NLO predictions for W W + jets

Giulia Zanderighi

Oxford Theoretical Physics & STFC

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Present status of QCD

✓ Thanks to LEP, Hera, and Tevatron QCD today *firmly established*

✓ Despite temporary discrepancies, theory successful in describing experimental data, currently *no major area of discrepancy* spanning energies from few MeV to few TeV

✗ However, the LHC brings a *new frontier in energy and luminosity*. Both at Tevatron and LHC we are seeing now a number of "excesses"

✗ Premier goals of the LHC

- ☛ discovery of the Higgs and New Physics
- ☛ identification of New Physics (requires precision measurements)

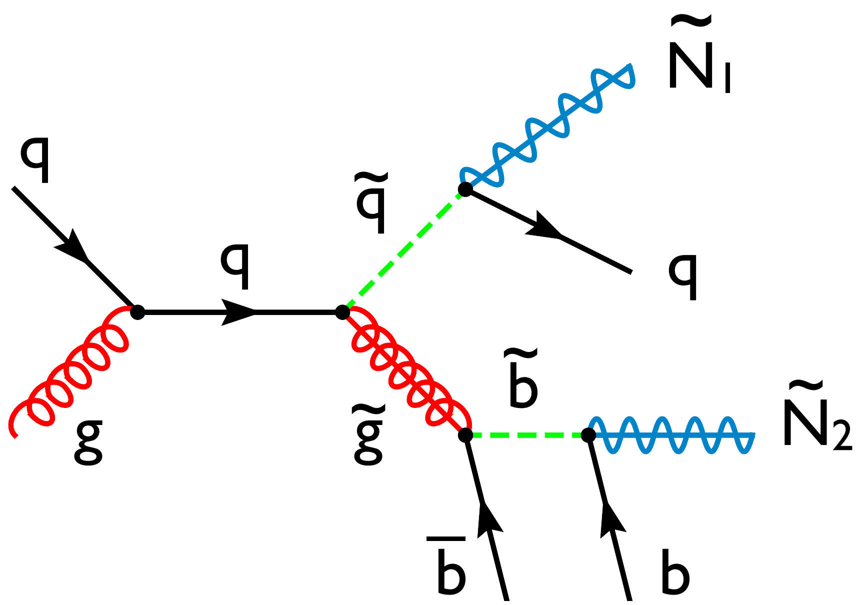
Solid understanding of backgrounds and relevant QCD corrections mandatory for interpretation of possible excesses

Multiparticle final states

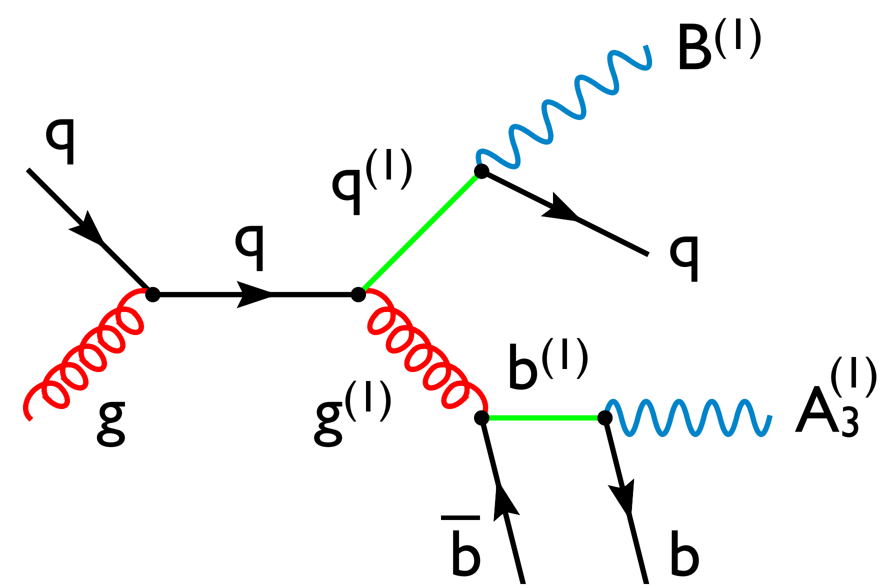
LHC's new regime in energy and luminosity implies that we will have a very large number of high-multiplicity events

- typical SM process is accompanied by radiation multi-jet events
- most signals involve pair-production and subsequent chain decays

SUSY:



UED:



More important than ever to describe high-multiplicity final states

Leading order

Status: fully automated, edge around outgoing 8 particles

Alpgen, CompHEP, CalcHEP, Helac, Madgraph, Helas, Sherpa, Whizard, ...

$\Rightarrow$ amazing progress in the last years [before only parton shower]

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Drawbacks of LO:

large scale dependences, sensitivity to cuts, poor modeling of jets, ...

Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by ±10% via change of Q ⇒ cross-section varies by ±40%

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When and why LO:

- always the fastest option, often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but *no precision*

Next-to-leading order

Benefits of next-to-leading order (NLO)

- reduce dependence on unphysical scales

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- through loop effects get indirect information about sectors not directly accessible

NLO: current status

- ☑ 2 → 2: all known (or easy) in SM and beyond
- ☑ 2 → 3: essentially all known today in the SM
- ☐ 2 → 4: *the frontier*

NLO cross sections available for a number of processes at LHC

✓ tt + bb [Bredenstein et al. '08; Bevilacqua et al. '09]

✓ W/Z + 3 jets [Berger et al. '09; (W) Ellis et al. '09]

✓ tt + 2 jets [Bevilacqua et al. '10]

✓ WW + bb [Denner et al. '08; Bevilacqua et al. '09]

✓ W^+W^+ + 2jets [Melia et al. '10]

✓ W^+W^- + 2jets [Melia et al. '10]

- ☐ 2 → 5: *the next frontier*

✓ dominant corrections to W + 4 jets [Berger et al. '09]

NLO: traditional approach

Numerical approaches:

- draw all possible Feynman diagrams (use automated tools)
- write one-loop amplitudes as $\sum$ (coefficients × tensor integrals)
- automated reduction of tensor integrals to scalar (known) ones

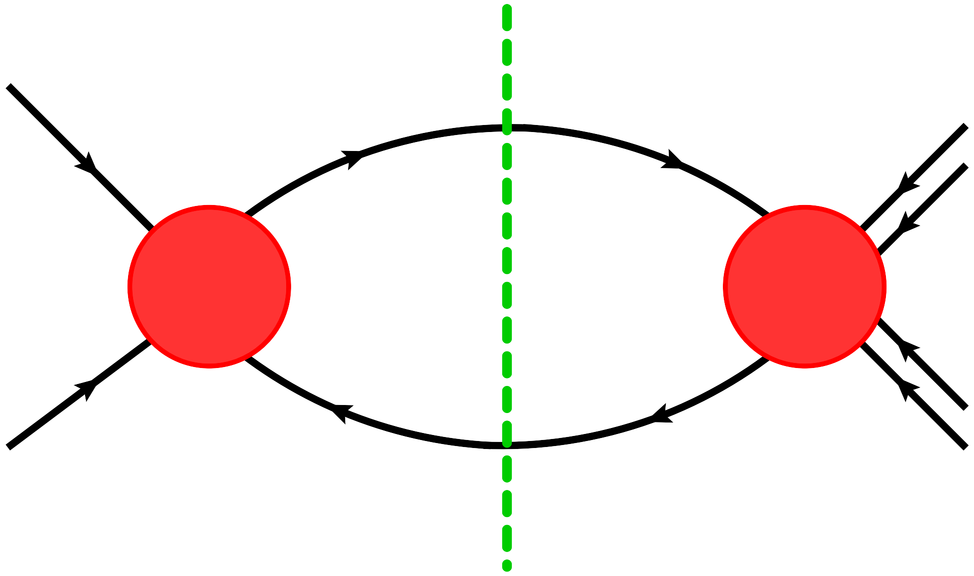
Problem solved in principle, but brute force approaches plagued by worse than factorial growth $\Rightarrow$ difficult to push methods beyond N=6 because of too high demand on computer power [+ issue of numerical instabilities]

Anastasiou, Andersen, Binoth, Ciccolini, Denner, Dittmaier, Ellis, Giele, Glover, Guffanti, Guillet, Heinrich, Karg, Kauer, Lazopoulos, Melnikov, Nagy, Pilon, Reiter, Roth, Passarino, Petriello, Sanguinetti, Schubert, Smillie, Soper, Uwer, Wieders, GZ ....

NLO without integration

<u>Unitarity in it's original form:</u>

use four-dimensional double cuts of amplitudes to classify the coefficients of discontinuities associated with physical invariants



$$T^\dagger - T = -2iT^\dagger T$$

$$\mathrm{Im}\, T^{1-\mathrm{loop}} = \sum_j c_j \, \mathrm{Im}\, I_j$$

[Landau '50s]

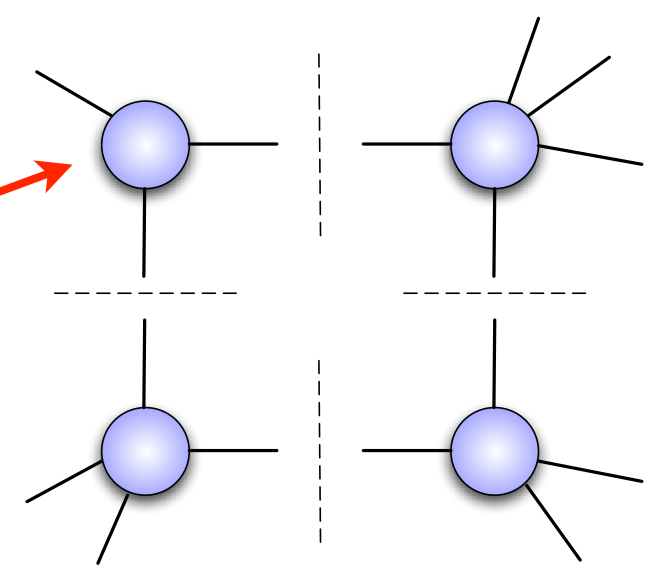
Framework applied to amplitudes in $\mathcal{N} = 1$ and $\mathcal{N} = 4$ SUSY Yang-Mills theories (no rational part) and to 5- and V+4- parton amplitudes

Clever tricks, but no full computational method, so impact limited

[Bern, Dixon, Dunbar, Kosower '94]

Breakthrough ideas

Enlightening idea that by considering quadruple cuts, one completely freeze the integration and one can extract coefficients of box integrals



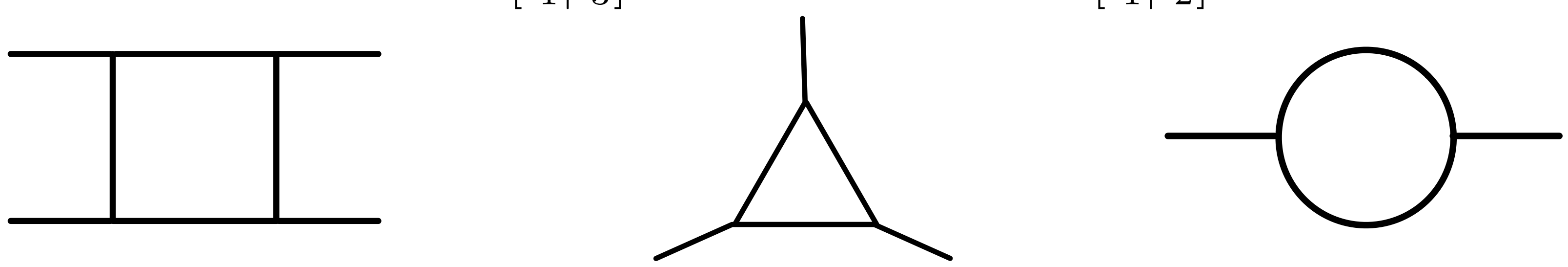
NB: non-zero because cut gives complex momenta

[Britto, Cachazo, Feng ’04]

Breakthrough ideas

Pure algebraic method to extract integral coefficients by making specific choices for the loop momentum and solving a system of equations. At the beginning method applied to each individual Feynman diagram.

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



[Ossola, Pittau, Papadopolous (OPP) '06]

<u>NB</u>: master integrals all known

't Hooft, Veltman '79; Bern, Dixon, Kosower '93, Duplancic, Nizic '02; Ellis, GZ '08 with public code QCDLoop [http://www.qcdloop.fnal.gov]

Generalized unitarity

I will briefly explain the method and remind of the main ideas behind it.
Second part of the seminar will concentrate on applications & recent results

References:

- Ellis, Giele, Kunszt '07	*[Unitarity in D=4]*
- Giele, Kunszt, Melnikov '08	*[Unitarity in D≠4]*
- Giele & GZ '08	*[All one-loop N-gluon amplitudes]*
- Ellis, Giele, Melnikov, Kunszt '08	*[Massive fermions, ttggg amplitudes]*
- Ellis, Giele, Melnikov, Kunszt, GZ '08	*[W+5p one-loop amplitudes]*
- Melia, Melnikov, Rontsch & GZ '10-'11	*[W^+W^+ +2 jets, W^+W^- + 2jets]*
- Melia, Nason, Rontsch & GZ '11	*[W^+W^+ +2 jets + Parton Shower]*

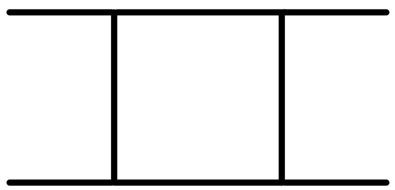
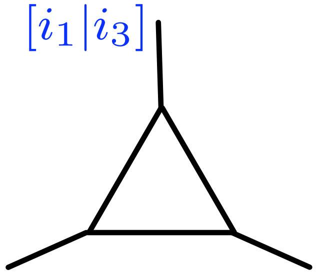
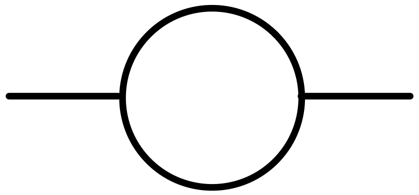
These papers heavily rely on previous work

- Bern, Dixon, Kosower '94	*[Unitarity, oneloop from trees]*
- Ossola, Pittau, Papadopoulos '06	*[OPP]*
- Britto, Cachazo, Feng '04	*[Generalized cuts]*
- *[....]*	

Decomposition of the one-loop amplitude

Suppose you did do a brute-force calculation, your result would read

$$\mathcal{A}_N^D = \sum_{[i_1|i_4]} d^D_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^D_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^D_{i_1 i_2} I^{(D)}_{i_1 i_2} \ ^*$$

* if non-vanishing masses: tadpole term; notation: $[i_1|i_m] = 1 \leq i_1 < i_2 \ldots < i_m \leq N$

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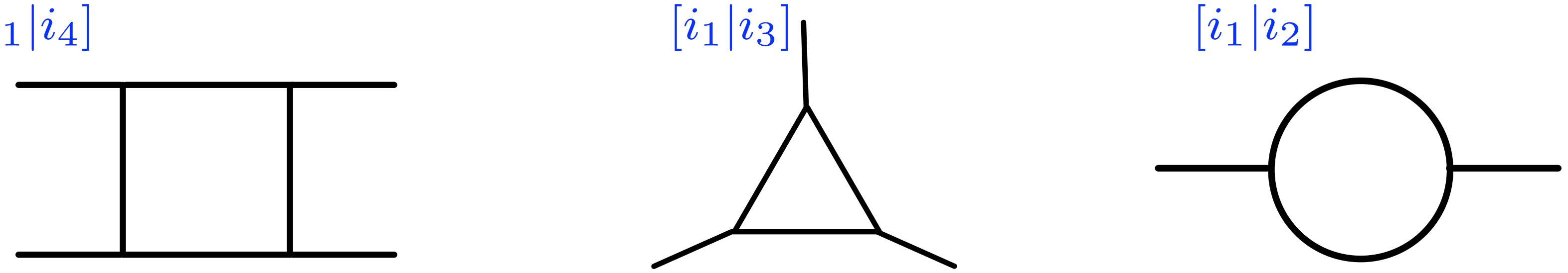

Remarks:

- higher point function can be reduced to boxes + vanishing terms
- coefficients depend in general on D (i.e. on ε)
- the above decomposition exists no matter how you compute $\mathcal{A}$
- box, triangles and bubble integrals all known analytically

['t Hooft & Veltman '79; Bern, Dixon Kosower '93, Duplancic & Nizic '02; Ellis & GZ '08 ⇒ http://www.qcdloop.fnal.gov]

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Cut-constructible and the rational part

When the coefficients are evaluated in D=4 one obtains the so-called cut-contructible part of the amplitude

($O(\varepsilon)$ contributions of the coefficients) × (poles of the integrals) give rise to the so called rational part of the amplitude

