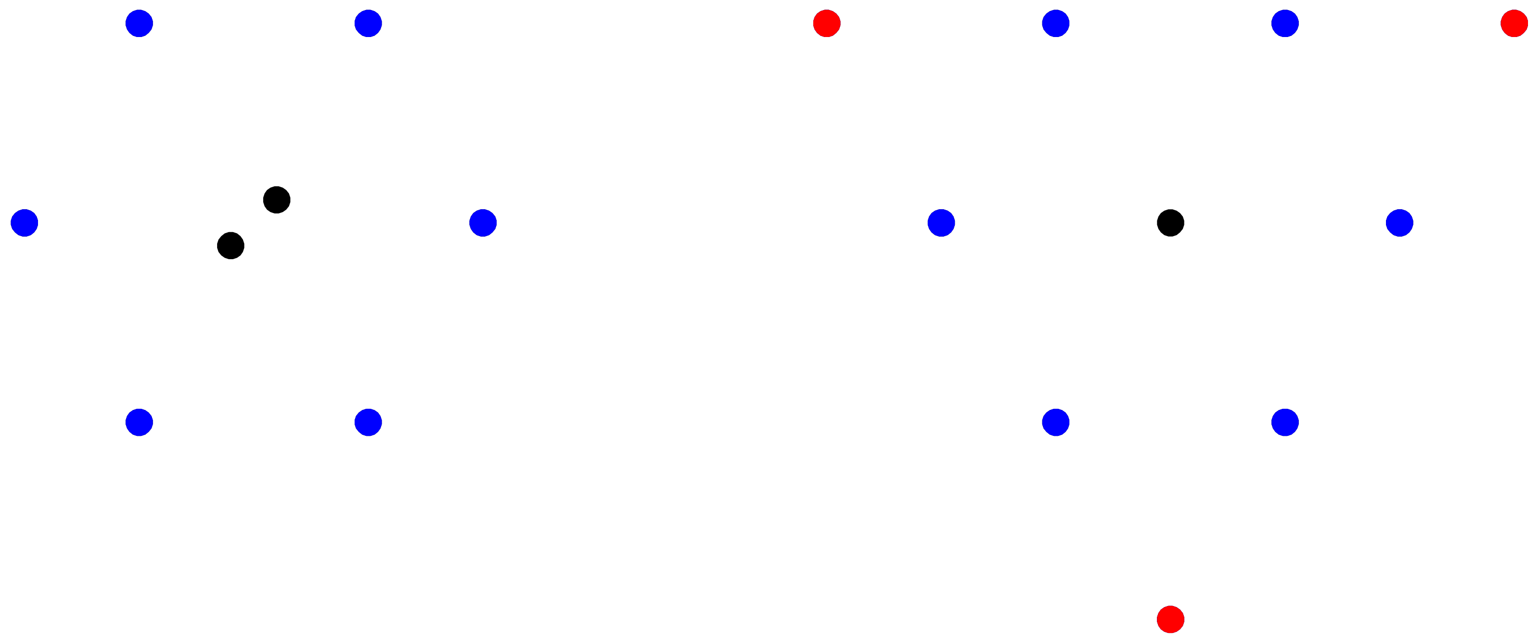
Multiplet Centre-of-Mass
Use octet baryons (X_N) to set scale for the other three multiplets.

Singlet Quantities



X_S so flat because we keep $m_u + m_d + m_s$ constant.
Choose initial m_0 to make X_S/X_N equal to physical value (red star).

SU(3) classification



The permutation group yields a lot of useful relationships, but can't capture the entire structure. No connection between Δ^{++}, uuu and Δ^{+}, uud.

SU(3) classification

- Classify physical quantities by $SU(3)$ and permutation group S_3 (which is a subgroup of SU(3)).
- Classify quark mass polynomials in same way.
- Quantity of Known Symmetry $=$ Polynomials of Matching Symmetry
- Taylor expansion about (m_0, m_0, m_0) strongly constrained by symmetry.

