Lattice QCD measurement of the strong coupling constant

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• Phenomenological considerations
• Hints of lattice QCD
• Hadronic and finite volume schemes
• Fixed gauge approach
Phenomenological considerations

A major activity in Particle Physics is nowadays the search of Higgs boson, whose the existence might explain the spontaneous symmetry breaking of $SU(2)_W \times U(1)_Y$ predicted by the Standard Model and observed in Nature.

[ATLAS, ’12; Lepton-Photon ’11]

ATLAS has excluded at 95% of CL the region $131 < m_H < 238$ GeV (and also the mass range $251 < m_H < 466$ GeV). Hint of a signal around 125 GeV, both for ATLAS and CMS, in $h \rightarrow \gamma\gamma$ and $h \rightarrow 4l$. 
Different modes of Higgs boson production:

**Gluon fusion**

![Gluon fusion diagram](image)

**Associated production with \( \bar{Q}Q \)**

![Associated production with \( \bar{Q}Q \) diagram](image)

**Higgs strahlung**

![Higgs strahlung diagram](image)

**Vector boson fusion**

![Vector boson fusion diagram](image)

Gluon fusion highly favored w.r.t. other Higgs production processes

Good hope to observe a SM Higgs, if existing, at LHC
Estimating as accurately as possible $\sigma_{gg \rightarrow H \rightarrow X}^{th}$ is an important ingredient to assess the detectors sensitivity to the Higgs physics. Several sources of uncertainty:

- NNNLO (QCD) and NNLO (EW) corrections
- factorisation scale uncertainties
- error on $H \rightarrow X$
- parton distribution functions and $\delta(\alpha_s)$

\[
\Delta \sigma_{gg \rightarrow H \rightarrow X}^{NNLO} \sim 20 - 25\% \text{ at LHC (}\sqrt{s} = 7 \text{ TeV)}, \text{ with 2-3 } \% \text{ from } \delta\alpha_s
\]
Plenty of $\alpha_s$ estimates based on experimental data analysis.

### Parton Distribution Function in DIS

\[
\alpha_s^{\text{NNLO, DIS}}(m_Z) = 0.1171(14) \quad 68\% \text{ CL}
\]

Event-shape in $e^+e^-$ collisions

\[\ldots\]
Phenomenological analysis of the $\tau$ decay into hadrons provides another way to extract $\alpha_s$. 

\[ R_\tau \equiv \frac{\Gamma[\tau^- \to \nu_\tau \text{ hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]} \]

\[ R_{\tau, V+A} = N_c |V_{ud}|^2 S_{EW} (1 + \delta_P + \delta_{NP}), \quad \delta_{NP} = -0.0059(14) \quad [\text{Davier et al, '08}] \]

\[ \delta_P = \sum_n K_n A^{(n)}(\alpha_s) = \sum_n (K_n + g_n)\alpha^n(m_\tau) \]

\[ A^{(n)} = \frac{1}{2\pi i} \int_{|s|=m_\tau^2} ds \left( \frac{\alpha_s(-s)}{\pi} \right)^n \left( 1 - \frac{s}{m_\tau^2} + \frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right) = \alpha^n(m_\tau) + O(\alpha^{n+1}(m_\tau)) \]

Fixed Order Perturbation Theory (FOPT) vs Contour Improved Perturbation Theory (CIPT): 
\[ \alpha_s(m_\tau)_{\text{CIPT}} = 0.344(14) \quad \alpha_s(m_\tau)_{\text{FOPT}} = 0.321(15) \quad [\text{A Pich, '11}] \]
Hints of lattice QCD

Discretisation of QCD in a finite volume of Euclidean space-time.

The lattice spacing $a$ is a non perturbative UV cut-off of the theory.

Fields: $\psi^i(x)$, $U_\mu(x) \equiv e^{iag_0A_\mu(x+a^\mu/2)}$.

Inputs: bare coupling $g_0(a) \equiv \sqrt{6/\beta}$, bare quark masses $m_i$.

