Lattice QCD measurement of the strong coupling constant

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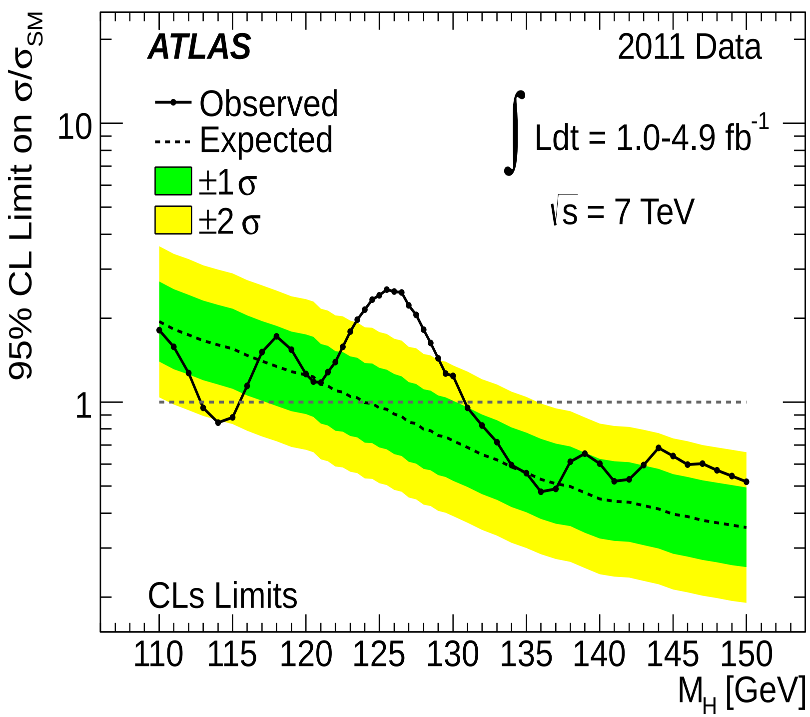
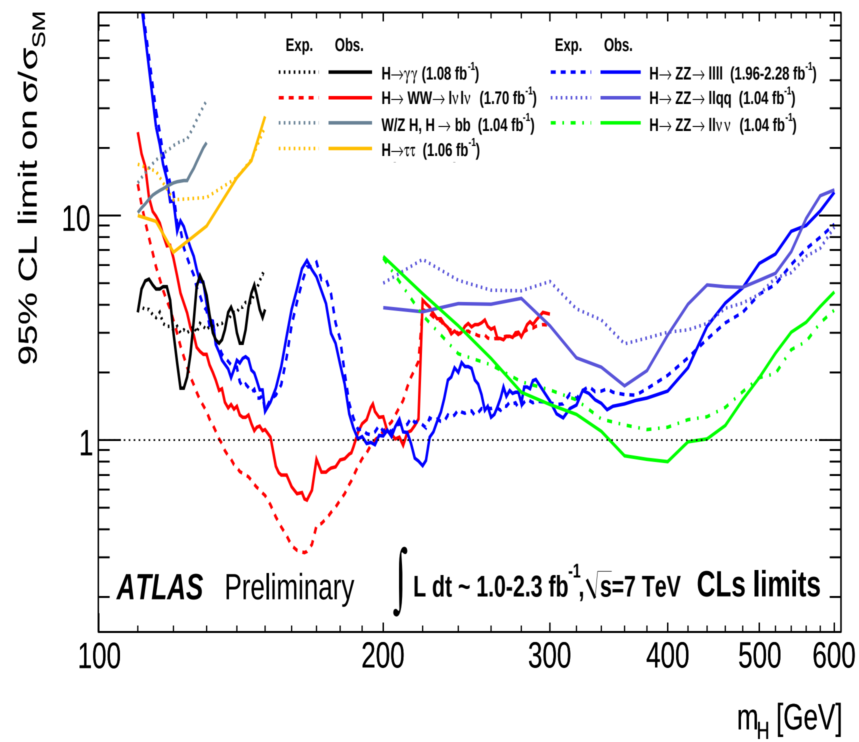
- Phenomenological considerations
- Hints of lattice QCD
- Hadronic and finite volume schemes
- Fixed gauge approach

Phenomenological considerations

A major activity in Particle Physics is nowadays the search of Higgs boson, whose the existence might explain the spontaneous symmetry breaking of $SU(2)_W \times U(1)_Y$ predicted by the Standard Model and observed in Nature.

[ATLAS, '12; Lepton-Photon '11]

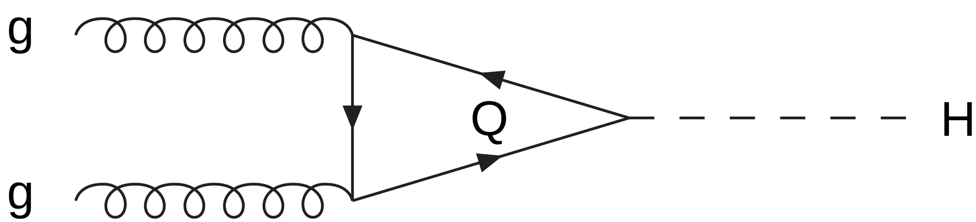




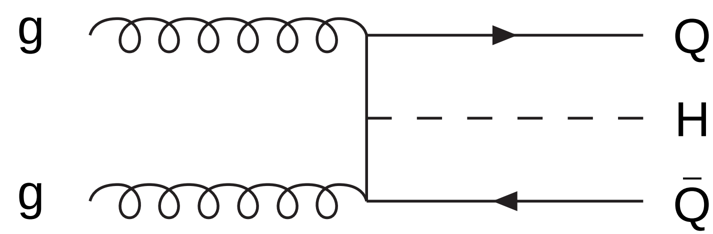
ATLAS has excluded at 95% of CL the region $131 < m_H < 238$ GeV (and also the mass range $251 < m_H < 466$ GeV). Hint of a signal around 125 GeV, both for ATLAS and CMS, in $h \to \gamma\gamma$ and $h \to 4l$.

Different modes of Higgs boson production:

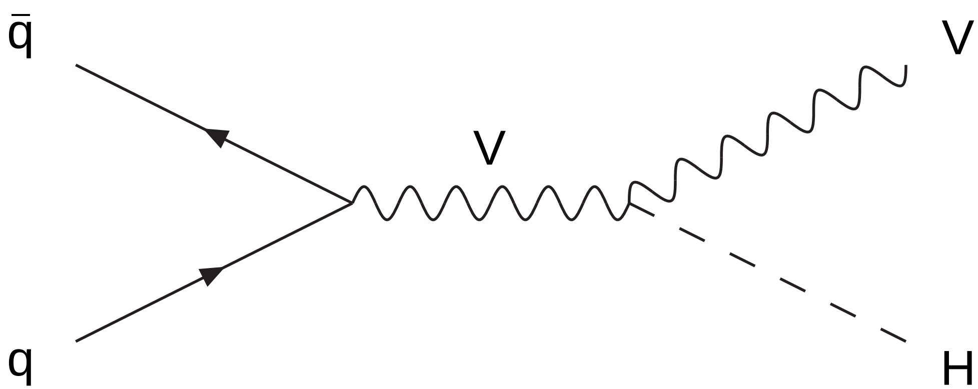
gluon fusion



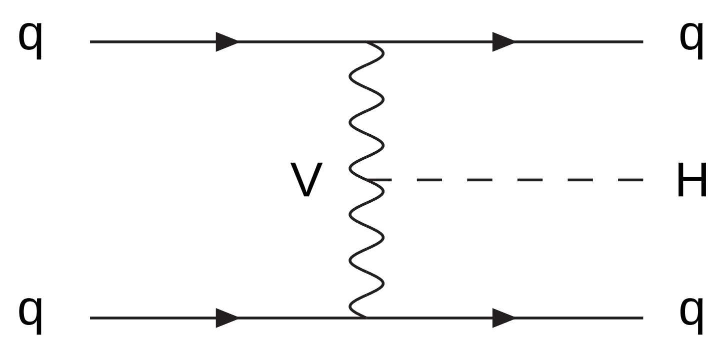
associated production with $Q\bar{Q}$



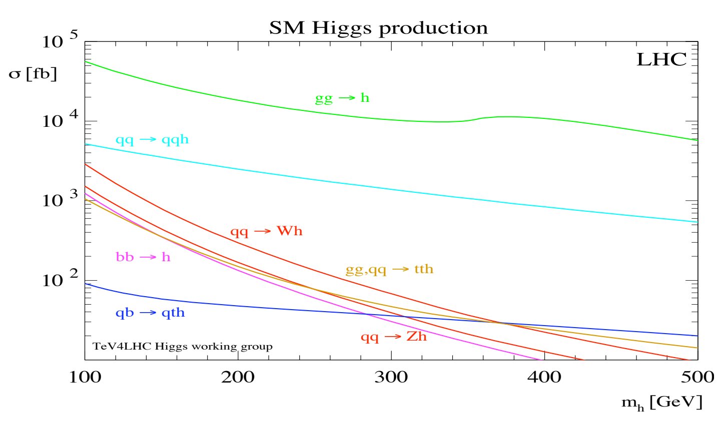
Higgs strahlung

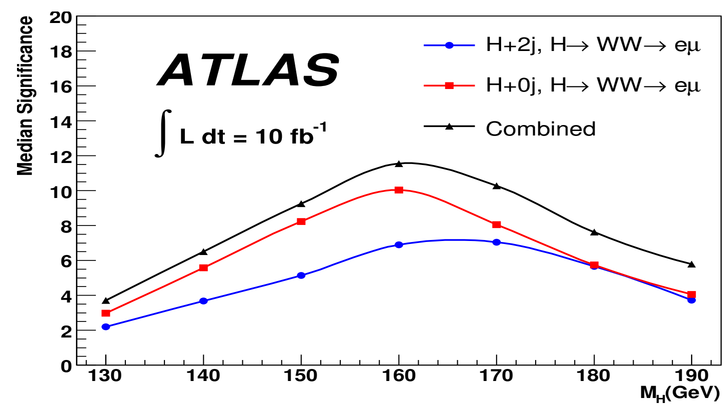


vector boson fusion



Gluon fusion highly favored w.r.t. other Higgs production processes
Good hope to observe a SM Higgs, if existing, at LHC