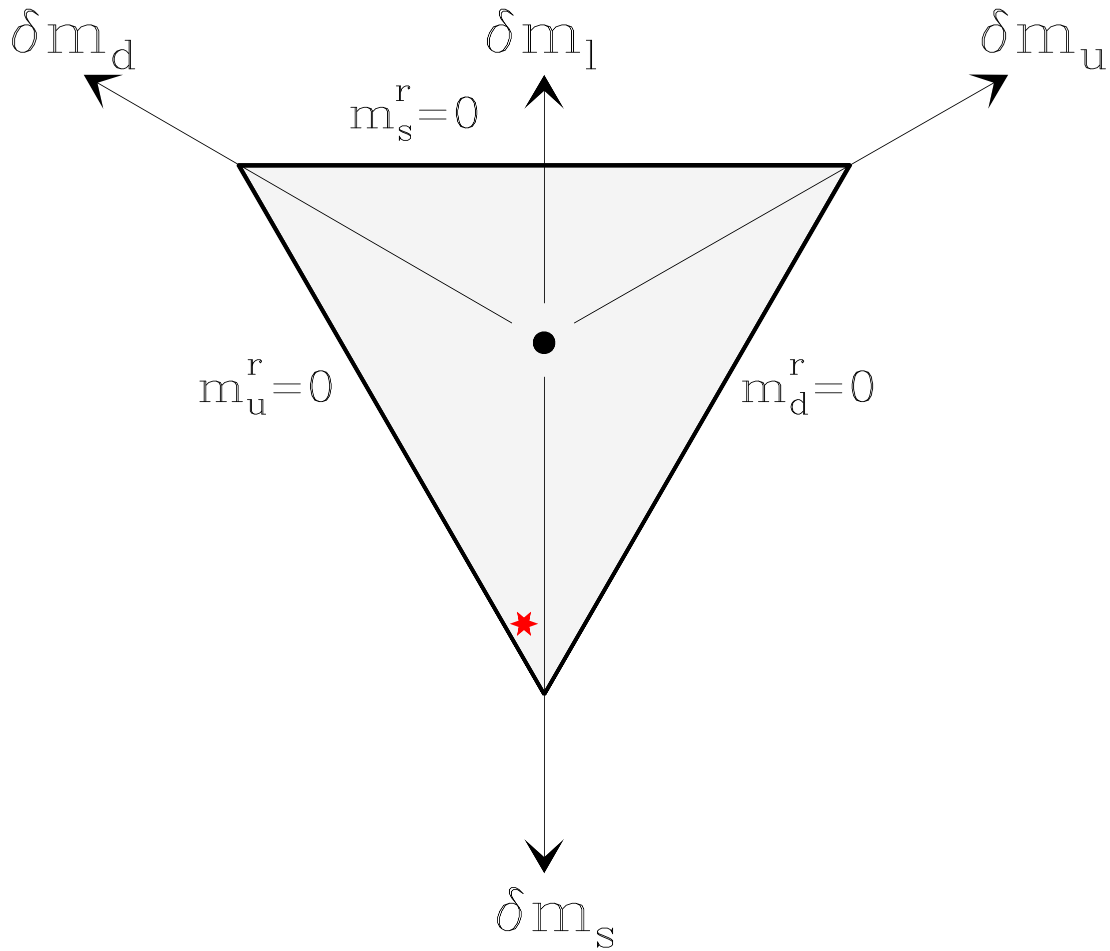
SU(3) classification

Polynomial		S_3	$SU(3)$		
1	✓	A_1	1		
$(\overline{m} - m_0)$		A_1	1		
δm_s	✓	E^+		8	
$(\delta m_u - \delta m_d)$	✓	E^-		8	
$(\overline{m} - m_0)^2$		A_1	1		
$(\overline{m} - m_0)\delta m_s$		E^+		8	
$(\overline{m} - m_0)(\delta m_u - \delta m_d)$		E^-		8	
$\delta m_u^2 + \delta m_d^2 + \delta m_s^2$	✓	A_1	1		27
$3\delta m_s^2 - (\delta m_u - \delta m_d)^2$	✓	E^+		8	27
$\delta m_s(\delta m_d - \delta m_u)$	✓	E^-		8	27

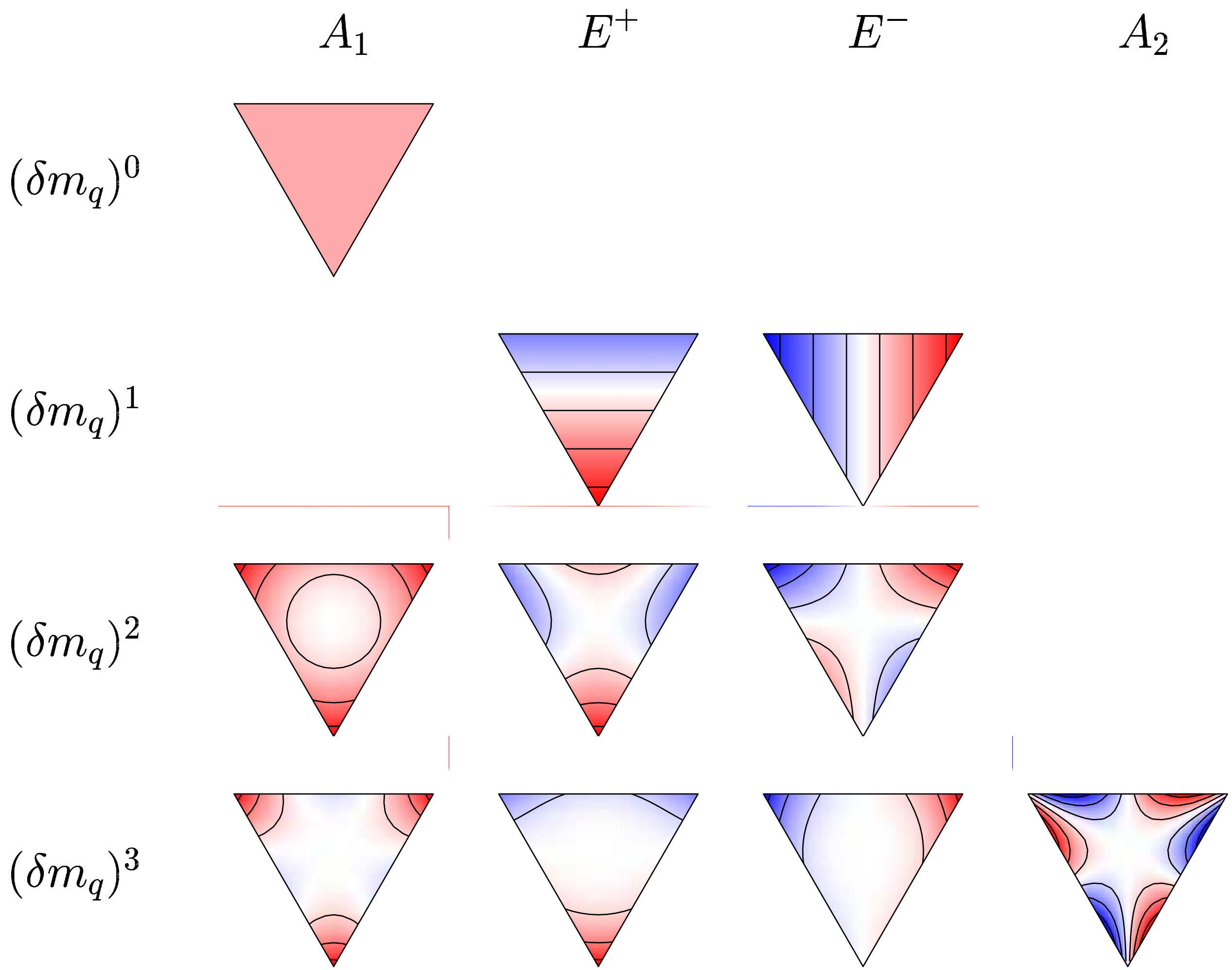
SU(3) classification

Polynomial		S_3	$SU(3)$					
$(\overline{m}-m_0)^3$		A_1	1					
$(\overline{m}-m_0)^2\delta m_s$		E^+		8				
$(\overline{m}-m_0)^2(\delta m_u-\delta m_d)$		E^-		8				
$(\overline{m}-m_0)(\delta m_u^2+\delta m_d^2+\delta m_s^2)$		A_1	1				27	
$(\overline{m}-m_0)\left[3\delta m_s^2-(\delta m_u-\delta m_d)^2\right]$		E^+		8			27	
$(\overline{m}-m_0)\delta m_s(\delta m_d-\delta m_u)$		E^-		8			27	
$\delta m_u\delta m_d\delta m_s$	✓	A_1	1				27	64
$\delta m_s(\delta m_u^2+\delta m_d^2+\delta m_s^2)$	✓	E^+		8			27	64
$(\delta m_u-\delta m_d)(\delta m_u^2+\delta m_d^2+\delta m_s^2)$	✓	E^-		8			27	64
$(\delta m_s-\delta m_u)(\delta m_s-\delta m_d)(\delta m_u-\delta m_d)$	✓	A_2			10	$\overline{10}$		64