Computation of Green functions of the theory from first principles:

\[
\langle O(U, \psi, \bar{\psi}) \rangle = \frac{1}{Z} \int DU D\psi D\bar{\psi} O(U, \psi, \bar{\psi}) e^{-S(U, \psi, \bar{\psi})}
\]

\[
Z = \int DU D\psi D\bar{\psi} e^{-S(U, \psi, \bar{\psi})}
\]

\[
S(U, \psi, \bar{\psi}) = S^{YM}(U) + \bar{\psi}_x M^{ij}_{xy}(U) \psi^j
\]

\[
Z = \int DU \text{Det}[M(U)] e^{-S^{YM}(U)} \equiv \int DU e^{-S_{\text{eff}}(U)}
\]

Monte Carlo simulation: $\langle O \rangle \sim \frac{1}{N_{\text{conf}}} \sum_i O\{U\}_i$:

we have to build the statistical sample $\{U\}_i$ in function of the Boltzmann weight $e^{-S_{\text{eff}}}$. Incorporating the quark loop effects hidden in $\text{Det}[M(U)]$ is particularly expensive in computer time. Crucial in the extraction of $\alpha_s$. 
Simulations set up

In the past years tremendous progresses have been made by the lattice community to perform simulations that are closer to the physical point.

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>( N_f )</th>
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<tr>
<td>ETMC</td>
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![Graph showing data points and parameters](image-url)
Improvements in algorithms

2001: \( N_{\text{op}} = k_1 \left( \frac{20 \text{ MeV}}{m_q} \right)^3 \left( \frac{L}{3 \text{ fm}} \right)^5 \left( \frac{0.1 \text{ fm}}{a} \right)^7 \) TFlops × years ("Berlin Wall")

2007: \( N_{\text{op}} = 0.01 k_1 \left( \frac{20 \text{ MeV}}{m_q} \right)^1 \left( \frac{L}{3 \text{ fm}} \right)^5 \left( \frac{0.1 \text{ fm}}{a} \right)^6 \) TFlops × years (deflation)

Regularisation of quarks and gluons with a smooth spectrum in the UV regime, mass shift, multi-step/Omelyan integrators, e/o preconditioning, domain decomposition, deflation,...

[M. Lüscher, '03] [M. Lüscher, '06]
Hadronic and finite volume schemes

Those schemes are based on hadronic quantities to fix the parameters (quark masses, renormalised coupling). It is not necessary to fix the gauge.

Hadronic schemes I [C. Davies et al, '08]

- $u/d$, $s$ and $c$ quark masses are tuned from $m_\pi$, $2m_K^2 - m_\pi^2$ and $m_\eta_c$ while the lattice spacing is extracted from the $\Upsilon$ spectrum.

- One computes Wilson loops $\mathcal{W}_{m,n}$ that one develops at short distance $r = a/d$ in perturbation theory: $\mathcal{W}_{m,n} = \sum_{i=1}^{\infty} c_i \alpha_V^i (d/a)$.

- $\alpha_V(q)$ defined by $V(q) = \frac{C_f 4 \pi \alpha_V(q)}{q^2}$ (one-gluon exchange part of the potential) [P. Lepage and P. McKenzie, '92].

- The series converges better by considering $\ln(\mathcal{W}_{m,n})$, $\ln(\mathcal{W}_{m,n}') \equiv \ln[\mathcal{W}_{m,n}/\mathcal{W}_{11}^{(m+n)/2}]$ (tadpole improvement), and even better, Creutz ratios $\ln \left( \frac{\mathcal{W}_{m,n+1}}{\mathcal{W}_{m,n}} \frac{\mathcal{W}_{m-1,n+1}}{\mathcal{W}_{m-1,n}} \right)$.

- One subtracts a gluon-condensate term $-\frac{\pi^2}{36} A^2 \langle \alpha_s G^2 / \pi \rangle$ to the lattice results to compare with perturbation theory.

- Running of $\alpha_V(d/a)$ to $\alpha_0 \equiv \alpha_V(7.5 \text{ GeV})$, conversion to $\alpha_{\overline{\text{MS}}}(m_Z, N_f = 5) = 0.1183(8)$. 
Hadronic schemes II [I. Allison et al, ’08, C. McNeile et al, ’10]

– Strategy to tune the bare quark mass parameters and $a$ already discussed.

– Consider $G(t) = a^6 \sum_x (a m_{0h})^2 \langle J_5(x, t) J_5(\vec{0}, 0) \rangle$, $J_5 = \bar{\psi}_c \gamma^5 \psi_c$.

– Define the $n^{th}$ moment of the correlator $G_n = (t/a)^n \sum_t G(t)$, its counterpart $G_n^{(0)}$ known at lowest order of perturbation theory, $R_4 \equiv G_4/G_4^{(0)}$ and $R_{n \geq 6} = \frac{a m_{0h}}{a m_{0h}} \left( \frac{G_n}{G_n^{(0)}} \right)^{1/(n-4)}$.