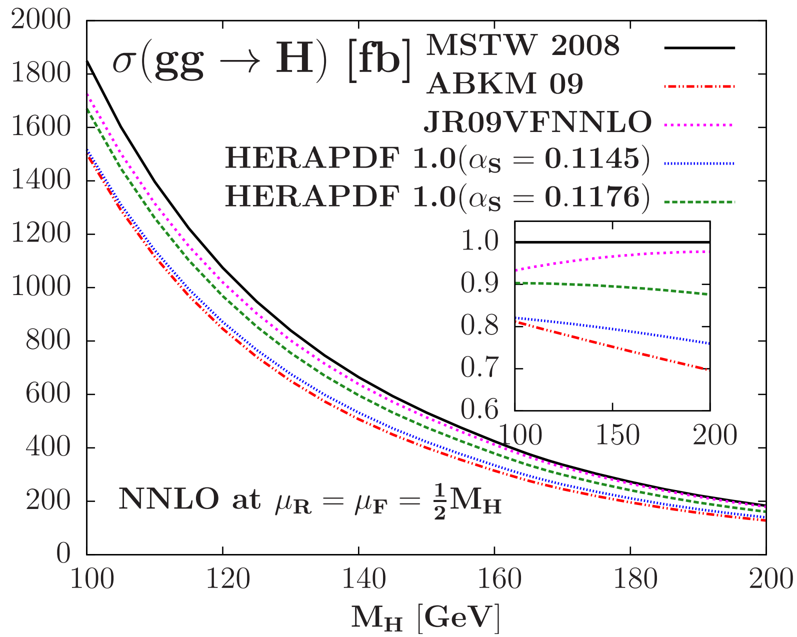




Estimating as accurately as possible $\sigma^{\rm th}_{gg\to H\to X}$ is an important ingredient to assess the detectors sensitivity to the Higgs physics. Several sources of uncertainty:

– NNNLO (QCD) and NNLO (EW) corrections
– factorisation scale uncertainties
– error on $H \to X$
– parton distribution functions and $\delta(\alpha_s)$

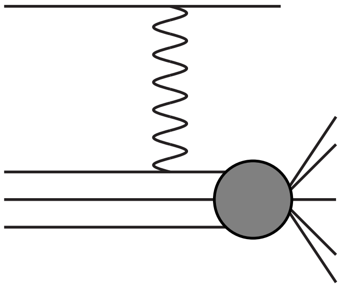
[J. Baglio et al, '11]

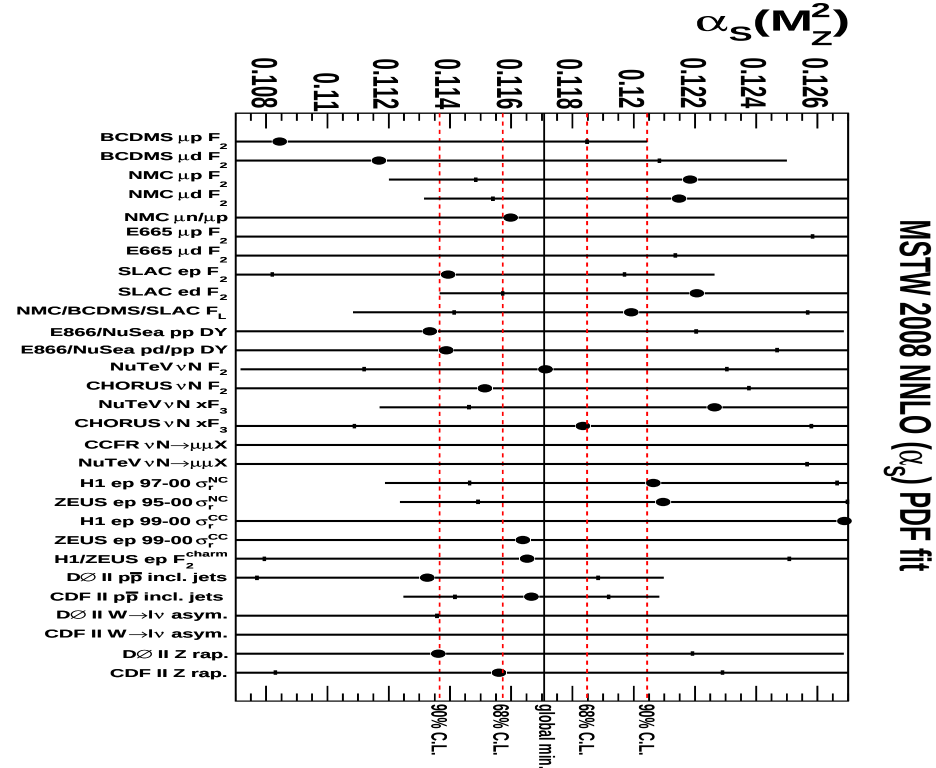


$\Delta\sigma^{\rm NNLO}_{gg\to H\to X} \sim 20 - 25\%$ at LHC ($\sqrt{s} = 7$ TeV), with 2-3 % from $\delta\alpha_s$

Plenty of α_s estimates based on experimental data analysis.

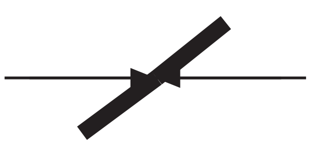
Parton Distribution Function in DIS

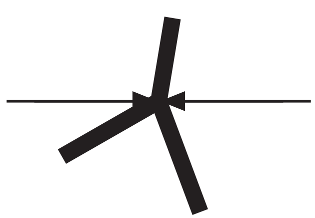




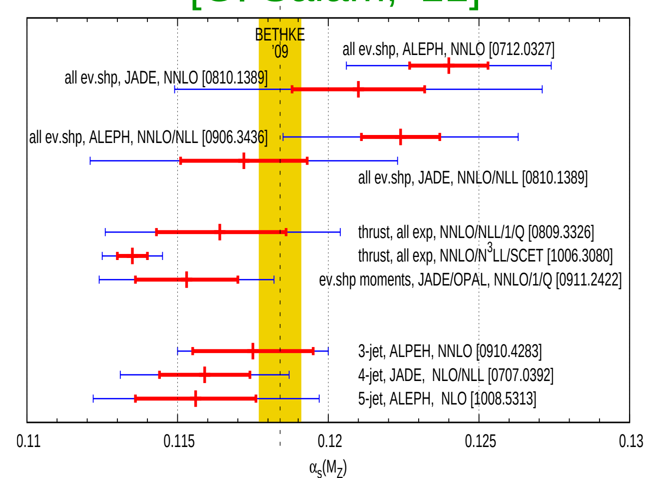
$$\alpha_s^{\mathrm{NNLO,\,DIS}}(m_Z) = 0.1171(14)$$ 68% CL

Event-shape in e^+e^- collisions

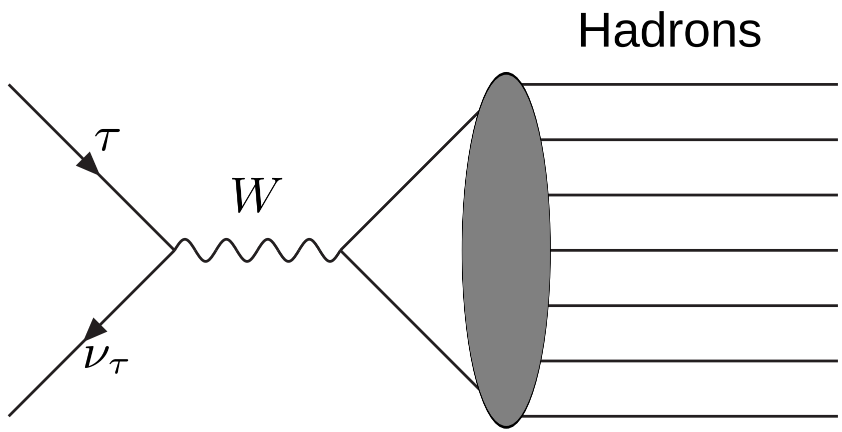


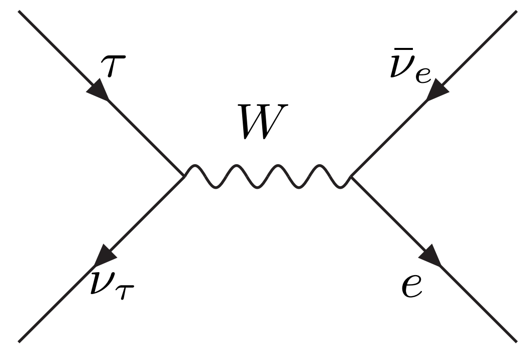


[G. Salam, '11]



Phenomenological analysis of the τ decay into hadrons provides another way to extract α_s.





$$R_\tau \equiv \Gamma[\tau^- \to \nu_\tau \text{ hadrons}]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]$$

$$R_{\tau,V+A} = N_c |V_{ud}|^2 S_{\mathrm{EW}}(1 + \delta_P + \delta_{NP}), \quad \delta_{NP} = -0.0059(14)$$ [Davier *et al*, '08]

$$\delta_P = \sum_n K_n A^{(n)}(\alpha_s) = \sum_n (K_n + g_n)\alpha^n(m_\tau)$$

$$A^{(n)} = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left(\frac{\alpha_s(-s)}{\pi}\right)^n \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8}\right) = \alpha^n(m_\tau) + \mathcal{O}(\alpha^{n+1}(m_\tau))$$

Fixed Order Perturbation Theory (FOPT) vs Contour Improved Perturbation Theory (CIPT):

$\alpha_s(m_\tau)_{\mathrm{CIPT}} = 0.344(14) \quad \alpha_s(m_\tau)_{\mathrm{FOPT}} = 0.321(15)$ [A Pich, '11]

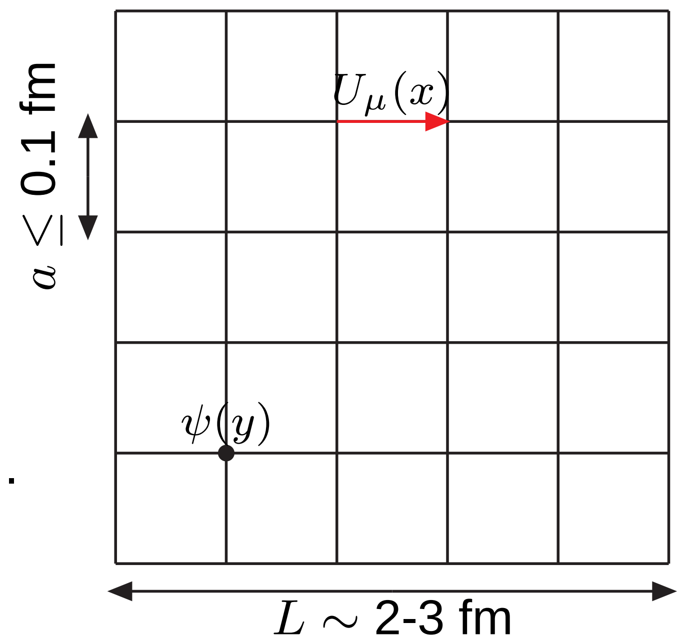
Hints of lattice QCD

Discretisation of QCD in a finite volume of Euclidean space-time.