Focus on cut-constructible part for the moment

☛ *the amplitude is known if the coefficients are known*

Cut-constructible and the rational part

Start from

$$\mathcal{A}_N^{\text{cut}} = \sum_{[i_1|i_4]} d_{i_1i_2i_3i_4} I^{(D)}_{i_1i_2i_3i_4} + \sum_{[i_1|i_3]} c_{i_1i_2i_3} I^{(D)}_{i_1i_2i_3} + \sum_{[i_1|i_2]} b_{i_1i_2} I^{(D)}_{i_1i_2} = \int \frac{d^Dl}{i(\pi)^{D/2}} \mathcal{A}_N^{\text{cut}}(l)$$

with

$$I^D_{i_1\cdots i_M} = \int \frac{d^Dl}{i(\pi)^{D/2}} \frac{1}{d_{i_1}\cdots d_{i_M}}$$

Focus on the integrand

$$\mathcal{A}_N^{\text{cut}}(l) = \sum_{[i_1|i_4]} \frac{\bar{d}_{i_1i_2i_3i_4}}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}} + \sum_{[i_1|i_3]} \frac{\bar{c}_{i_1i_2i_3}}{d_{i_1}d_{i_2}d_{i_3}} + \sum_{[i_1|i_1]} \frac{\bar{b}_{i_1i_2}}{d_{i_1}d_{i_2}}$$

Get cut numerators by taking residues: i.e. set inverse propagator = 0
In D=4 up to 4 constraints on the loop momentum (4 onshell propagators)
⇒ get up to box integrals coefficients

Integral coefficients

E.g. for a box coefficient, find the solution to

$$d_i(l_{ijkl}) = d_j(l_{ijkl}) = d_k(l_{ijkl}) = d_l(l_{ijkl})$$

then

$$\bar{d}_{ijkl}(l_{ijkl}) = \mathrm{Res}\left(\mathcal{A}_N(l)\right) = \left(d_i(l_{ijkl})d_j(l_{ijkl})d_k(l_{ijkl})d_l(l_{ijkl})\mathcal{A}_N(l)\right)|_{l=l_{ijkl}}$$

For lower point coefficients same procedure, but need to subtract higher-point contributions

$$\bar{c}_{ijk}(l) = \mathrm{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

$$\bar{b}_{ij}(l) = \mathrm{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

$$\bar{a}_i(l) = \mathrm{Res}_i\left(\mathcal{A}_N(l) - \sum_{j\neq i}\frac{\bar{b}_{ij}(l)}{d_i d_j} - \frac{1}{2!}\sum_{j,k\neq i}\frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{3!}\sum_{j,k,l\neq i}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

Construction of the box residue

Decompose loop momentum as

$$l^\mu = V_4^\mu + \alpha_1 \, n_1^\mu$$

V_4: constructed using 3 external vectors $\Rightarrow$ physical space

n_l: spans orthogonal space $\Rightarrow$ trivial space

α_l: determined so as to fulfill the unitarity conditions

Explicitly: find two complex solutions

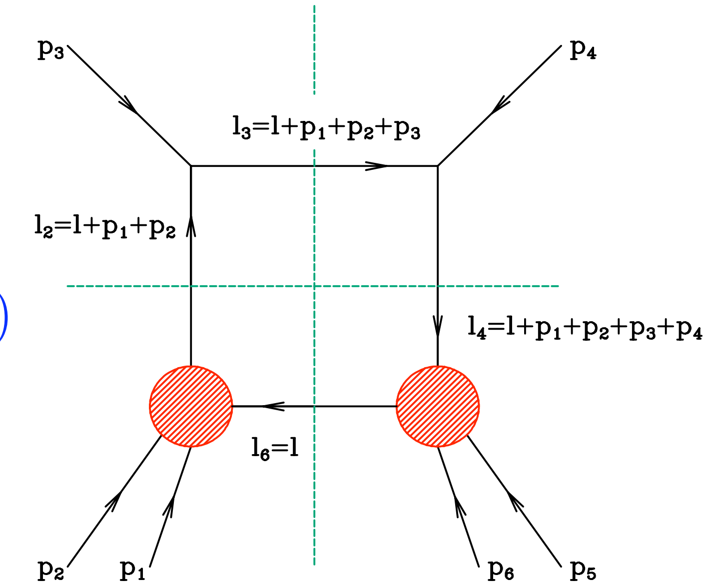
$$l_\pm^\mu = V_4^\mu \pm i \sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

Definition of V_4: Ellis, Giele, Kunszt 0708.2398

Construction of the box residue

Four cut propagators are onshell
⇒ the amplitude factorizes into 4 tree-level amplitudes

$$
\begin{aligned}
\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^\pm)\Big) &= \mathcal{M}^{(0)}(l_i^\pm; p_{i+1}, \ldots, p_j; -l_j^\pm) \times \mathcal{M}^{(0)}(l_j^\pm; p_{j+1}, \ldots, p_k; -l_k^\pm) \\
&\times \mathcal{M}^{(0)}(l_k^\pm; p_{k+1}, \ldots, p_l; -l_l^\pm) \times \mathcal{M}^{(0)}(l_l^\pm; p_{l+1}, \ldots, p_i; -l_i^\pm)
\end{aligned}
$$



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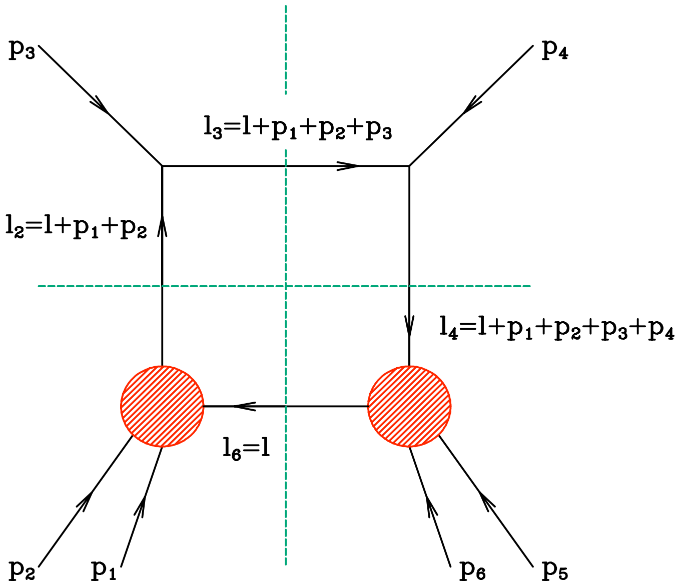


<u>Remarks:</u>

- implicit sum over two helicity states of the four cut gluons
- tree-level three-gluon amplitudes are non-zero because the cut gluons have complex momenta

Construction of the box residue

Residual dependence on loop momentum enters only through component in the trivial space

$$\overline{d}_{ijkl}(l) \equiv \overline{d}_{ijkl}(n_1 \cdot l)$$

Use

$$(n_1 \cdot l)^2 \sim n_1^2 = 1$$

Then the maximum rank is one and the most general form is

$$\overline{d}_{ijkl}(l) = d^{(0)}_{ijkl} + d^{(1)}_{ijkl}\, l \cdot n_1$$

Using the two solutions of the unitarity constraint one obtains

$$\begin{aligned} d^{(0)}_{ijkl} &= \frac{\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) + \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2} \\ d^{(1)}_{ijkl} &= \frac{\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) - \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2i\sqrt{V_4^2 - m_l^2}} \end{aligned}$$

Construction of the triangle residue

Decompose loop momentum as

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

V_3: constructed using 2 external vectors $\Rightarrow$ physical space

n_1, n_2: span orthogonal space $\Rightarrow$ trivial space

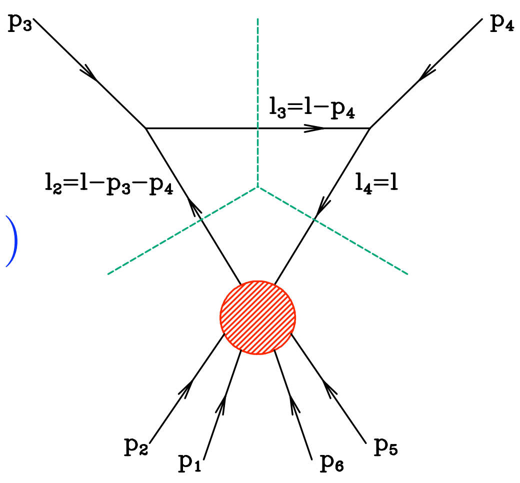
α_1, α_2: determined so as to fulfill the unitarity conditions

Explicitly: find an infinite number of solutions

$$l^\mu_{\alpha_1 \alpha_2} = V_3^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu\,, \quad \forall \alpha_1, \alpha_2 \quad \text{with} \quad \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

Construction of the triangle residue

Three cut propagators are onshell
$\Rightarrow$ the amplitude factorizes into 3 tree-level amplitudes



$$\mathrm{Res}_{ijk}\Big(\mathcal{A}_N(l^{\alpha_1\alpha_2})\Big) = \mathcal{M}^{(0)}(l_i^{\alpha_1\alpha_2}; p_{i+1}, \ldots, p_j; -l_j^{\alpha_1\alpha_2}) \times \mathcal{M}^{(0)}(l_j^{\alpha_1\alpha_2}; p_{j+1}, \ldots, p_k; -l_k^{\alpha_1\alpha_2})$$
$$\times\ \mathcal{M}^{(0)}(l_k^{\alpha_1\alpha_2}; p_{k+1}, \ldots, p_i; -l_i^{\alpha_1\alpha_2})$$

The maximum rank is three, taking into account all constraints the most general form is

$$\overline{c}_{ijk}(l) = c_{ijk}^{(0)} + c_{ijk}^{(1)} s_1 + c_{ijk}^{(2)} s_2 + c_{ijk}^{(3)}(s_1^2 - s_2^2) + s_1 s_2 (c_{ijk}^{(4)} + c_{ijk}^{(5)} s_1 + c_{ijk}^{(6)} s_2)\,, \qquad s_i \equiv (l \cdot n_i)$$

Make 7 choices of α_1, α_2 and find all 7 coefficients

For bubble and tadpole coefficients proceed in the same way.

Final result: cut-constructible part

Spurious terms integrate to zero

$$\int [d\,l]\ \frac{\bar{d}_{ijk}(l)}{d_i d_j d_k d_l} = d^{(0)}_{ijkl} \int [d\,l]\ \frac{1}{d_i d_j d_k d_l} = d_{ijkl} I_{ijkl}$$

$$\int [d\,l]\ \frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} = c^{(0)}_{ijk} \int [d\,l]\ \frac{1}{d_i d_j d_k} = c_{ijk} I_{ijk}$$

$$\int [d\,l]\ \frac{\bar{b}_{ij}(l)}{d_i d_j} = b^{(0)}_{ijk} \int [d\,l]\ \frac{1}{d_i d_j} = b_{ij} I_{ij}$$

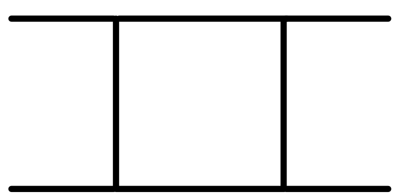
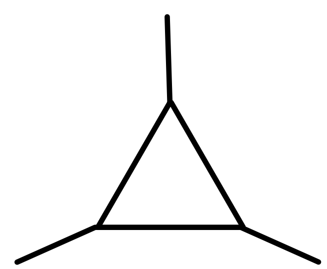
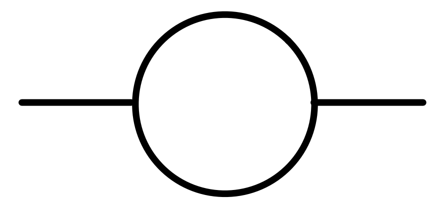
The final result for the cut constructible part then reads

$$\boxed{\mathcal{A}_N^{\rm cut} = \sum_{[i_1|i_4]} d^{(0)}_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} I^{(D)}_{i_1 i_2}}$$

One-loop virtual amplitudes

Cut constructible part can be obtained by taking residues in D=4

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$

Rational part: can be obtained with D ≠ 4

Generic D dependence

Two sources of D dependence

dimensionality of loop momentum D

# of spin eigenstates/ polarization states D_s

Keep D and D_s distinct

$$\mathcal{A}^{D} \Rightarrow \mathcal{A}^{(D,D_s)}$$

Two key observations

1. External particles in D=4 ⇒ no preferred direction in the extra space

$$\mathcal{N}(l) = \mathcal{N}(l_4, \tilde{l}^2) \qquad \tilde{l}^2 = -\sum_{i=5}^{D} l_i^2$$

$\mathcal{N}$: numerator function

☛ in arbitrary D up to 5 constraints ⇒ get up to pentagon integrals

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2. Dependence of $\mathcal{N}$ on D_s is linear (or two-parameter form)

$$\mathcal{N}^{D_s}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

☛ evaluate at any D_{s1}, D_{s2} ⇒ get $\mathcal{N}_0$ and $\mathcal{N}_1$, i.e. , full $\mathcal{N}$

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☛ evaluate at any D_{s1}, D_{s2} ⇒ get $\mathcal{N}_0$ and $\mathcal{N}_1$, i.e. , full $\mathcal{N}$

Choose D_{s1}, D_{s2} integer ⇒ suitable for numerical implementation

[D_s = 4 - 2ε ‘t-Hooft-Veltman scheme, D_s = 4 FDH scheme]

In practice

- Start from

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

- Use unitarity constraints to determine the coefficients, computed as products of tree-level amplitudes with complex momenta in higher dimensions

- Berends-Giele recursion relations are natural candidates to compute tree level amplitudes: they are very fast for large N and very general (spin, masses, complex momenta)

Berends, Giele '88

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Berends, Giele '88

☺ Generalized unitarity: very simple, efficient, general, transparent method, straightforward to implement/automate

Final result

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$
$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2} d^{(2)}_{i_1 i_2 i_3 i_4} \; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4} d^{(4)}_{i_1 i_2 i_3 i_4} \; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$
$$+ \sum_{[i_1|i_3]} \left(c^{(0)}_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} - \frac{D-4}{2} c^{(9)}_{i_1 i_2 i_3} \; I^{(D+2)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b^{(0)}_{i_1 i_2} \; I^{(D)}_{i_1 i_2} - \frac{D-4}{2} b^{(9)}_{i_1 i_2} \; I^{(D+2)}_{i_1 i_2} \right)$$

Cut-constructible part:

$$\mathcal{A}^{CC}_N = \sum_{[i_1|i_4]} d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \; I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \; I^{(4-2\epsilon)}_{i_1 i_2}$$

Rational part:

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m^2_{i_1} + m^2_{i_2}}{2} \right) b^{(9)}_{i_1 i_2}$$

Vanishing contributions: $\mathcal{A} = \mathcal{O}(\epsilon)$

Scalar integrals $I^{(D)}_{i_1 i_2 \ldots}$ all known

‘t Hooft & Veltman ‘79; Bern, Dixon Kosower ’93, Duplancic & Nizic ’02;
Ellis & GZ ’08, public code ⇒ http://www.qcdloop.fnal.gov

The F90 Rocket program

Rocket science!