– Those $R_n$ have an expression in continuum perturbation theory: $R_4 = r_4(\alpha_{MS}, \mu/m_h)$ and $R_{n \geq 6} = \frac{r_n(\alpha_{MS}, \mu/m_h)}{2 m_h(\mu)/m_{\eta h}}$.

– Power corrections to the perturbative expression of moments are taken into account by including a factor $1 + d_n \langle \alpha_s G^2 / \pi \rangle / (2 m_h)^4$.

Despite the errant structure of cut-off effects in $R_n$, one can extract from that analysis $\alpha_{MS}(m_Z, N_f = 5) = 0.1183(7)$. 

![Graph](image-url)
Finite volume scheme

Partition function: \[ \mathcal{Z}[C, C'] = \langle C' | e^{-H^T} | C \rangle \] [K. Symanzik, '81]

\( C(x_0 = 0) \) and \( C'(x_0 = T) \) are 2 field configurations that are given.

The Schrödinger Functional \( \mathcal{Z} \) is renormalisable with Yang-Mills theories. [M. Lüscher et al, '92]

The associated renormalisation scheme is of finite volume kind and regularisation independent:

\[
\Gamma(B) \equiv - \ln \mathcal{Z}[C, C'] = g_0^{-2} \Gamma_0[B] + \Gamma_1[B] + g_0^2 \Gamma_2[B] + \ldots \quad \left. \frac{\delta S}{\delta \Phi} \right|_{\Phi = B} = 0
\]

\[
C^{(')} \equiv C^{(')}(\eta) \quad \bar{g}^2(L) = \left[ \frac{\partial \Gamma_0(B)}{\partial \eta} \right] / \left[ \frac{\partial \Gamma(B)}{\partial \eta} \right] \bigg|_{\eta=0} \quad \bar{g}^2(L) = \left. \left\langle \frac{\partial S}{\partial \eta} \right\rangle \right|_{\eta=0}
\]
The running of the coupling constant is obtained from step scaling functions
\[ \sigma(u) = \lim_{a/L \to 0} \Sigma(u, a/L), \quad \Sigma(u, a/L) = \bar{g}^2(2L)\bar{g}^2(L) = u; \]
\[ \sigma(u) \] is an integrated \( \beta \) function at discrete points.

Several tuning simulations are necessary to fix \( \bar{g}^2(L) = u \) for a given \( L/a \).

Computation of the RGI \( \Lambda \) scale [M. Lüscher et al, '93]:

\[ \Lambda_{SF} = \frac{1}{L} \left[ \beta_0 \bar{g}(L) \right]^{-\frac{\beta_1}{2\beta_0^2}} \exp \left( -\frac{1}{2\beta_0 \bar{g}(L)} \right) \exp \left[ -\int_0^{\bar{g}(L)} dg \left( \frac{1}{\beta(g)} + \frac{1}{\beta_0 g^3} - \frac{\beta_1}{\beta_0^2 g} \right) \right] \]

\[ \beta(g) = -g^3(\beta_0 + \beta_1 g^2 + \beta_2 g^4 + \cdots) \]
Starting from \((L_{\text{max}}, \bar{g}^2(L_{\text{max}}))\), one uses the step scaling functions to follow the RG flow and reach \((L \equiv 2^{-n} L_{\text{max}}, \bar{g}^2(L))\).

One can finally extract \(\Lambda_{\text{SF}} L_{\text{max}}\) and, more interesting for phenomenology, \(\Lambda_{\overline{\text{MS}}}\).

One needs a physical input to convert the numbers obtained on the lattice to physical units. Ambiguity: \(r_0, f_K\) or \(f_\pi\) at \(N_f = 2\), \(m_\Omega\) at \(N_f = 2 + 1\).

\[\Lambda_{\overline{\text{MS}}} = 340(30) \text{ MeV} \quad \alpha_s(m_Z) = 0.12047(81)(48)(-173)\]

\[r_0 \Lambda_{\overline{\text{MS}}}^{N_f=2} = 0.73(3)(5) \quad \text{[F. Knechtli and B. Leder, '10]}\]

\[\Lambda_{\overline{\text{MS}}}^{N_f=2} = 316(26)(17) \text{ MeV} \quad \text{[M. Marinkovic et al, '11]}\]
Fixed gauge approach

Another very popular way to extract $\alpha_s$ is the analysis of Green functions. It is necessary to fix the gauge so that $\langle O_{q,G,F} \rangle \neq 0$.

