Solving the η-Problem

Inflection Point Inflation

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Introductions

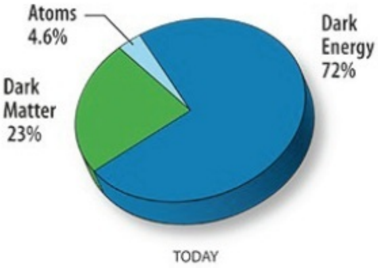
Introductory Materiel

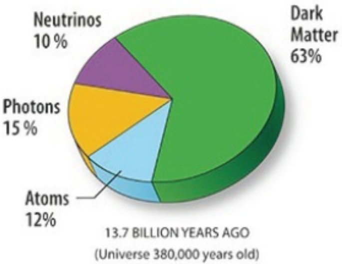
The work presented here is based on several papers:

1. Inflection point inflation within supersymmetry by K. Enqvist A. Mazumdar & PS JCAP 1006:020,2010, arXiv:1004.3724
2. Inflection point inflation: WMAP constraints and a solution to the fine-tuning problem S. Hotchkiss, A. Mazumdar & S. Nadathur arXiv:1101.6046
3. Super-Hubble Supergravity Inflation by A. Mazumdar, S. Nadathur, & PS. arXiv:1105.0430

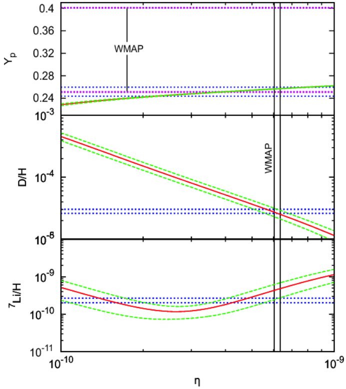
What do we need to explain?

The Matter content of the Universe is:

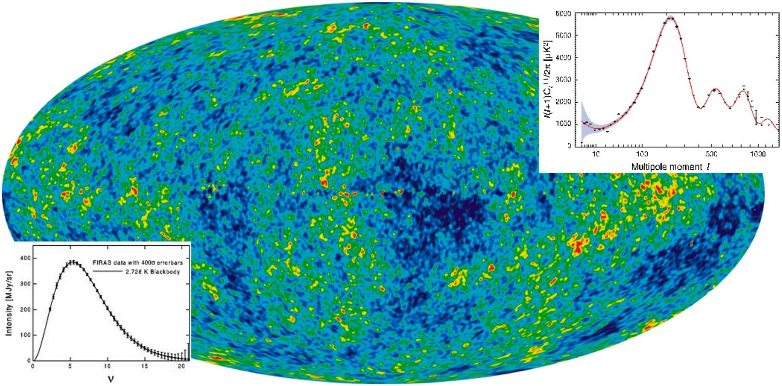




Of which the baryonic matter is explained by BBN:



And finally we have the CMB measurements:



The Standard Cosmology

The standard explanation of these observations comes in three parts:

- Inflation flattens the universe and embeds Quantum fluctuations into the CMB
- Reheating produces the matter and radiation content of the universe, this includes dark matter. This means that the inflaton decay must excite the SM degrees of freedom.
- The matter content eventually ends up as baryons and DM undergoing BBN, this puts many constraints on the Particle Content.

Inflation

For a homogeneous universe, the metric is given by

FRW-Metric

$$ds^2 = dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2[d\Omega_2]^2\right) \quad (1)$$

Via (the trace of) the Einstein Field Equations this yields an acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (2)$$

which is negative for most known forms of matter, however a (damped) scalar field has $\rho = -p = V(\phi)$ and causes the universe to accelerate.

slow-roll inflation

Under the assumption of slow roll, the friedmann equation is reduced to

$$H^2 \approx \frac{V(\phi)}{3M_{Pl}^2} \tag{3}$$

and the EQM is

$$3H\dot{\phi} \approx -\frac{dV(\phi)}{d\phi}. \tag{4}$$

We can define the "slow-roll paramaters" by

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta = M_{Pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right) \tag{5}$$

Inflationary Perturbations

A perturbation in the field (in fourier space) grows according to the perturbed wave equation:

$$(\delta\ddot{\phi}_k) + 3H(\delta\dot{\phi}_k) + \left(\frac{k}{a}\right)^2 \delta_k + \frac{1}{2}m^2\delta\phi_k = 0 \tag{6}$$

This equation shows already two important features- for slow rolling inflation the effective mass is small compared to the wave number of the perturbation inside the horizon. As the perturbation grows the wave number declines and the EQM are dominated by the inflaton mass, which is negligible. Hence the perturbations are frozen in.

