The Tight Knot Spectrum in QCD

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Knots, Links, and Physics

Examples of Knots and Links in Physics

Knots and Tight Knots: Theory

Tight Knots/Links in QCD

Other Possible Tight Knot Applications

Introduction: Tight Knots and Physics

- Many Potential Physical Systems can be Knotted or Linked, some tightly
- Examples in Several Areas of Physics

List of Examples

CLASSICAL PHYSICS/BIOPHYSICS

- Plasma Physics
- DNA
- MacroBiology

QUANTUM PHYSICS

- QCD Flux Tubes (tight)
- Superconductors (tight)
- Superfluids and Superfluid Turbulence
- Atomic Condensates
- Cosmic Strings (tight?)

UNIVERSALITY for Tight Quantum Case

Plasma Phys: Magnetic Flux Tubes

- Tubes under Tension Contract
- Minimum Length Determined by Topology
- Taylor States in Plasma
- Differs for QCD where radius is fixed

Plasma Physics

L. Woltjer, PNAS, 44, 489 (1958)

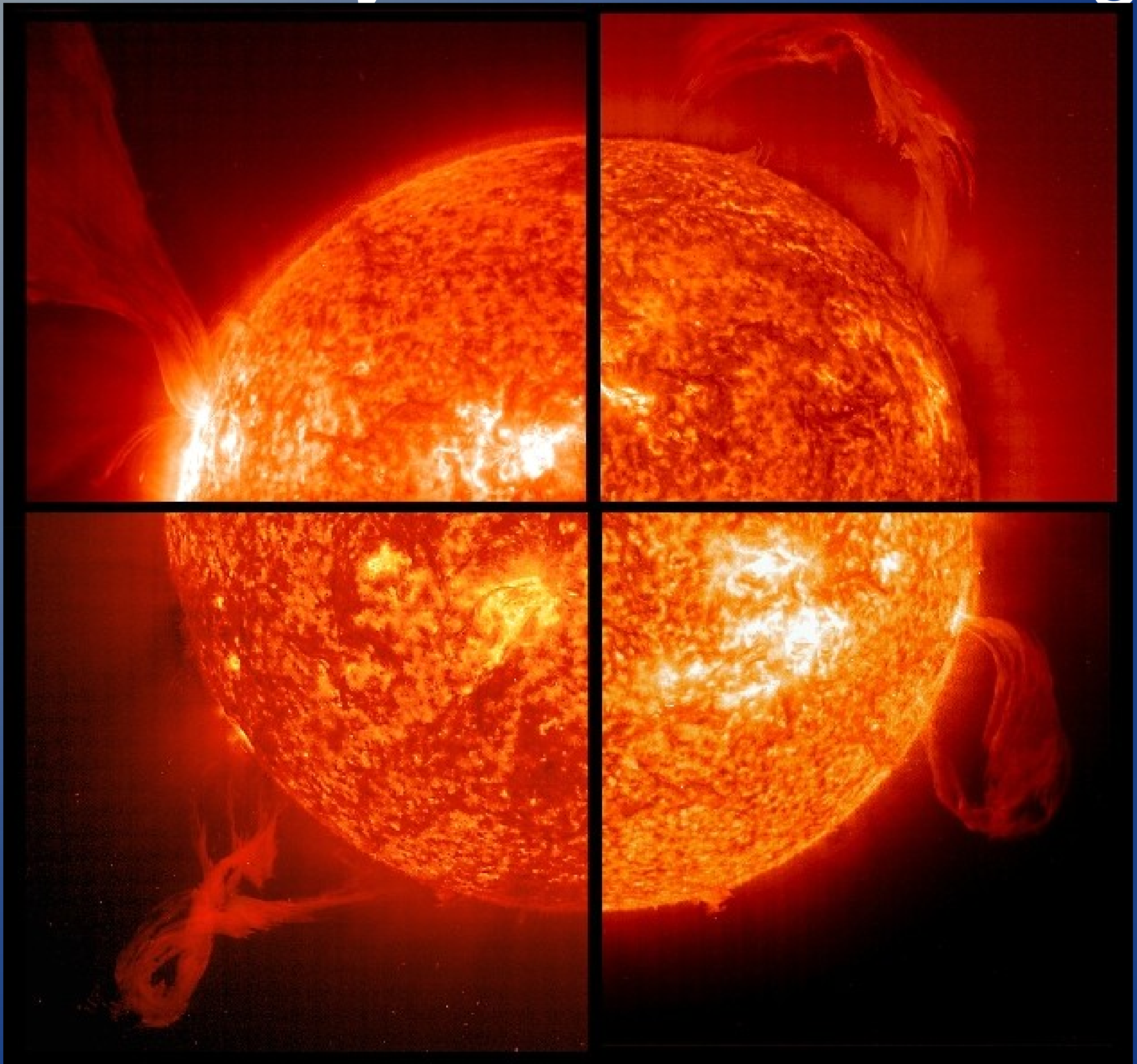
H. K. Moffatt, J. Fluid Mech. 35, 117 (1969)

J. B. Taylor, PRL, 33, 1139 (1974)

Biology

A. Stasiak, et al., Nature 384, 122 (1996)

Plasma Physics: SOHO images



Helicity Conservation:

Energy $E = \frac{1}{2}\int B^2 dV$ Helicity $H = \int B \cdot dA$

Minimize Energy E Holding H Fixed

Find $J = \lambda B$ or $\nabla \times B = \lambda B$

Force Free Configurations

See e.g., P. M. Bellan, ``Spheromaks,'' (2000)

Knot / Link stabilty

Conserved quantum numbers

Gaussian linking--Hopf link, trefoil knot

Genealized linking--Borromean rings, etc.

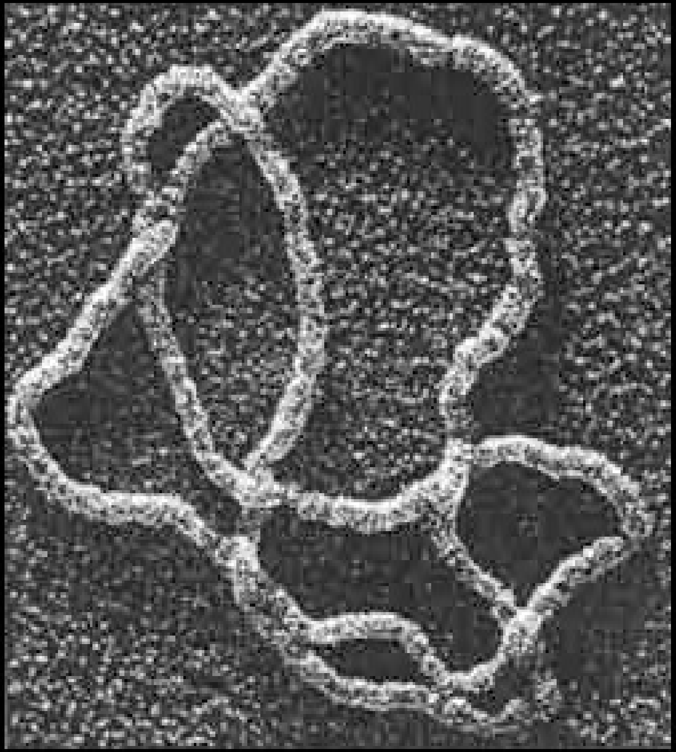
Tightly Knotted DNA

Tight knots first discussed and lengths estimated in:

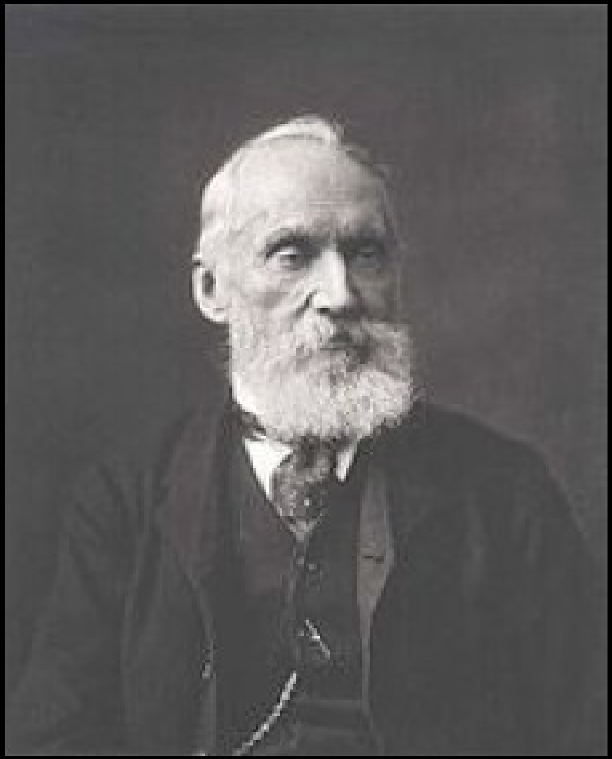
Katritch, V., Bednar, J., Michoud, D., Scharein, R.G., Dubochet, J. & Stasiak, A. (1996) Nature

Katritch, V., Olson, W.K., Pieranski, P., Dubochet, J. & Stasiak, A. (1997) Nature.

Knotted DNA



Knots and Particle Physics



Lord Kelvin: Modeled “elementrary” atoms as knotted fluid vortices in the aether.

Flux Tubes in Quantum Chromodynamics

- Glueballs as Tight Knots and Links
- Quantized Flux
- Knot "Energy" ($E_K = L/r$) Length Proportional to Particle Mass
- Semi-classical model at the level of liquid drop model of nucleus or QCD bag model

Tight Knots and Links in QCD

Roman V. Buniy and TWK, “A model of glueballs,” Phys. Lett. B576, 127 (2003)

R. Buniy and TWK,“Glueballs and the universal energy spectrum of tight knots and links,” Int. J. Mod. Phys. A20, 1252 (2005)

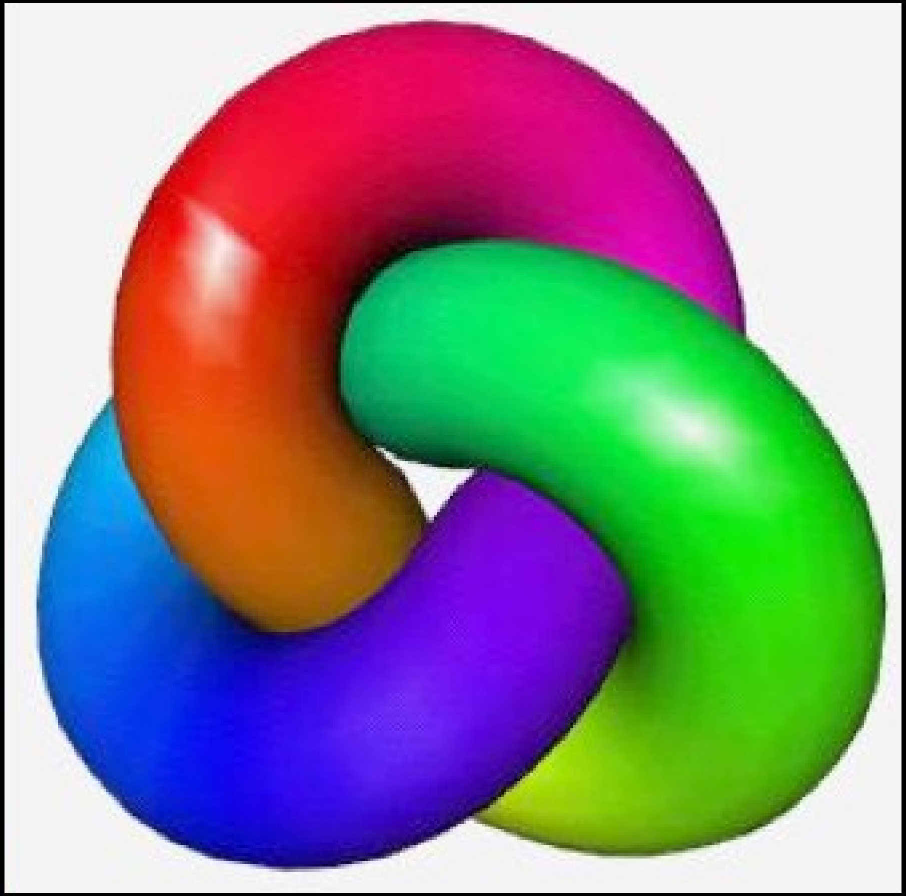
R. Buniy, J. Cantarella, TWK and E. Rawdon, “The tight knot spectrum in QCD,” arXiv:1212.xxx

T. Ashton, J. Cantarella, M. Piatek, E. Rawdon,“Knot Tightening By Constrained Gradient Descent,” arXiv:1002.1723

Chromoelectric Flux

- Flux between q, qbar pair is chromoelectric
- Open for normal mesons
- Closed in our case
- Knotted and/or linked

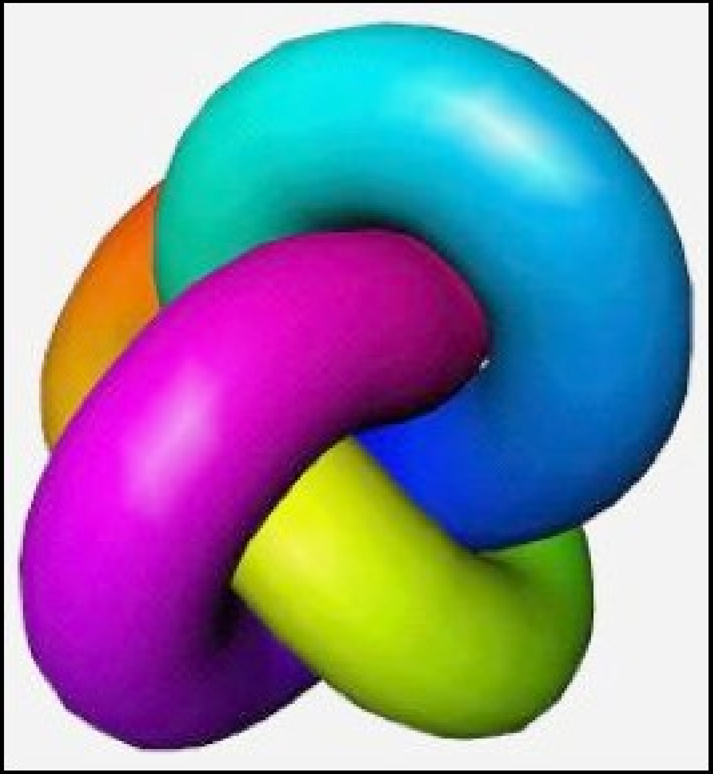
Tight Trefoil (3_1 knot)