**3-gluon vertex** [B. Alles *et al*, '97; Ph. Boucaud *et al*, '98, '01]

$$A_\mu(x + \hat{\mu}/2) = \left[ \frac{U_\mu(x) - U_\mu^\dagger(x)}{2i a_0} \right]_{\text{traceless}} A_\mu(p) = \int e^{ipx} A_\mu(x)$$

- Consider $G_{\mu\nu}^{(2)}(p) = \langle A_\mu(p) A_\nu(p) \rangle \equiv G(p^2) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$ and

  $$G_{\mu\nu\rho}^{(3)}(p_1, p_2, p_3) = \langle A_\mu(p_1) A_\nu(p_2) A_\rho(p_3) \rangle$$

  $$\equiv \Gamma_{\alpha\beta\gamma}(p_1, p_2, p_3) G_{\alpha\mu}^{(2)}(p_1) G_{\beta\nu}^{(2)}(p_2) G_{\gamma\rho}^{(2)}(p_3)$$

- RI-MOM renormalisation scheme: $Z_3^{-1}(\mu) G(p)|_{p^2 = \mu^2} = \frac{1}{\mu^2}$ and

  $$\sum_{\alpha=1}^4 G_{\alpha\beta\alpha}(p,0,-p) = 6i Z_1^{-1}(p) g_0 p_\beta \text{ (MOM)}.$$  

- The renormalised coupling is then defined by

  $$g_R(\mu) = Z_3^{3/2}(\mu) Z_1^{-1}(\mu) g_0.$$

- One fits $\alpha_s^{\text{Latt}}(\mu^2) = \alpha_{s,\text{pert}}(\mu^2) \left( 1 + \frac{c}{\mu^2} \right)$

- From configurations with $N_f = 2$ Wilson fermions, it was found $\alpha_s(m_Z) = 0.113(3)(4)$. 

[Diagram of 3-gluon vertex]
**Ghost-gluon vertex** [A. Sternbeck *et al*, '07; Ph. Boucaud *et al*, '08; B. Blossier *et al*, '11, '12]

In Landau gauge, bare gluon and ghost propagators read

\[
\left( G^{(2)} \right)^{ab \mu \nu}_{p^2}(p^2, \Lambda) = \frac{G(p^2, \Lambda)}{p^2} \delta_{ab} \left( \delta_{\mu \nu} - \frac{p_\mu p_\nu}{p^2} \right) \quad \left( F^{(2)} \right)^{ab}_{p^2}(p^2, \Lambda) = -\delta_{ab} \frac{F(p^2, \Lambda)}{p^2}
\]

Renormalized dressing functions \( G_R \) and \( F_R \) are defined through

\[
G_R(p^2, \mu^2) = \lim_{\Lambda \to \infty} Z_3^{-1}(\mu^2, \Lambda) G(p^2, \Lambda) \quad F_R(p^2, \mu^2) = \lim_{\Lambda \to \infty} \tilde{Z}_3^{-1}(\mu^2, \Lambda) F(p^2, \Lambda)
\]

\[
G_R(\mu^2, \mu^2) = F_R(\mu^2, \mu^2) = 1
\]

The amputated ghost-gluon vertex is given by

\[
\tilde{\Gamma}^{abc\nu}_{\mu}(q, k; q-k) = -\Gamma^{abc\nu}_{\mu} = ig_0 f^{abc}_{\nu} (q_\nu H_1(q, k) + (q-k)_\nu H_2(q, k))
\]

The renormalised vertex is \( \tilde{\Gamma}_R = \tilde{Z}_1 \Gamma \) \quad MOM prescription:

\[
(H_1^R(q, k) + H_2^R(q, k)) \bigg|_{q^2=\mu^2} = \lim_{\Lambda \to \infty} \tilde{Z}_1(\mu^2, \Lambda) (H_1(q, k; \Lambda) + H_2(q, k; \Lambda)) \bigg|_{q^2=\mu^2} = 1
\]
In terms of $H_1$ and $H_2$ scalar form factors, one has

$$g_R(\mu^2) = \lim_{\Lambda \to \infty} \frac{Z_3(\mu^2, \Lambda) Z_3^{1/2}(\mu^2, \Lambda) g_0(\Lambda^2)}{Z_1(\mu^2, \Lambda^2)} \left( H_1(q, k; \Lambda) + H_2(q, k; \Lambda) \right)_{q^2 \equiv \mu^2}$$

$$= \lim_{\Lambda \to \infty} g_0(\Lambda^2) \frac{Z_3^{1/2}(\mu^2, \Lambda^2) Z_3(\mu^2, \Lambda^2)}{Z_1(\mu^2, \Lambda^2)}$$