SU(3) classification



SU(3) classification



SU(3) classification

The only quantities with a non-zero slope at symmetric point are flavour octet quantities.
(only applies on $m_u + m_d + m_s = const$ line.)
Often slopes highly constrained:
Decuplet baryons - 4 particles; but 1 slope parameter.
Octet baryons - 4 particles; but 2 slopes.
Octet mesons - 3 particles; but 1 slope parameter.

$2 + 1$ Simulation

Tree-level Symanzik glue, $\beta = 5.50$

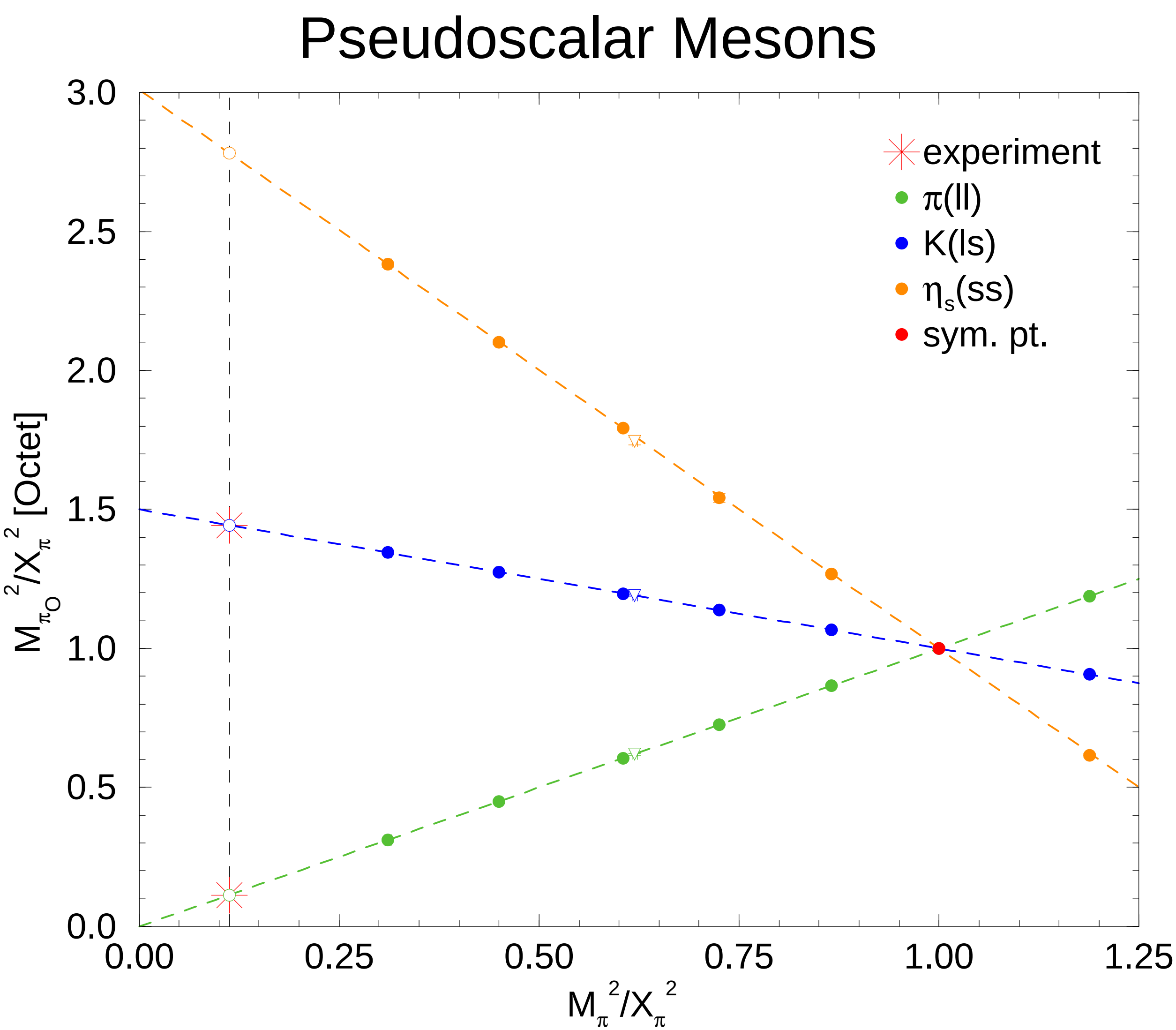
Clover Fermions, non-pert c_{SW}.

To to keep the action highly local, the hopping terms use a stout smeared link (‘fat link’) with $\alpha = 0.1$ ‘mild smearing’ for the Dirac kinetic term and Wilson mass term.