Eruca sativa = Rocket = roquette = arugula = rucola

Recursive unitarity calculation of one-loop amplitudes

<u>So far computed one-loop amplitudes:</u>

- ✓ N-gluons
- ✓ qq + N-gluons
- ✓ qq + W + N-gluons
- ✓ qq + QQ + W
- ✓ tt + N-gluons [Melnikov,Schulze]
- ✓ tt + qq + N-gluons [Melnikov,Schulze]
- ✓ qq WW + N g
- ✓ qq WW qq + 1 g

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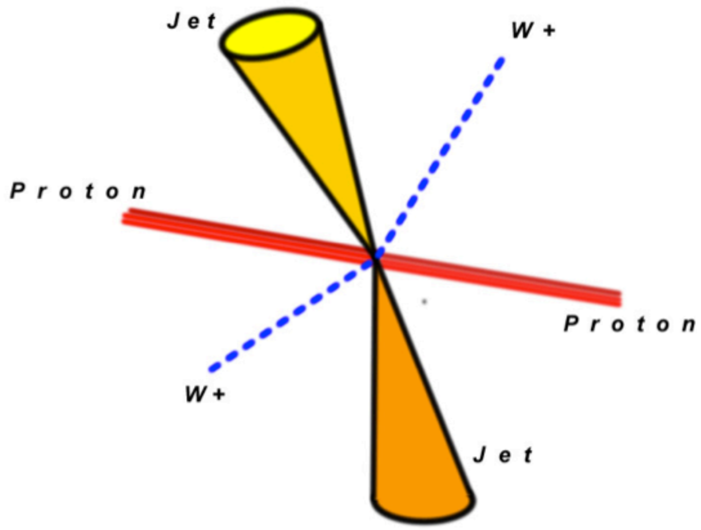
In perspective, for gluons:
N = 6 $\Rightarrow$ 10860 diags.
N = 7 $\Rightarrow$ 168925 diags.
Computed up to N=20

W^+W^+ plus dijets



Melia, Melnikov, Rontsch, GZ '11

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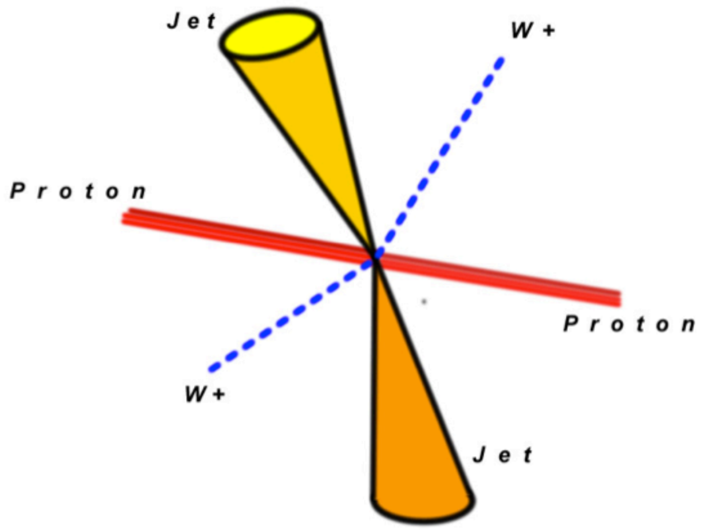

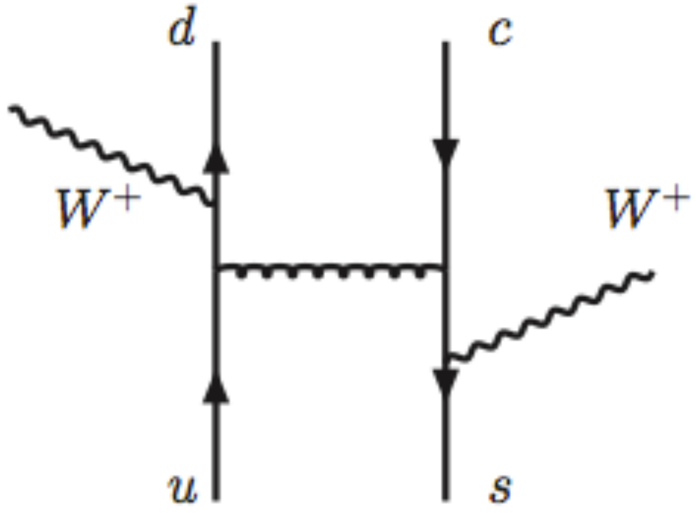

W^+W^+ production is a peculiar SM process

Melia, Melnikov, Rontsch, GZ '11

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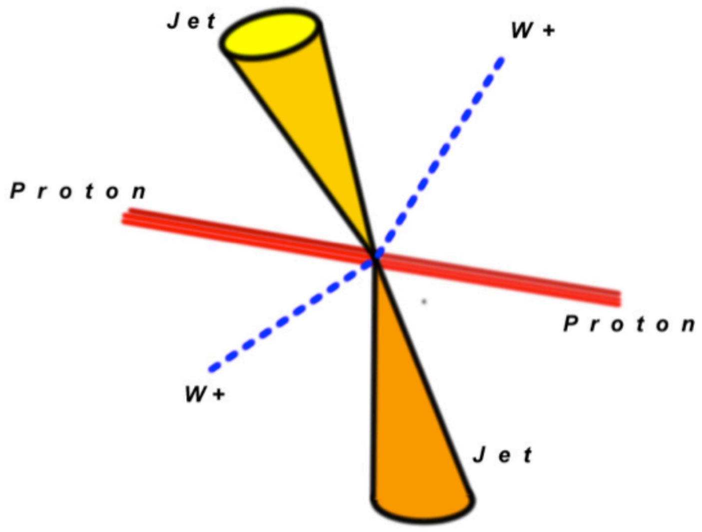

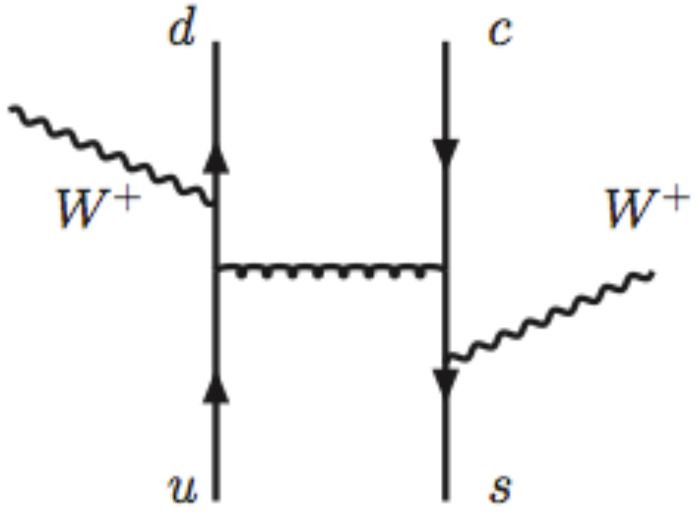

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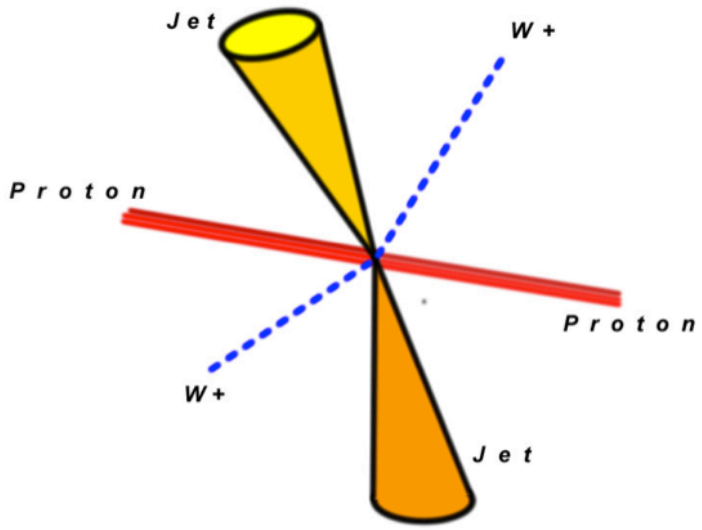

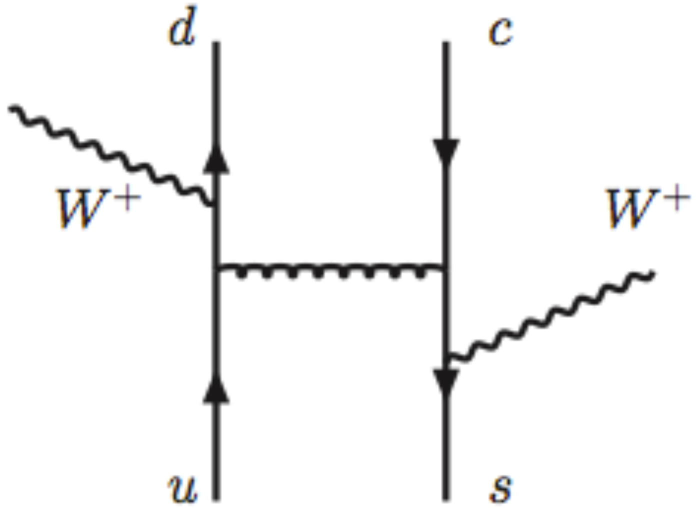

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Melia, Melnikov, Rontsch, GZ '11

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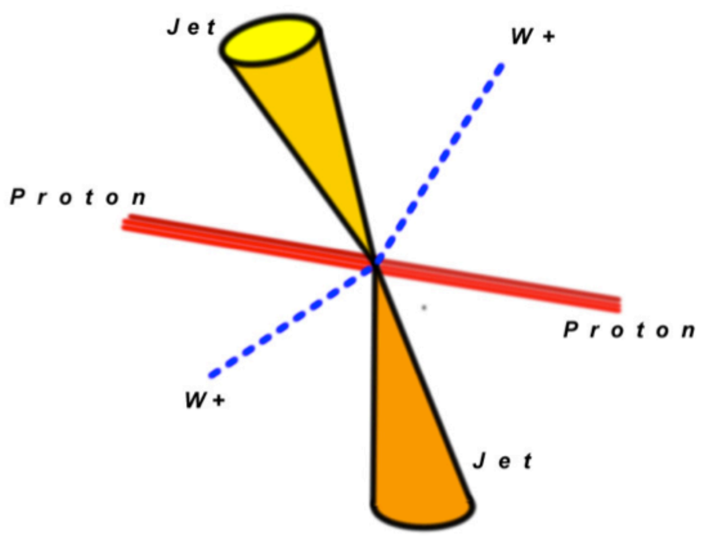

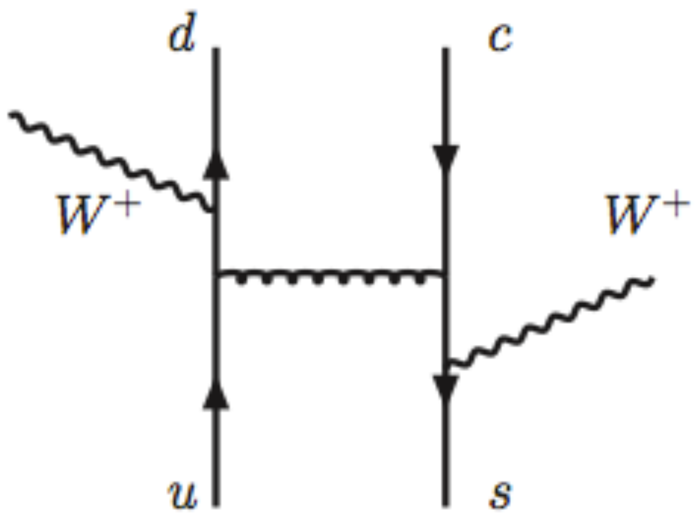

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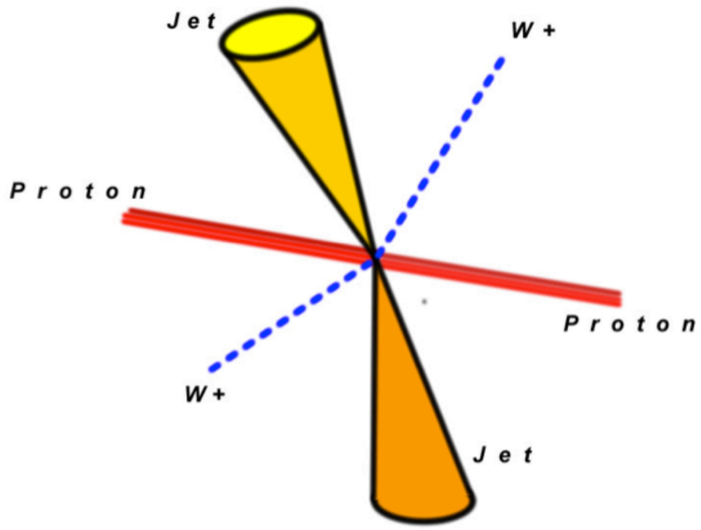

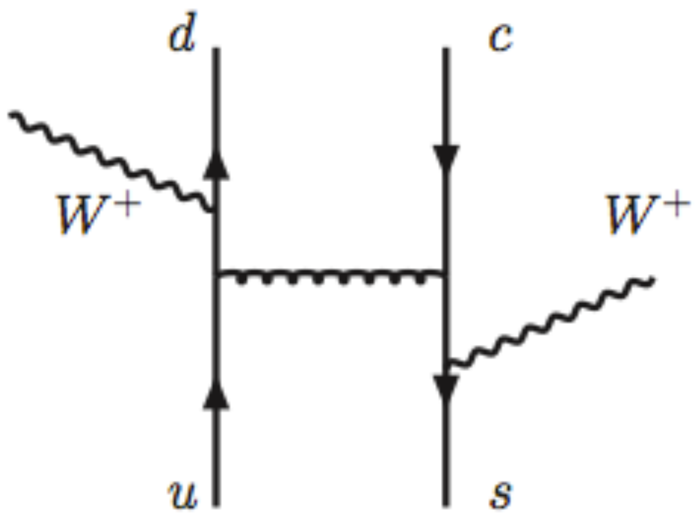

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Melia, Melnikov, Rontsch, GZ '11

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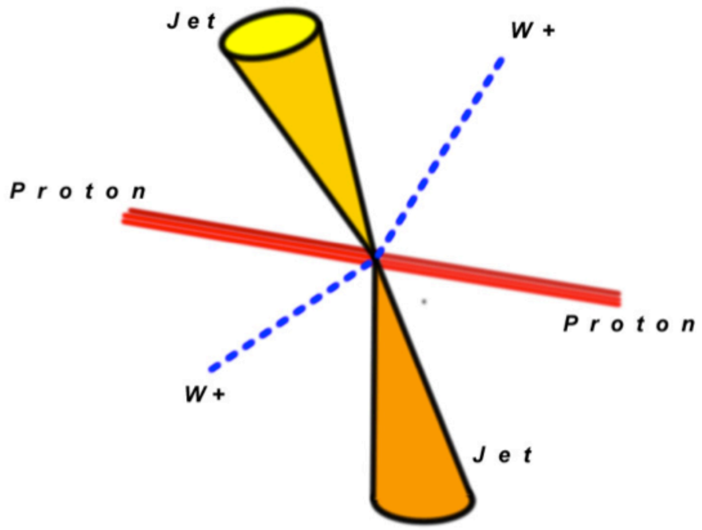

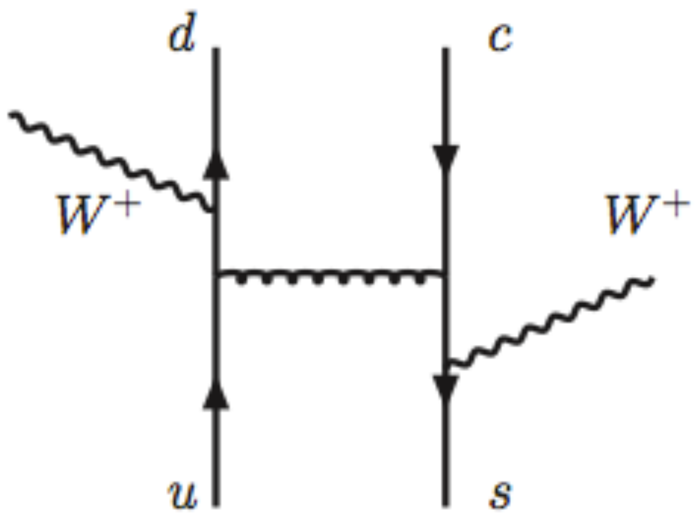

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[background to double parton scattering, models with doubly charged Higgs, di-quark production, R-parity violating smoun production . . .]