From E. Rawdon webpage

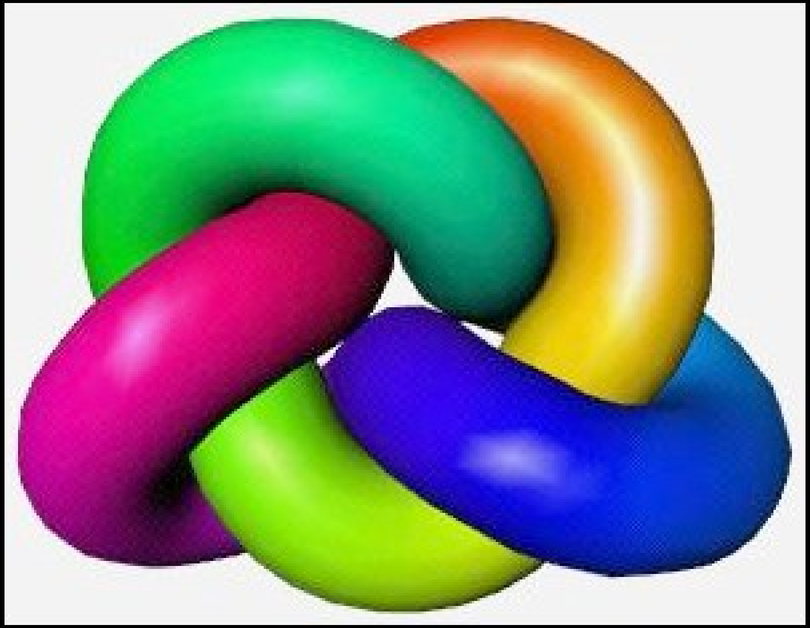
rope length 16.435

Tight Figure Eight Knot (4_1 knot)



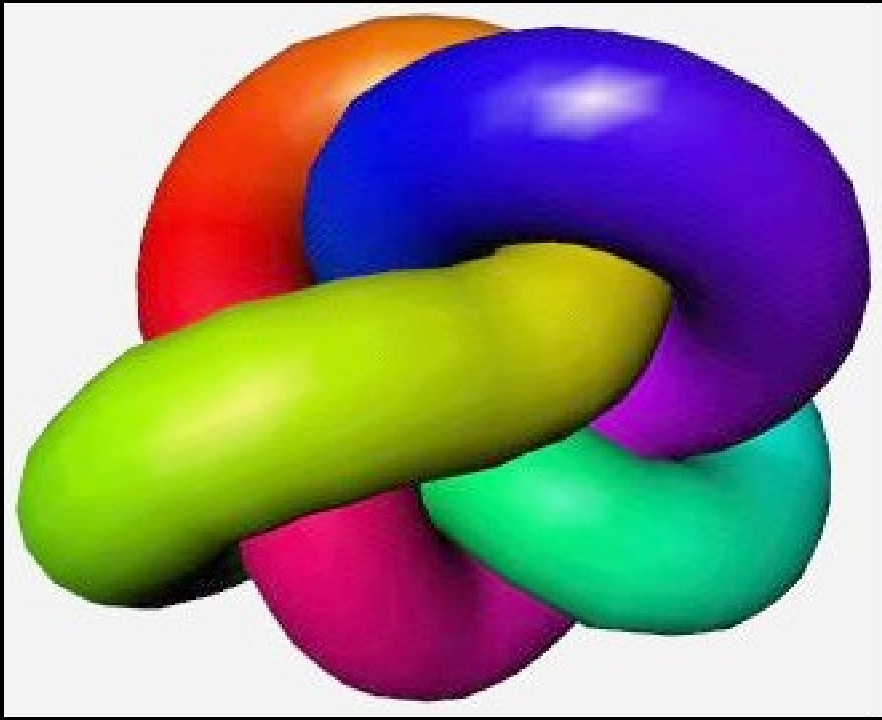
rope length 21.2313

Tight 5_1 Knot



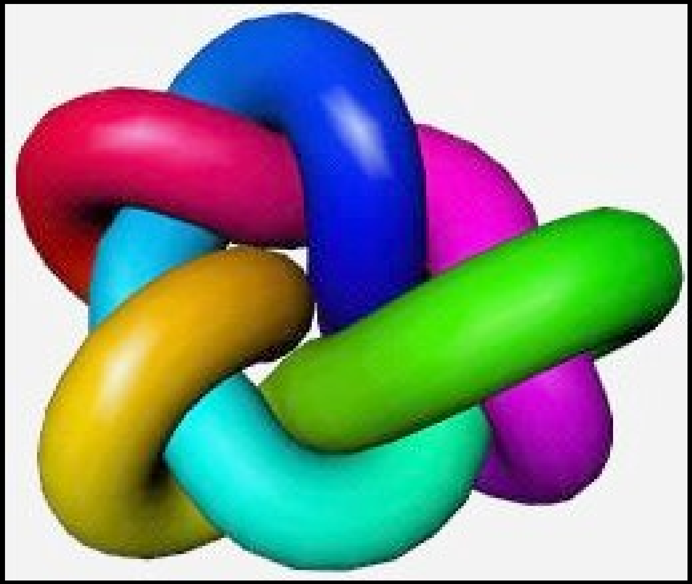
rope length 23.8431

Tight 5_2 Knot



rope length 25.5724

Tight 8_1 Knot



rope length 35.9874

Counting knots

n = number of crossings

n	prime knots	prime alternating knots	prime nonalternating knots	torus knots	satellite knots
Sloane	A002863	A002864	A051763	A051764	A051765
3	1	1	0	1	0
4	1	1	0	0	0
5	2	2	0	1	0
6	3	3	0	0	0
7	7	7	0	1	0
8	21	18	3	1	0
9	49	41	8	1	0
10	165	123	42	1	0
11	552	367	185	1	0
12	2176	1288	888	0	0
13	9988	4878	5110	1	2
14	46972	19536	27436	1	2
15	253293	85263	168030	2	6
16	1388705	379799	1008906	1	10

Hoste et al. 1998

N. Sloane, The On-Line Encyclopedia of Integer Sequences!

Types of knots and links

- Prime knots
- Composite knots (connected sums of prime knots)
- Prime links
- Composite links
- Physics should allow all types !

Non-q-qbar bosonic hadrons

(1) hybrids—bound states of quarks and gluons, like $q\bar{q}G$ with $J^{PC} = 0^{-+}, 1^{-+}, 1^{--}, 2^{-+}, \ldots$

(2) exotics—for example, $qq\bar{q}\bar{q}$ and $qqq\bar{q}\bar{q}\bar{q}$ with $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

(3) glueballs—states with no valence quarks at all, composed of pointlike or collective glue, e.g., closed flux tubes, with $J^{PC} = 0^{++}, 1^{++}, 2^{++}, \ldots$

Glueballs

- Hadrons - Strong Interactions
- No Valence Quarks, f_J states ($J^{PC} = J^{++}$)
- Do not Decay Directly to Photons

- Decay in Knotted Flux Tube Model of Glueballs via QM processes:
  1. String (Tube) Breaking
  2. Reconnection
  3. Tunneling

Summary of model assumptions:

(1) There is a one-to-one correspondence between f states and tightly knotted and linked chromo-electric flux tubes.

(2) The flux is quantized with one flux quantum per tube.

(3) Knotted and linked flux tubes are stabilized by topological quantum numbers.

Summary of model assumptions:

(4) The tube diameter is in the ~ 0.1 fm range.

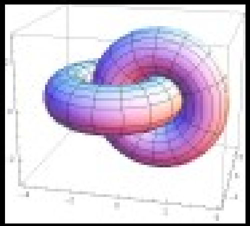
(5) The quantity J in an f_J or f'_J state is the intrinsic angular momentum of the associated knotted soliton.

(6) The relaxation to a tight state configuration (via processes where no topology change is involved) is faster than its decay rate (via processes with topology change) for an f state, i.e., $\tau_{\text{relax}} \ll \tau_{\text{decay}}$.

Identify:

Lightest glueball candidate --Shortest Knot/Link

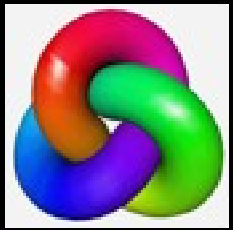
$f_0(600)$ <---> Hopf link



Identify:

Next Lightest -- Next Shortest

$f_0(980)$ <---> trefoil knot



Identify:

etc. etc.

2012 Refit with New Data

- New Glueball Data $J^{PC} = J^{++}$ States
  Particle Data Group 2011
- New Tight Prime Knot/Link Data
- New! Composite Knots and Links
- Expect continuum of glueballs by ~3 GeV from knot counts

R. Buniy, J. Cantarella, TWK and E. Rawdon,
"The tight knot spectrum in QCD," arXiv:1212.xxxx

Helicity Conservation & Force Free Configurations

Generalize to relativistic case and to QCD:

Chromoelectric helicity, higher order invariants

Caveats

Until August 2012 $\sigma = f_0(600)$

$$M_{f_0(600)} = 800 \pm 400\ MeV$$

Now $\sigma \to f_0(500)$

$$M_{f_0(500)} = 475 \pm\ 75MeV$$

$M_{f_0(500)}$ and $M_{f_0(1370)}$ extracted from partial wave analysis. Others from invariant mass plots.

Mixing with light $qq\bar{q}\bar{q}$ and $qqq\bar{q}\bar{q}\bar{q}$, etc. states?

Rope lengths from Ashton et al.

Link	Rop_p	Rop	Previous
2^2_1	25.1439	25.1388	
3_1	32.7490	32.7448	32.7433864 (–%)
4_1	42.0997	42.0928	42.1158845 (0.05%)
4^2_1	40.0247	40.0169	
5_1	47.2156	47.2016	47.51 (0.64%)
5_2	49.4840	49.4704	49.73 (0.52%)
5^2_1	49.7874	49.7723	
6_1	56.7316	56.7150	57.11 (0.69%)
6_2	57.0451	57.0271	57.44 (0.71%)
6_3	57.8602	57.8435	58.48 (1.08%)
6^2_1	54.4068	54.3893	
6^2_2	56.7132	56.7028	
6^2_3	58.1161	58.1044	
6^3_1	57.8334	57.8170	
6^3_2	58.0300	58.0145	
6^3_3	50.5865	50.5745	
7_1	61.4319	61.4109	61.89 (0.77%)
7_2	63.9165	63.8956	65.36 (2.24%)
7_3	63.9539	63.9327	64.35 (0.64%)
7_4	64.2960	64.2724	65.63 (2.06%)
7_5	65.2802	65.2609	65.70 (0.66%)
7_6	65.7183	65.7012	66.17 (0.7%)
7_7	65.6316	65.6108	66.09 (0.72%)
7^2_1	64.2585	64.2353	
7^2_2	65.0467	65.0274	
7^2_3	65.3743	65.3561	
7^2_4	65.0971	65.0759	
7^2_5	66.2400	66.2186	
7^2_6	66.3494	66.3372	
7^2_7	55.5451	55.5311	
7^2_8	57.8043	57.7948	
7^3_1	65.8275	65.8090	
8_1	71.0484	71.0241	71.44 (0.58%)
8_2	71.4327	71.4107	71.91 (0.69%)

Link	Rop_p	Rop	Previous
8_3	71.1880	71.1655	71.56 (0.55%)
8_4	72.0301	72.0049	72.41 (0.55%)
8_5	72.2100	72.1878	72.70 (0.7%)
8_6	72.5005	72.4791	72.93 (0.61%)
8_7	72.2447	72.2204	72.63 (0.56%)
8_8	73.3533	73.3334	73.88 (0.73%)
8_9	72.4717	72.4461	72.96 (0.7%)
8_{10}	73.4279	73.4095	73.86 (0.6%)
8_{11}	73.5029	73.4802	76.70 (4.19%)
8_{12}	74.0291	74.0098	74.61 (0.8%)
8_{13}	72.8291	72.8045	73.29 (0.66%)
8_{14}	73.9226	73.8991	74.93 (1.37%)
8_{15}	74.3344	74.3134	74.82 (0.67%)
8_{16}	74.9213	74.8962	75.47 (0.76%)
8_{17}	74.5276	74.5071	75.08 (0.76%)
8_{18}	74.9420	74.9252	75.44 (0.68%)
8_{19}	61.0734	61.0430	61.35 (0.5%)
8_{20}	63.1530	63.1146	64.11 (1.55%)
8_{21}	65.5504	65.5298	65.91 (0.57%)
8^2_1	68.5198	68.4884	
8^2_2	71.1823	71.1587	
8^2_3	72.7498	72.7291	
8^2_4	72.6102	72.5908	
8^2_5	74.0039	73.9826	
8^2_6	73.2932	73.2502	
8^2_7	74.4165	74.3885	
8^2_8	73.7849	73.7702	
8^2_9	74.0620	74.0386	
8^2_{10}	73.6890	73.6684	
8^2_{11}	73.0115	72.9899	
8^2_{12}	74.0194	73.9140	
8^2_{13}	74.1685	74.1501	
8^2_{14}	73.7000	73.6775	
8^2_{15}	64.3305	64.3086	
8^2_{16}	66.8434	66.8315	
8^3_1	72.2883	72.2649	
8^3_2	72.9544	72.9360	
8^3_3	74.9366	74.9139	
8^3_4	77.8544	77.8314	
8^3_5	73.4286	73.4061	
8^3_6	74.7680	74.7468	
8^3_7	60.6065	60.5888	

Curvature Corrections

- We have physical knots, not ideal knots
- Flux is uniform across cross sections of straight tubes
- But, knotted and linked tubes are curved
- Flux is not uniform across cross sections of curved tubes
- Correct energy for curvature