Symmetric point $\kappa_0 = 0.12090$

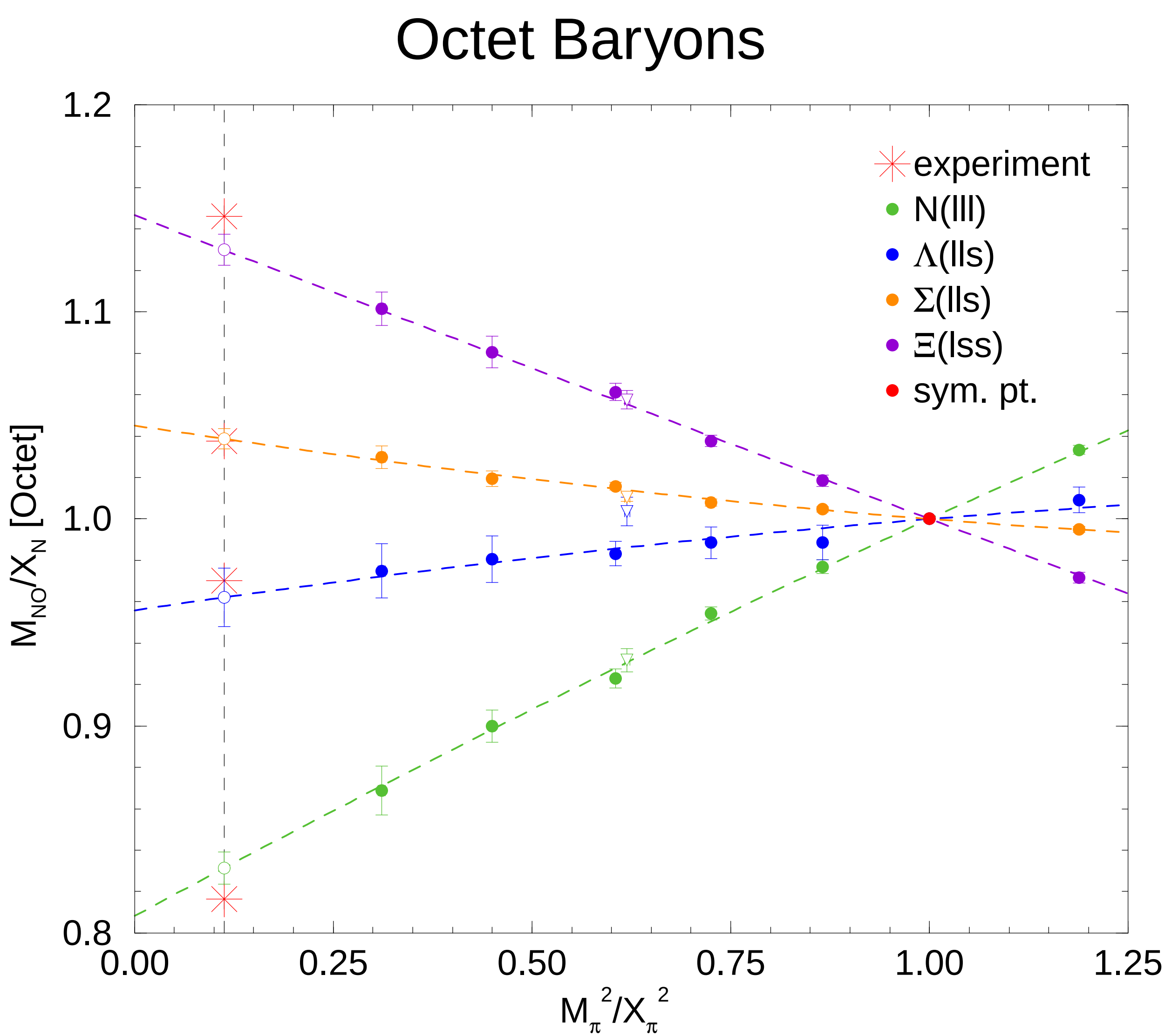
$24^3 \times 48$ lattices and $32^3 \times 64$ lattices

Fan Plot



Pseudoscalar Mesons
experiment
π(ll)
K(ls)
η_s(ss)
sym. pt.
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00
1.25
$M_{\pi_O}^2/X_\pi^2$ [Octet]
M_π^2/X_π^2

Fan Plot


Octet Baryons
experiment
N(lll)
Λ(lls)
Σ(lls)
Ξ(lss)
sym. pt.
1.2
1.1
1.0
0.9
0.8
0.00
0.25
0.50
0.75
1.00
1.25
M_{NO}/X_N [Octet]
M_π^2/X_π^2

Inverted World

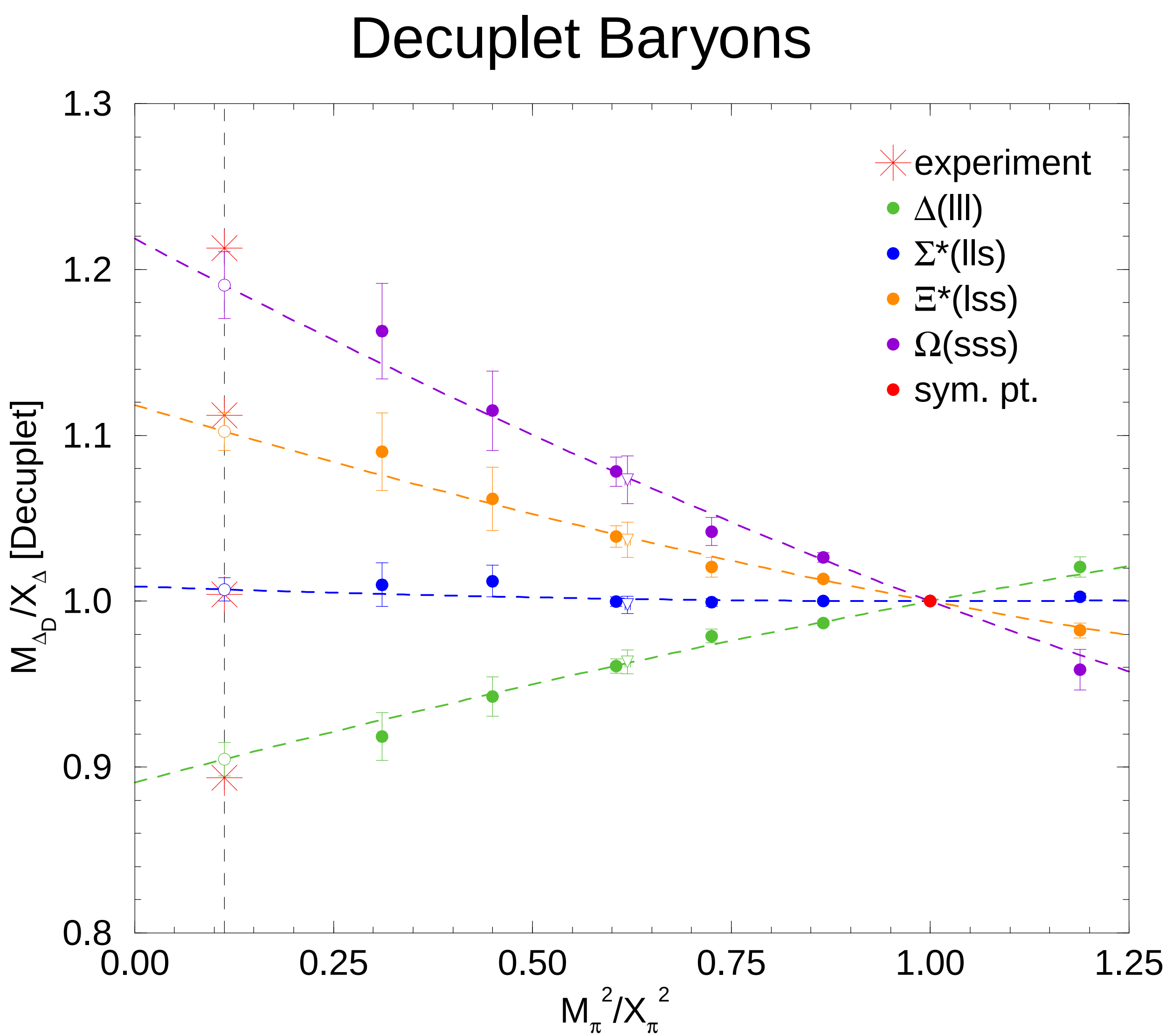
To increase our lever-arm, and to check that singlet quantities have zero slope at the symmetric point, we have measured one point on the "other side" of the symmetrical point, a point with $m_s < m_u, m_d$.