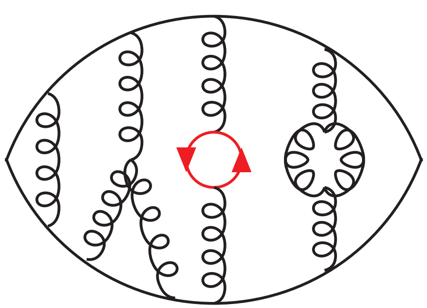
The lattice spacing a is a non perturbative UV cut-off of the theory.

Fields: $\psi^i(x)$, $U_\mu(x) \equiv e^{iag_0 A_\mu(x+\frac{a\hat{\mu}}{2})}$.

Inputs: bare coupling $g_0(a) \equiv \sqrt{6/\beta}$, bare quark masses m_i.



Computation of Green functions of the theory from first principles:



$$\langle O(U,\psi,\bar{\psi})\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, O(U,\psi,\bar{\psi}) e^{-S(U,\psi,\bar{\psi})}$$

$$\mathcal{Z} = \int \mathcal{D}U\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi} e^{-S(U,\psi,\bar{\psi})}$$

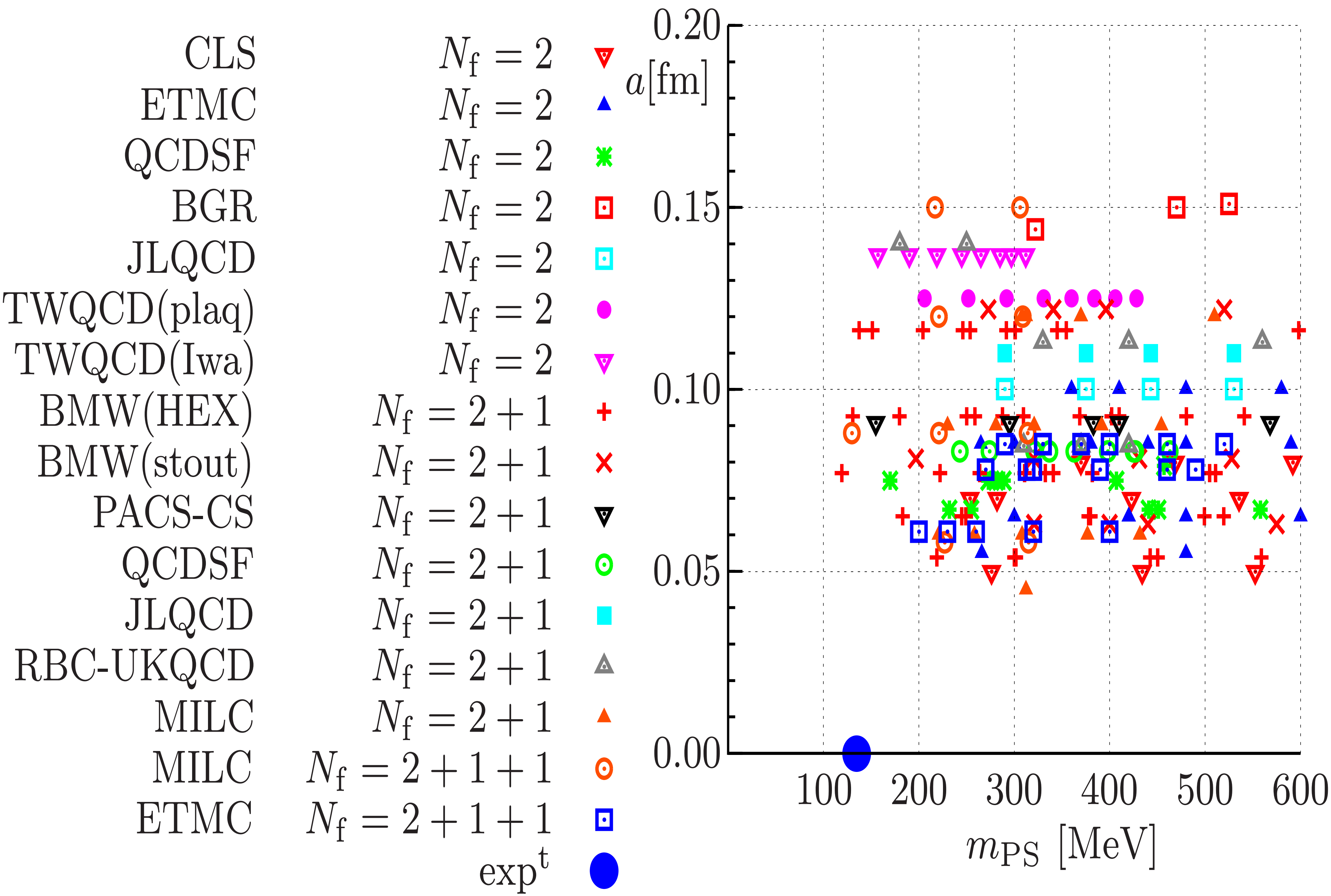
$$S(U,\psi,\bar{\psi}) = S^{\mathrm{YM}}(U) + \bar{\psi}^i_x M^{ij}_{xy}(U)\psi^j_y$$

$$\mathcal{Z} = \int \mathcal{D}U\, \mathrm{Det[M(U)]} e^{-S^{\mathrm{YM}}(U)} \equiv \int \mathcal{D}U e^{-S_{\mathrm{eff}}(U)}$$

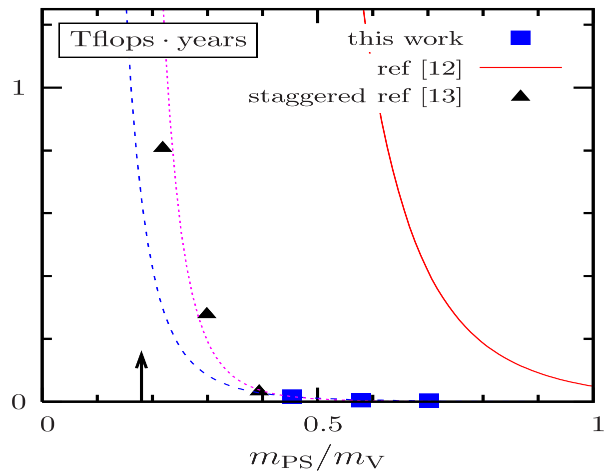
Monte Carlo simulation: $\langle O\rangle \sim \frac{1}{\mathrm{N_{conf}}} \sum_i O(\{U\}_i)$: we have to build the statistical sample $\{U\}_i$ in function of the Boltzmann weight $e^{-S_{\mathrm{eff}}}$. Incorporating the quark loop effects hidden in $\mathrm{Det}[M(U)]$ is particularly expensive in computer time. Crucial in the extraction of α_s.

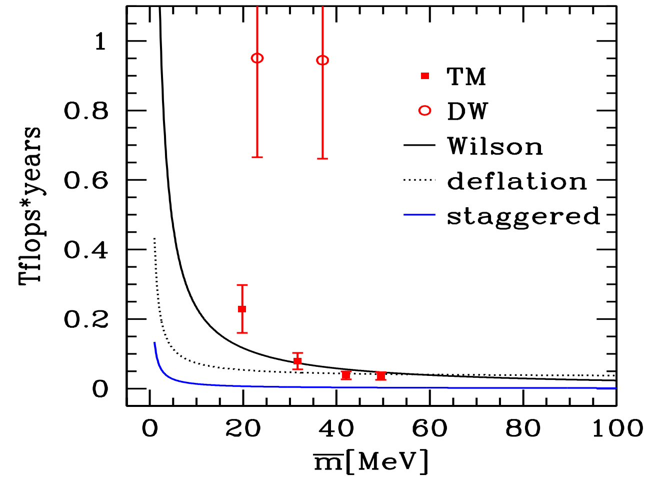
Simulations set up

In the past years tremendous progresses have been made by the lattice community to perform simulations that are closer to the physical point.