Curvature Corrections for a Toroidal Solenoid

Flux

$$\Phi = \int_D B dz d\rho$$

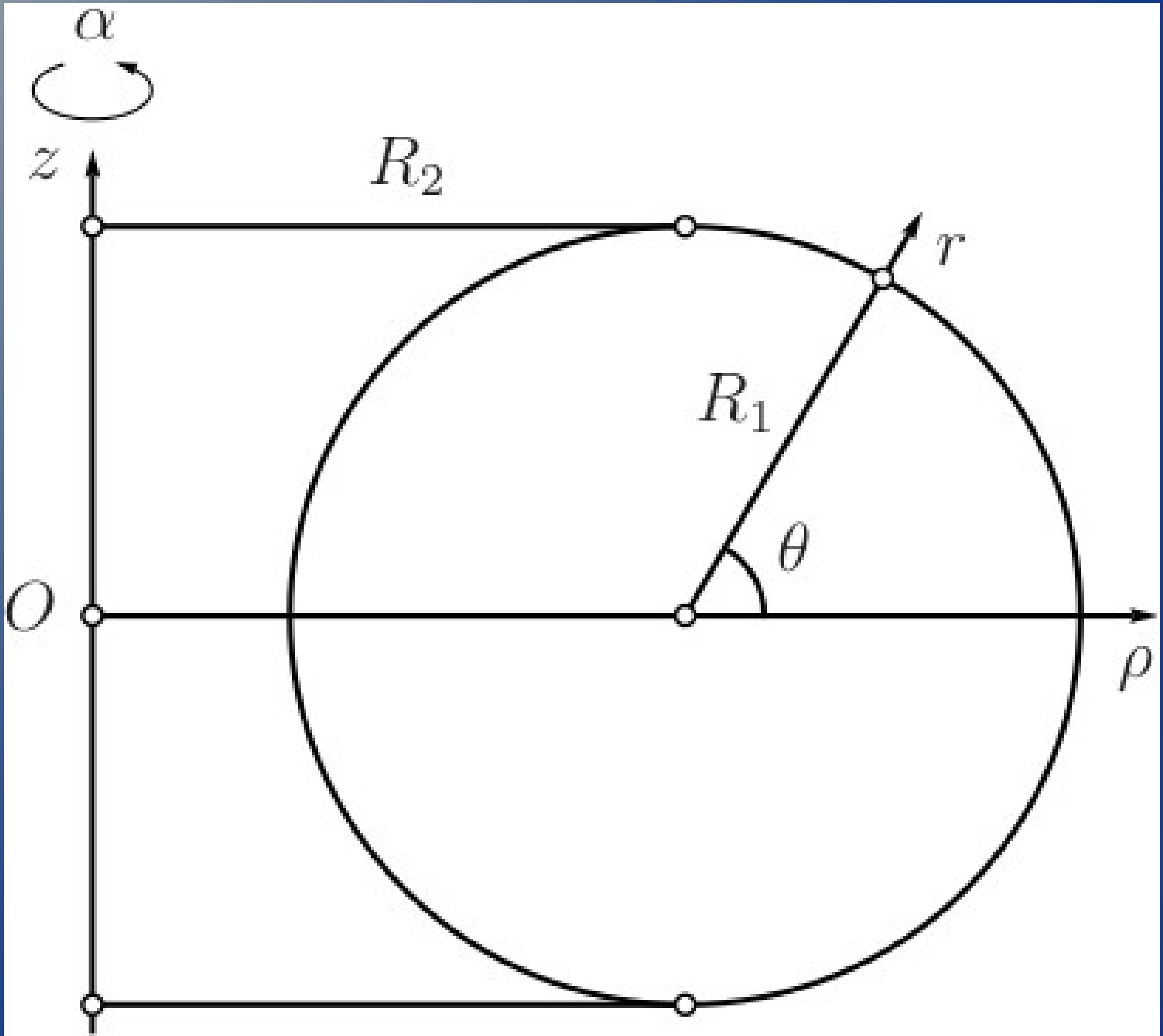
Energy

$$W = \frac{1}{2} \int B^2 \rho dz d\rho d\alpha$$

Hold flux Φ fixed and vary the field

$$\delta(W - \lambda\Phi) = 0$$

Toroidal Solenoid-Cross Section



Curvature Corrections for a Toroidal Solenoid

Variation of B gives

$$\int_D (2\pi B\rho - \lambda)\delta B dz d\rho = 0$$

λ is a Lagrange multiplier.

Vanishes for arbitrary δB iff

$$B(\rho) = \frac{\lambda}{2\pi\rho}$$

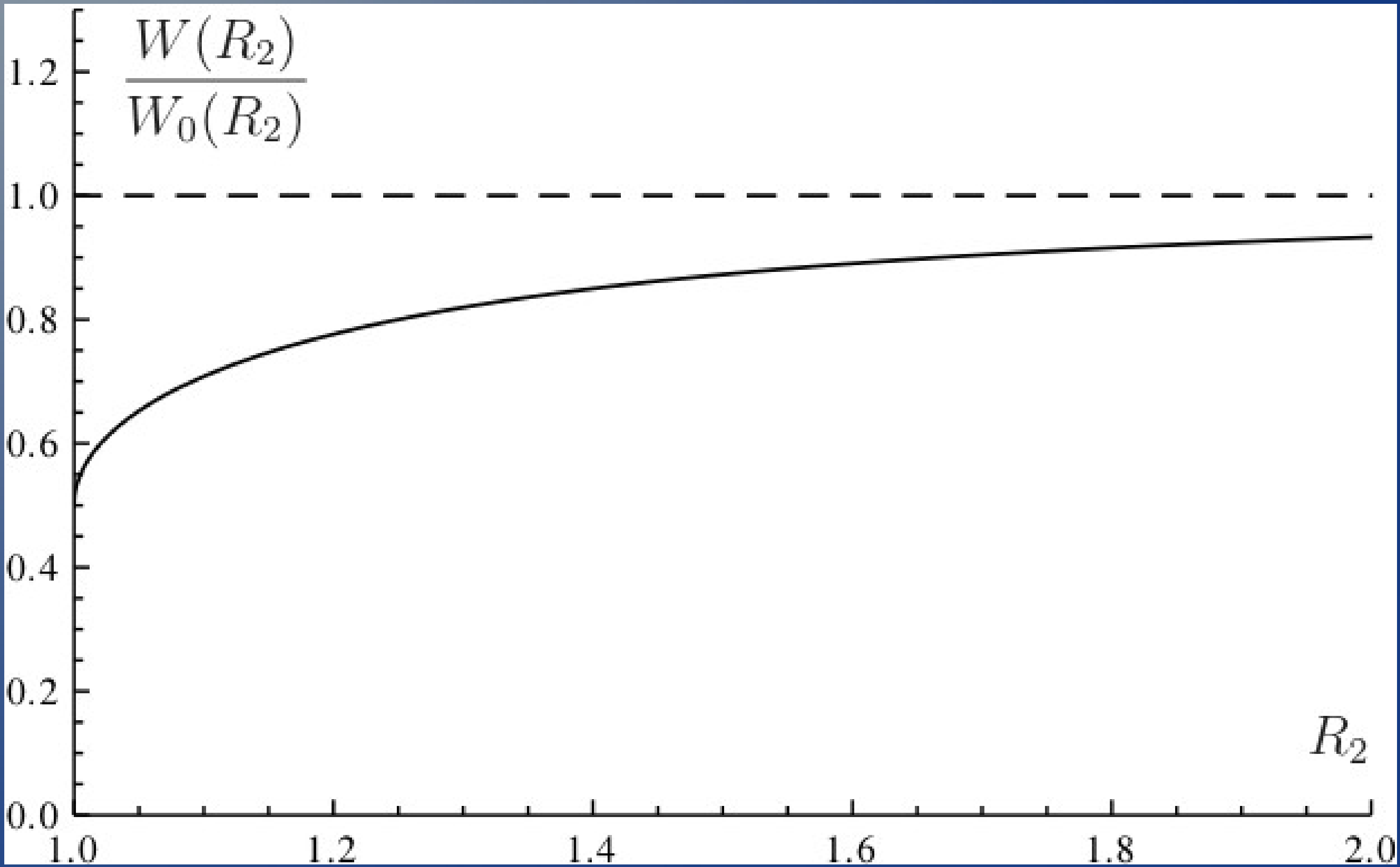
leads to

$$W(R_2) = \frac{\Phi^2}{2\left[R_2 - (R_2^2 - R_1^2)^{1/2}\right]}$$

Compare straight cylinder of length $2\pi R_2$

$$W_0(R_2) = \frac{\Phi^2 R_2}{R_1^2}$$

Curvature Correction as a Function of R_2



Curvature Corrections for a Toroidal Solenoid

Each wedge of a link is corrected by a factor (Link with c components ith wedge)

$$r_{ic} = \frac{E_{ic}(R_{2,ic})}{E_{0,ic}(R_{2,ic})}.$$

Full model of a knotted flux tube K:

- Sum of n solenoidal wedges
- Minimize the energy in each wedge

$$W_K = \sum_{i=1}^{n} W(R_{2,i}),$$

Curvature Corrections for a Toroidal Solenoid

Example: We can calculate exactly for:

Chain of three unknots $2_1^2 \# 2_1^2$

Length $6\pi + 2 \approx 20.8496$

$R_2 = 2R_1$ in curved regions.

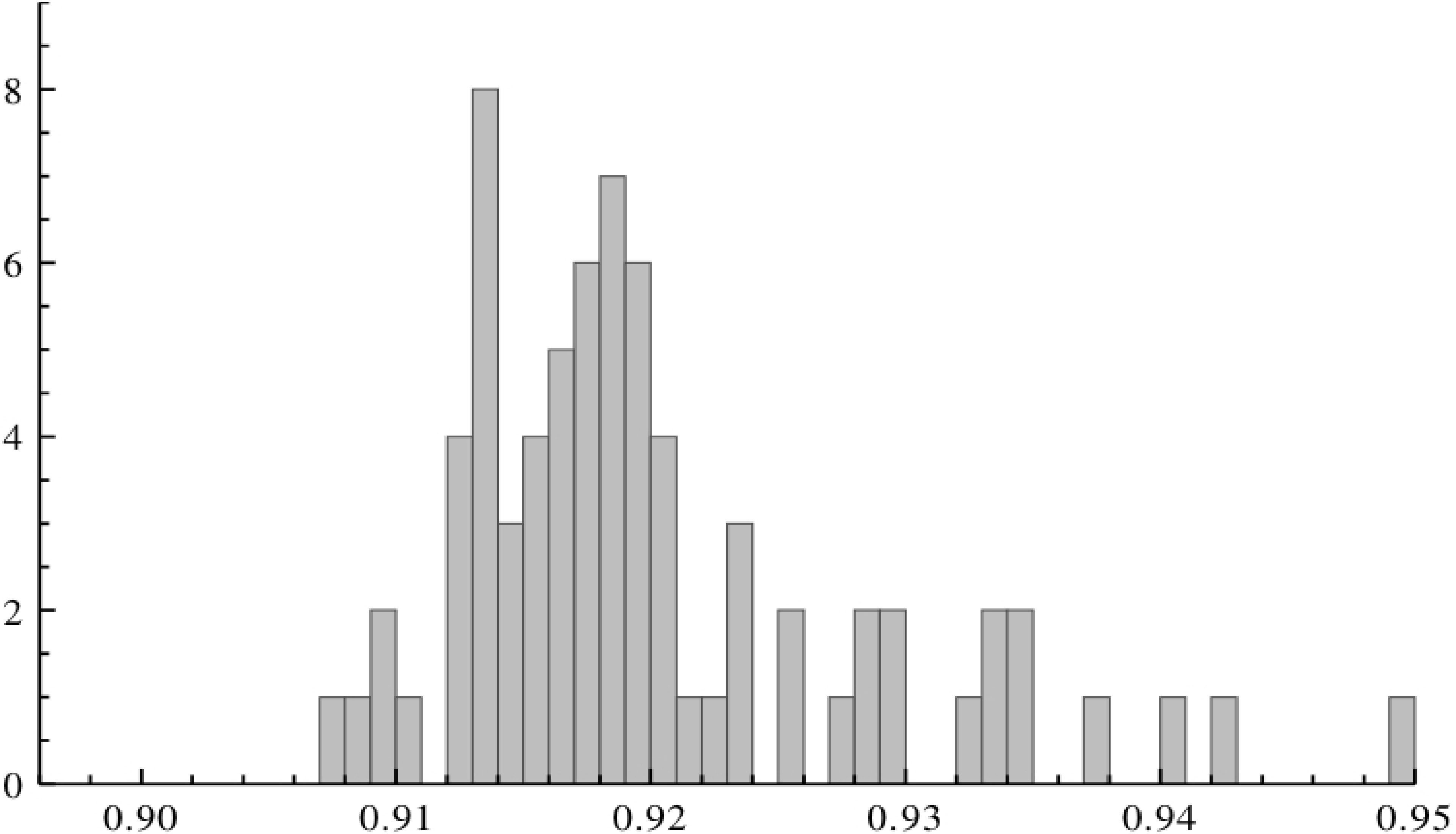
$R_2 \to \infty$ in straight sections.

Corrected value

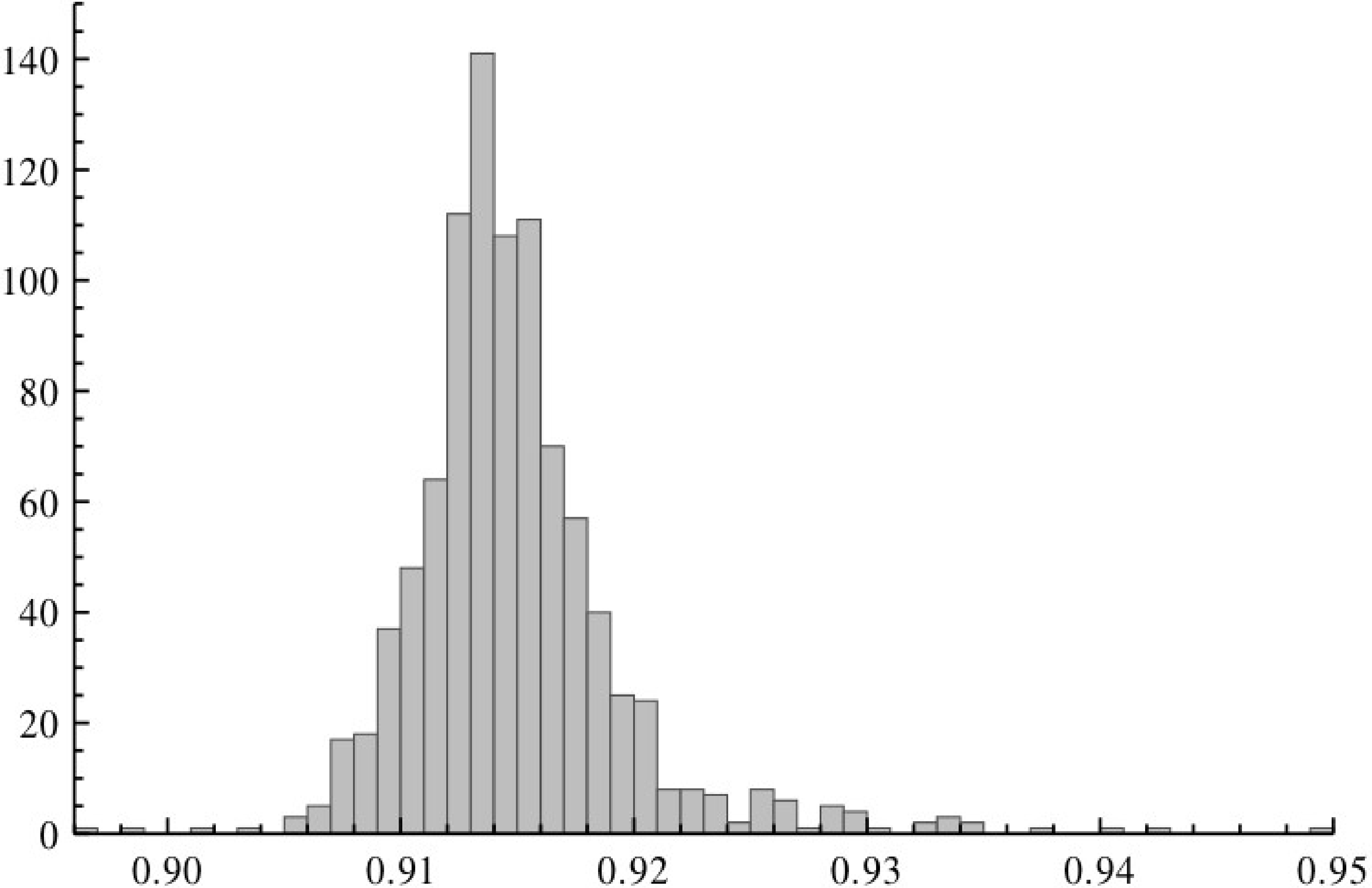
$$E(2_1^2 \# 2_1^2) = \frac{1}{4(2-\sqrt{3})} 6\pi + 2$$