Here the Ξ is the lightest octet baryon, N the heaviest.

Ω the lightest decuplet baryon, and Δ the heaviest.

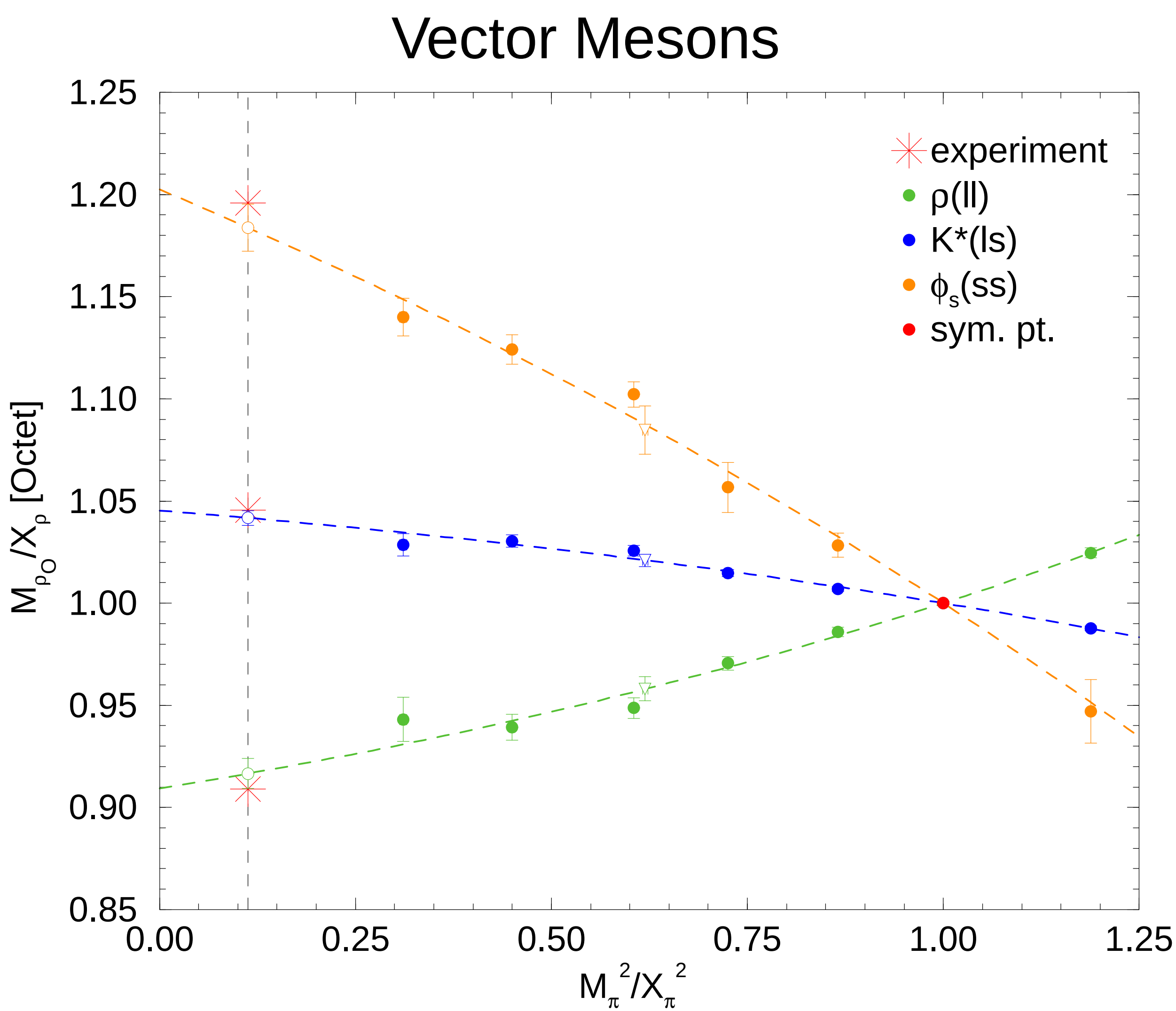
Weak decays would go in the direction $u \to s$, the proton would decay to a Σ or Λ, and then to a Ξ.