From the perturbed equation of motion we can calculate the perturbation spectrum:

$$\mathcal{P}_R = \frac{1}{24\pi^2 M_{Pl}^4} \frac{V}{\epsilon} \tag{7}$$

and the spectral tilt,

$$n_s - 1 = -6\epsilon + 2\eta \tag{8}$$

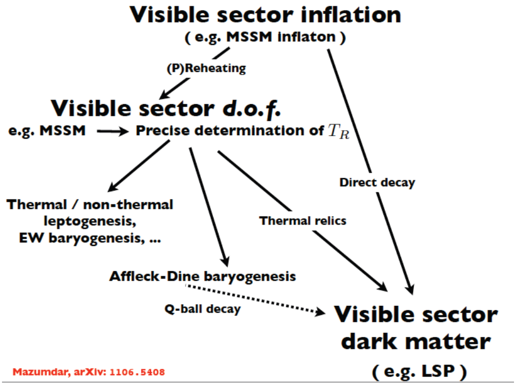
Reheating

In the simplest model the field oscillates about its minimum and decays into the SM+Dark matter. This puts strong constraints on the Particle Physics.

Constraints

- The vacuum state of the inflaton is the SM either before or after the EW phase transition.
- The couplings to DM are of the same order as the SM couplings.
- The DM decouples long before BBN.
- There is some baryogenesis.

The Supersymettry Project



Supersymettry offers a chance to put all the pieces together in a consistent framework.

Flat Directions

Directions where the gradient of the potential will (classically) vanish are generic in supersymettry. Consider a toy model with a superpotential:

$$W = S(\phi^2 - \phi_0^2) \qquad S = \frac{\sigma}{\sqrt{2}} \qquad \phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}} \tag{9}$$

which has a potential

$$V(\sigma, \phi) = \phi_0^4 + \phi_1^2(\sigma^2 - \phi_0^2) + \phi_2^2(\sigma^2 + \phi_0^2) + \frac{(\phi_1^2 + \phi_2^2)^2}{4} \tag{10}$$

and for $\sigma > \phi_0$ the field sits at $\phi_1 = \phi_2 = 0$ and is flat in the σ direction.

In practice flat directions are lifted, either by loop corrections or by soft supersymettry breaking terms.
In the toy model above, a loop correction will have the schematic form

$$\phi_0^4 \rightarrow \phi^4(1 + \alpha \ln(\frac{\sigma}{M})) \tag{11}$$

where alpha includes a guage coupling, and these have been used to produce both F-term and D-term inflation. We are more concerned with the soft supersymettry breaking terms. In general a flat direction will be lifted my some non renormalisable operator $W = \lambda\phi^n/nM^{n-3}$

Typical Flat Direction Potential

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 - A\frac{\lambda\phi^6}{6M_p^3} + \frac{\lambda^2\phi^{10}}{M_P^6} \tag{12}$$

With a little foresight I will choose the parameterisation

$$\frac{A^2}{40m_\phi^2} = 1 - 4\alpha^2 \tag{13}$$

and some very simple calculus later:

The fine-tuning problem

$$V'(\phi) = 4\alpha^2 m_\phi^2\phi_0 \qquad \text{hence} \qquad \epsilon \approx \frac{M^2}{\phi_0^2}\alpha^4 \tag{14}$$

The Slow-Roll Paramater is related to observation by:

Density Perturbations

$$P_R^{1/2} \approx 10^{-5} \approx \frac{H}{M\sqrt{\epsilon}} \tag{15}$$

The physical insight then, is that by lifting the potential, and increasing H, one can lower the fine tuning on ϵ.

An Example

Suppose that we choose the flat direction mass to be O(100GeV), then:

ϕ_0	V_0	α^2
$7.5 \times 10^{13} GeV$	$1.5 \times 10^{31} (GeV)^4$	10^{-22}
$7.5 \times 10^{13} GeV$	$10^{40} (GeV)^4$	3×10^{-9}

This is the maximum lift for mass O(100GeV), as for a lift greater than $m^2 M_{Plank}^2$ the flat direction dynamics are irrelevant.

Embedding

In practice, one can calculate explicitly the Non Renormalizable operators that lift the flat directions in the MSSM. In particular we require that the lowest dimension operator should not contain any other fields.
LLe and uud are the only viable candidates.
However, inflation in supersymettry tends to be broken due to the η problem. That is the fact that supergravity predicts O(H) corrections to the potential via the Kahler potential.