or

$$E(2_1^2 \# 2_1^2) \approx 0.933013(6\pi + 2) \approx 19.4529$$



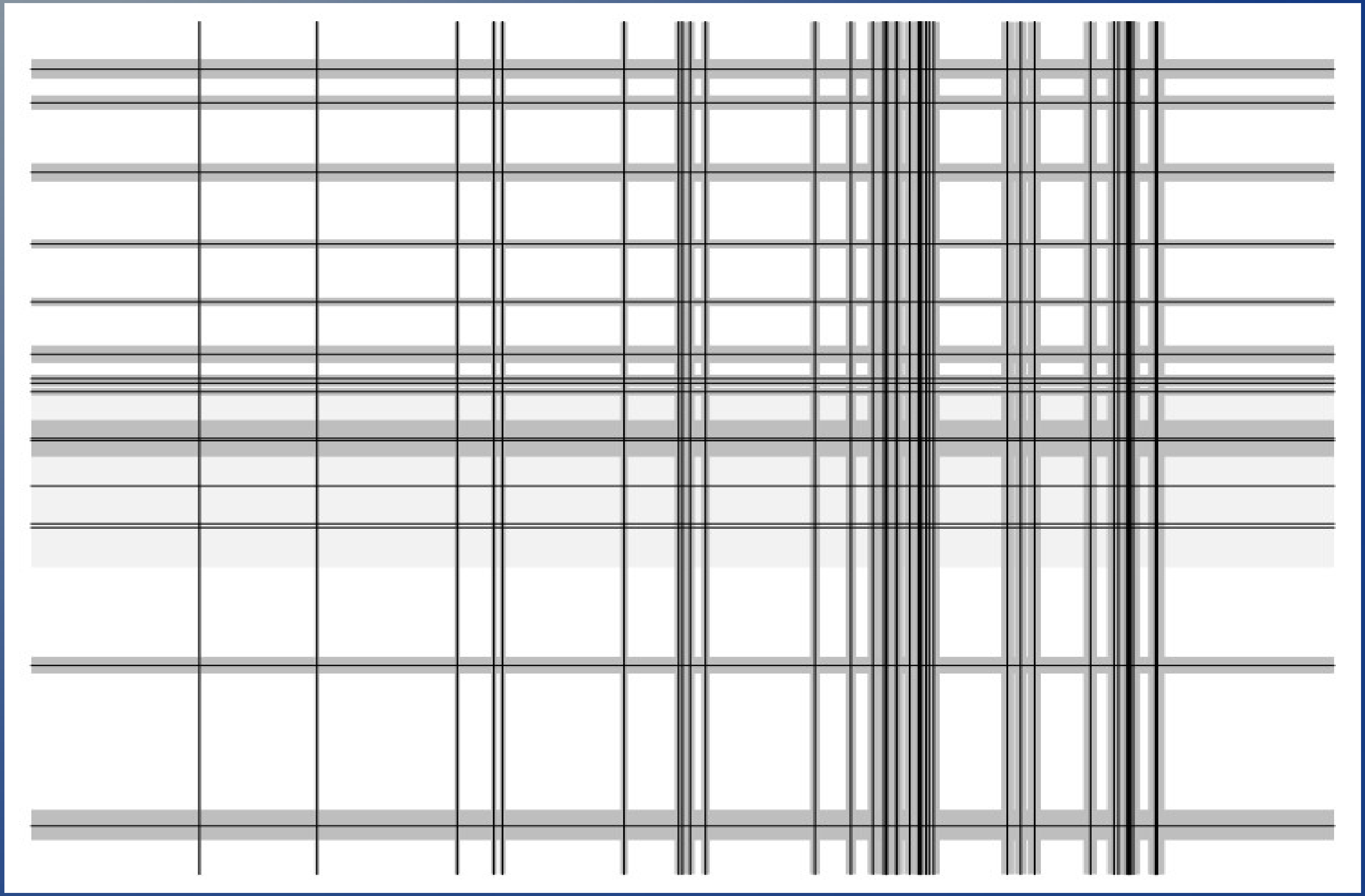
Histogram of the magnitudes of the curvature corrections to the first 72 knot and link lengths.



Histogram of the magnitudes of the curvature corrections to all 945 currently tabulated knot and link lengths.

Fit f_J's to knots

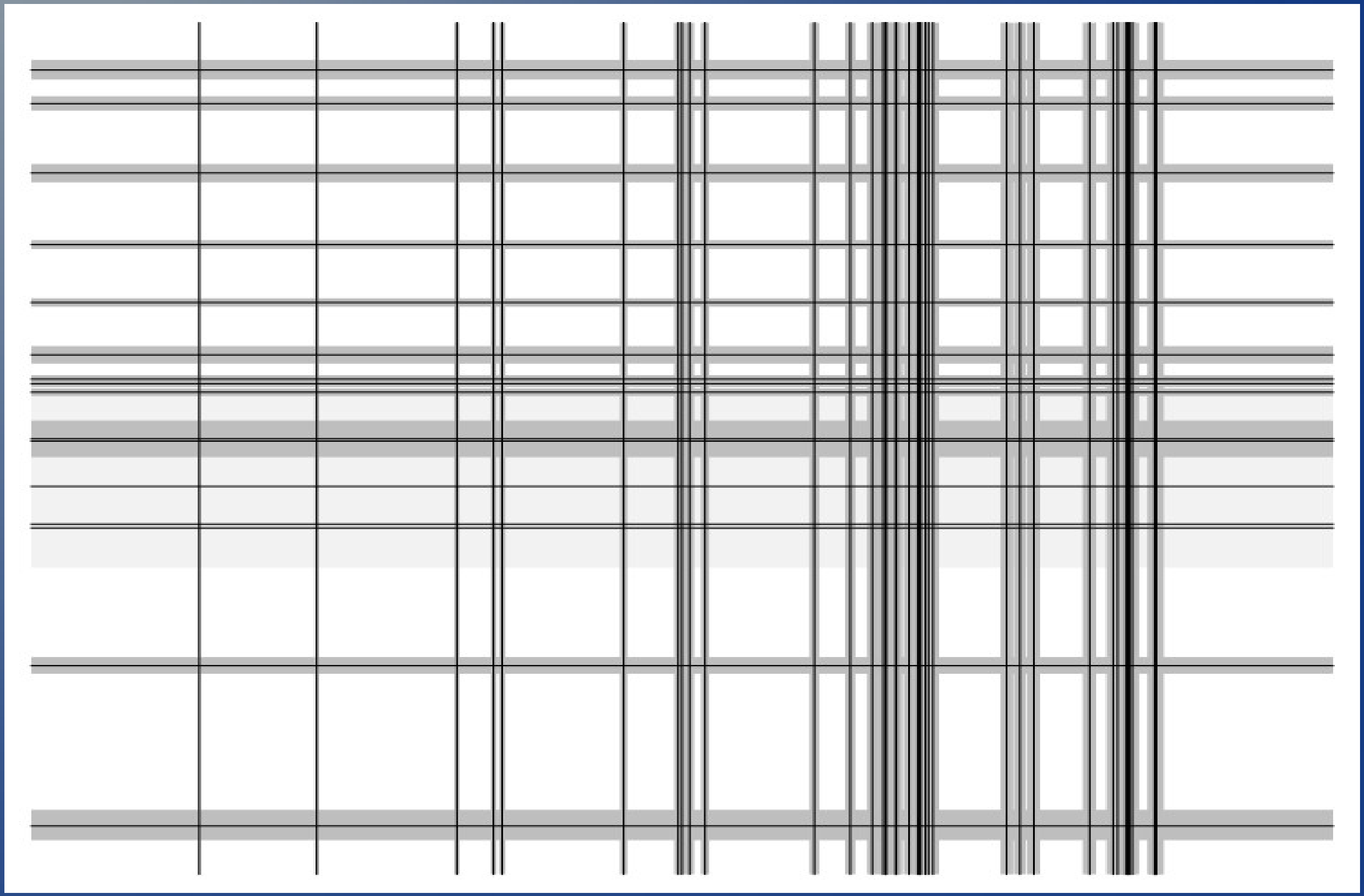
Curvature corrections included in fit



Glueball Mass

Knot Length

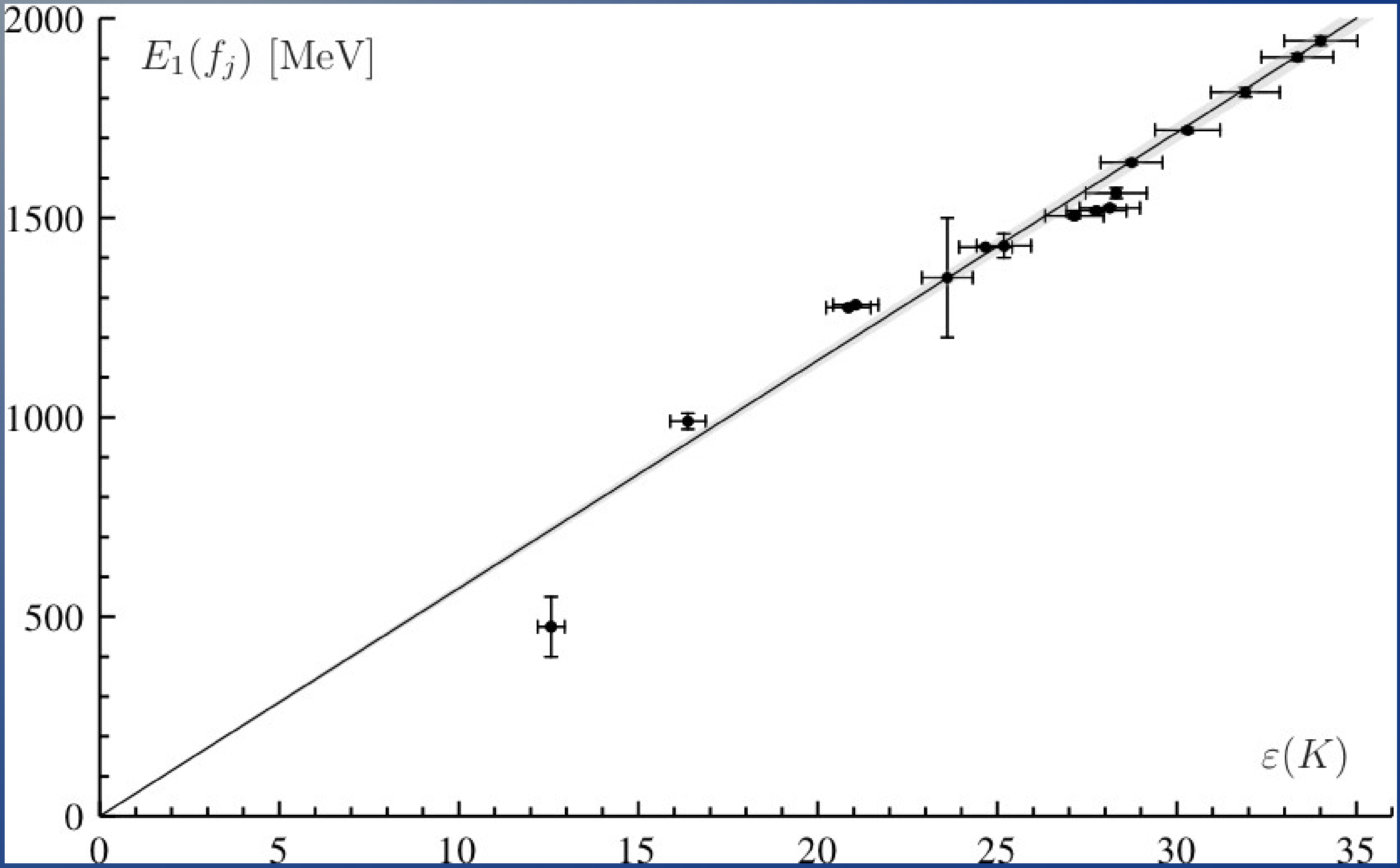
The Glueball Tartan



Fit

- We fit one set of sporadic numbers with another
- I.e., f-state masses <---> knot lengths
- Either one-parameter: slope
- Or two parameters: slope and intercept