Improvements in algorithms



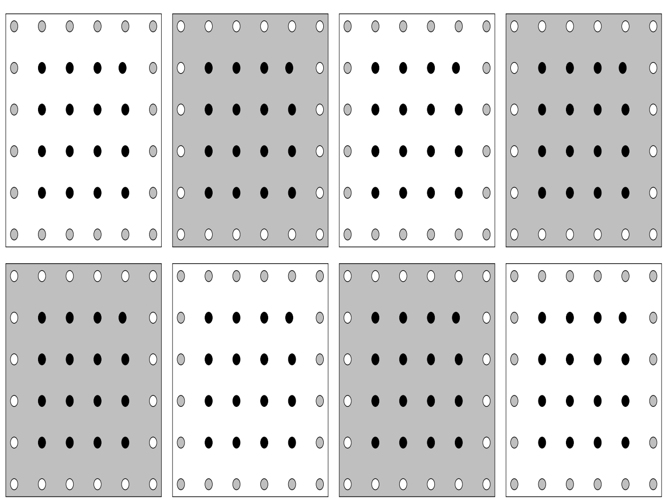


2001: $N_{op} = k_1 \left(\frac{20\,\mathrm{MeV}}{\bar{m}_q}\right)^{3} \left(\frac{L}{3\,\mathrm{fm}}\right)^{5} \left(\frac{0.1\,\mathrm{fm}}{a}\right)^{7}$ TFlops $\times$ years ("Berlin Wall")

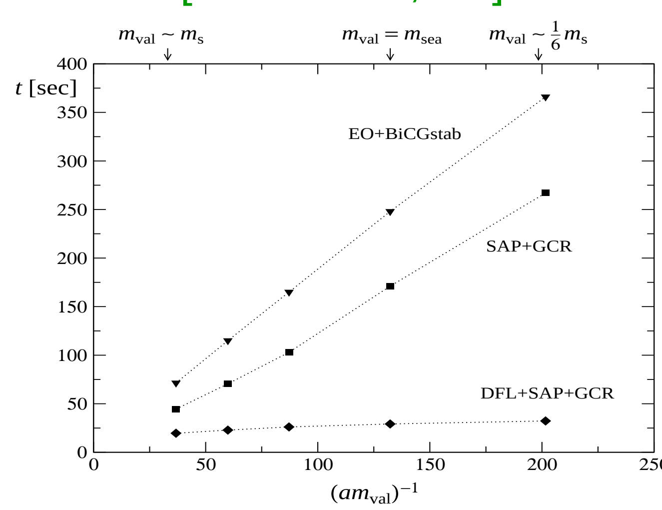
2007: $N_{op} = 0.01\,k_1 \left(\frac{20\,\mathrm{MeV}}{\bar{m}_q}\right)^{1} \left(\frac{L}{3\,\mathrm{fm}}\right)^{5} \left(\frac{0.1\,\mathrm{fm}}{a}\right)^{6}$ TFlops $\times$ years (deflation)

Regularisation of quarks and gluons with a smooth spectrum in the UV regime, mass shift, multi-step/Omelyan integrators, e/o preconditionning, domain decomposition, deflation,...

[M. Lüscher, '03]



[M. Lüscher, '06]



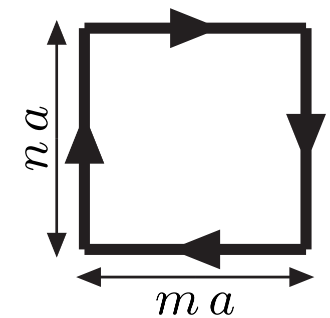
Hadronic and finite volume schemes

Those schemes are based on hadronic quantities to fix the parameters (quark masses, renormalised coupling). It is not necessary to fix the gauge.

Hadronic schemes I [C. Davies *et al*, '08]

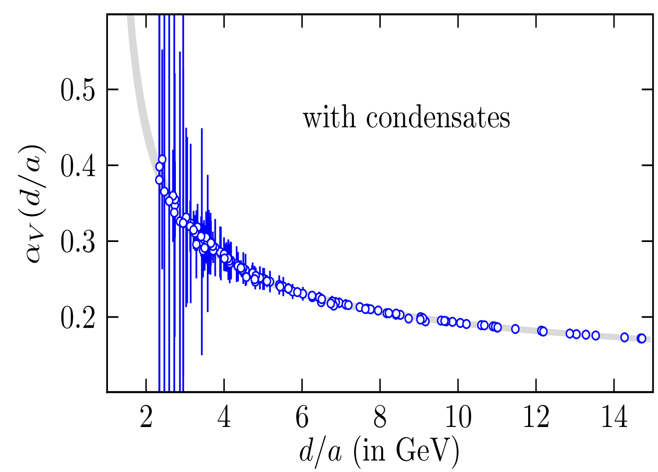
– u/d, s and c quark masses are tuned from m_π, $2m_K^2 - m_\pi^2$ and m_{η_c} while the lattice spacing is extracted from the Υ spectrum.

– One computes Wilson loops $\mathcal{W}_{m\,n}$ that one develops at short distance $r = a/d$ in perturbation theory: $\mathcal{W}_{m\,n} = \sum_{i=1}^{\infty} c_i \alpha_V^i(d/a)$.



– $\alpha_V(q)$ defined by $V(q) = \frac{C_f 4\pi\alpha_V(q)}{q^2}$ (one-gluon exchange part of the potential) [P. Lepage and P. McKenzie, '92].

– The series converges better by considering $\ln(\mathcal{W}_{m\,n})$, $\ln(\mathcal{W}'_{m\,n}) \equiv \ln[\mathcal{W}_{m\,n}/\mathcal{W}_{1\,1}^{(m+n)/2}]$ (tadpole improvement), and even better, Creutz ratios $\ln\left(\frac{\mathcal{W}_{m\,n+1}}{\mathcal{W}_{m\,n}}\frac{\mathcal{W}_{m-1\,n}}{\mathcal{W}_{m-1\,n+1}}\right)$.



– One subtracts a gluon-condensate term $-\frac{\pi^2}{36}\mathcal{A}^2\langle\alpha_s G^2/\pi\rangle$ to the lattice results to compare with perturbation theory.

– Running of $\alpha_V(d/a)$ to $\alpha_0 \equiv \alpha_V(7.5\,\text{GeV})$, conversion to $\alpha^{\overline{\text{MS}}}(m_Z, \text{N}_\text{f} = 5) = 0.1183(8)$.

Hadronic schemes II [I. Allison *et al*, '08, C. McNeile *et al*, '10]

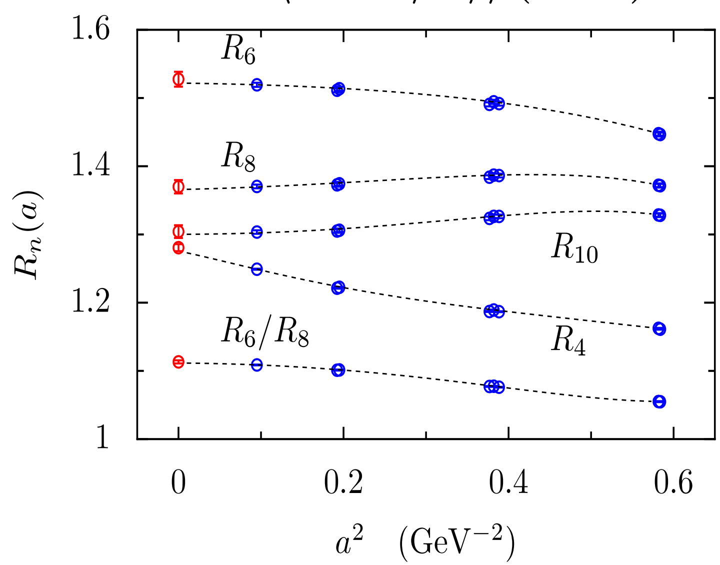
– Strategy to tune the bare quark mass parameters and a already discussed.

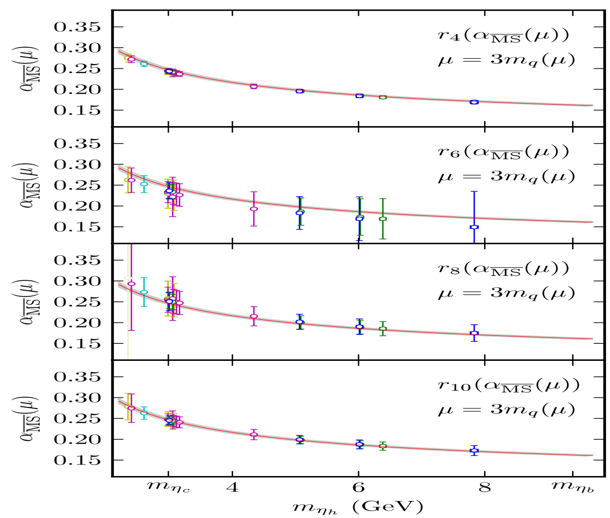
– Consider $G(t) = a^6 \sum_{\vec{x}} (am_{0h})^2 \langle J_5(\vec{x},t) J_5(\vec{0},0)\rangle$, $J_5 = \bar{\psi}_c \gamma^5 \psi_c$.

– Define the n^{th} moment of the correlator $G_n = (t/a)^n \sum_t G(t)$, its counterpart $G_n^{(0)}$ known at lowest order of perturbation theory, $R_4 \equiv G_4/G_4^{(0)}$ and $R_{n\geq 6} = \frac{am_{\eta_h}}{2am_{0h}} \left(\frac{G_n}{G_n^{(0)}}\right)^{1/(n-4)}$.

– Those R_n have an expression in continuum perturbation theory: $R_4 = r_4(\alpha_{\overline{MS}}, \mu/m_h)$ and $R_{n\geq 6} = \frac{r_n(\alpha_{\overline{MS}}, \mu/m_h)}{2m_h(\mu)/m_{\eta_h}}$.

– Power corrections to the perturbative expression of moments are taken into account by including a factor $1 + d_n \langle \alpha_s G^2/\pi\rangle/(2m_h)^4$.





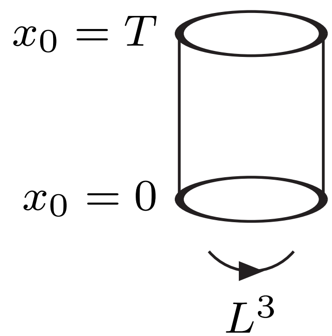
Despite the errant structure of cut-off effects in R_n, one can extract from that analysis $\alpha_{\overline{MS}}(m_Z, \mathrm{N_f} = 5) = 0.1183(7)$.

Finite volume scheme

Partition function: $\mathcal{Z}[C, C'] = \langle C' | e^{-H\,T} | C \rangle$ [K. Symanzik, '81]

$C(x_0 = 0)$ and $C'(x_0 = T)$ are 2 field configurations that are given.