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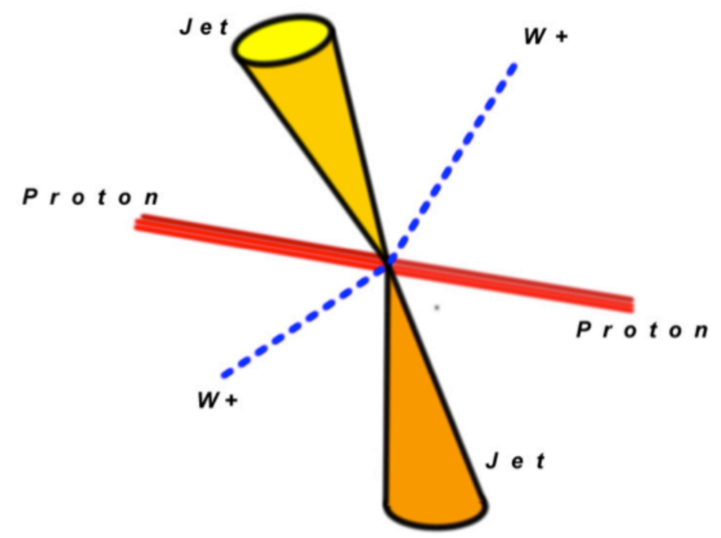

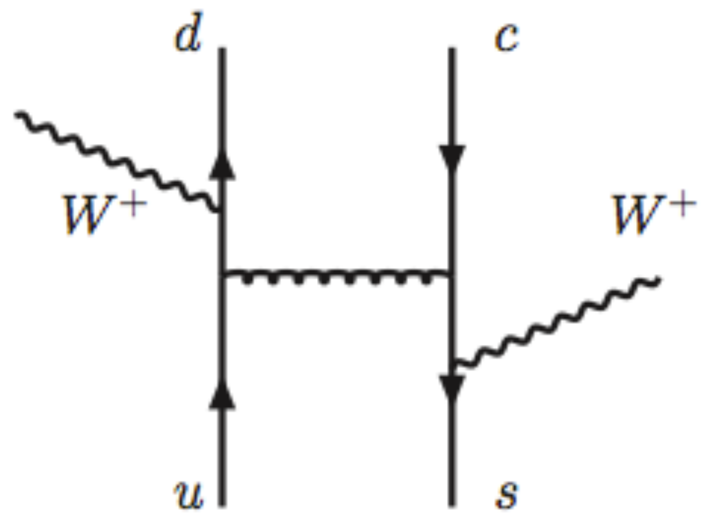

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- the cross-section is around 6 fb at 14 TeV (3 fb at 7 TeV)

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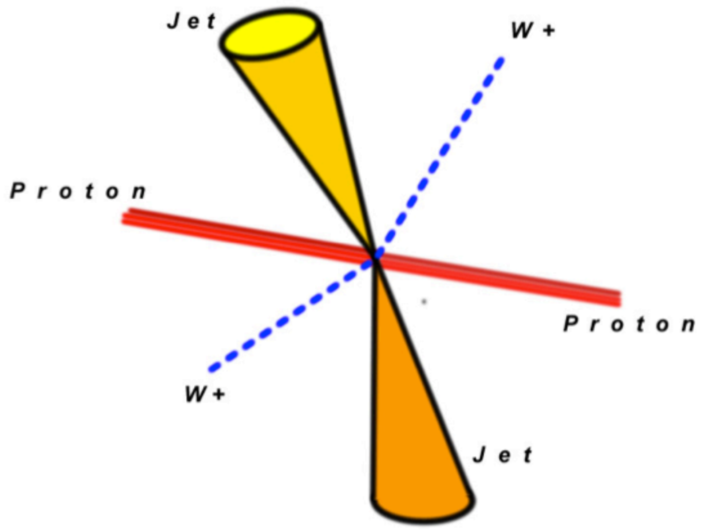

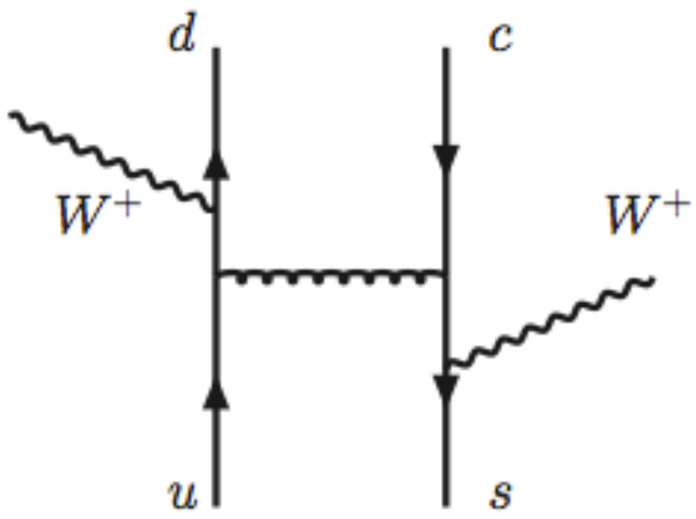

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- the cross-section is around 6 fb at 14 TeV (3 fb at 7 TeV)
- W^-W^- + dijet is roughly 40% the size

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Setup

<u>Cuts and input parameters</u>

- pp collision at 14 TeV with decay to $e^+\mu^+$ (full $l^+l^+ \sim$ twice as large)
- jets reconstructed using anti-k_T with R = 0.4
- use MTSW08LO, $\alpha_s(M_Z)$= 0.139, and MSTW08NLO, $\alpha_s(M_Z)$ = 0.120
- EW input

  M_W = 80.419 GeV, Γ_W = 2.141 GeV α_{QED} = 1/128.802 $\sin^2\theta_W$ = 0.222
- Cuts:

  $p_{T,l}$ > 20 GeV, $|\eta_l| < 2.4$, $p_{t,miss}$ > 30 GeV, no jet cut

- Real + virtual implemented in the MCFM parton level integrator

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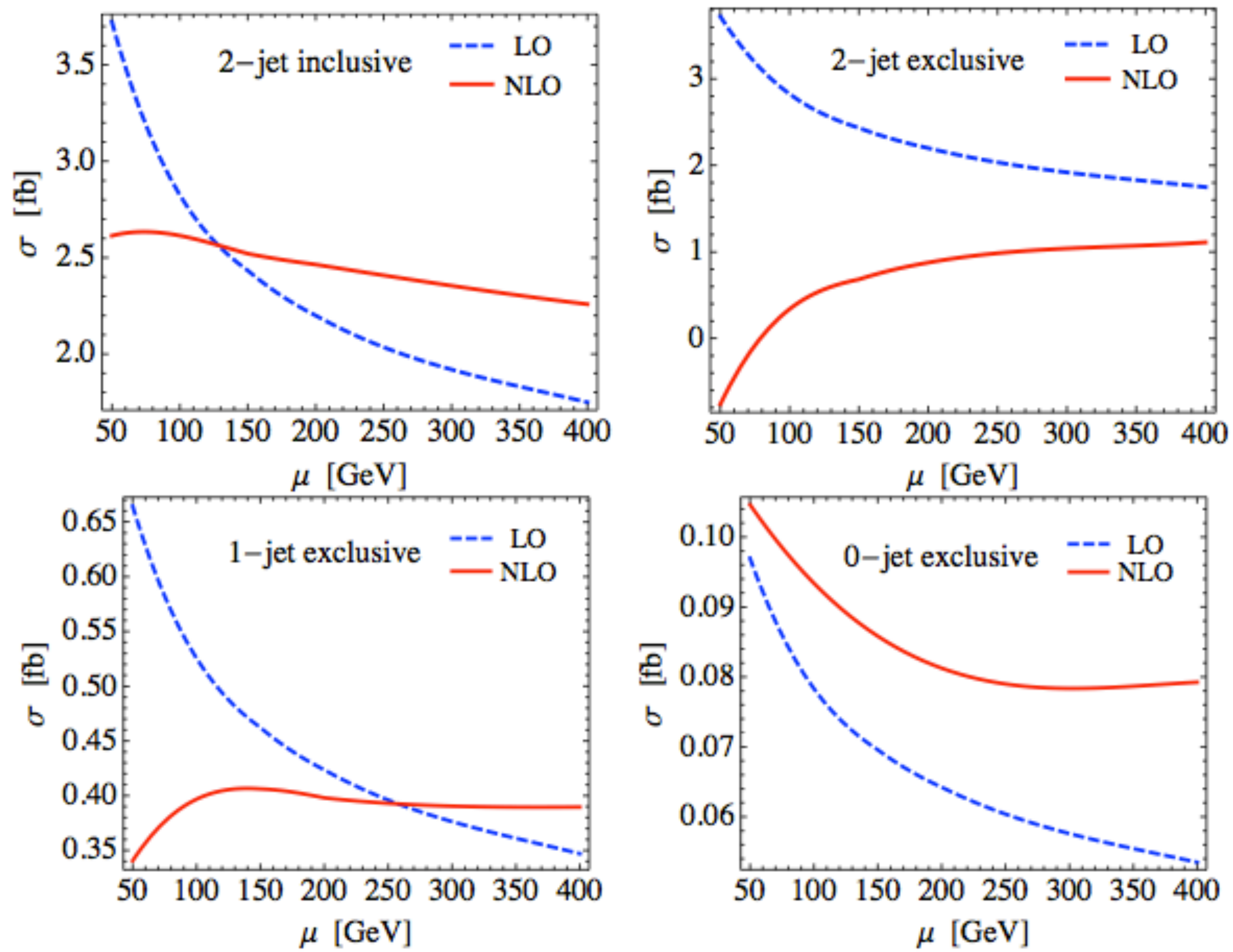
Campbell, Ellis

Inclusive and exclusive cross-sections



$$\mu = \mu_R = \mu_F$$

$$\sigma^{\rm LO} = 2.7 \pm 1.0\,{\rm fb}$$

$$\sigma^{\rm NLO} = 2.4 \pm 0.2\,{\rm fb}$$

~ 60 l^+l^- events for 10 fb^{-1}

- scale dependence reduced significantly at NLO
- 2-jet inclusive cross-section considerably larger than 2-jet exclusive
  $\Rightarrow$ most events have a relatively hard third jet

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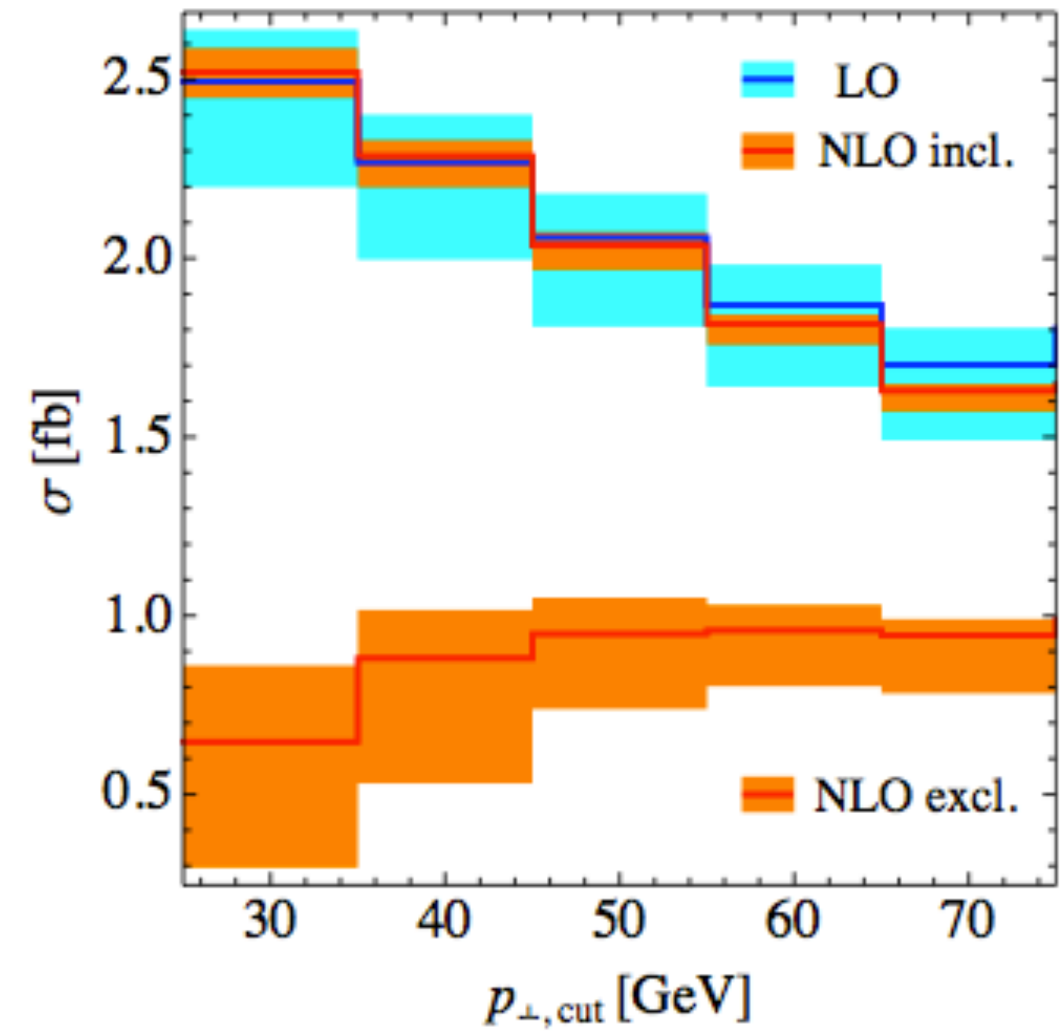

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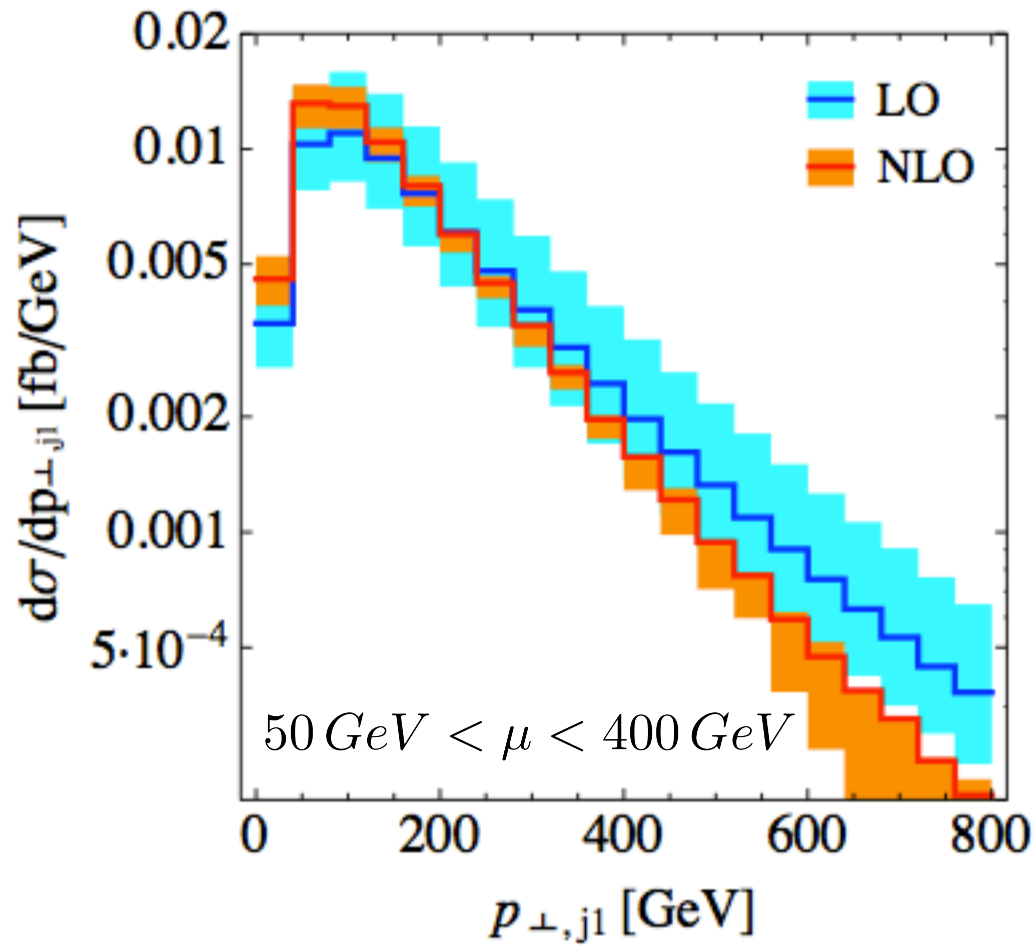
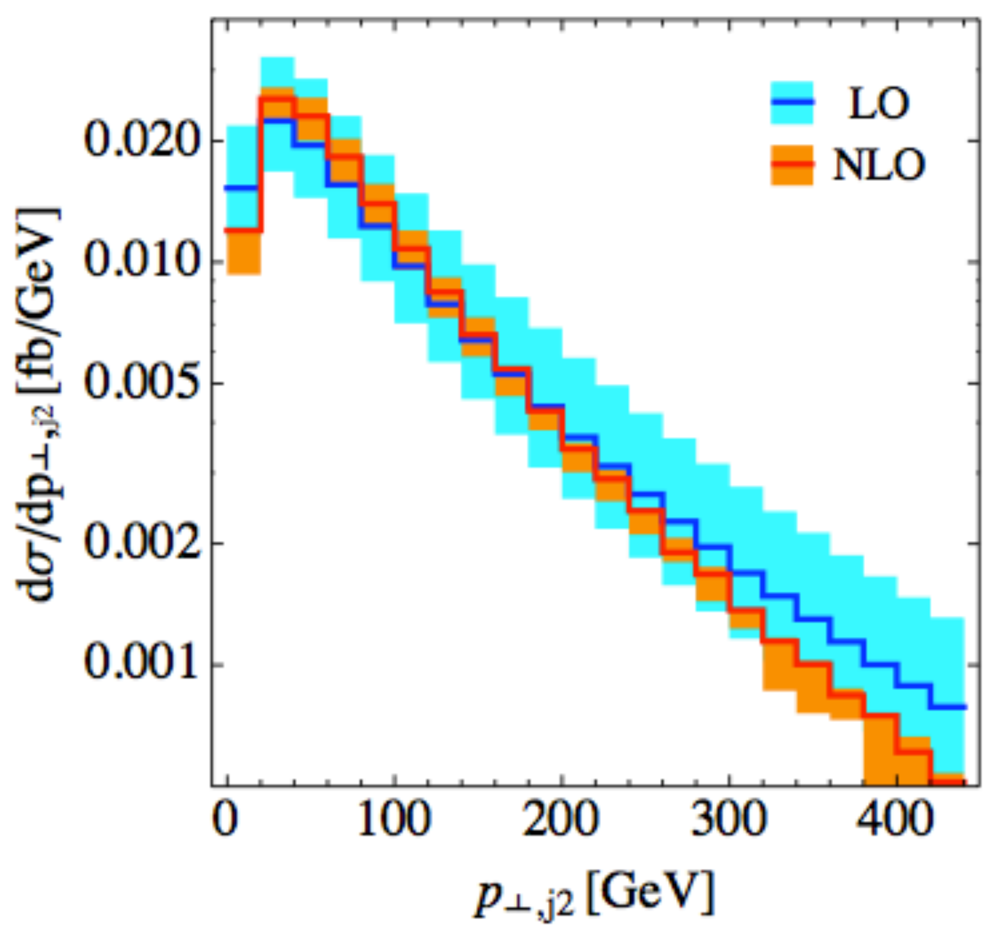
Kinematic distributions