f_J's vs knot lengths



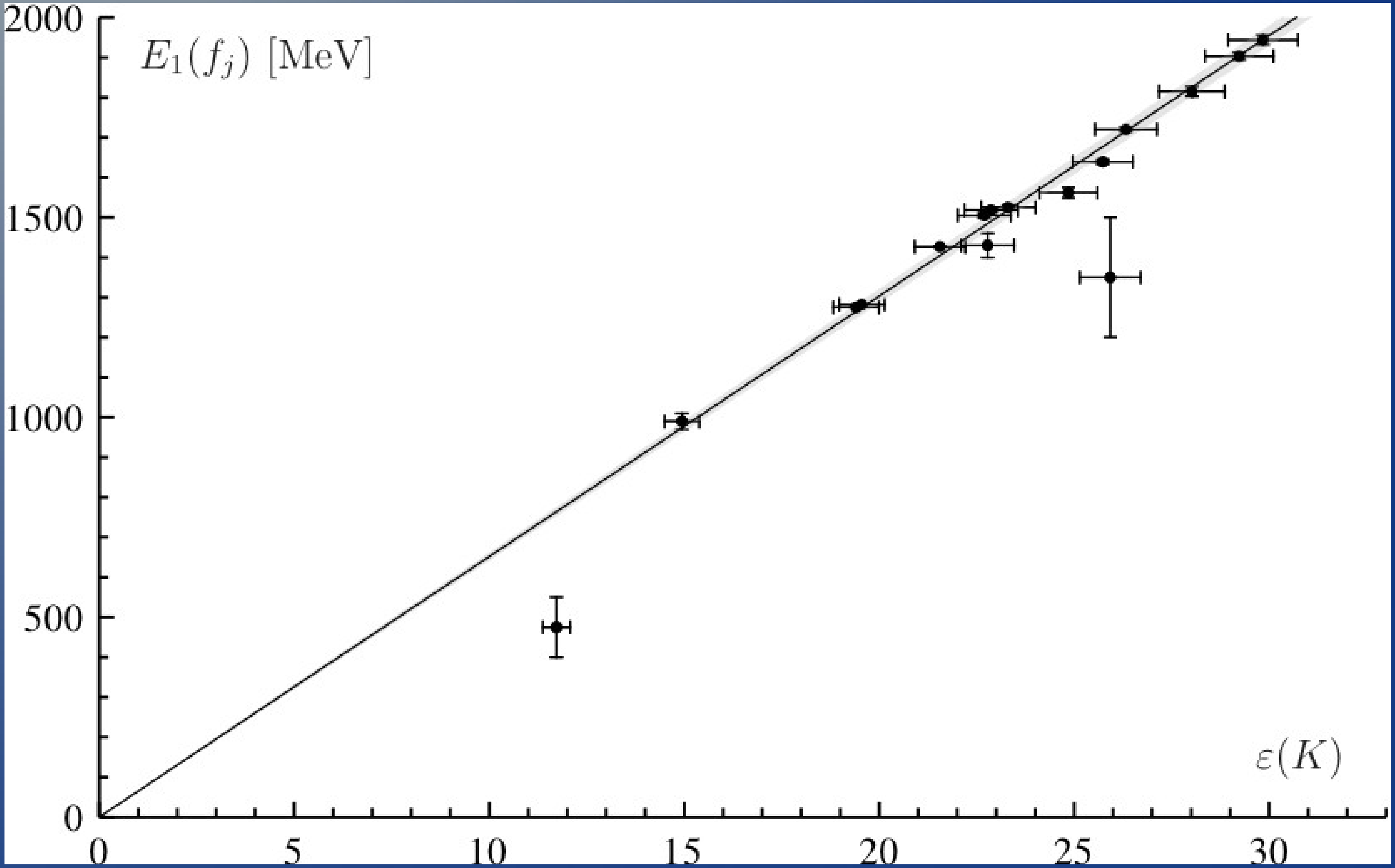
One parameter fit with uncorrected knot lengths

Options

- Linear fit: 1-1 in order of mass/length
- Move f states with large error bars out of order to improve fit to:

1. Account for tension in data by leaving gaps
2. Find best fit

f_J's vs knot lengths

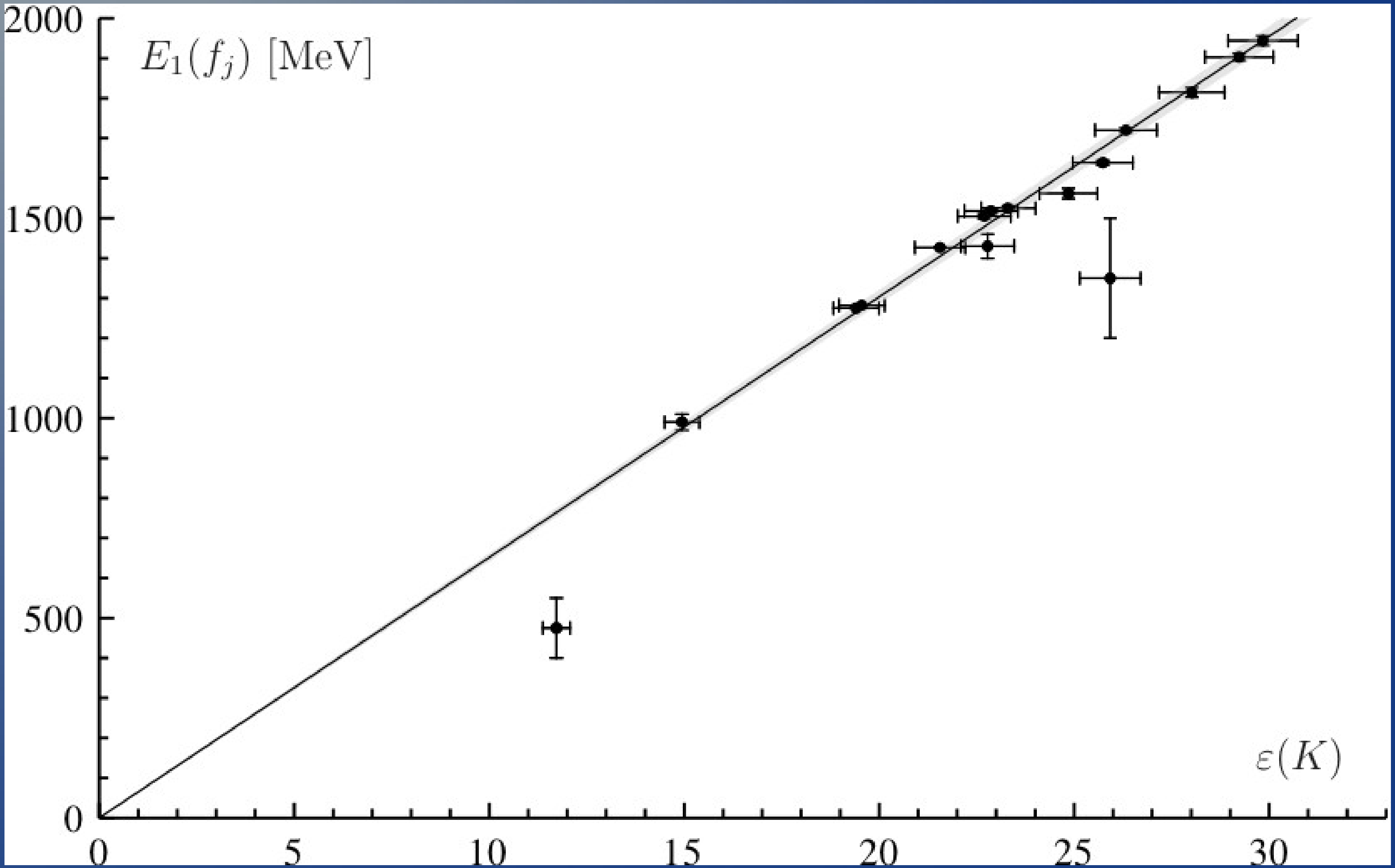


One parameter fit with curvature corrected knot lengths

Best qualified fit

- This previous figure is best fit allowing for resolution of the tension in the data at ~1220 MeV
- Free knot i.e., 4^2_1 unassigned and used to predict a state near 1200 MeV

f_J's vs knot lengths

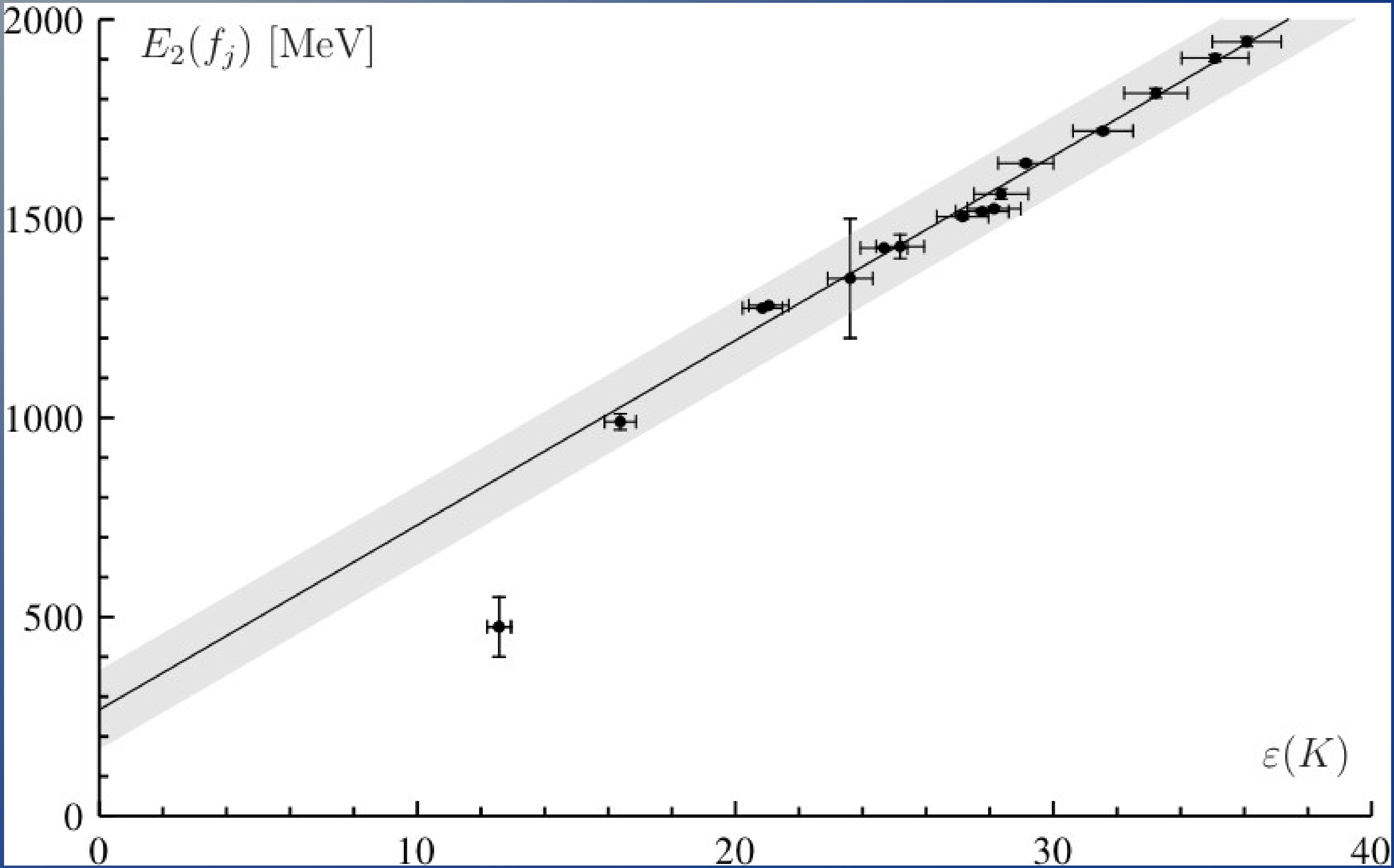


One parameter fit with curvature corrected knot lengths

One parameter curvature corrected vs uncorrected fits

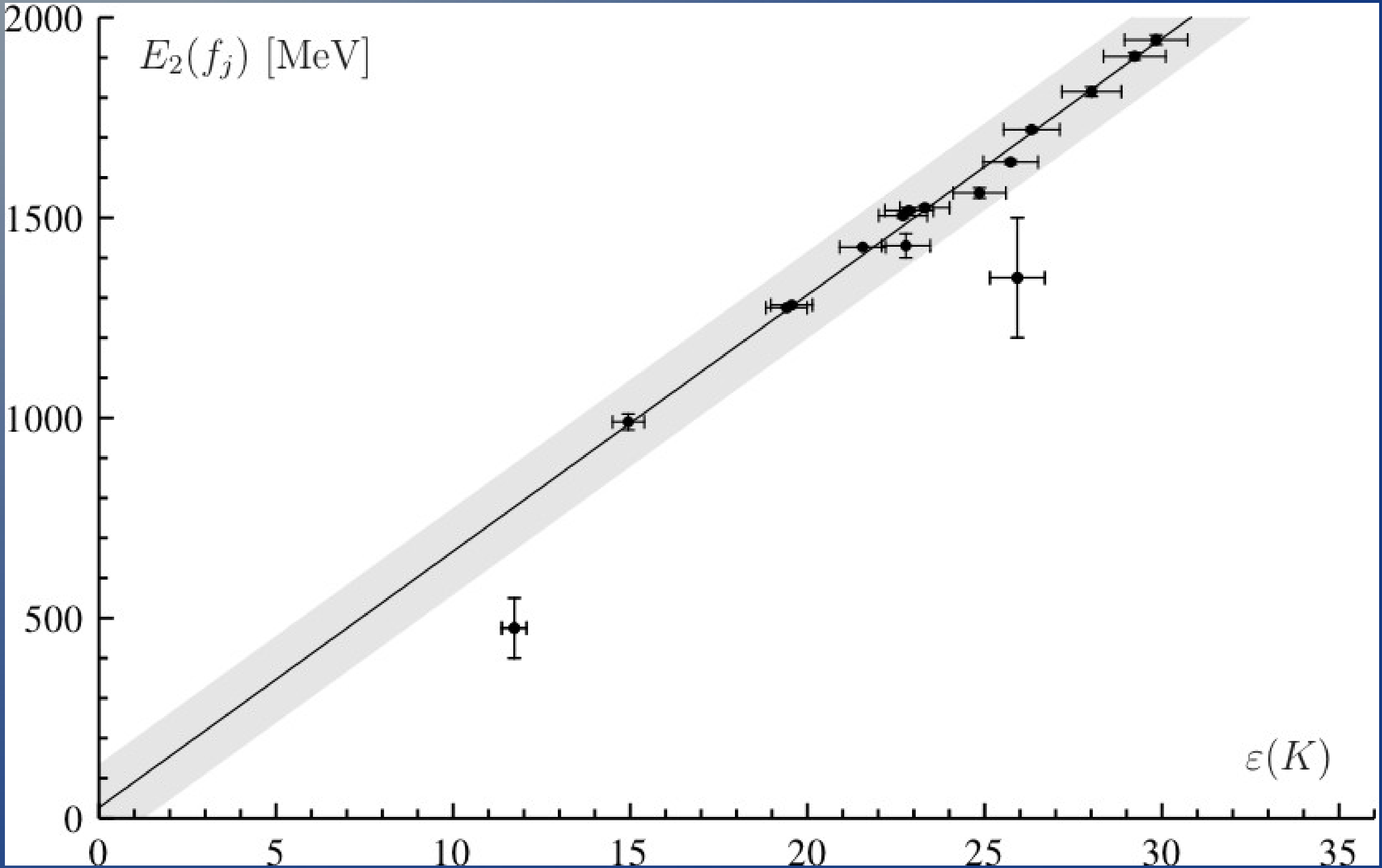
- f(1270) and f(1285) poorly fit without curvature correction
- Fit of f(1270) and f(1285) considerably improved with curv. correction