The case of MOM scheme with zero incoming ghost momentum corresponds to a kinematical configurations where the non-renormalisation theorem by Taylor applies [J. Taylor, '71]

$$\tilde{\Gamma}^{abc}_{\nu}(-q, 0; q) = ig_0 f^{abc} (H_1(q, 0) + H_2(q, 0)) q_\nu \quad H_1(q, 0; \Lambda) + H_2(q, 0; \Lambda) = 1 \quad \tilde{Z}_1(\mu^2) = 1$$

Taylor scheme: $\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2, \Lambda^2) F^2(\mu^2, \Lambda^2)$

😊 Only gluon and ghost propagators are involved to extract $\alpha_T(\mu^2)$, the ghost-gluon vertex is not required. $\tilde{Z}_1$ is equal to 1 only in Taylor scheme.

Several steps are necessary to get from lattice simulations $\alpha_T(\mu^2)$ with a good control on systematic errors.
ETMC $N_f=2+1+1$ ensembles: $a^\beta=2.1 \sim 0.06$ fm, $a^\beta=1.95 \sim 0.08$ fm, $a^\beta=1.9 \sim 0.09$ fm, $m_\pi \in [250-325]$ MeV

Landau gauge is obtained by standard methods: minimisation of $A^\mu A_\mu$, overrelaxation, Fourier acceleration

Ghost propagator $F^{(2)}(x-y)\delta^{ab} = \left\langle \left[ \left( \mathcal{M}^{\text{lat}}_{FP} \right)^{-1} \right] \right\rangle$, lattice Faddeev-Popov $\mathcal{M}^{\text{lat}}_{FP}$ defined by

$$\mathcal{M}^{\text{lat}}_{FP} \omega = \frac{1}{V} \sum_\mu \left\{ G^{ab}_\mu(x) \left( \omega^b(x + e_\mu)\omega^b(x) - (x \leftrightarrow x - e_\mu) \right) + \frac{1}{2} f^{abc} \omega^b(x + e_\mu) A^c_\mu \left( x + \frac{e_\mu}{2} \right) - \omega^b(x - e_\mu) A^c_\mu \left( x - \frac{e_\mu}{2} \right) \right\}$$

$$G^{ab}_\mu(x) = -\frac{1}{2} \text{Tr} \left[ \left\{ t^a, t^b \right\} \left( U_\mu(x) + U_\mu^\dagger(x) \right) \right] \quad A^c_\mu \left( x + \frac{e_\mu}{2} \right) = \text{Tr} \left[ t^c \left( U_\mu(x) - U_\mu^\dagger(x) \right) \right]$$

It is crucial to properly eliminate lattice artifacts: $O(a^2 p^2)$ (O(4) invariant) but remove also $H(4)$ invariant artifacts [F. de Soto and C. Roiesnel, ’07]:

$$\alpha_T^{\text{Latt}} \left( a^2 p^2, a^2 \frac{p^{[4]}}{p^2}, \ldots \right) = \hat{\alpha}_T(a^2 p^2) + \left. \frac{\partial \alpha_T^{\text{Latt}}}{\partial \left( a^2 \frac{p^{[4]}}{p^2} \right) \left| a^2 \frac{p^{[4]}}{p^2} = 0 \right.} \right| a^2 \frac{p^{[4]}}{p^2} + \ldots \frac{a^2 p^{[4]}}{p^2} = \sum_i p_i^{[4]}$$
"fishbone" structure clearly visible for $F$ small statistical errors on $\hat{\alpha}_T(p^2)$

$F(p^2)$

$\hat{\alpha}_T(p^2)$

$a \sim 0.08 \text{ fm}$

$a \sim 0.06, 0.08 \text{ fm}$

Remaining $O(a^2p^2)$ artifacts are taken into account by fitting data according to the formula

$$\hat{\alpha}_T(a^2p^2) = \alpha_T(p^2) + c_{a2p2} a^2 p^2 + O(a^4)$$

Power corrections to the OPE read

$$\alpha_T(\mu^2) = \alpha_T^{\text{pert}}(\mu^2) \left[ 1 + \frac{9}{\mu^2} R(\alpha_T^{\text{pert}}(\mu^2), \alpha_T^{\text{pert}}(q_0^2)) \left( \frac{\alpha_T^{\text{pert}}(\mu^2)}{\alpha_T^{\text{pert}}(q_0^2)} \right)^{1-\gamma^A_0/\beta_0} g_T^2(q_0^2) \langle A^2 \rangle R_{q_0^2}^2 \right]$$