- scale dependence reduced significantly at NLO
- LO result overshoot at high p_T. Characteristic effect of using a fixed rather than a dynamical scale in the LO calculation

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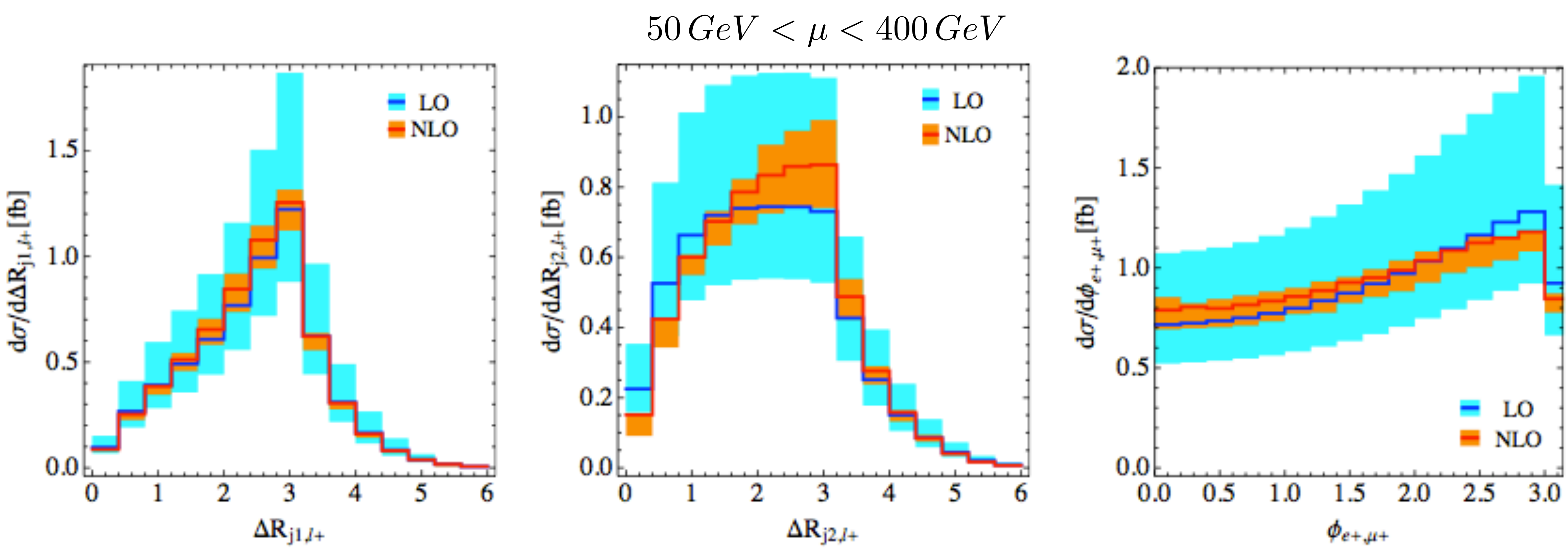
Kinematic distributions



- broad angular distribution between jet and lepton, peaked at $\Delta R = 3$. NLO enhances the peak slightly.
- leptons prefer to be back to back (less so at NLO)
- in double parton scattering lepton directions are uncorrelated -- cut on ϕ_{ll} could reduce the background

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NLO + Parton Shower

- NLO describes the effect of at most one additional parton in the final state. Quite far from realistic LHC events that involve a large number of particles in the final state
- NLO accurate for inclusive observables, but not so much for exclusive ones, sensitive to the complex structure of LHC events
- recently the QCD production of W^+W^+ calculation was implemented in the POWHEG-BOX, this allow to maintain NLO accuracy while generating exclusive, realistic events
- the code is publicly available http://powheg-box.mib.infn.it
- this is the first $2 \rightarrow 4$ scattering process to be know at this accuracy

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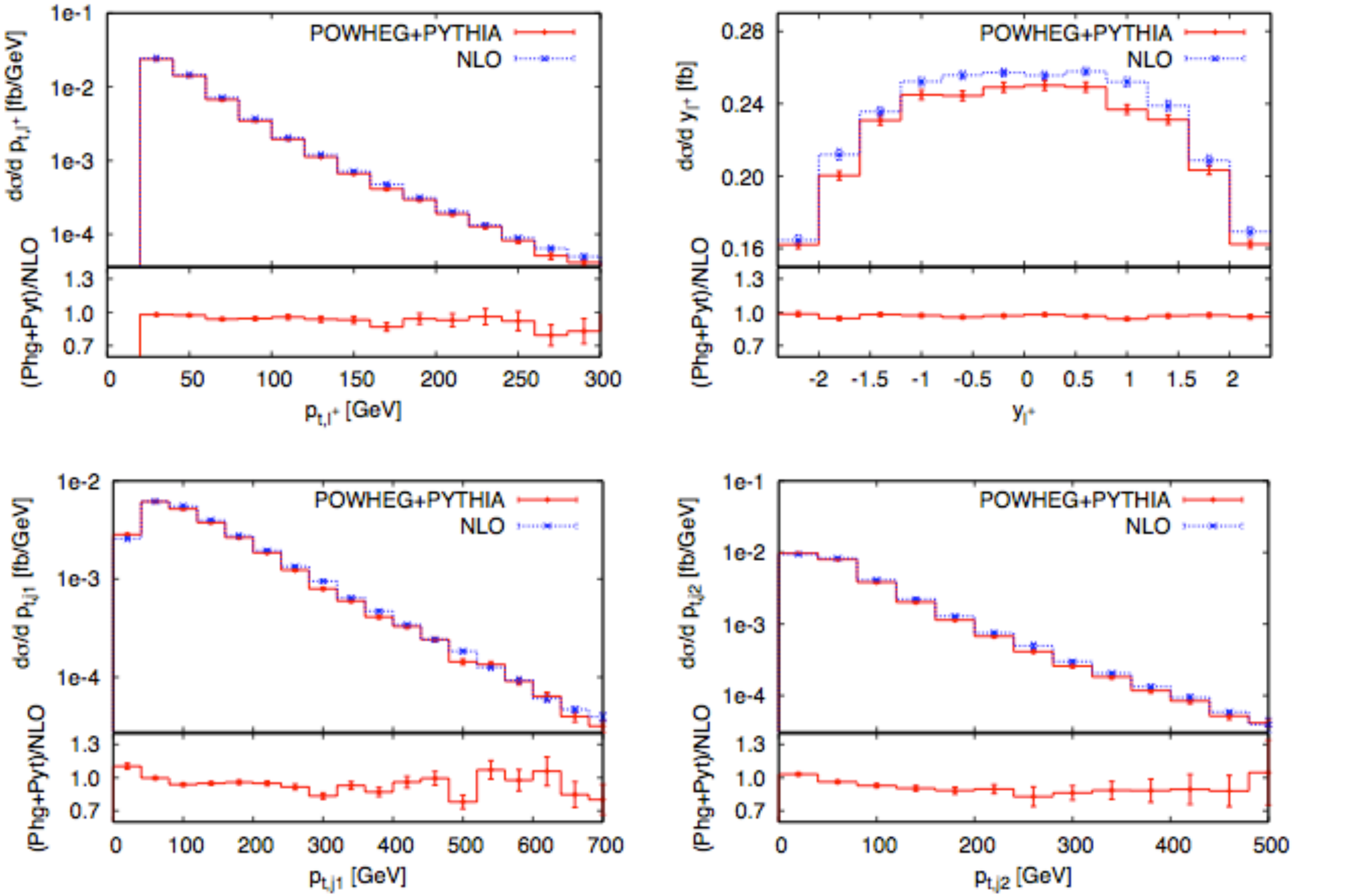
NLO vs NLO+PS: inclusive distributions



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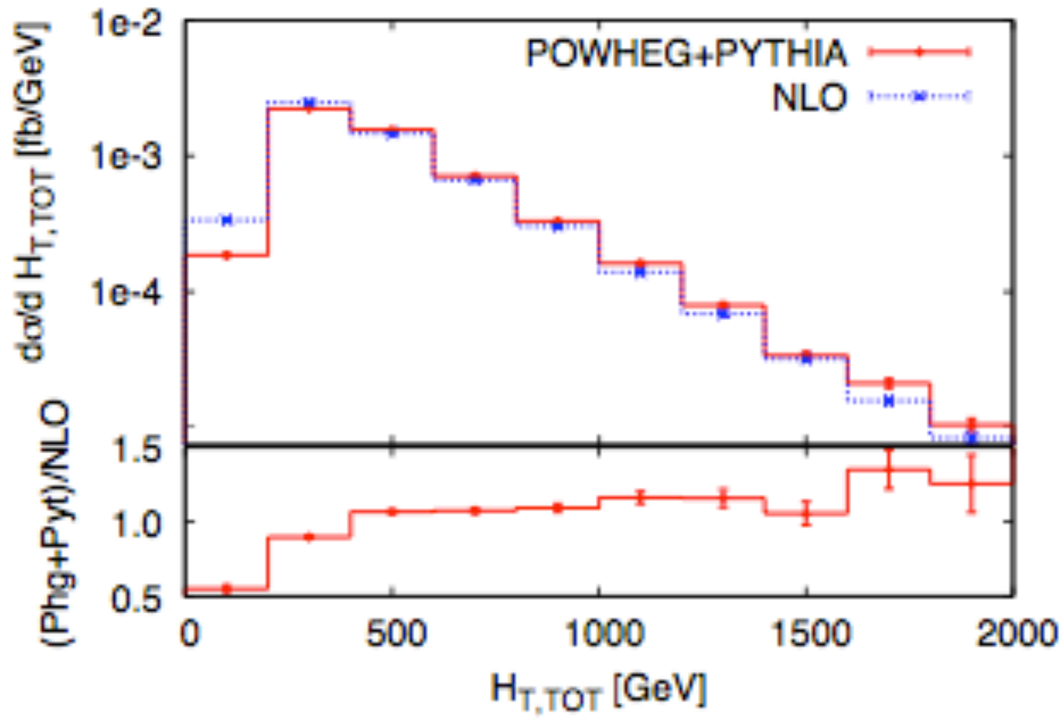
NLO vs NLO+PS: $H_{T,TOT}$



$$H_{\mathrm{T,TOT}} = p_{\mathrm{t,e^+}} + p_{\mathrm{t,\mu^+}} + p_{\mathrm{t,miss}} + \sum_j p_{\mathrm{t,j}}$$

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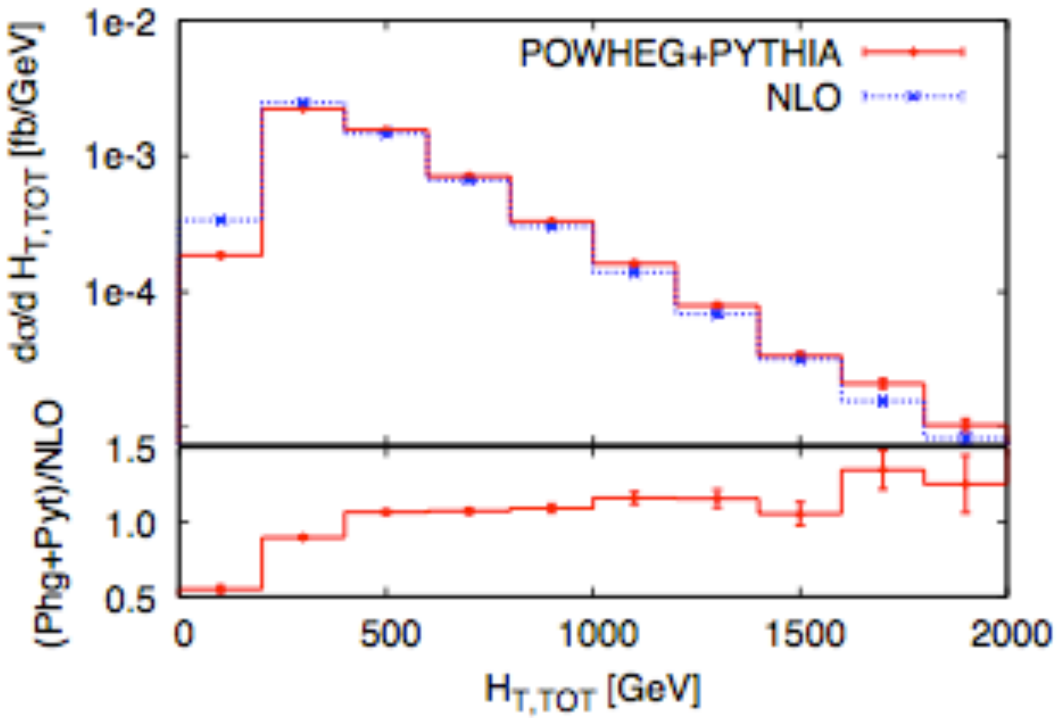