f_J's vs knot lengths



Two parameter fit with uncorrected knot lengths

f_J's vs knot lengths

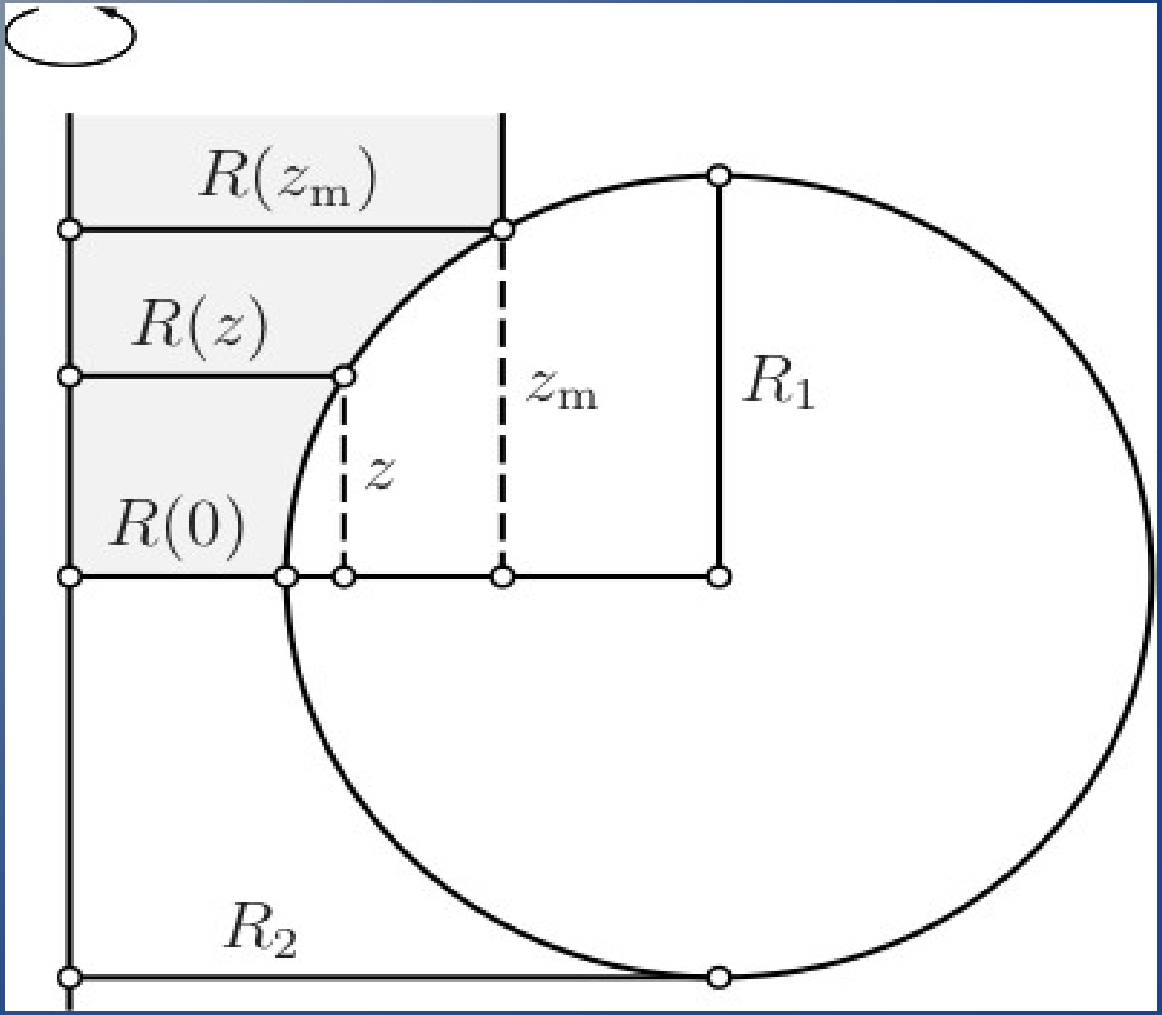


Two parameter fit with uncorrected knot lengths

Two parameter fits

- Two parameter fit does not pass through origin for no curv. correction
- Curv. Corrected two parameter fit is consistent with zero intercept

Other Corrections



Toroidal flux tube (minor radius R1, major radius R2) constricting a cylindrical flux tube of radius R(z)

Other Corrections

Constriction

Balance tension with magnetic pressure

Estimate 5% correction for Hopf link

QCD may be more complicated—confinement effects, etc.

Other Corrections

Distortion of tube cross section

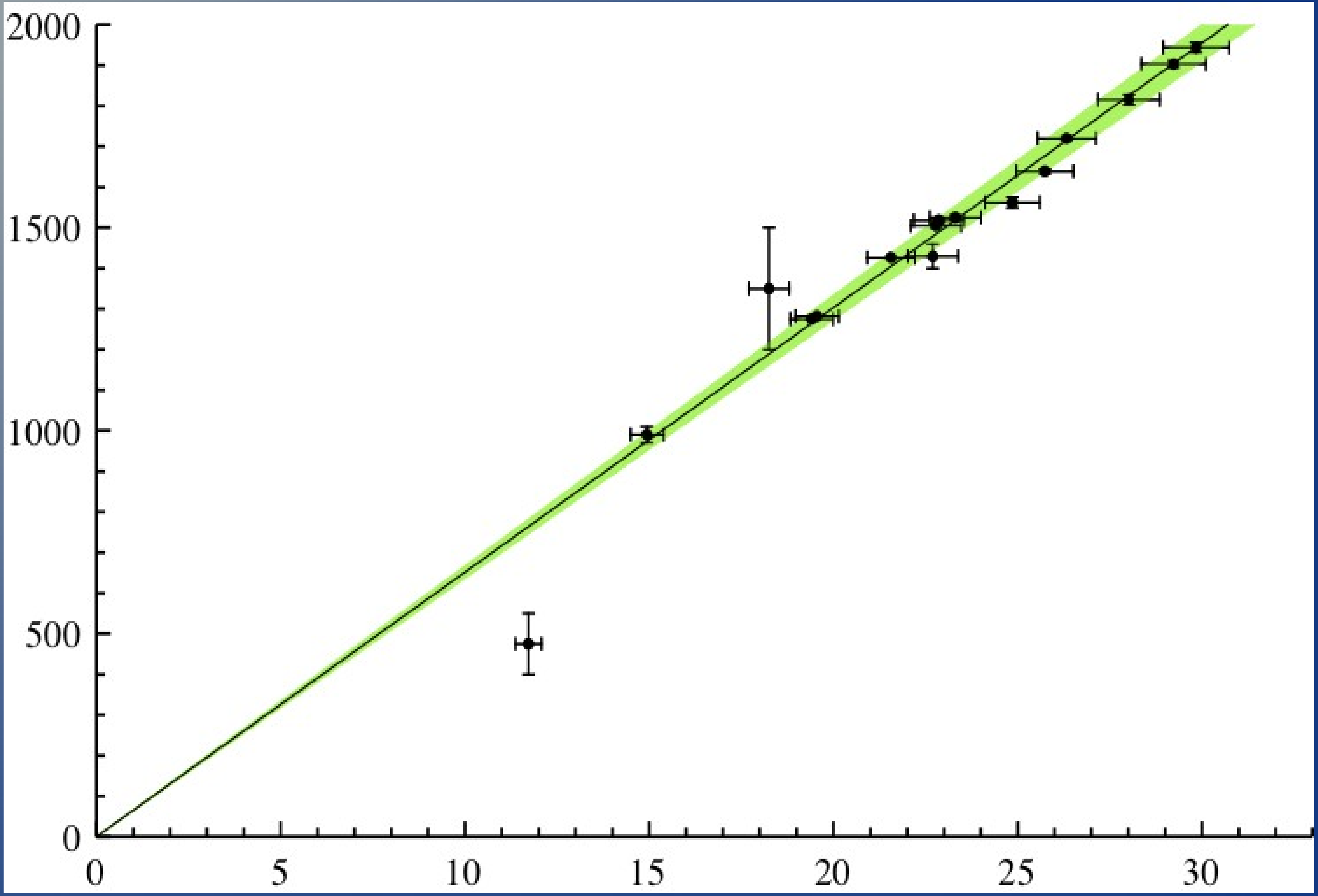
E.g., wrap a rope tightly around a post and it's
Cross section distorts

Estimate few % corrections

Corrections to Knot Lengths

- Curvature corrections ~ few % variation
- Constriction corrections few %
- Distortion corrections few %
- QCD effects ?

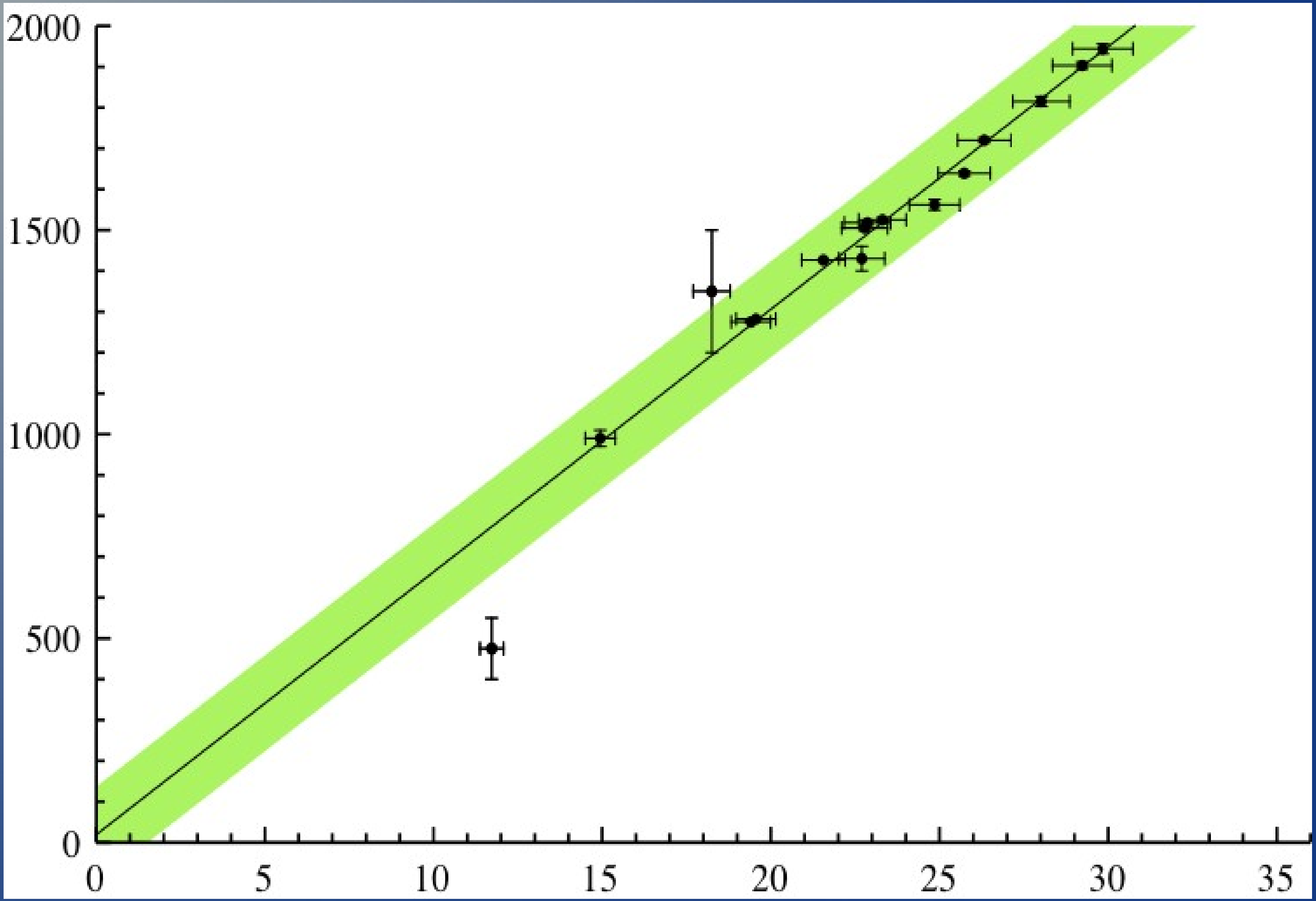
- Conservative assumption:
  3% error on knot energy
- Note: only the spread in corrections is important