The Schrödinger Functional $\mathcal{Z}$ is renormalisable with Yang-Mills theories.
[M. Lüscher et al, '92]



The associated renormalisation scheme is of finite volume kind and regularisation independent:

$$\Gamma(B) \equiv -\ln \mathcal{Z}[C, C'] = g_0^{-2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots \qquad \left.\frac{\delta S}{\delta \Phi}\right|_{\Phi = B} = 0$$

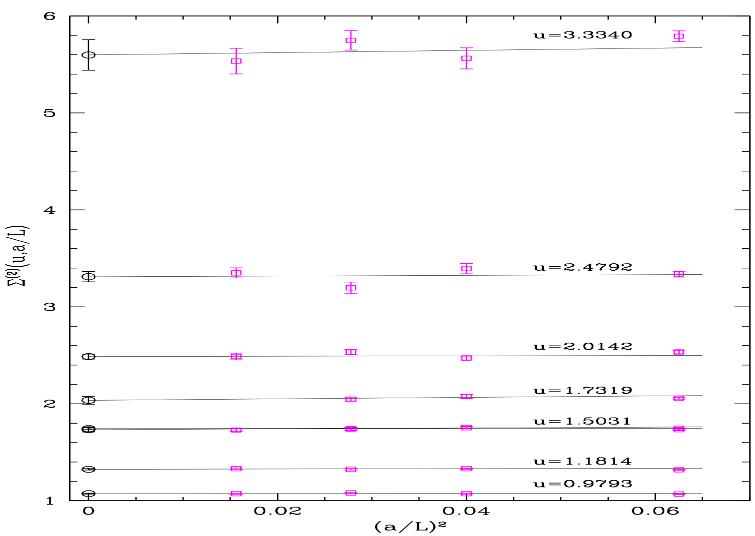
$$C^{(\prime)} \equiv C^{(\prime)}(\eta) \qquad \bar{g}^2(L) = \left[\frac{\partial \Gamma_0(B)}{\partial \eta}\right] \Big/ \left.\left[\frac{\partial \Gamma(B)}{\partial \eta}\right]\right|_{\eta=0} \qquad \bar{g}^2(L) = \left.\left\langle \frac{\partial S}{\partial \eta} \right\rangle\right|_{\eta=0}$$

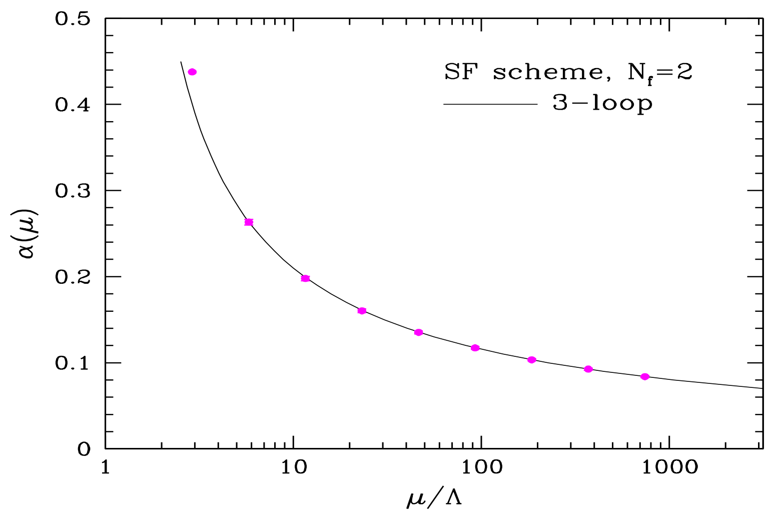
The running of the coupling constant is obtained from step scaling functions

$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, a/L)$, $\Sigma(u, a/L) = \bar{g}^2(2L)_{\bar{g}^2(L)=u}$; $\sigma(u)$ is an integrated β function at discrete points.

Several tuning simulations are necessary to fix $\bar{g}^2(L) = u$ for a given L/a.

[M. Della Morte *et al*, '04]





Computation of the RGI Λ scale [M. Lüscher *et al*, '93]:

$$\Lambda_{\rm SF} = \frac{1}{L}\left[\beta_0 \bar{g}(L)\right]^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{2\beta_0 \bar{g}(L)}\right) \exp\left[-\int_0^{\bar{g}(L)} dg \left(\frac{1}{\beta(g)} + \frac{1}{\beta_0 g^3} - \frac{\beta_1}{\beta_0^2 g}\right)\right]$$

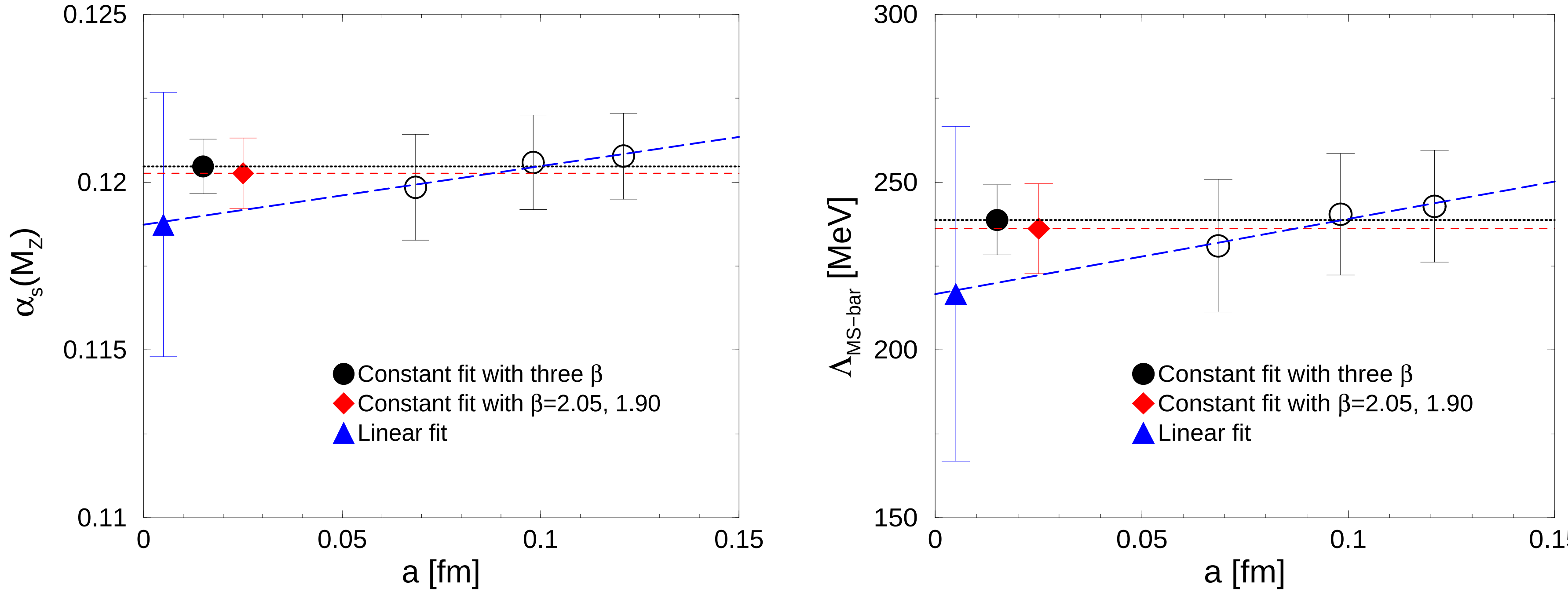
$$\beta(g) = -g^3(\beta_0 + \beta_1 g^2 + \beta_2 g^4 + \cdots)$$

Starting from $(L_{\max}, \bar{g}^2(L_{\max}))$, one uses the step scaling functions to follow the RG flow and reach $(L \equiv 2^{-n} L_{\max}, \bar{g}^2(L))$.

One can finally extract $\Lambda_{\rm SF} L_{\max}$ and, more interesting for phenomenology, $\Lambda_{\overline{\rm MS}}$.

One needs a physical input to convert the numbers obtained on the lattice to physical units. Ambiguity: r_0, f_K or f_π at $\rm N_f = 2$, m_Ω at $N_f = 2+1$.

[S. Aoki *et al*, '09]



$\Lambda_{\overline{\rm MS}}^{\rm N_f=3} = 340(30)$ MeV $\quad \alpha_s(m_Z) = 0.12047(81)(48)(^{\ 0}_{-173})$

$r_0 \Lambda_{\overline{\rm MS}}^{\rm N_f=2} = 0.73(3)(5)$ [F. Knechtli and B. Leder, '10]

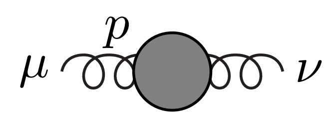
$\Lambda_{\overline{\rm MS}}^{\rm N_f=2} = 316(26)(17)$ MeV [M. Marinkovic *et al*, '11]

Fixed gauge approach

Another very popular way to extract α_s is the analysis of Green functions. It is necessary to fix the gauge so that $\langle O_{q,G,F} \rangle \neq 0$.