Potentials

The corrections can be calculated from the familiar equation:

Supergravity

$$V = e^{\frac{K}{M^2}} \left[(D_\Phi W) K^{\Phi\Phi} (D_{\bar{\Phi}} \bar{W}) - \frac{3}{M^2} |W|^2 \right] \tag{16}$$

We can understand the η problem easily, by supposing we had some potential which generates inflation in global supersymettry, then we can write

$$V = e^{\frac{K}{M^2}} \left[H_{flat}^2 M_{Pl}^2 \left(1 + f(\frac{\phi}{M}) \right) \right] \tag{17}$$

which for a minimal kahler potential $K = |\phi|^2$ generates a mass term of size H.

A repeat performance

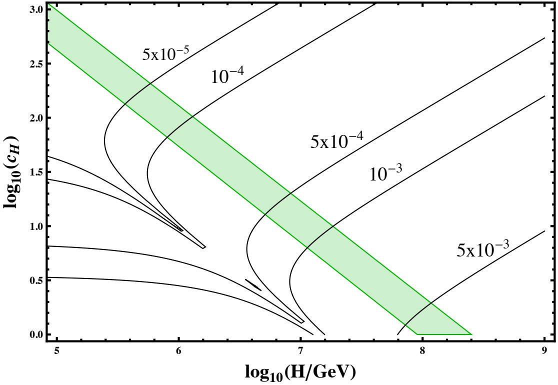
Just as the NR operators cancelled the mass contributions to the flat direction to produce a flat potential, so an appropriate choice of kahler potential can generate an inflection point.
Phemomenologically we are looking for a potential of the form:

Potential

$$V = 3H^2M^2 + \frac{c_H}{2}H^2\phi^2 - \frac{a_H\lambda H}{nM^{n-3}}\phi^n + \lambda^2\frac{\phi^{2n-2}}{M^{2(n-3)}} \quad (18)$$

Numerics

We can investigate this phenomenological potential numerically:



Building Blocks

The right question

In what sense is the Phenomenological potential generic?

- The first term just represents some energy density.
- The mass term is from the exponential of the Kahler potential, and any kahler potential term containing the minmal kahler potential will have mass terms.
- The final term just comes from the non-renormalisable operator which is assumed to lift the flat directions in a high energy limit.

The A-Term

The A-term is built out of cross terms between the superpotential field with the vacuum energy, and the flat direction, multiplied by the cross terms of the Kahler potential. We can make the following general statements:

- If a term linear in H exists, it dominates over all higher order terms which are suppressed by $\frac{H}{M} \lll 1$.
- If a term linear in H exists then it is accompanied by a phase angle, and by judicious choice I can give it a negative sign.
- Simple examples with these properties do exist.

Beyond these statements you just have to work them out (tedious I know!).

An example

$K = K_{min} + \frac{b}{2M_{Pl}}(\bar{I}\phi\phi + \bar{\phi}\bar{\phi}I)$ and $W = M_I^2 I + \frac{\lambda}{6}\frac{\phi^6}{M_{Pl}^3}$

$$V = 3H^2 M_{Pl}^2 + 3(1+b^2)H^2\phi^2 - b2\sqrt{3}\lambda H\cos(\theta)\frac{\phi^n}{M_{Pl}^{n-3}} + \lambda^2\frac{\phi^{2n-2}}{M_{Pl}^{2n-6}} \quad (19)$$

and our condition for a Suitable POI is:

$$\frac{a_H^2}{8(n-1)c_H} = 1 + O(\text{finetuning}) \quad (20)$$

and for $n = 6$ yields $b \approx 1.118$ where we can read off the tuning from our graph. Note that fixing the form of c_H and a_H has taken away one degree of freedom and given us a point on our graph. In general there are many interaction terms that will add a correction to the mass term without disturbing the A-Term.

A few subtlies

The toy model above has a slight issue, in that inflation doesnt actually come to an end properly - it transitions from slow roll to fast roll. This is more an issue with the toy model than with the theory - Our previous toy model had a flat direction only for $\sigma > \phi_0$, we simply assume there is some such transition shortly after slow roll ends.
The field I is assumed to be small throughout, this might not be trivial to achieve in practice, as it could end up oscilating or reheating the universe.

Conclusion

To recap:

- An inflection point can be lifted to relax the fine tuning.
- Supergravity corrections can generate inflationary dynamics on a flat direction via a source term where the source is virtually any vacuum energy density.
- This requires an appropriate choice of Kahler potential, with fine tuning of about $10^{-6} - 10^{-8}$.
- Highlight: This bypasses the eta problem entirely as we no longer require the supergravity corrections to be small.