Fan Plot



Decuplet Baryons
experiment
Δ(lll)
Σ*(lls)
Ξ*(lss)
Ω(sss)
sym. pt.
1.3
1.2
1.1
1.0
0.9
0.8
0.00
0.25
0.50
0.75
1.00
1.25
M_{Δ_D}/X_Δ [Decuplet]
M_π^2/X_π^2

Fan Plot



Vector Mesons
experiment
ρ(ll)
K*(ls)
ϕ_s(ss)
sym. pt.
M_{ρ_O}/X_ρ [Octet]
M_π^2/X_π^2
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.00
0.25
0.50
0.75
1.00
1.25

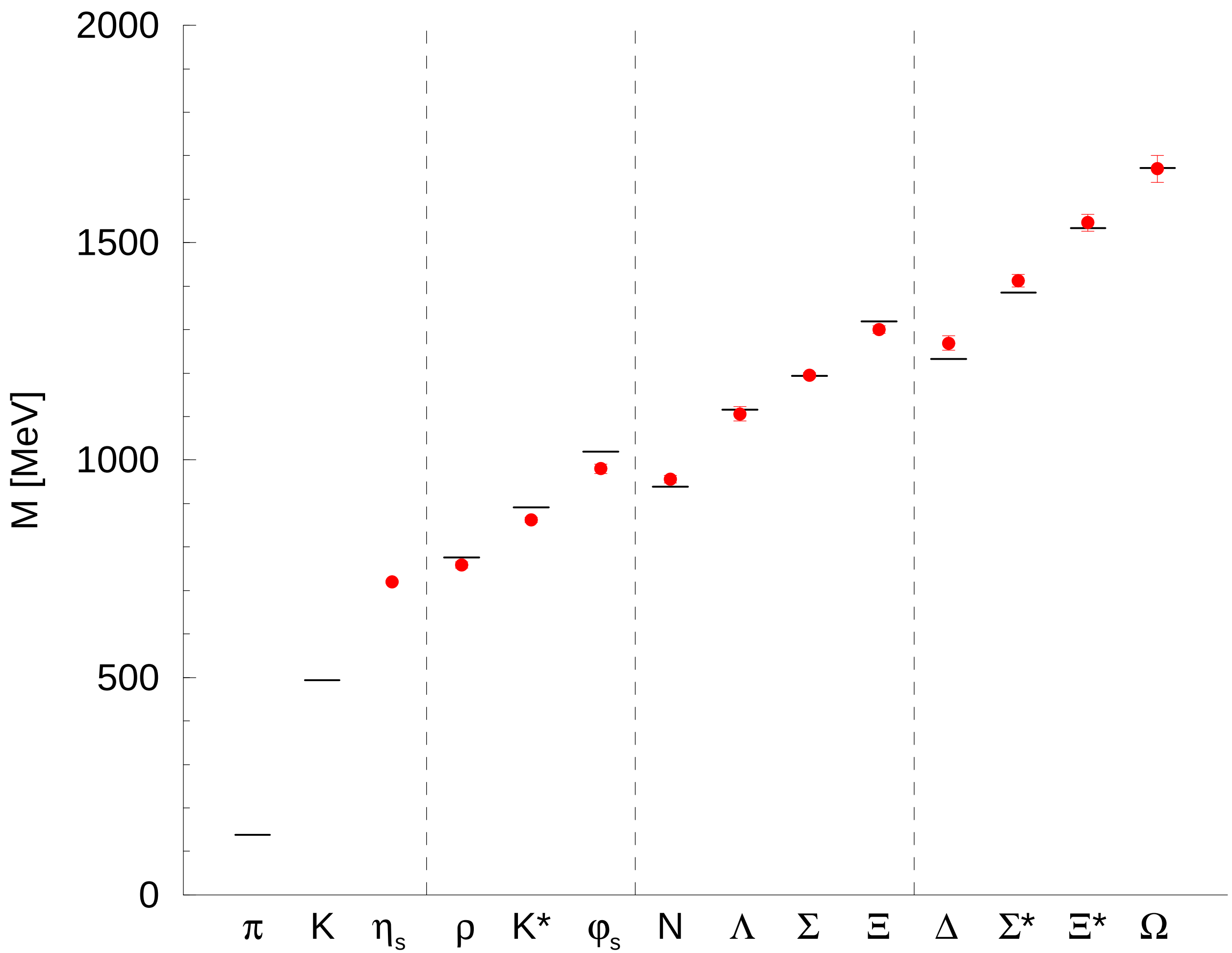
Fan Plot

Fan plots fitted with linear plus quadratic terms.

Find quadratic terms small

(An observation, not an assumption)