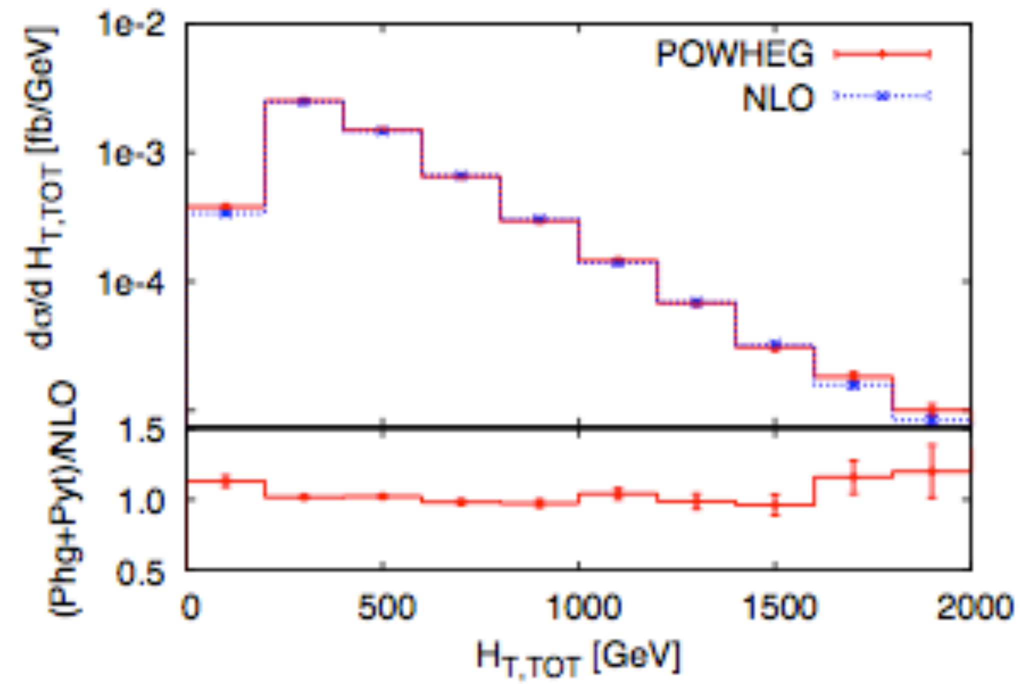

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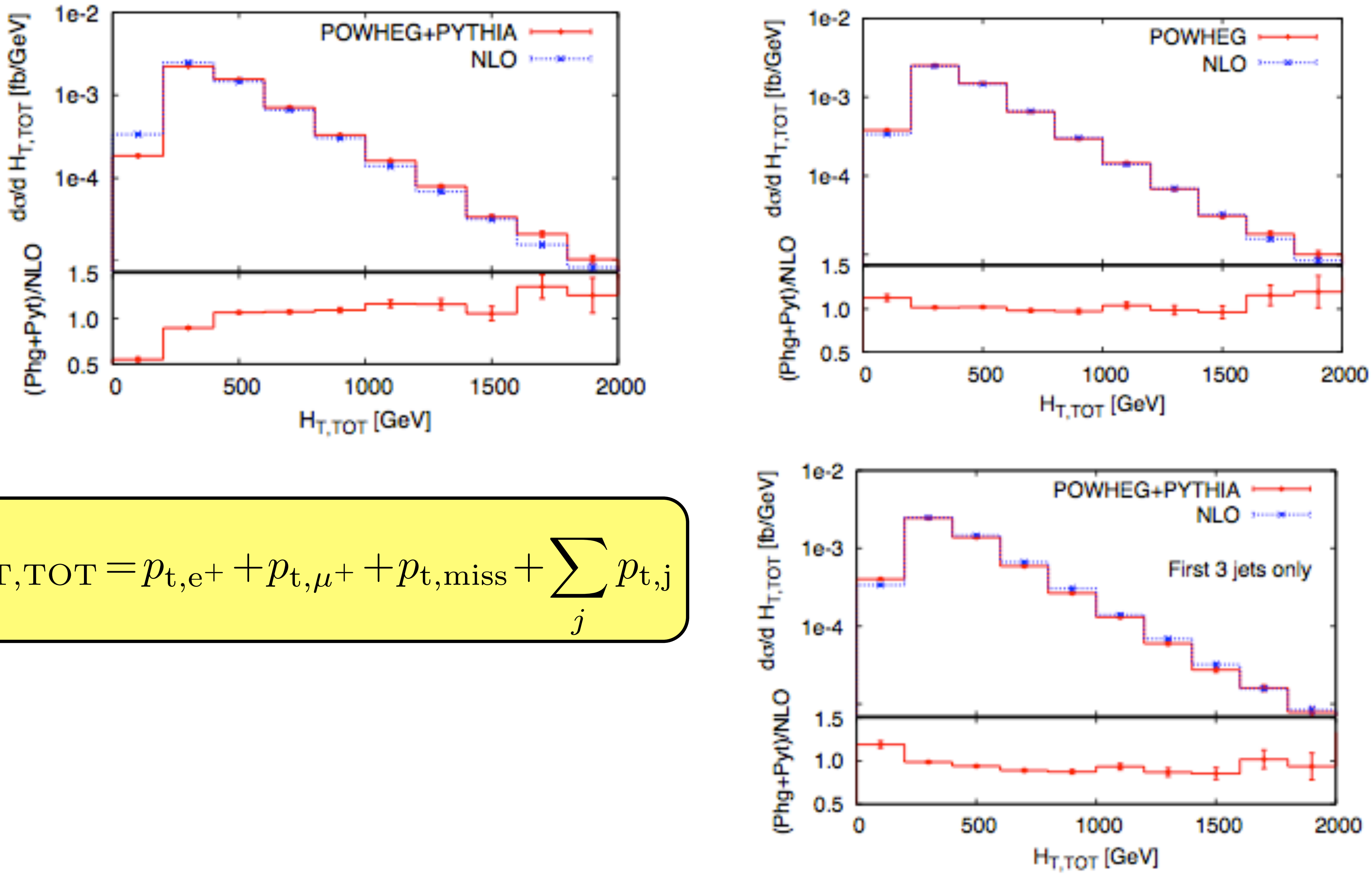

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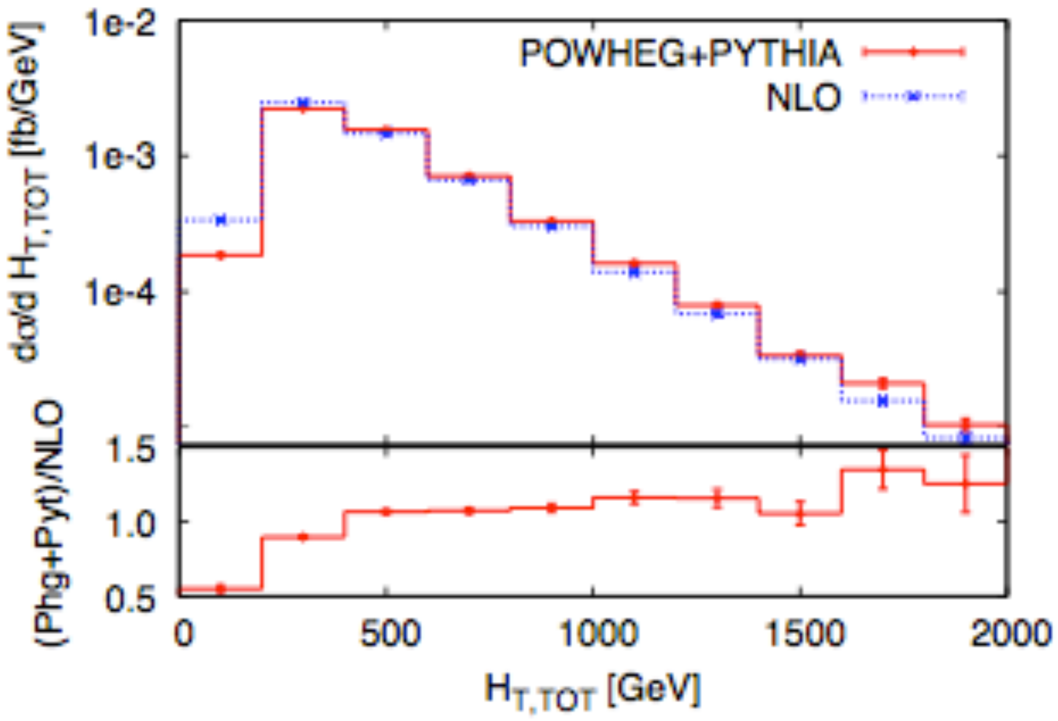

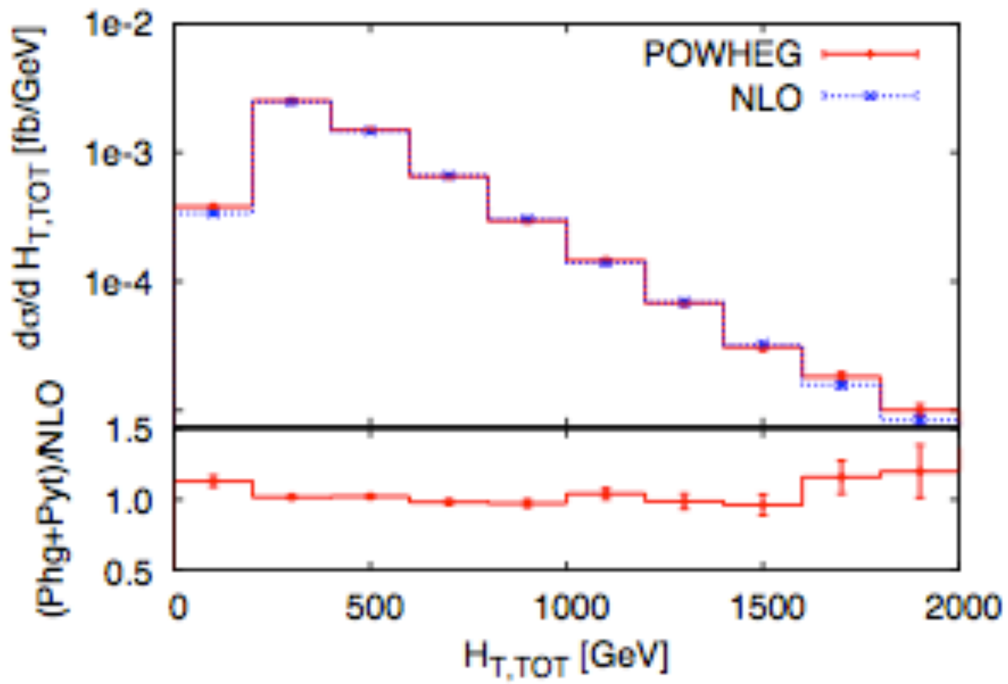

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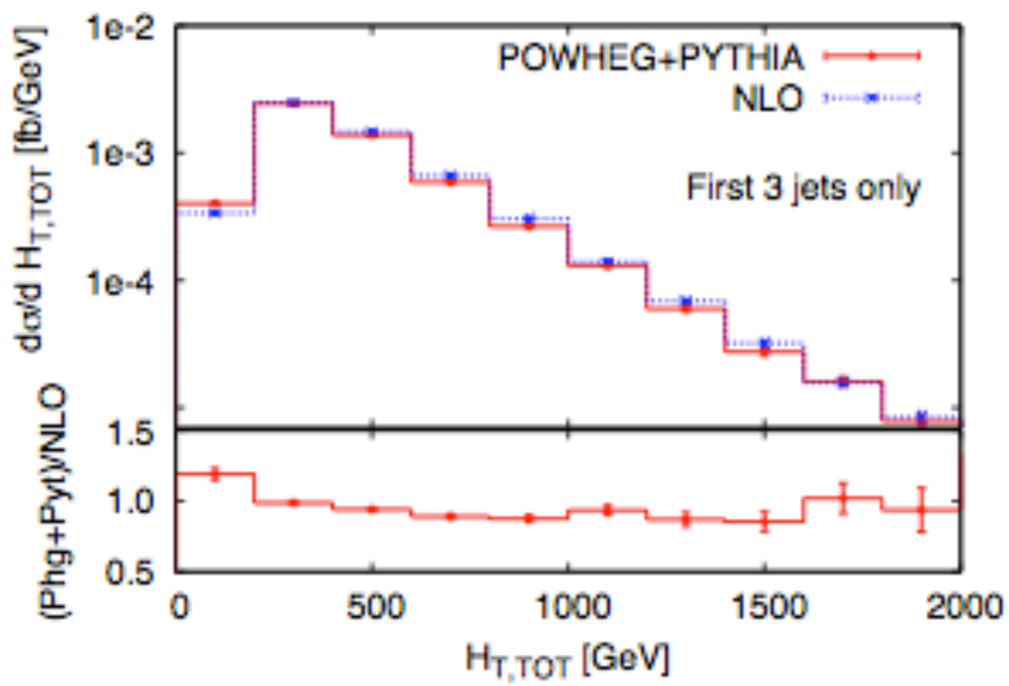

Details of the observable definition can be important

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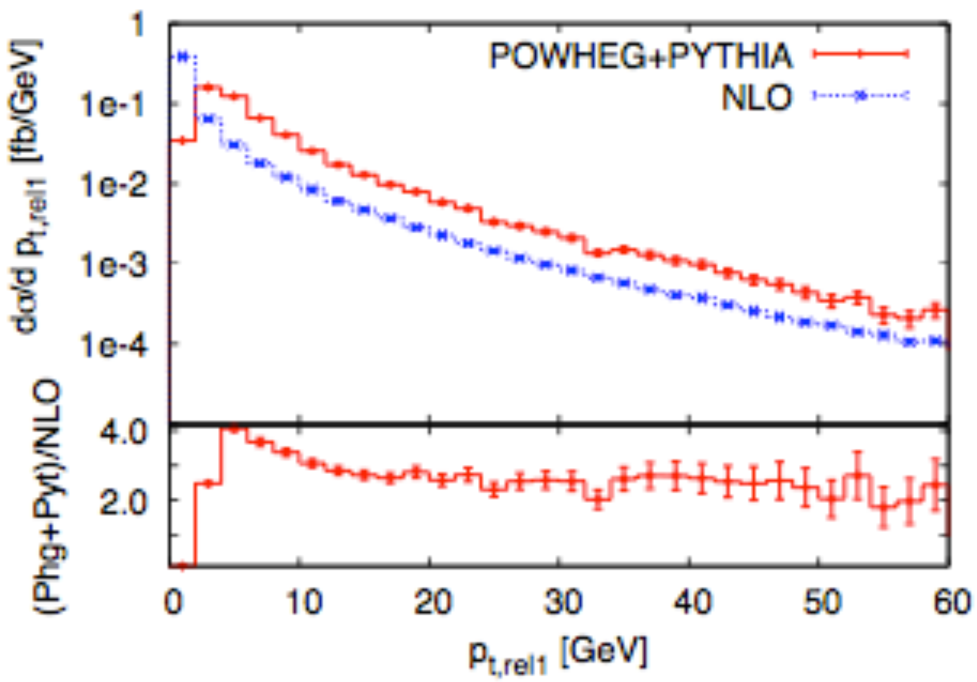
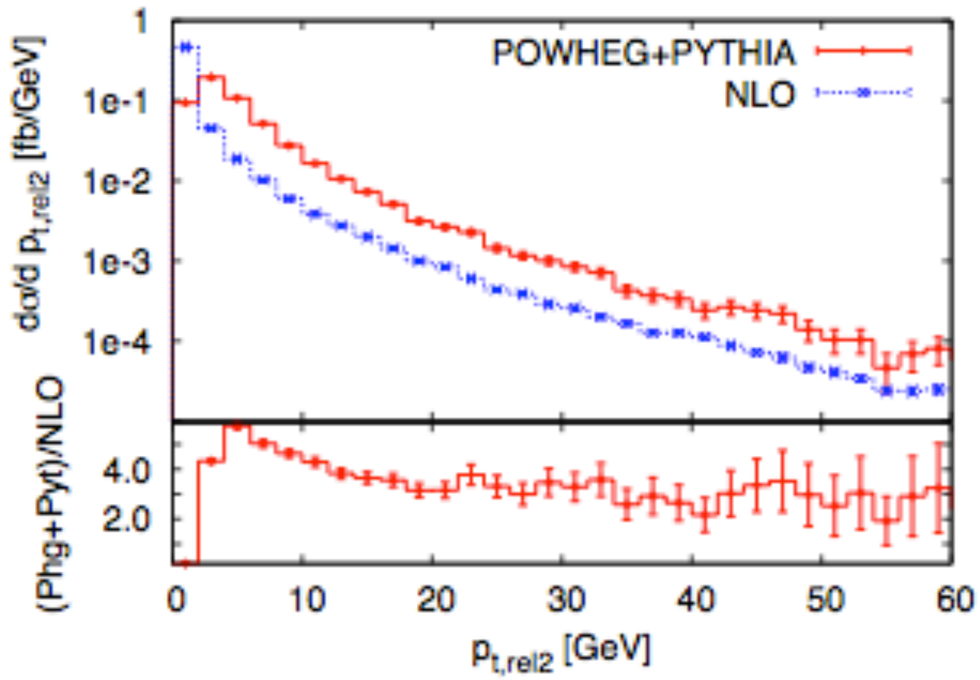
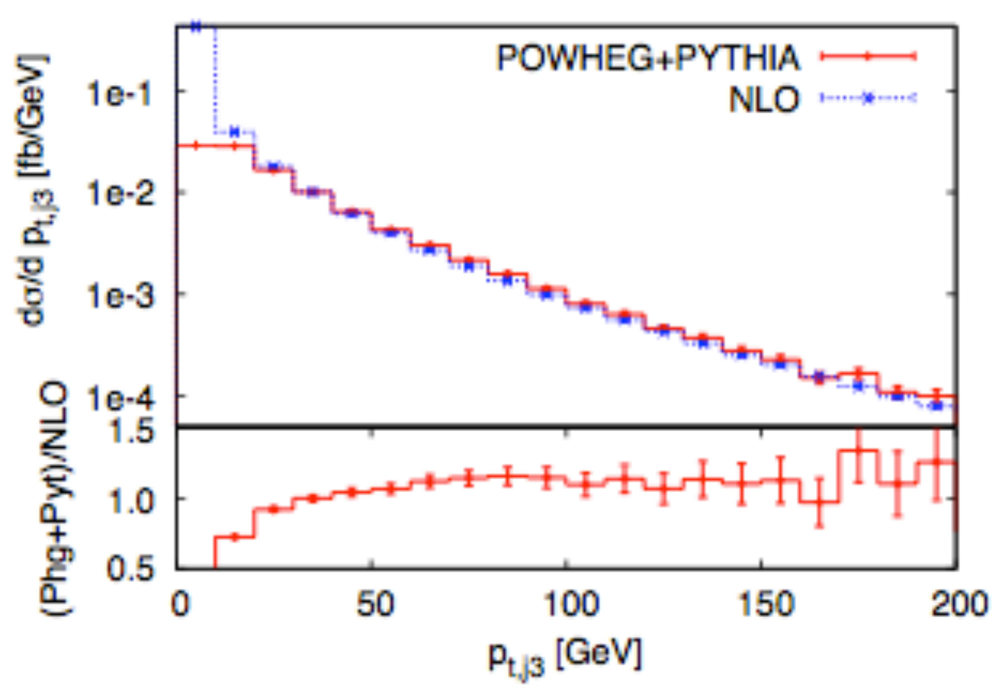
NLO vs NLO+PS: exclusive distributions