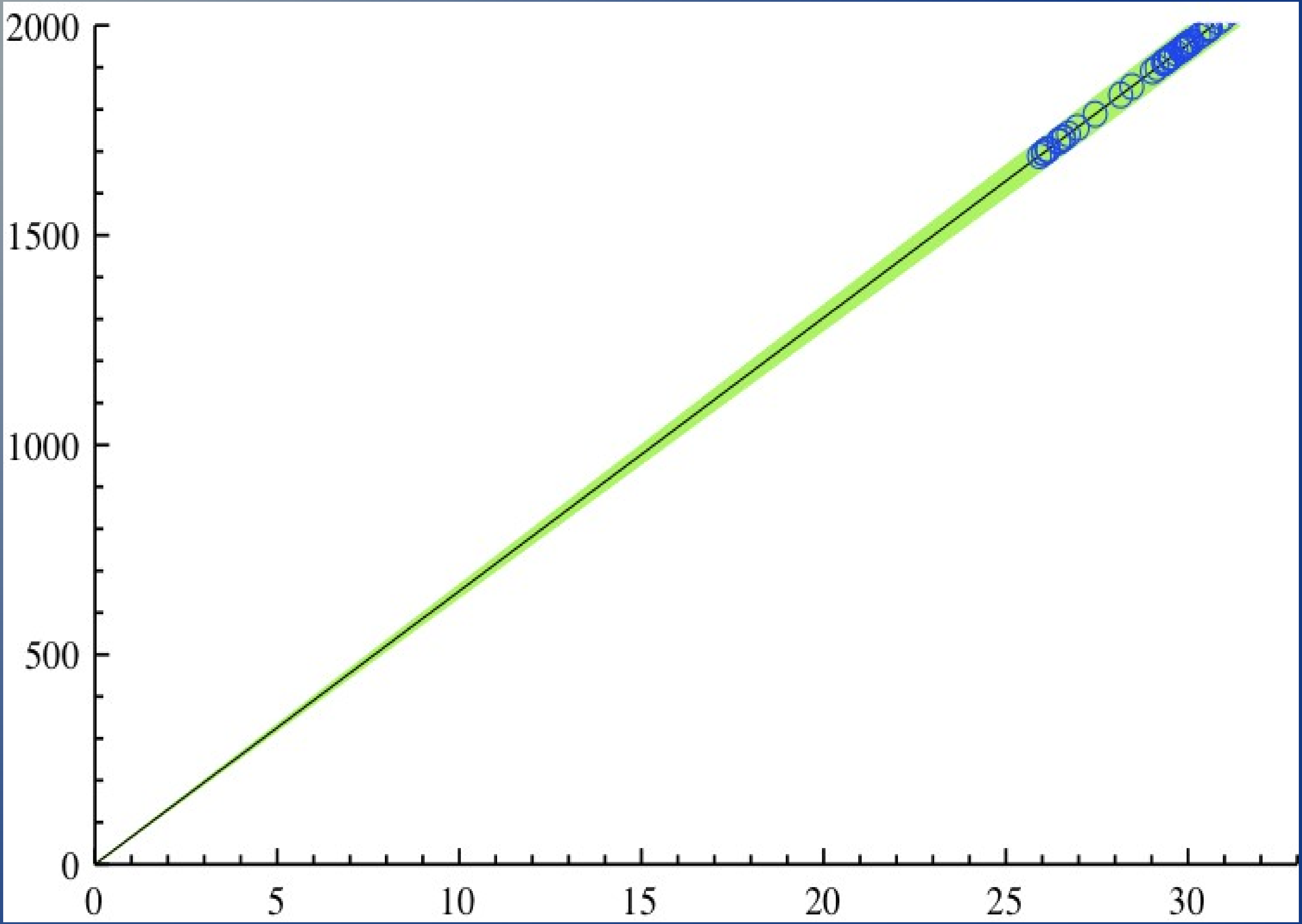
one parameter fit—curvature corrected

Best unqualified fit

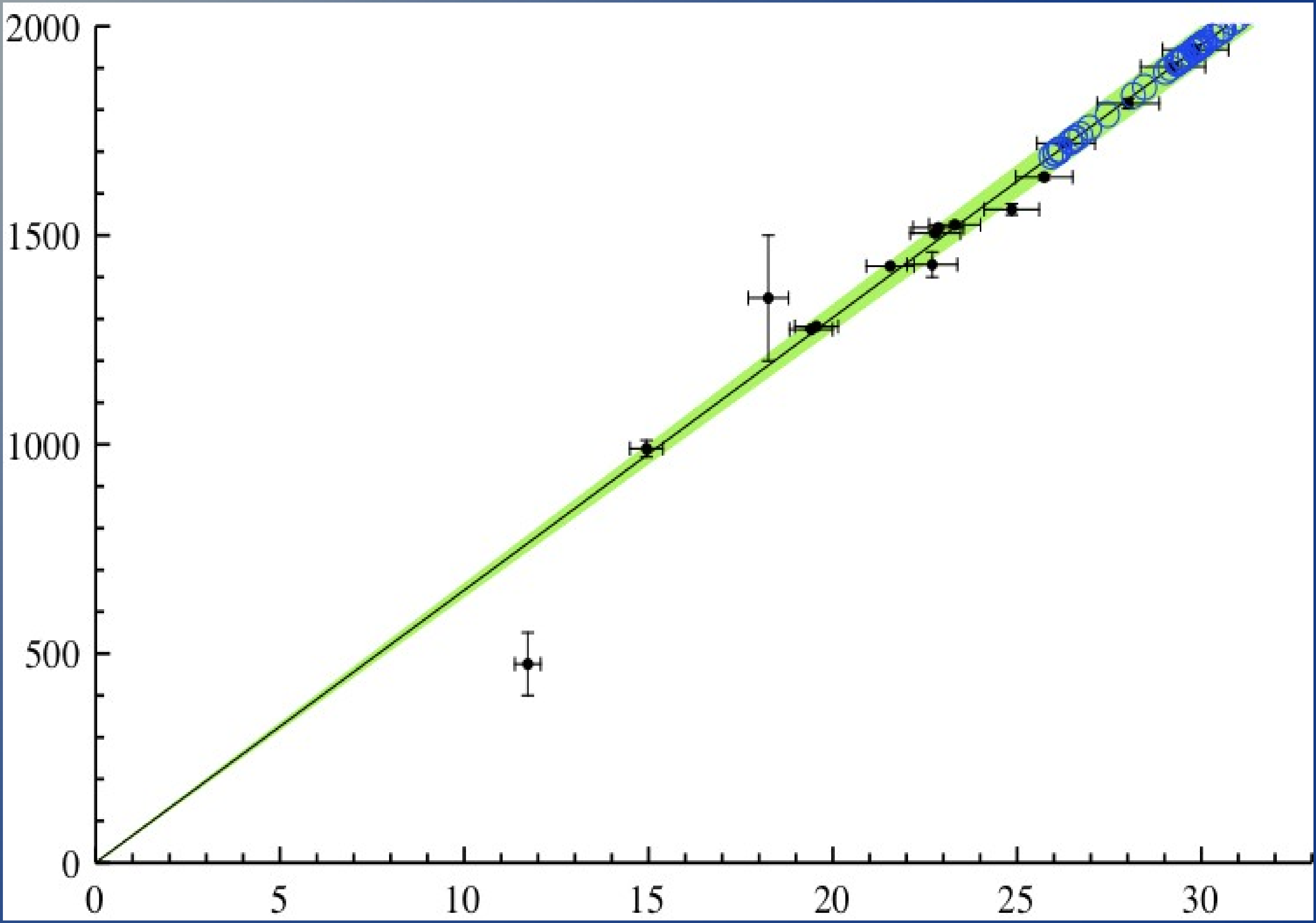
- The previous figure is the best overall fit
- f(1350) was reordered



two parameter fit—curvature corrected



Predictions for masses



Fit plus predictions

RESULTS and PREDICTIONS for 1-Parameter Fit

(1) Knots and f_J states are in one-to-one for first 12 knots/links and first 12 states

(2) $\chi^2 = 28$

(3) $R^2 = 0.9994$

(4) $\Lambda_{tube} = 65.2 \pm 0.8$ MeV or ± 1.5 %

Compare to Λ_{QCD} where

$\Lambda_{\bar{M}\bar{S}} = 217 \pm 12$ MeV or $\pm 5\%$

2-Parameter Fit

(1) Knots and f_J states are in one-to-one for first 12 knots/links and first 12 states

(2) $\chi^2 = 28$ unchanged

(3) $\Lambda_{tube} = 65.2 \pm 0.8$ MeV (unchanged)

(4) Intercept $= 19 \pm 114$ MeV
Consistent with zero. I.e., model is not improved by adding an extra parameter

Quality of the fit

TABLE I: Statistical tests of the model. Recalling that p is bounded $0 \leq p \leq 1$ and $p < 0.01$ implies poor correlation, $0.01 < p < 0.05$ implies moderate correlation and $0.1 < p$ implies strong correlation, we see that all these tests strongly support the model.

Goodness-of-fit test	p-value	Variance test	p-value
Pearson χ^2	0.66	Brown-Forsythe	0.74
Kolmogorov-Smirnov	0.95	Fisher Ratio	0.69
Cramer-von Mises	0.96	Levene	0.74
Anderson-Darling	0.97	Siegel-Tukey	0.82
Kuiper	0.99		
Watson U Square	0.90		

State	Mass	K^a	$\varepsilon(K)^b$	$E(K)^c$
$f_0(500)$	475 ± 75	2_1^2	11.724	764
$f_0(980)$	990 ± 20	3_1	14.943	974
$f_0(1370)$	1350 ± 150	4_1^2	18.250	1189
$f_2(1270)$	1275.1 ± 1.2	4_1	19.411	1265
$f_1(1285)$	1282.1 ± 0.6	$2_1^2\#2_1^2$	19.556	1274
$f_1(1420)$	1426.4 ± 0.9	5_1	21.559	1405
$f_2(1430)$	≈ 1430	$2_1^2\#3_1$	22.697	1479
$f_0(1500)$	1505 ± 6	5_2	22.779	1484
$f_1(1510)$	1518 ± 5	5_1^2	22.866	1490
$f_2'(1525)$	1525 ± 5	6_3^3	23.309	1519
$f_2(1565)$	1562 ± 13	6_1^2	24.854	1619
$f_2(1640)$	1639 ± 6	7_7^2	25.735	1677
		6_2^2	25.924	1689
		6_1	26.025	1696
		$2_1^2\#4_1^2$	26.046	1697
		$3_1\#3_{1m}$	26.135	1703
		$3_1\#3_1$	26.151	1704
		6_2	26.158	1704
$f_0(1710)$	1720 ± 6	6_1^3	26.327	1715
		$(2_1^2\#2_1^2\#2_1^2)_{\mathrm{kc}}$	26.449	1723
		$2_1^2\#4_1$	26.466	1724
		6_3	26.567	1731

Fit and predicted masses for 1st 22 knots/links

Predicted f_J's from knot energies

- No new states below ~1680 MeV

  (Not inconsistent with one new state at ~1195 MeV upon reording f(1370))

- Five new states 1690—1705 MeV range
- Many new states above ~ 1720 MeV
- Estimated to 3% accuracy

From one parameter fit with curvature corrected knot lengths

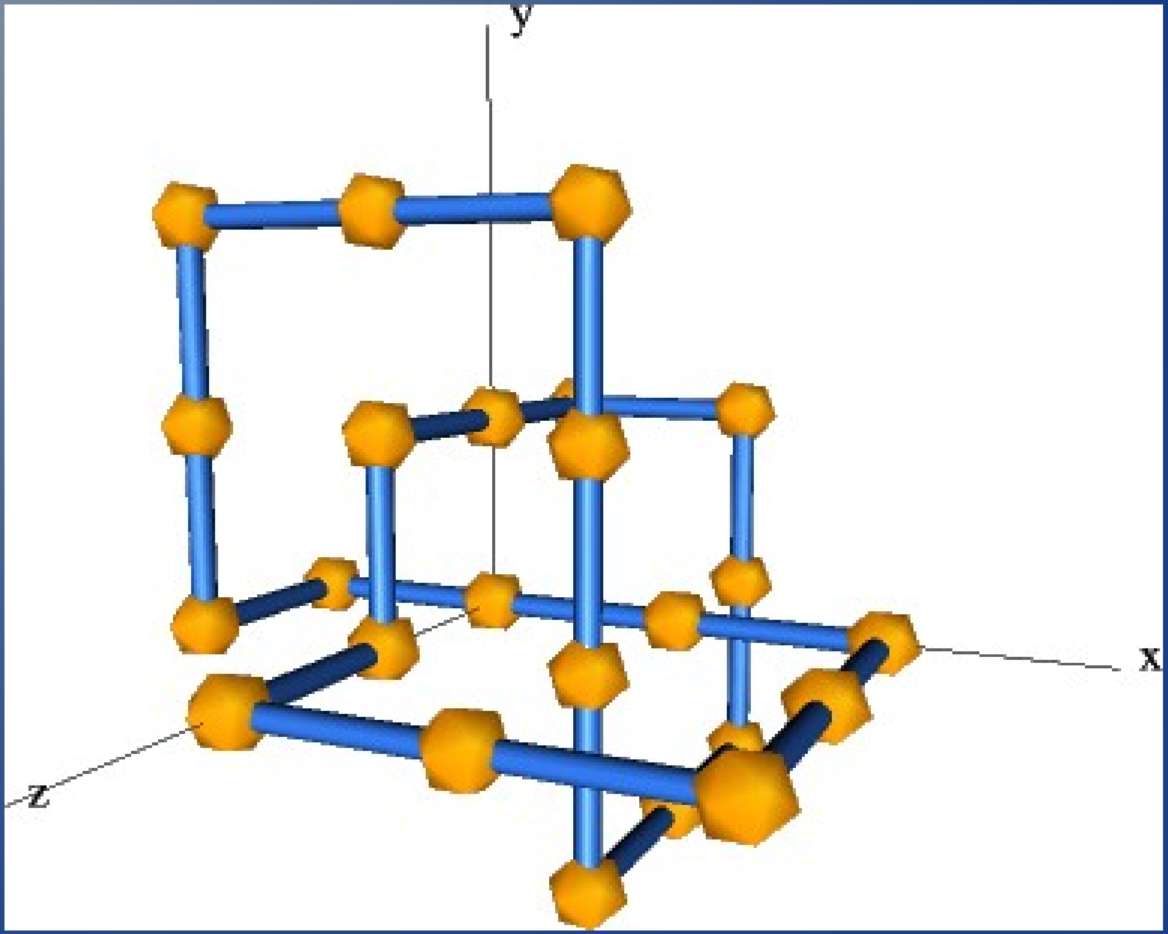
Comments about Predictions

- There is tension in the data near f(1710), f(1810), and some other high mass

  f-states.
- 50 years of HEP data to reconcile
- Multiple states possible in these regions
- Attempts should be made to resolve them

Smallest knot on a cubic lattice

From knotplot, Rob Scharein



Minimal trefoil hits 24 vertices, Y. Diao (1993)

Trefoil energy

- Minimal lattice trefoil energy = 24
- Minimal lattice trefoil energy ~ 16.44
- Lattice could predict knot energies on the high side

Lattice

Conventional glueball on lattice--difficult problem

Lattice knotted and linked flux tubes--open problem

CONCLUSIONS

CLASSICAL SYSTEMS

- Systems can Knot and Link
- Possibly Tight
- Varying Tube Radius and Energy Spectrum

CONCLUSIONS

QUANTUM SYSTEMS

- Knots and Links
- Fixed Tube Radius (Quantized Flux)
- Tight implies "Quantized Lengths" for tight knots
- Quantized Energy

- Universal Spectra

One Parameter per System - the Slope

Predictions for QCD !