Wilson coefficient ($q_0 = 10 \text{ GeV}$): $R(\alpha, \alpha_0) = (1 + 1.18692\alpha + 1.45026\alpha^2 + 2.44980\alpha^3) \times (1 - 0.54994\alpha_0 - 0.13349\alpha_0^2 - 0.10955\alpha_0^3)$

Finally $\alpha_T^{\text{pert}}$ is expressed in function of $\Lambda_T$ with $\frac{\Lambda_{\text{MS}}}{\Lambda_T} = \exp \left( -\frac{507 - 40N_f}{792 - 48N_f} \right)$
Data from different masses and lattice spacings merged by rescaling the momenta to a common unit. Rescaling factors enter the data fit, as well as $a(\beta)\Lambda_{\text{MS}}$, $a^2(\beta)g^2\langle A^2 \rangle$ and $c_a a^2 p^2$; $a$-independence of $c_a a^2 p^2$ and the smallness of higher order cut-off effects is checked:

$$\alpha_{\text{Latt}}^{\beta=2.1}(a(1.95)p) - \alpha_{\text{Latt}}^{\beta=1.95}(a(1.95)p) = \left( \frac{a^2(2.1)}{a^2(1.95)} - 1 \right) c_a a^2 p^2 a^2(1.95) p^2 + o(a^2(1.95) p^2)$$

At low $ap$ the precise interpolating $O(a^0)$ formula to match data from different $a$ is not crucial.
Our data confirm, once more, the necessity to include a (gauge-artifact) power correction.

\[ a(1.95)p \leq 1.5, \text{ need to include a } \frac{d}{p^6} \text{ term (at this point, no physical interpretation)} \]

<table>
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<tr>
<th>( \Lambda_{MS}^{N_f=4} ) (MeV)</th>
<th>( g^2(q_0^2)\langle A^2 \rangle_{q_0^6}^R ) (GeV(^2))</th>
<th>( d^{1/6} ) (GeV)</th>
<th>( \alpha(m_Z) )</th>
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\[ N_f = 2 + 1 + 1 \]

**ETMC ’12**

**N_f = 2 + 1**

**PACS-CS ’11**

**N_f = 2 + 1**

**HPQCD ’10**

**DIS**

**MSTW ’08**

**WA ’10**

**WA ’’12**

\[ \alpha_s(m_Z) \]

**WA**: PDG world average

**WA ’**: world average replacing \( N_f = 2 + 1 \) lattice results by \( N_f = 2 + 1 + 1 \)
A last word about $\alpha(m_\tau)$

Nice agreement between lattice data and phenomenological analyses: $\alpha_s(m_\tau) = 0.339(13)$
Outlook

- As the production at LHC of energetic particles like the Higgs boson comes from $pp$ collisions, it is welcome to reduce as much as possible the QCD part of the uncertainty on their theoretical production rate, in order to well estimate the detectors sensitivity to their physics.

- Beyond the uncertainty on PDF’s and intermediate ingredients in the computations, like the factorisation scale, a non negligible part of the theoretical error comes from $\alpha_s$.

- Several experimental ways to determine $\alpha_s$ exist: DIS, event-shape. Quite large uncertainty $\sim 2.5\%$ for the latter (hadronisation effects,...). $\tau$ decay analysis by OPE (discrepancy of $1.5\sigma$ between CIPT and FOPT, $6\%$ of uncertainty).

- Lattice QCD is appropriate to extract $\alpha_s$. Popular methods consists in studying short-distance related quantities ($Q\bar{Q}$ potential at small $r$, moments of 2-pt $c\bar{c}$ correlators) or defining a renormalised coupling such that integration of $\beta$ function at discrete points is possible.

- A complementary approach is the analysis of gluon and ghost Green functions: three gluons vertex and ghost gluon vertex (small statistical error). For the first time $\alpha_s$ has been extracted without an ad-hoc treatment of the charm threshold ($N_f = 2 + 1 + 1$ ensembles). Data indicate the need to subtract a $1/p^2$ power correction to the OPE below 5 GeV and a $1/p^6$ term below 3 GeV.