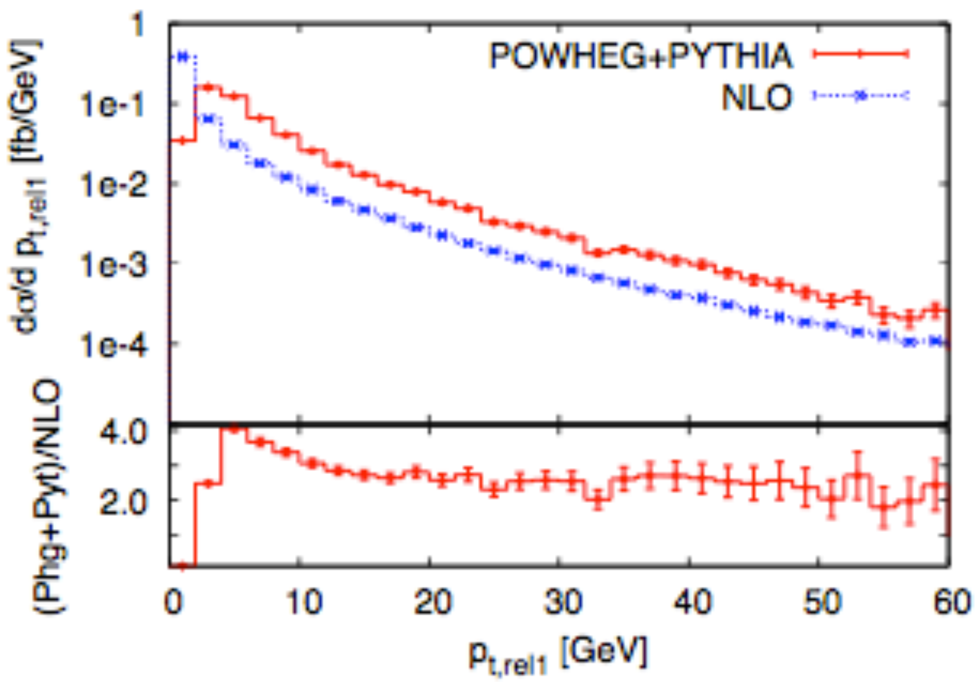


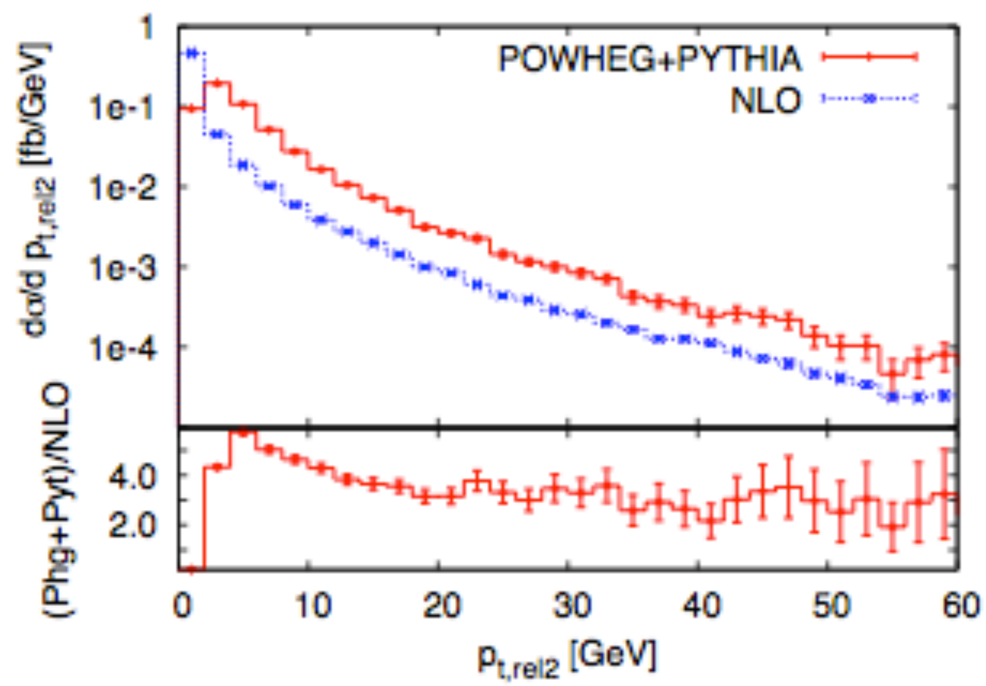
$$p_{\mathrm{t,relj}} = \sum_{i \in j} \frac{|\boldsymbol{k}_i \wedge \boldsymbol{p}_j|}{|\boldsymbol{p}_j|}$$



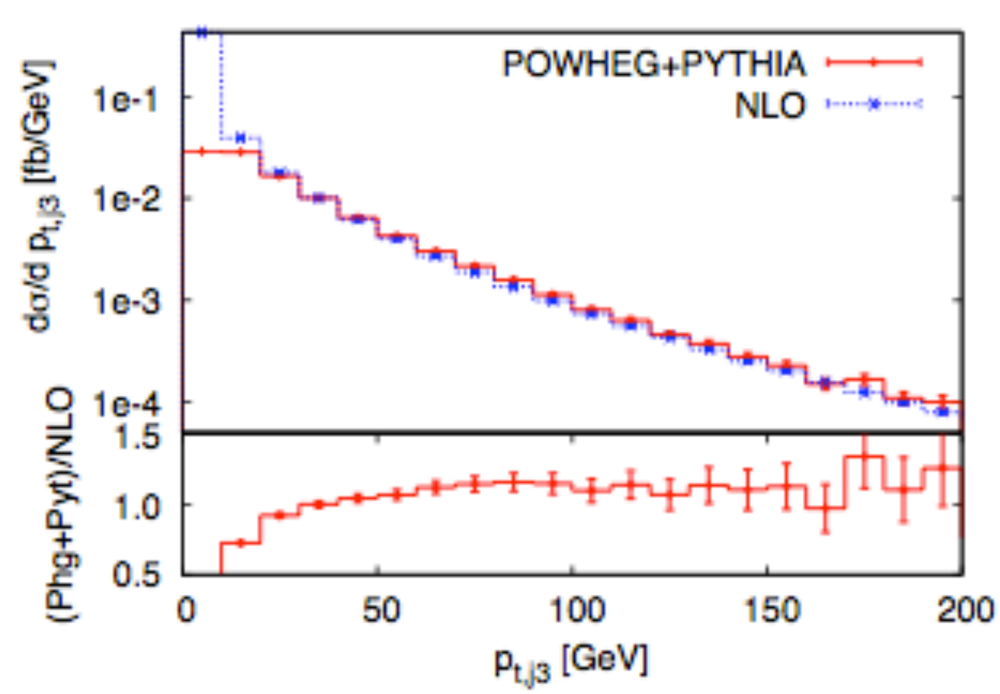
Important differences between NLO and NLO+PS

NLO vs NLO+PS: exclusive distributions





$$p_{\mathrm{t,relj}} = \sum_{i \in j} \frac{|\boldsymbol{k}_i \wedge \boldsymbol{p}_j|}{|\boldsymbol{p}_j|}$$

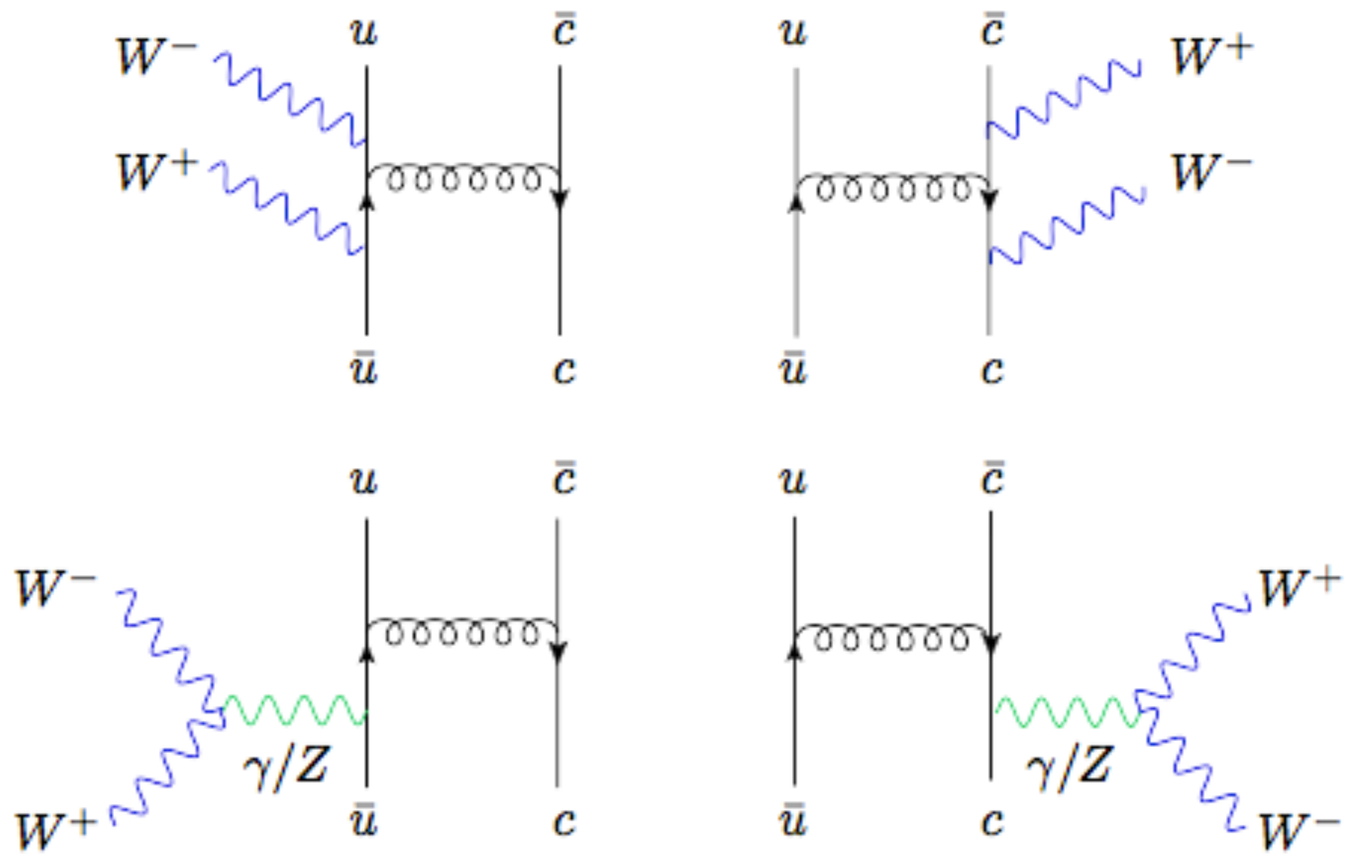


Important differences between NLO and NLO+PS

Melia, Nason, Rontsch, GZ '11

W^+W^- plus dijets

- compared to W^+W^+ more challenging calculation
- larger number of subprocesses and of primitive amplitudes required
- important background to intermediate mass/heavy Higgs searches + New Physics searches



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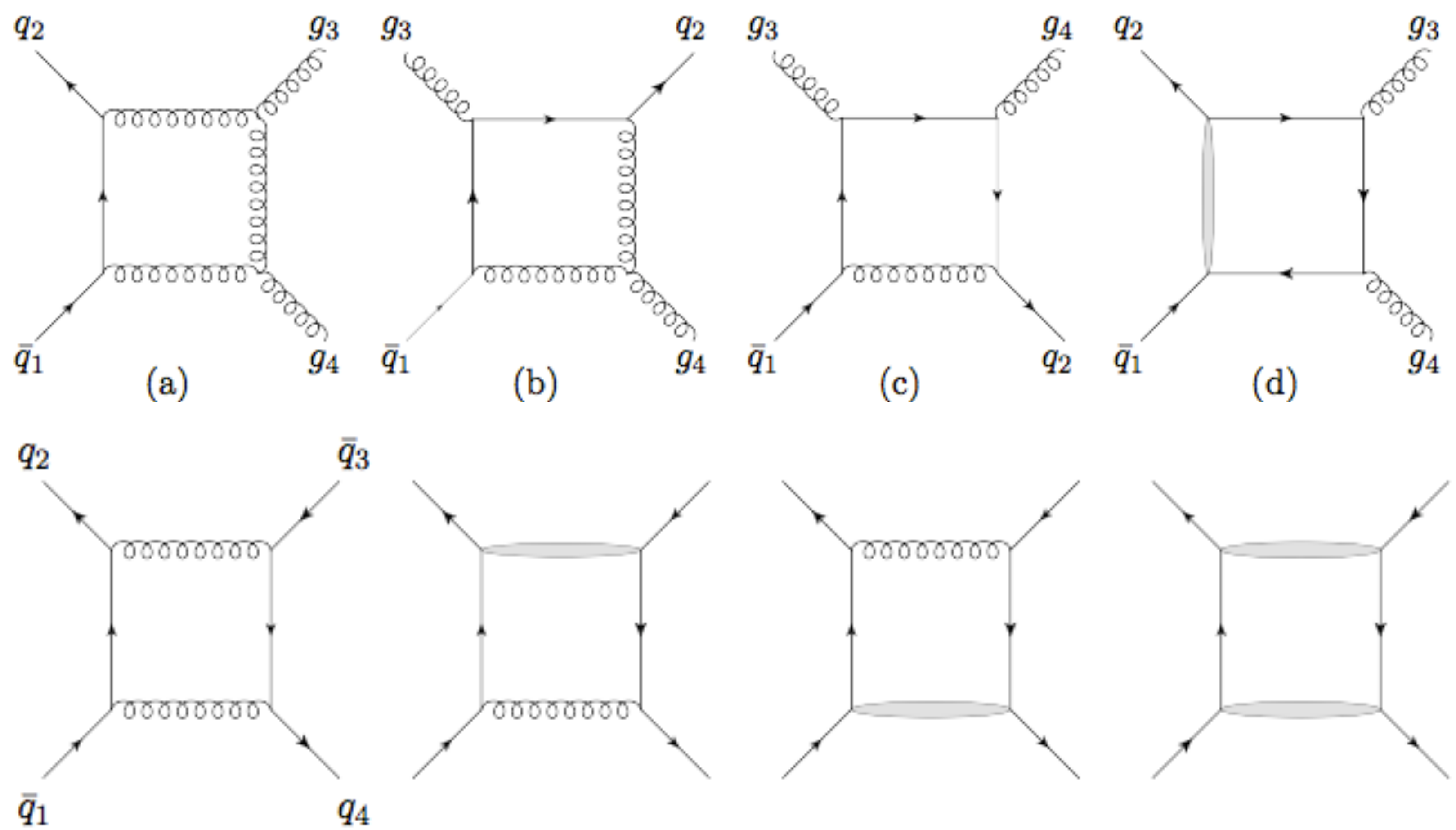

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W^+W^- plus dijets: Tevatron

At the Tevatron this process is important background to Higgs plus dijets production. For m_H = 160 GeV, with standard CFD Higgs search cuts:

$$\boxed{\sigma^{\mathrm{NLO}}_{H(\to \mathrm{WW} \to \mathrm{lept})+2\mathrm{j}} \sim 0.2\mathrm{fb}}$$