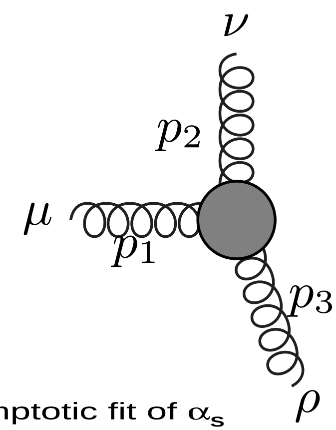
3-gluon vertex [B. Alles *et al*, '97; Ph. Boucaud *et al*, '98, '01]

$A_\mu(x+\hat{\mu}/2) = \left[\frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0}\right]_{\text{traceless}} \qquad A_\mu(p) = \int e^{ipx} A_\mu(x)$



– Consider $G^{(2)}_{\mu\nu}(p) = \langle A_\mu(p) A_\nu(p) \rangle \equiv G(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$ and

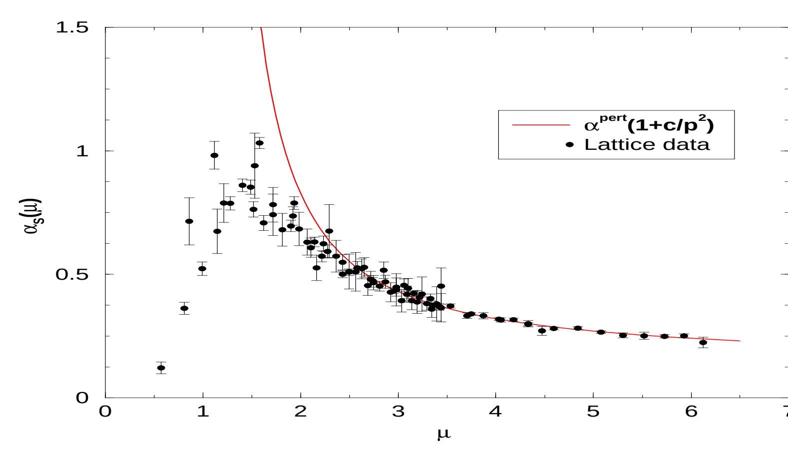
$$G^{(3)}_{\mu\nu\rho}(p_1,p_2,p_3) = \langle A_\mu(p_1) A_\nu(p_2) A_\rho(p_3) \rangle$$
$$\equiv \Gamma_{\alpha\beta\gamma}(p_1,p_2,p_3) G^{(2)}_{\alpha\mu}(p_1) G^{(2)}_{\beta\nu}(p_2) G^{(2)}_{\gamma\rho}(p_3)$$



– RI-MOM renormalisation scheme: $Z_3^{-1}(\mu) G(p)|_{p^2=\mu^2} = \frac{1}{\mu^2}$ and

$\frac{\sum_{\alpha=1}^{4} G^{(3)}_{\alpha\beta\alpha}(p,0,-p)}{G^2(p)G(0)} = 6iZ_1^{-1}(p) g_0 p_\beta$ ($\widetilde{\text{MOM}}$).

Asymptotic fit of α_s



– The renormalised coupling is then defined by

$g_R(\mu) = Z_3^{3/2}(\mu) Z_1^{-1}(\mu) g_0$.

– One fits $\alpha_s^{\text{Latt}}(\mu^2) = \alpha_{s,\text{pert}}(\mu^2)\left(1 + \frac{c}{\mu^2}\right)$

– From configurations with $\mathrm{N_f} = 2$ Wilson fermions, it was found $\alpha_s(m_Z) = 0.113(3)(4)$.

Ghost-gluon vertex [A. Sternbeck *et al*, '07; Ph. Boucaud *et al*, '08; B. Blossier *et al*, '11, '12]

In Landau gauge, bare gluon and ghost propagators read

$$\left(G^{(2)}\right)^{ab}_{\mu\nu}(p^2,\Lambda) = \frac{G(p^2,\Lambda)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \qquad \left(F^{(2)}\right)^{ab}(p^2,\Lambda) = -\delta_{ab}\,\frac{F(p^2,\Lambda)}{p^2}$$

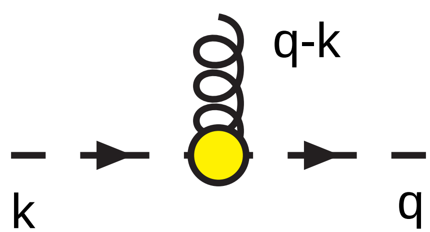
Renormalized dressing functions G_R and F_R are defined through

$$G_R(p^2,\mu^2) \;=\; \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda) \qquad F_R(p^2,\mu^2) \;=\; \lim_{\Lambda\to\infty} \widetilde{Z}_3^{-1}(\mu^2,\Lambda)\,F(p^2,\Lambda)$$

$$G_R(\mu^2,\mu^2) = F_R(\mu^2,\mu^2) = 1$$

The amputated ghost-gluon vertex is given by

$$\widetilde{\Gamma}^{abc}_\nu(-q,k;q-k) =$$



$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

The renormalised vertex is $\widetilde{\Gamma}_R = \widetilde{Z}_1\Gamma$ MOM prescription:

$$\left(H_1^R(q,k) + H_2^R(q,k)\right)\Big|_{q^2=\mu^2} = \lim_{\Lambda\to\infty}\widetilde{Z}_1(\mu^2,\Lambda)\,\left(H_1(q,k;\Lambda) + H_2(q,k;\Lambda)\right)\big|_{q^2=\mu^2} = 1$$

In terms of H_1 and H_2 scalar form factors, one has

$$g_R(\mu^2) = \lim_{\Lambda\to\infty} \widetilde{Z}_3(\mu^2,\Lambda) Z_3^{1/2}(\mu^2,\Lambda) g_0(\Lambda^2) \left(H_1(q,k;\Lambda) + H_2(q,k;\Lambda) \right)\Big|_{q^2\equiv\mu^2}$$

$$= \lim_{\Lambda\to\infty} g_0(\Lambda^2) \frac{Z_3^{1/2}(\mu^2,\Lambda^2)\widetilde{Z}_3(\mu^2,\Lambda^2)}{\widetilde{Z}_1(\mu^2,\Lambda^2)}$$

The case of MOM scheme with zero incoming ghost momentum corresponds to a kinematical configurations where the non-renormalisation theorem by Taylor applies [J. Taylor, '71]

$$\widetilde{\Gamma}_\nu^{abc}(-q,0;q) = i g_0 f^{abc} \left(H_1(q,0) + H_2(q,0) \right) q_\nu \quad H_1(q,0;\Lambda) + H_2(q,0;\Lambda) = 1 \quad \widetilde{Z}_1(\mu^2) = 1$$

Taylor scheme: $\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2,\Lambda^2) F^2(\mu^2,\Lambda^2)$

☺ Only gluon and ghost propagators are involved to extract $\alpha_T(\mu^2)$, the ghost-gluon vertex is not required. $\widetilde{Z_1}$ is equal to 1 only in Taylor scheme.

Several steps are necessary to get from lattice simulations $\alpha_T(\mu^2)$ with a good control on systematic errors.

ETMC $\mathrm{N_f}$=2+1+1 ensembles: $a^{\beta=2.1} \sim 0.06$ fm, $a^{\beta=1.95} \sim 0.08$ fm, $a^{\beta=1.9} \sim 0.09$ fm, $m_\pi \in$ [250-325] MeV

Landau gauge is obtained by standard methods: minimisation of $A^\mu A_\mu$, overrelaxation, Fourier acceleration

Ghost propagator $F^{(2)}(x-y)\delta^{ab} = \left\langle \left[\left(\mathcal{M}_{FP}^{\mathrm{lat}} \right)^{-1} \right] \right\rangle$, lattice Faddeev-Popov $\mathcal{M}_{FP}^{\mathrm{lat}}$ defined by

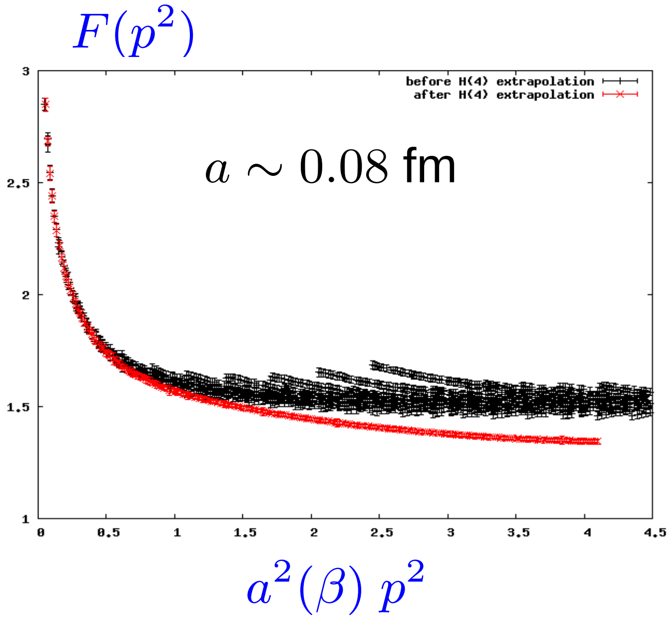
$$\begin{aligned} \mathcal{M}_{FP}^{\mathrm{lat}} \omega \quad &= \quad \frac{1}{V} \sum_\mu \left\{ G_\mu^{ab}(x) \left(\omega^b(x+e_\mu)\omega^b(x) - (x \leftrightarrow x - e_\mu) \right) \right. \\ & \left. + \frac{1}{2} f^{abc} \omega^b(x+e_\mu) A_\mu^c \left(x + \frac{e_\mu}{2} \right) - \omega^b(x-e_\mu) A_\mu^c \left(x - \frac{e_\mu}{2} \right) \right\} \end{aligned}$$

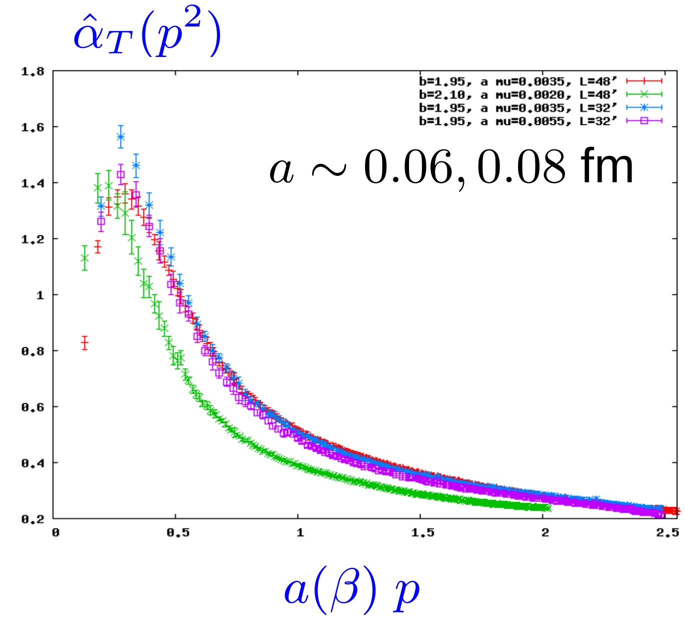
$$G_\mu^{ab}(x) = -\frac{1}{2} \mathrm{Tr} \left[\left\{ t^a, t^b \right\} \left(U_\mu(x) + U_\mu^\dagger(x) \right) \right] \quad A_\mu^c \left(x + \frac{e_\mu}{2} \right) = \mathrm{Tr} \left[t^c \left(U_\mu(x) - U_\mu^\dagger(x) \right) \right]$$