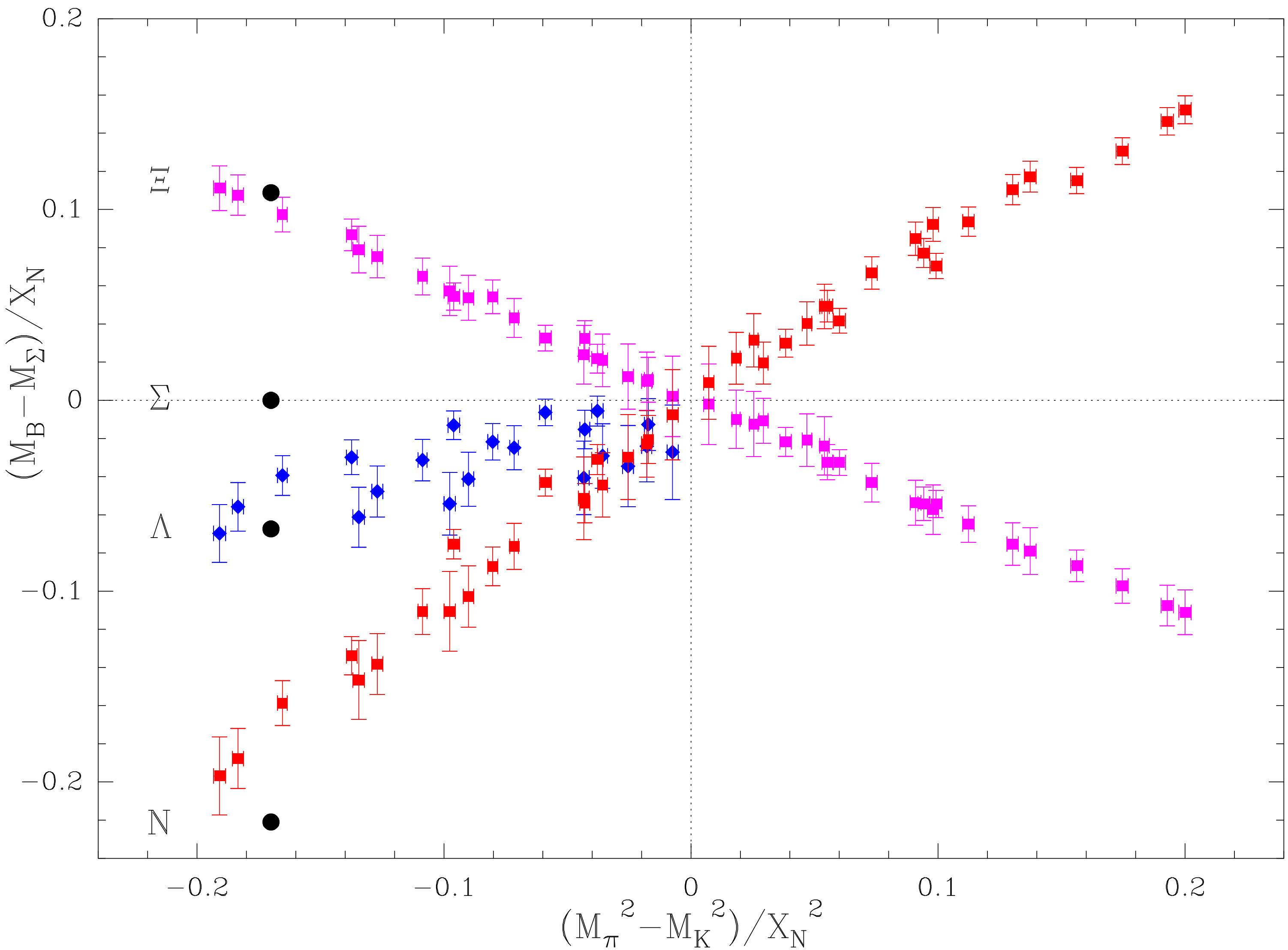
Hadron Spectrum



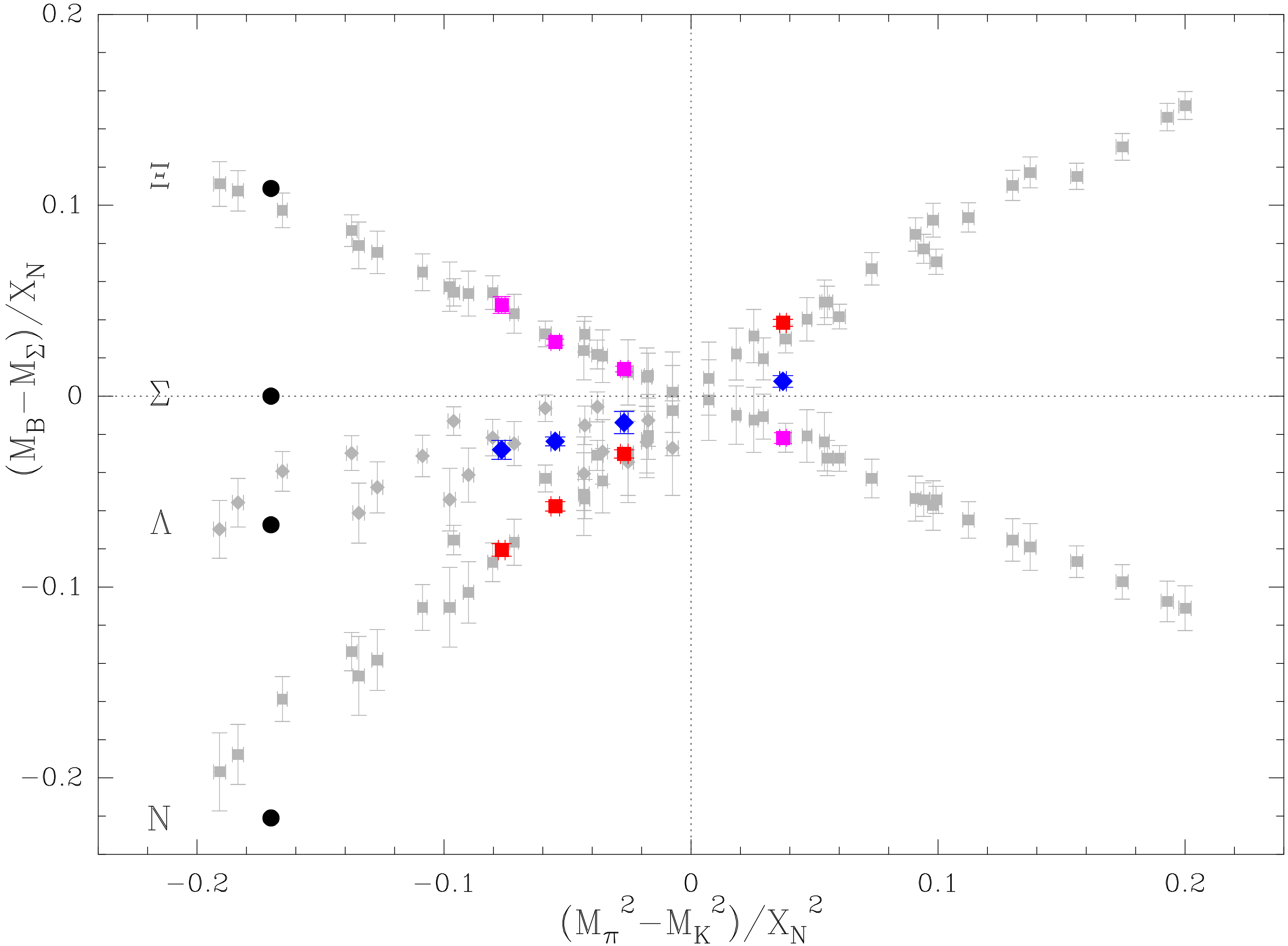
Partial Quenching

Partial quenching (making measurements with valence quarks which have masses different from the sea quarks used to generate a configuration) works well along the line $\overline{m}_{sea} = const$. The argument is very similar to the one we gave earlier, the effects of making the u_{sea} and d_{sea} lighter is largely cancelled by the effect of making the s_{sea} heavier. The cancellation is perfect at the symmetric point. On our trajectory, the error from partial quenching is quadratic in the quark mass; normally partial quenching errors are linear in m_q.