Ellis, Campbell, Williams '10

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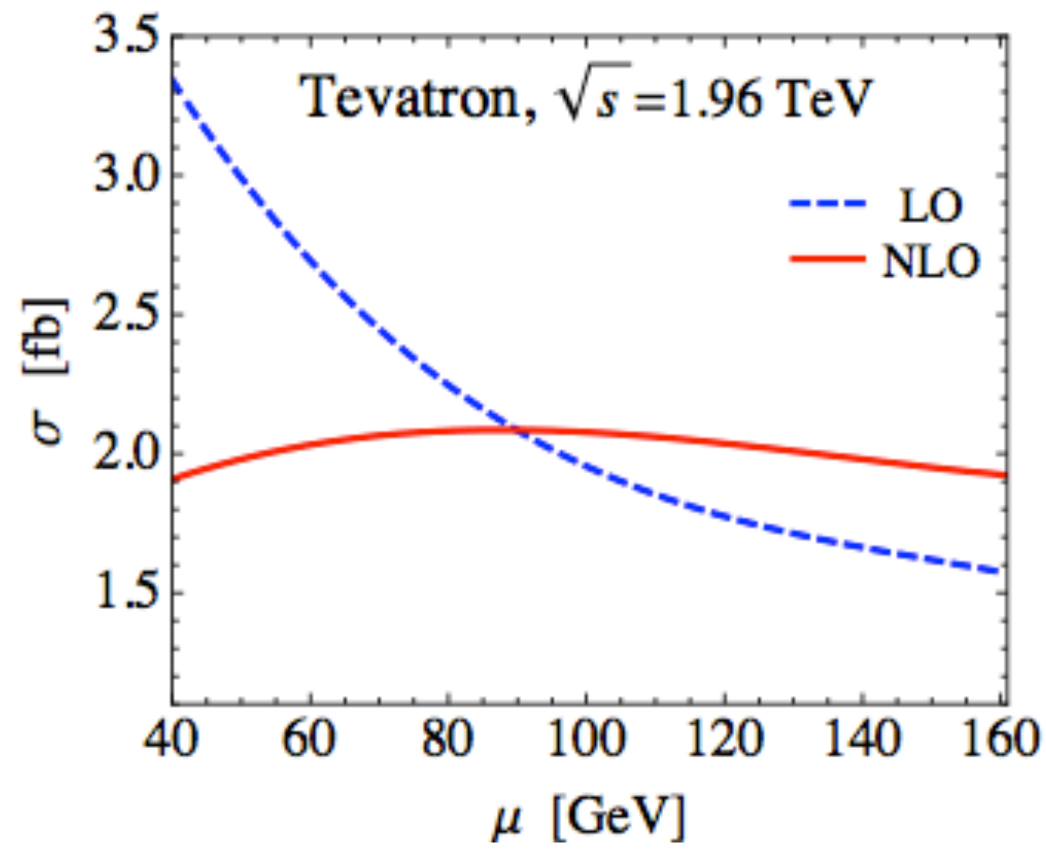
Ellis, Campbell, Williams '10



$$\sigma^{\rm LO}_{WW(\to lept)+2j} \sim 2.5 \pm 0.9\,{\rm fb}$$

$$\sigma^{\rm NLO}_{WW(\to lept)+2j} \sim 2.0 \pm 0.1\,{\rm fb}$$

At LO the uncertainty of W^+W^- + 2j cross-section is larger than the signal

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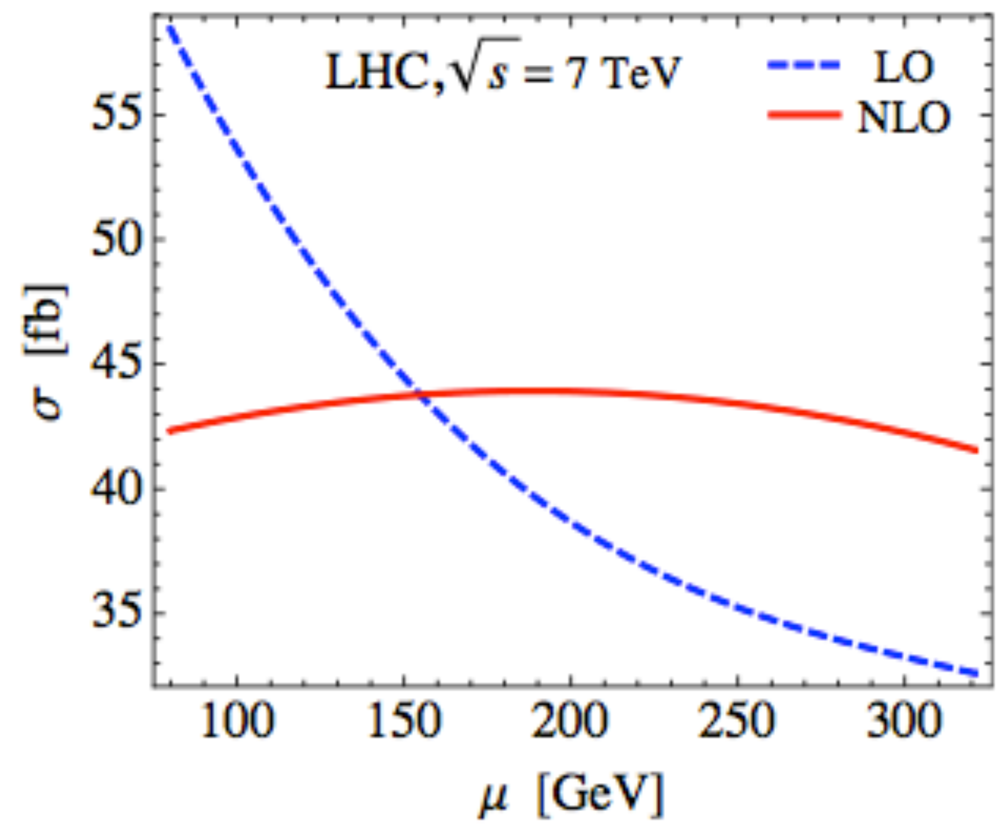
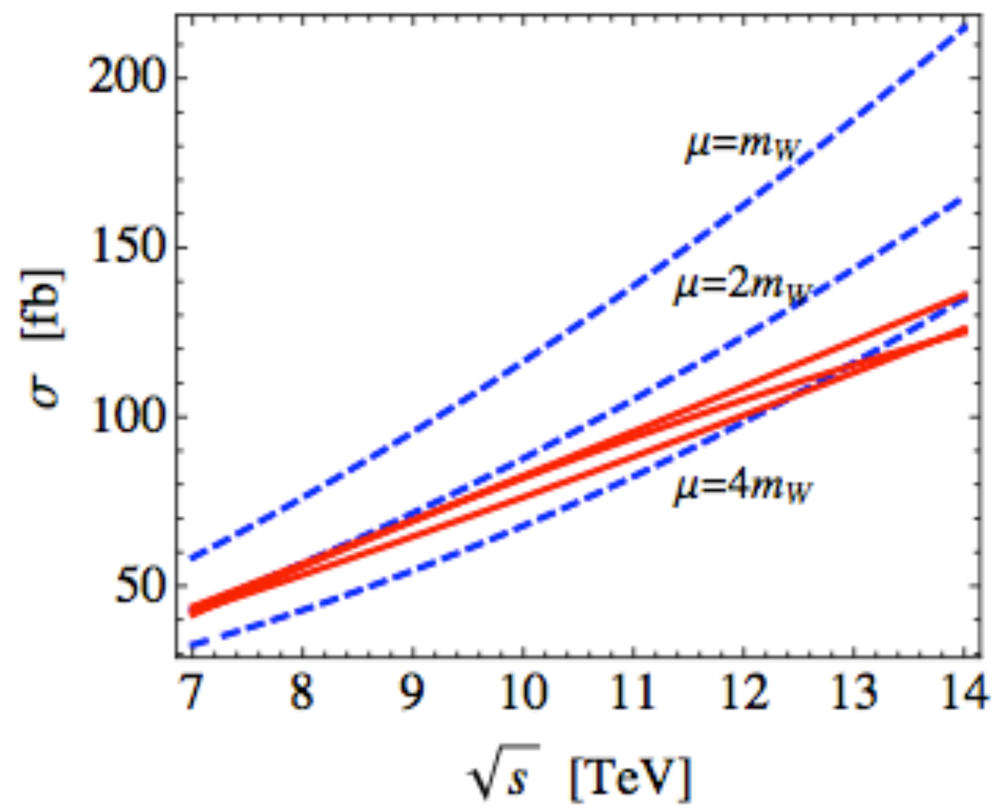
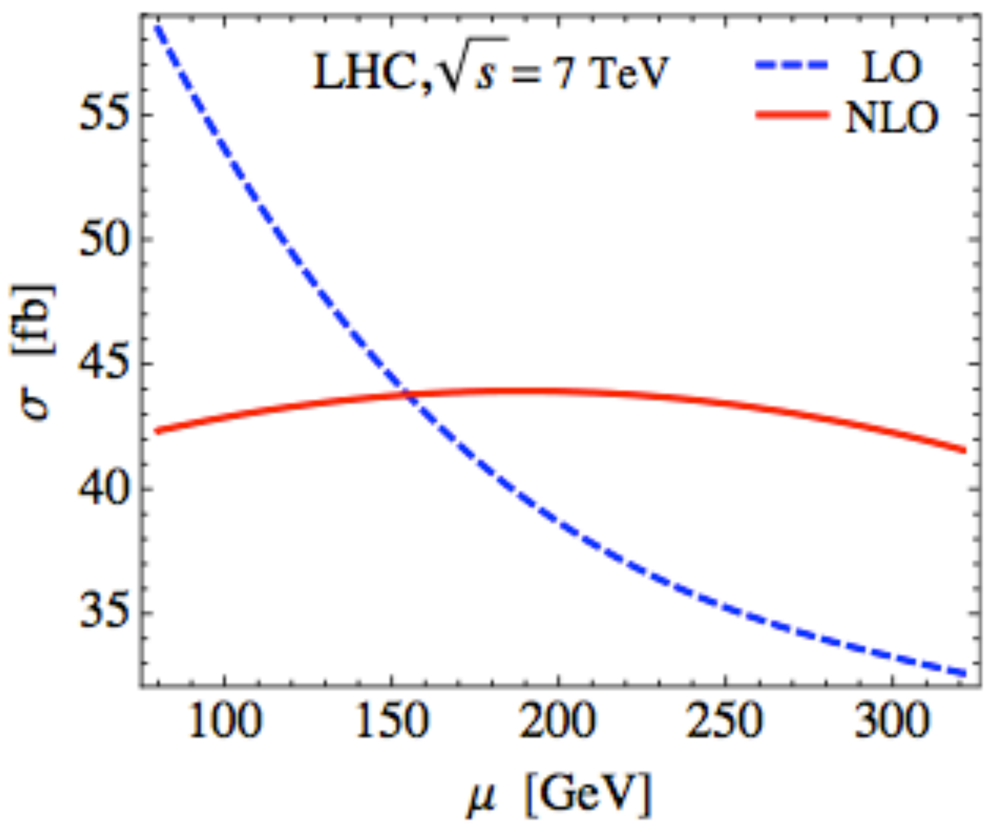
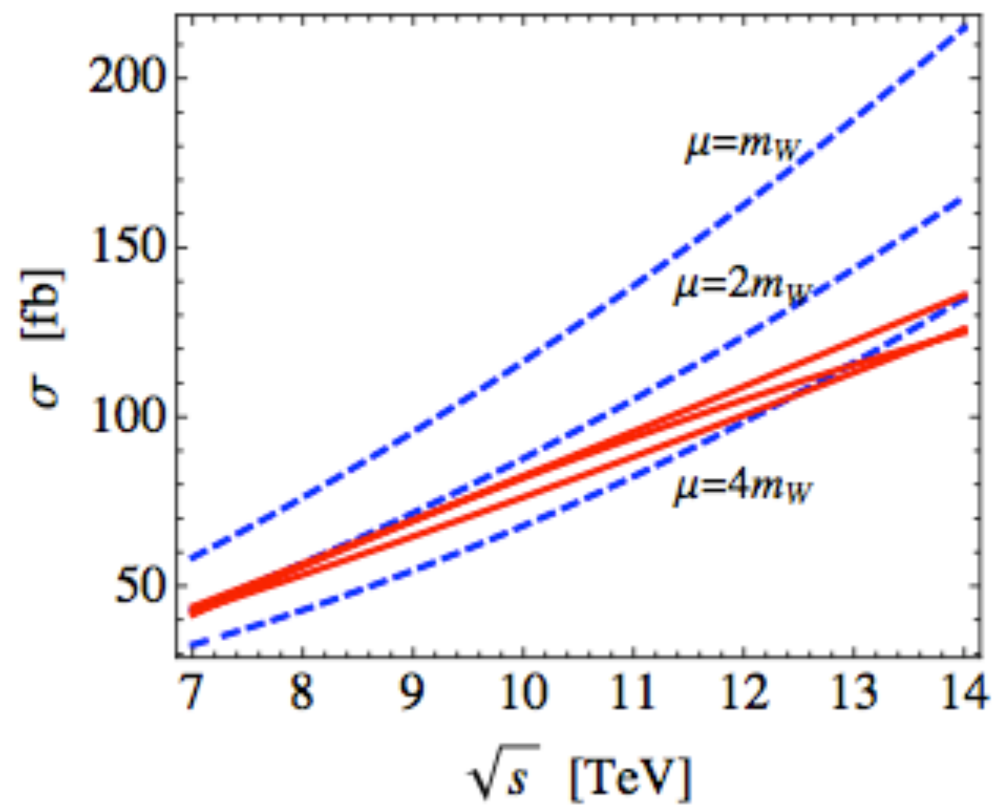
W^+W^- plus dijets: LHC





W^+W^- plus dijets: LHC





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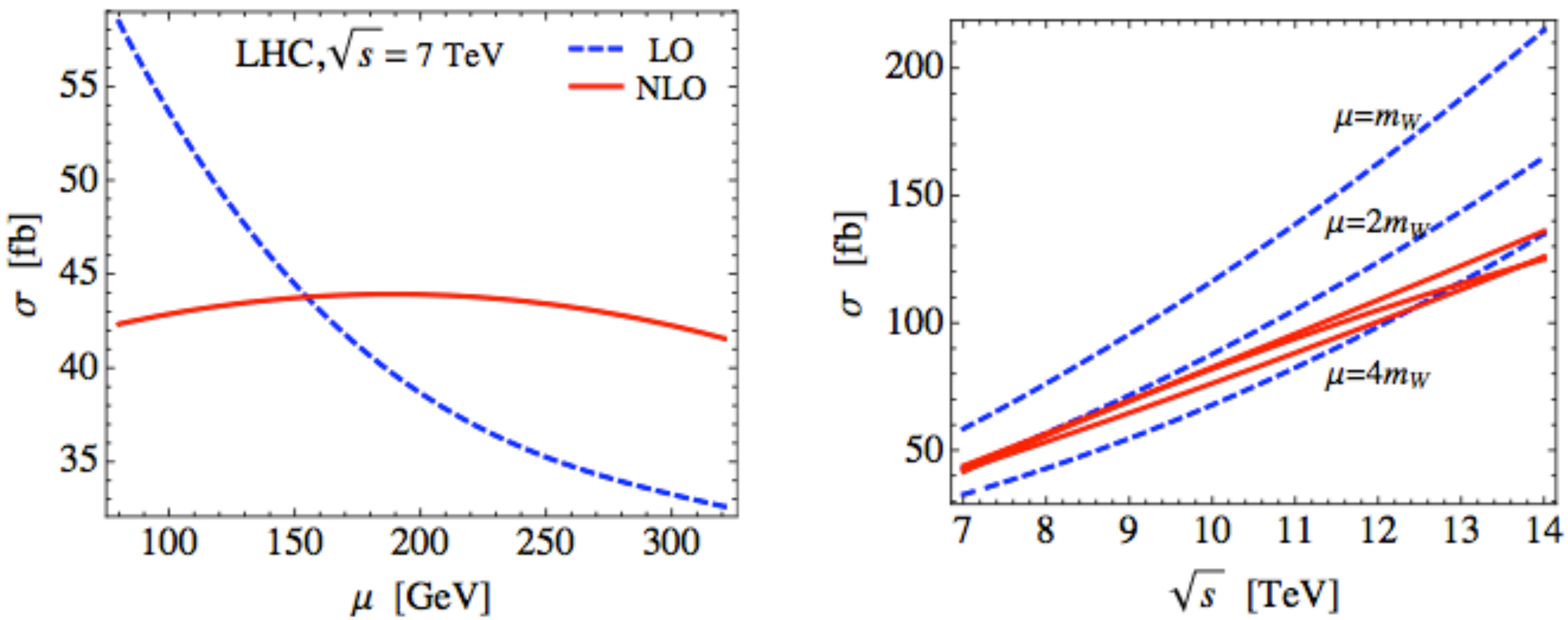
W^+W^- plus dijets: LHC



- NLO cross section grows almost linearly with energy
- "optimal scale choice" depends on collider energy
- as at the Tevatron, after inclusion of NLO corrections, cross-section know to around 10% accuracy

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Conclusions

D-dimensional unitarity is a very powerful tool for NLO calculations

- needs as input only tree level amplitudes (computed with Berends-Giele recursion relations)
- simple, efficient, general and transparent method
- suitable for automation

$\Rightarrow$ a number of highly non-trivial calculations performed with this method

- W^+W^+ plus dijet production + merging to parton shower
- W^+W^- plus dijet production
- also: W + 3 jets, tt, tt+ 1jet

For a pedagogical review see
One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts, Ellis, Kunszt, Melnikov, GZ to appear soon