It is crucial to properly eliminate lattice artifacts: $\mathcal{O}(a^2p^2)$ (O(4) invariant) but remove also H(4) invariant artifacts [F. de Soto and C. Roiesnel, '07]:

$$\alpha_T^{\mathrm{Latt}} \left(a^2p^2, a^2 \frac{p^{[4]}}{p^2}, \ldots \right) \; = \; \widehat{\alpha}_T(a^2p^2) \; + \; \left. \frac{\partial \alpha_T^{\mathrm{Latt}}}{\partial \left(a^2 \frac{p^{[4]}}{p^2} \right)} \right|_{a^2 \frac{p^{[4]}}{p^2}=0} a^2 \frac{p^{[4]}}{p^2} \; + \; \ldots \quad p^{[4]} = \sum_i p_i^4$$

"fishbone" structure clearly visible for F small statistical errors on $\hat{\alpha}_T(p^2)$





Remaining $\mathcal{O}(a^2p^2)$ artifacts are taken into account by fitting data according to the formula

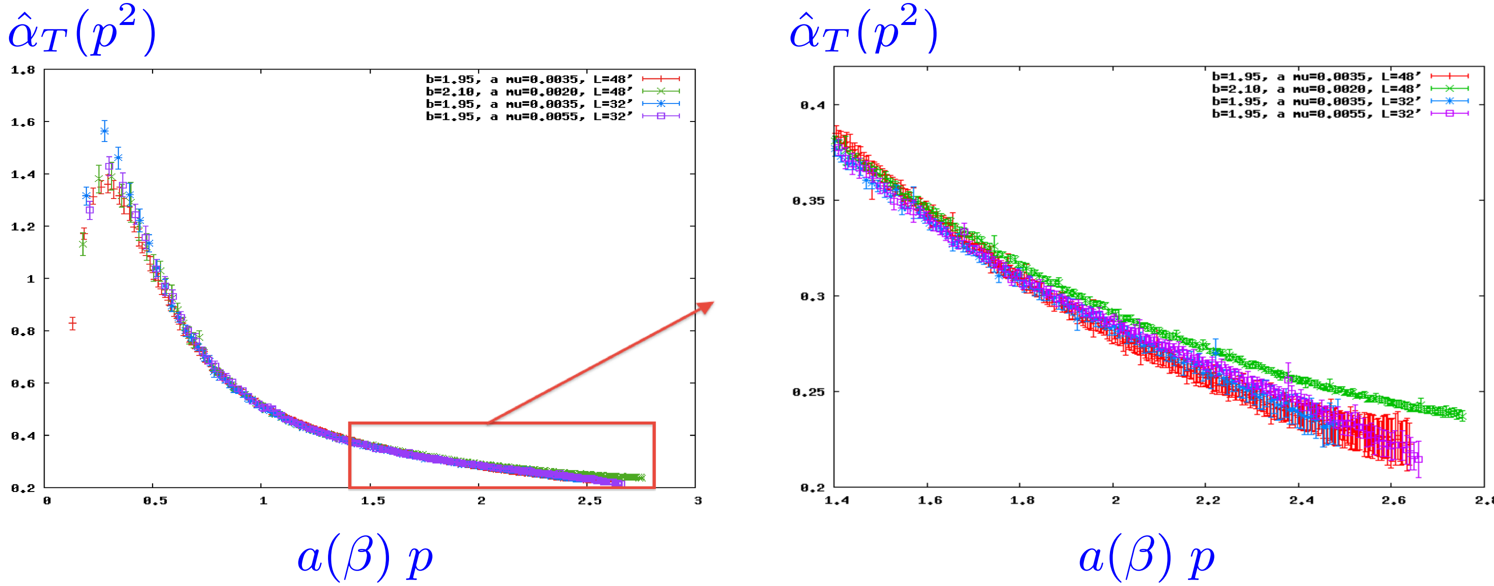
$$\widehat{\alpha}_T(a^2p^2) \; = \; \alpha_T(p^2) \; + \; c_{a2p2}\, a^2p^2 \; + \; \mathcal{O}(a^4)$$

Power corrections to the OPE read

$$\alpha_T(\mu^2) = \alpha_T^{\rm pert}(\mu^2) \left[1 + \frac{9}{\mu^2}\, R\left(\alpha_T^{\rm pert}(\mu^2), \alpha_T^{\rm pert}(q_0^2)\right) \left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)} \right)^{1-\gamma_0^{A^2}/\beta_0} \frac{g_T^2(q_0^2)\langle A^2 \rangle_{q_0^2}^R}{4(N_C^2-1)} \right]$$

Wilson coefficient ($q_0 = 10\,$GeV): $R\left(\alpha, \alpha_0\right) = \left(1 + 1.18692\alpha + 1.45026\alpha^2 + 2.44980\alpha^3\right)$
$\times \left(1 - 0.54994\alpha_0 - 0.13349\alpha_0^2 - 0.10955\alpha_0^3\right)$

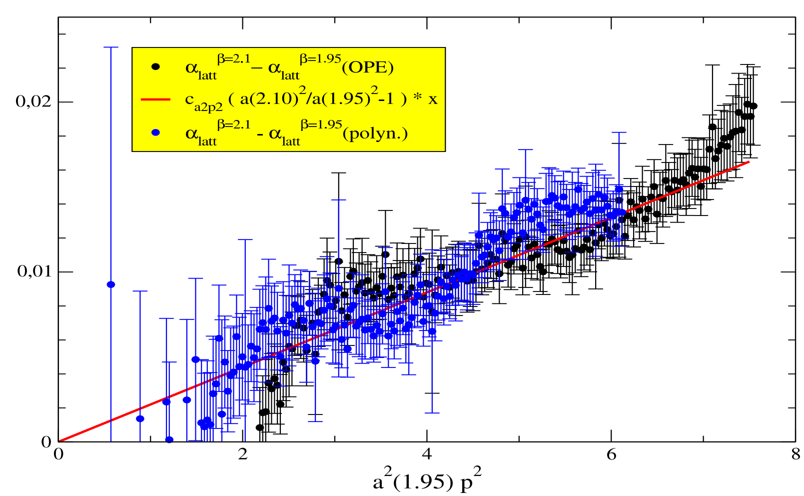
Finally $\alpha_T^{\rm pert}$ is expressed in function of Λ_T with $\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} = \exp\left(-\frac{507-40N_f}{792-48N_f}\right)$



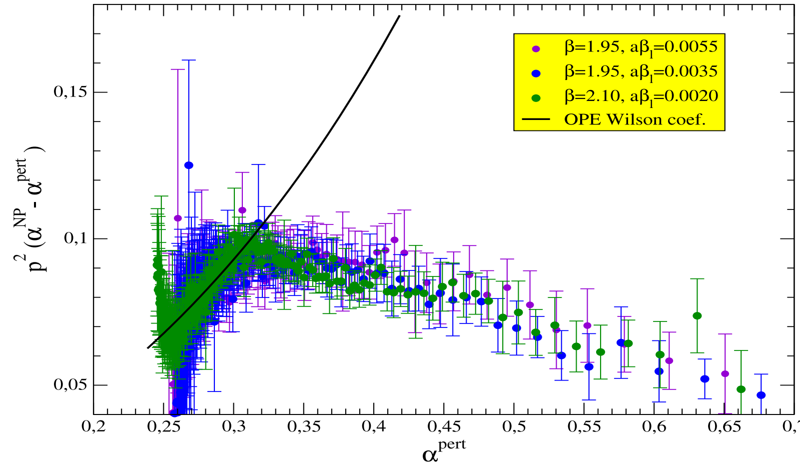
Data from different masses and lattice spacings merged by rescaling the momenta to a common unit. Rescaling factors enter the data fit, as well as $a(\beta)\Lambda_{\overline{\mathrm{MS}}}$, $a^2(\beta)g^2\langle A^2\rangle$ and $c_{a^2p^2}$; a-independence of $c_{a^2p^2}$ and the smallness of higher order cut-off effects is checked:

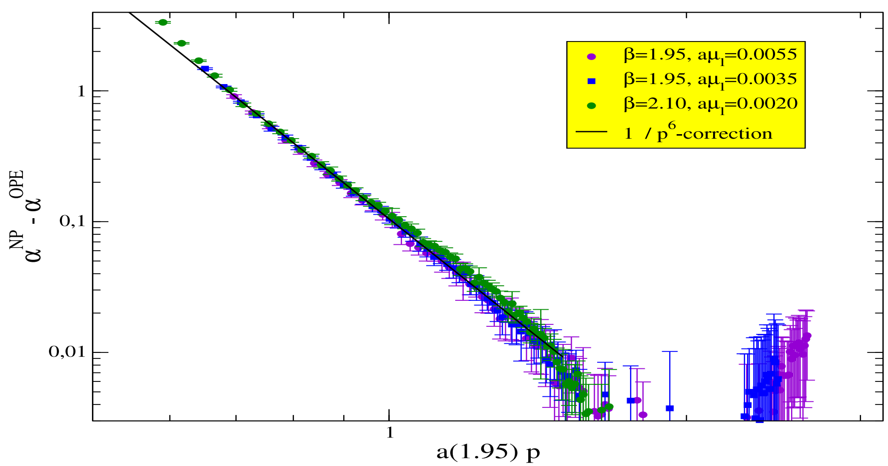
$$\alpha_{\mathrm{Latt}}^{\beta=2.1}\left(a(1.95)p\right) - \alpha_{\mathrm{Latt}}^{\beta=1.95}\left(a(1.95)p\right) = \left(\frac{a^2(2.1)}{a^2(1.95)} - 1\right) c_{a2p2}\ a^2(1.95)p^2 + o\left(a^2(1.95)p^2\right)$$

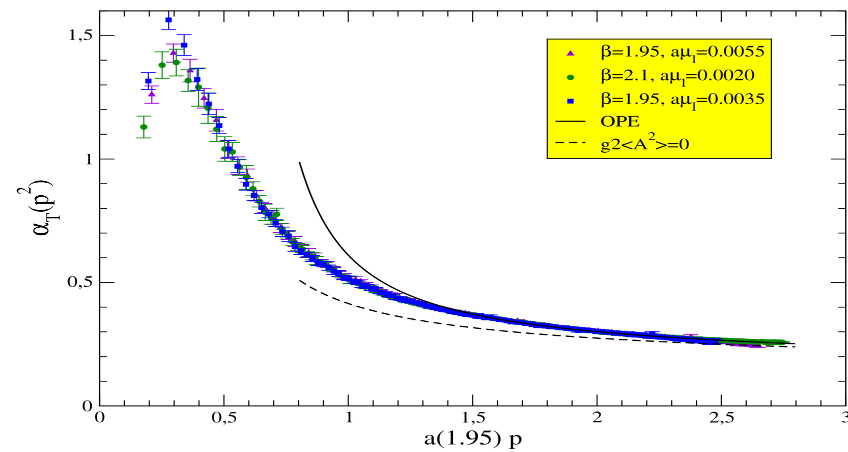
At low ap the precise interpolating $\mathcal{O}(a^0)$ formula to match data from different a is not crucial.