Partial Quenching



Partial Quenching



Partial Quenching

As an example, let us look at the Ω made with three quarks with $\kappa_{val} = 0.12080$.

We have measured this combination on 4 different backgrounds, 3 with the same value for $(m_u + m_d + m_s)_{sea}$, and one lying off the trajectory, with a larger value of $(m_u + m_d + m_s)_{sea}$.

κ_l^{sea}	κ_s^{sea}	κ^{val}	aM_Ω	
0.12100	0.12070	0.12080	$0.610(7)$	PQ
0.12095	0.12080	0.12080	$0.605(4)$	full
0.12090	0.12090	0.12080	$0.608(7)$	PQ
0.12080	0.12080	0.12080	$0.642(10)$	full

On trajectory, PQ and full results agree, but not off the trajectory.

Matrix Elements

Apply the same idea to matrix elements and transition amplitudes. Well known that at the symmetry point only 3 independent amplitudes

$\langle p|(\bar{u}u + \bar{d}d + \bar{s}s)|p\rangle,\ \langle p|(\bar{u}u + \bar{d}d - 2\bar{s}s)|p\rangle,\ \langle p|(\bar{u}u - \bar{d}d)|p\rangle$

needed to predict all matrix elements within the octet, eg

$\langle \Xi^-|\bar{s}d|\Sigma^-\rangle$

Away from symmetry point corrections $\delta m_l, \delta m_l^2, \delta m_l^3$. Have many more amplitudes than free parameters, so group analysis greatly constrains fits and extrapolations.

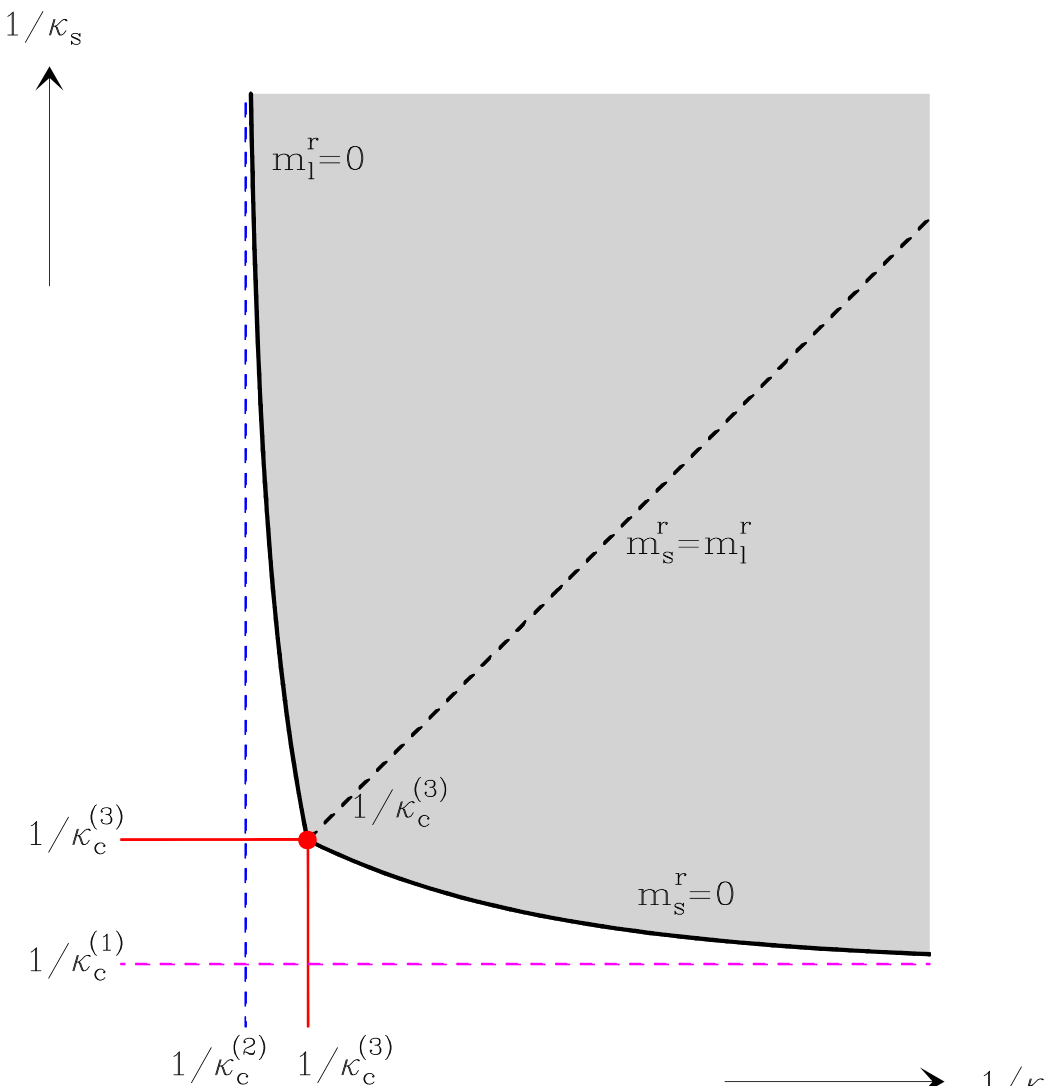
Extensions

- 1 + 1 + 1 spectrum, mixings, isospin splittings.
- $SU(4)$ — charmed spectrum.
- Matrix elements. Quark Mass corrections to predictions of unbroken $SU(3)$.
- Links to $2+1$ flavour chiral perturbation theory.

Conclusions

- Extrapolating from lattice simulations to the physical quark masses is made much easier by keeping $m_u + m_d + m_s$ constant.
- Flavour SU(3) analysis strongly constrains Taylor expansions in quark masses.
- Partial Quenching errors reduced if $\overline{m}$ constant.
- Apply the same idea to matrix elements and transition amplitudes.

Extra



Allowed Region