Our data confirm, once more, the necessity to include a (gauge-artifact) power correction.



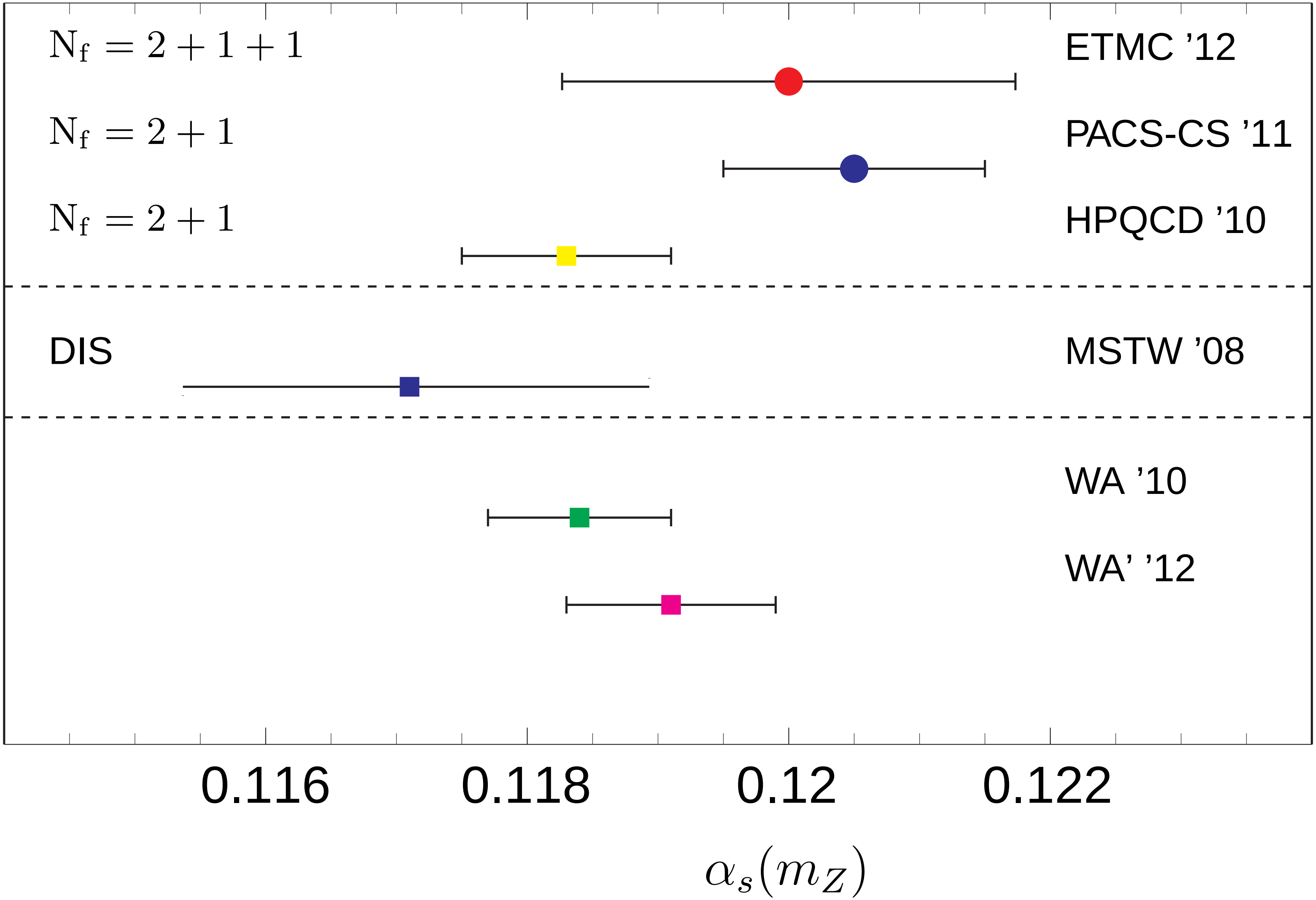




$a(1.95)p \leq 1.5$, need to include a d/p^6 term (at this point, no physical interpretation)

$\Lambda_{\overline{\rm MS}}^{N_f=4}$ (MeV)	$g^2(q_0^2)\langle A^2\rangle^R_{q_0^2}$ (GeV^2)	$d^{1/6}$ (GeV)	$\alpha(m_Z)$
316(13)	4.5(4)		0.1198(9)
324(17)	3.8(1.0)	1.72(3)	0.1203(11)

Collection of results

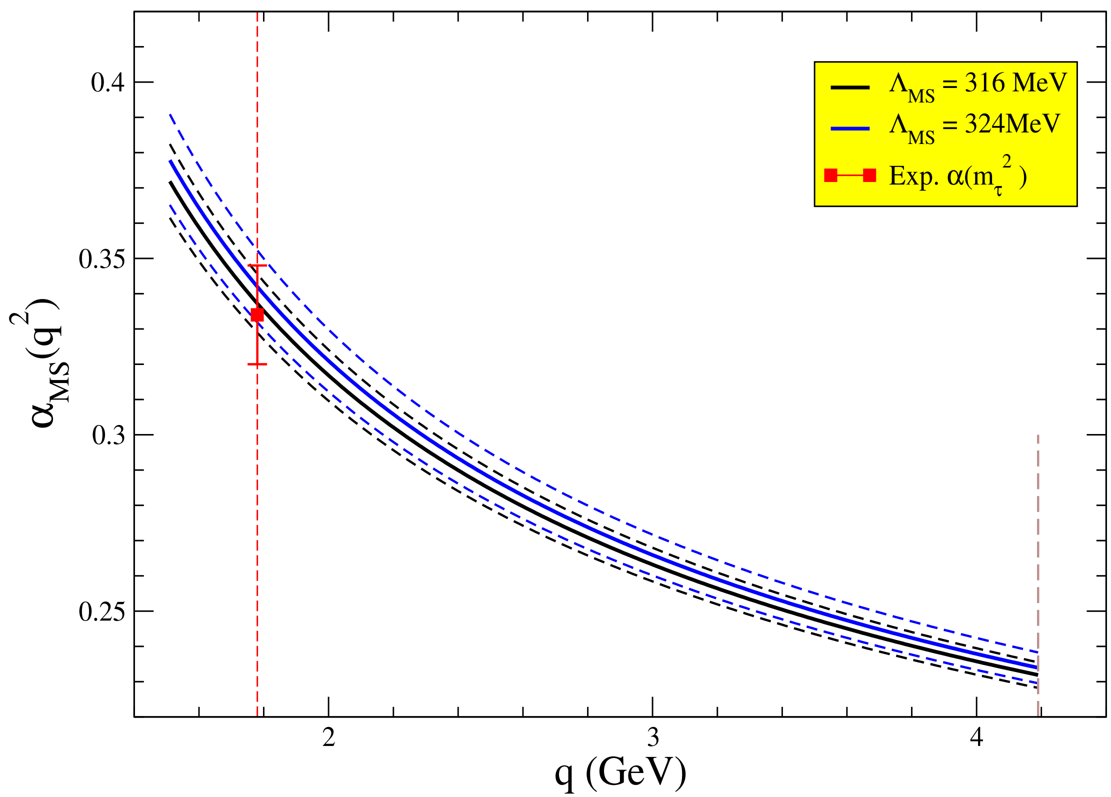


WA: PDG world average

WA': world average replacing $N_f = 2+1$ lattice results by $N_f = 2+1+1$

A last word about $\alpha(m_\tau)$

Nice agreement between lattice data and phenomenological analyses: $\alpha_s(m_\tau) = 0.339(13)$



Outlook

- As the production at LHC of energetic particles like the Higgs boson comes from $p\,p$ collisions, it is welcome to reduce as much as possible the QCD part of the uncertainty on their theoretical production rate, in order to well estimate the detectors sensitivity to their physics.

- Beyond the uncertainty on PDF's and intermediate ingredients in the computations, like the factorisation scale, a non negligible part of the theoretical error comes from α_s.

- Several experimental ways to determine α_s exist: DIS, event-shape. Quite large uncertainty $\sim 2.5\%$ for the latter (hadronisation effects,...). τ decay analysis by OPE (discrepancy of 1.5 σ between CIPT and FOPT, 6% of uncertainty).

- Lattice QCD is appropriate to extract α_s. Popular methods consists in studying short-distance related quantities ($\mathrm{Q\bar{Q}}$ potential at small r, moments of 2-pt $c\bar{c}$ correlators) or defining a renormalised coupling such that integration of β function at discrete points is possible.

- A complementary approach is the analysis of gluon and ghost Green functions: three gluons vertex and ghost gluon vertex (small statistical error). For the first time α_s has been extracted without an *ad-hoc* treatment of the charm threshold ($\mathrm{N_f} = 2+1+1$ ensembles). Data indicate the need to subtract a $1/p^2$ power correction to the OPE below 5 GeV and a $1/p^6$ term below 3 GeV.