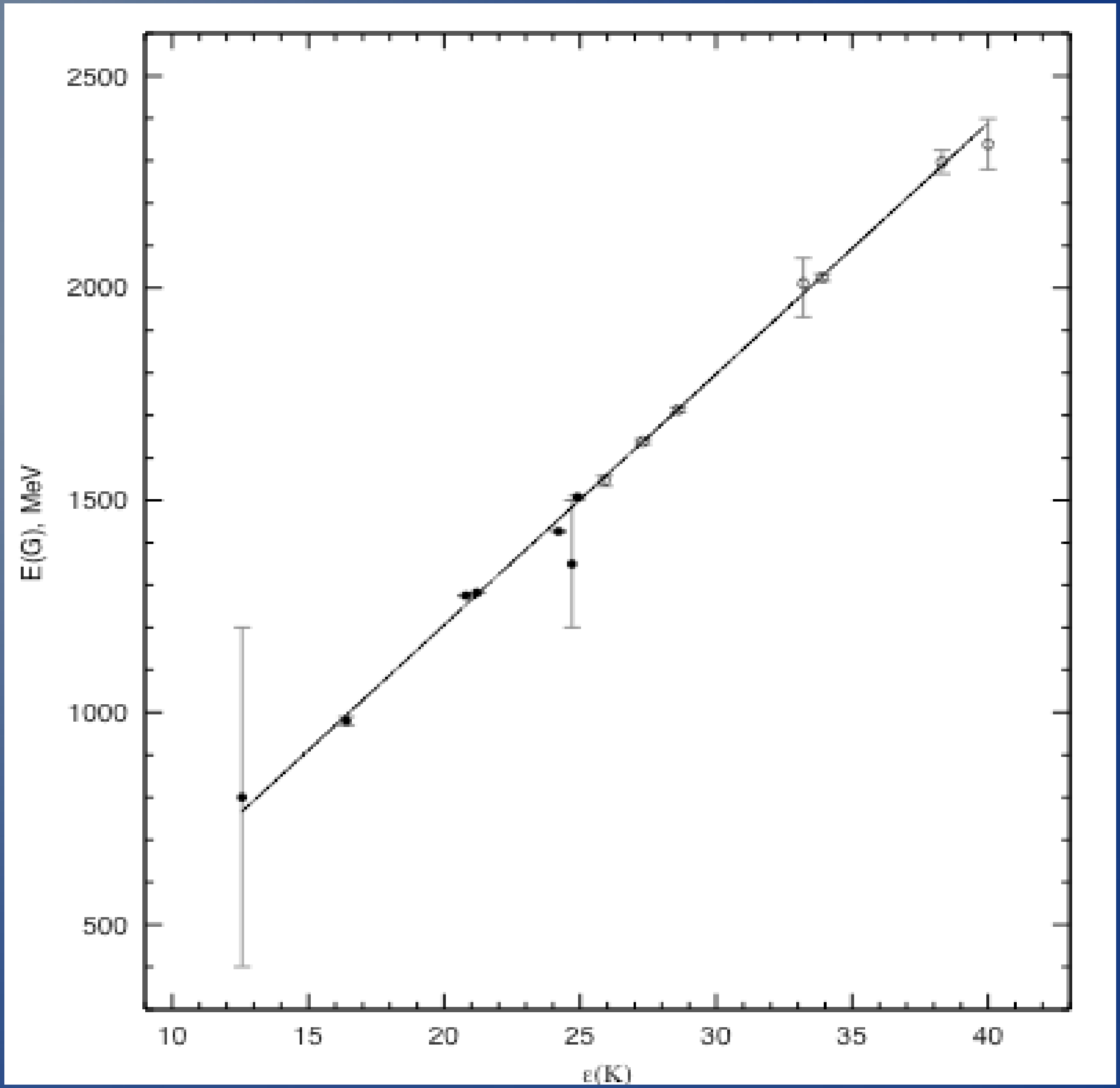
END

Now fit with only f_0 data

More conservative

No interpretation of $J > 0$

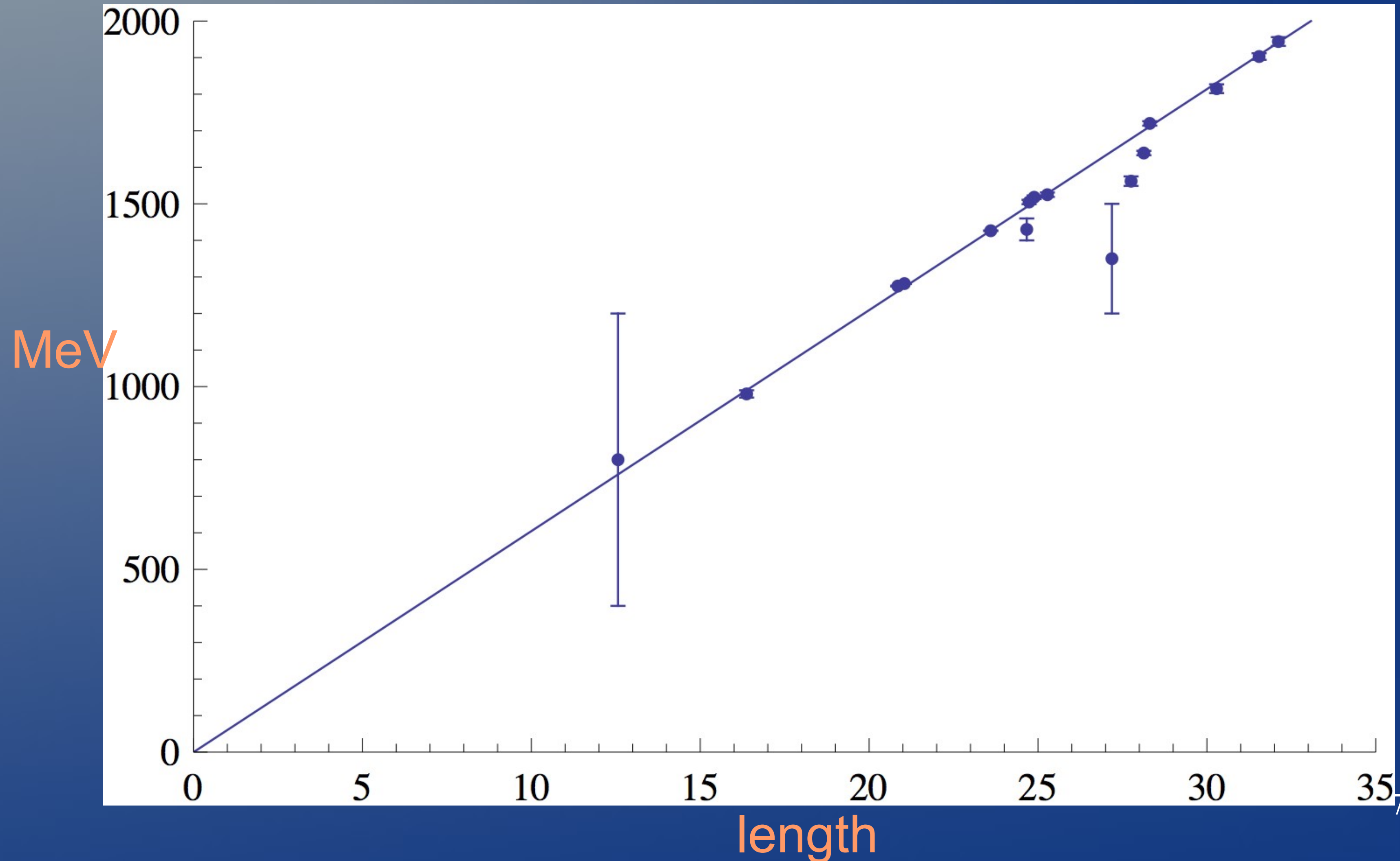
Only 6 states, so more predictions



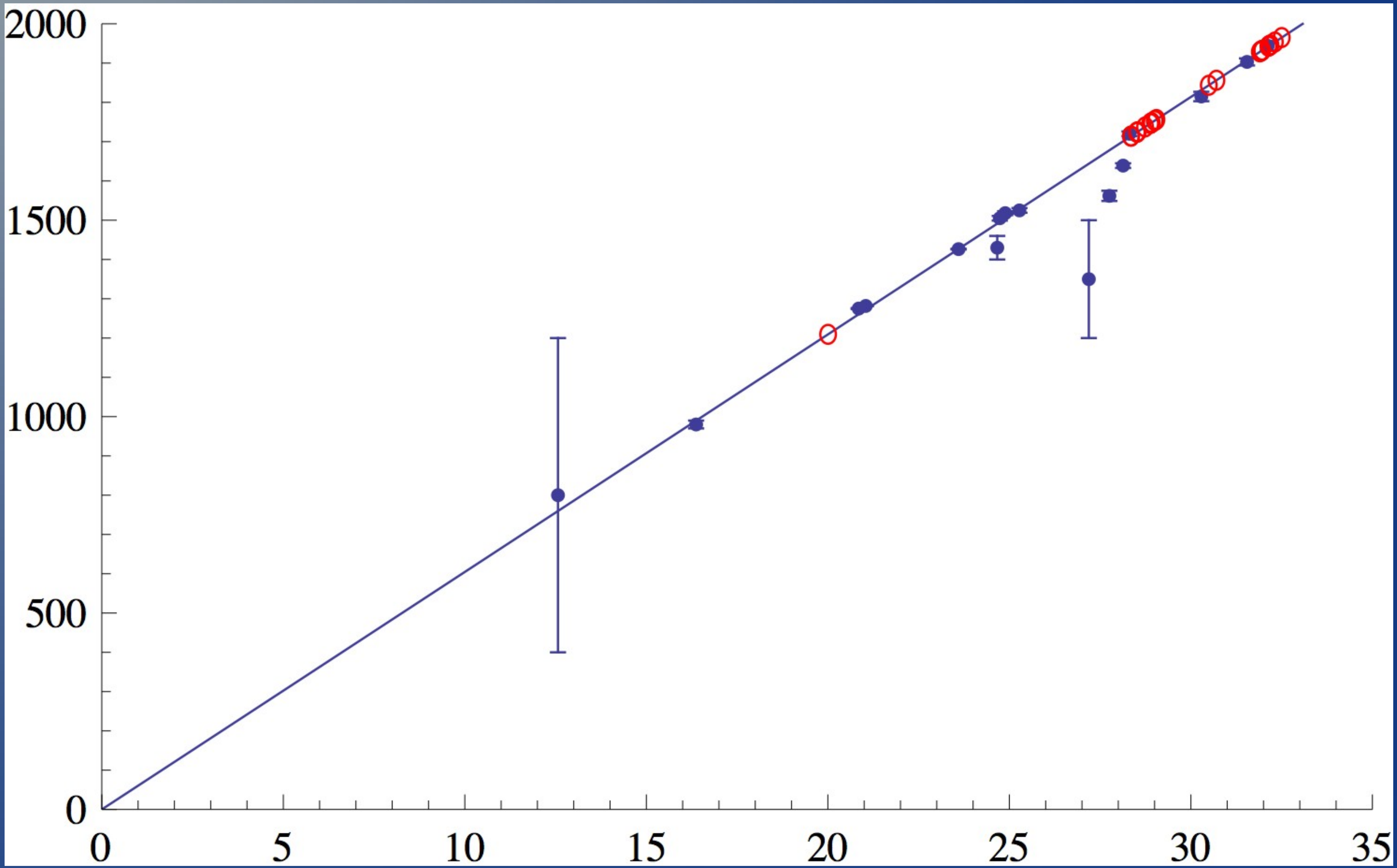
Knot energy vs glueball mass, 2003 results

Fit of all f_J data of mass < 2 GeV/c^2

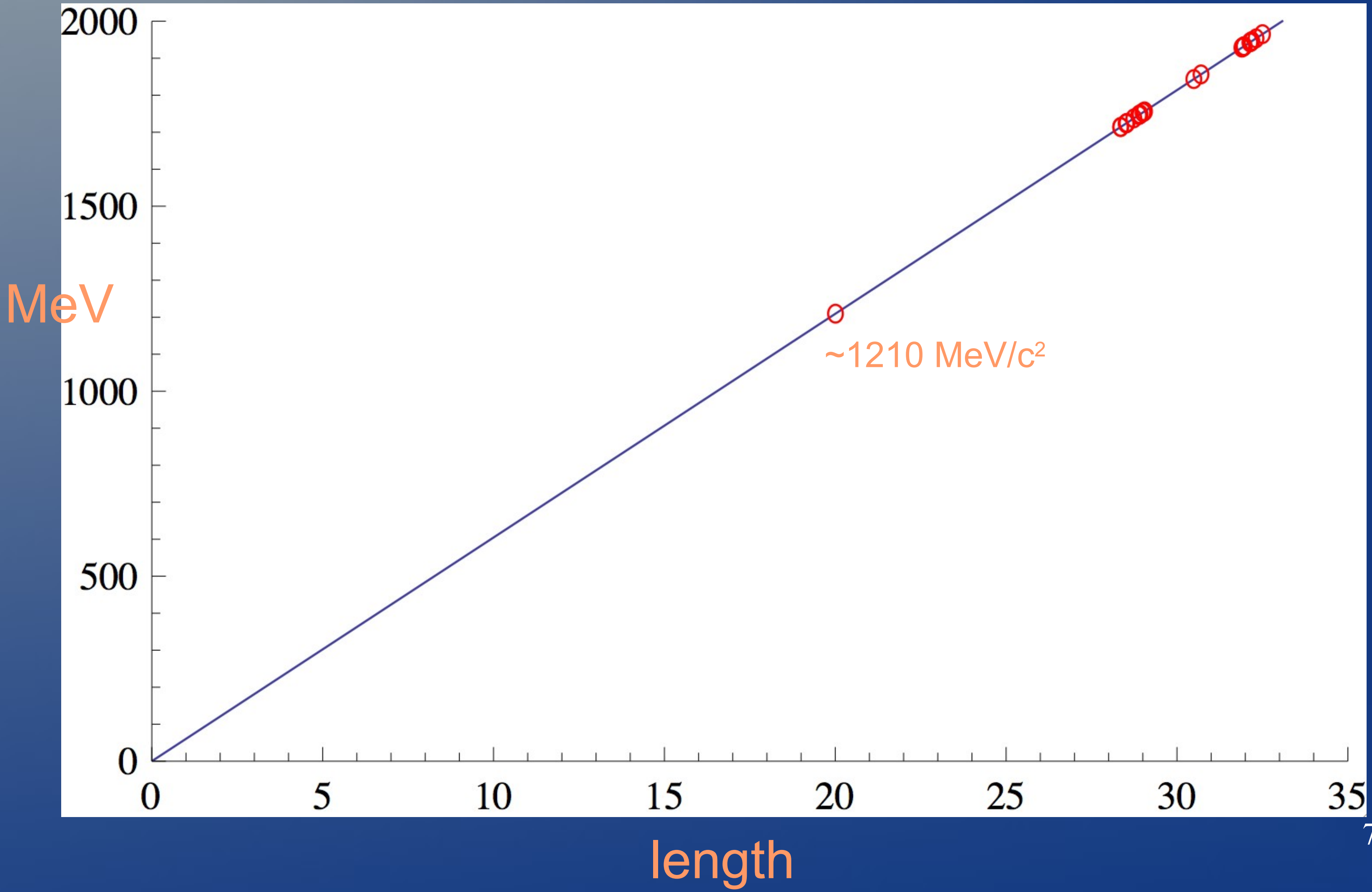
(Assume all knots and links known)



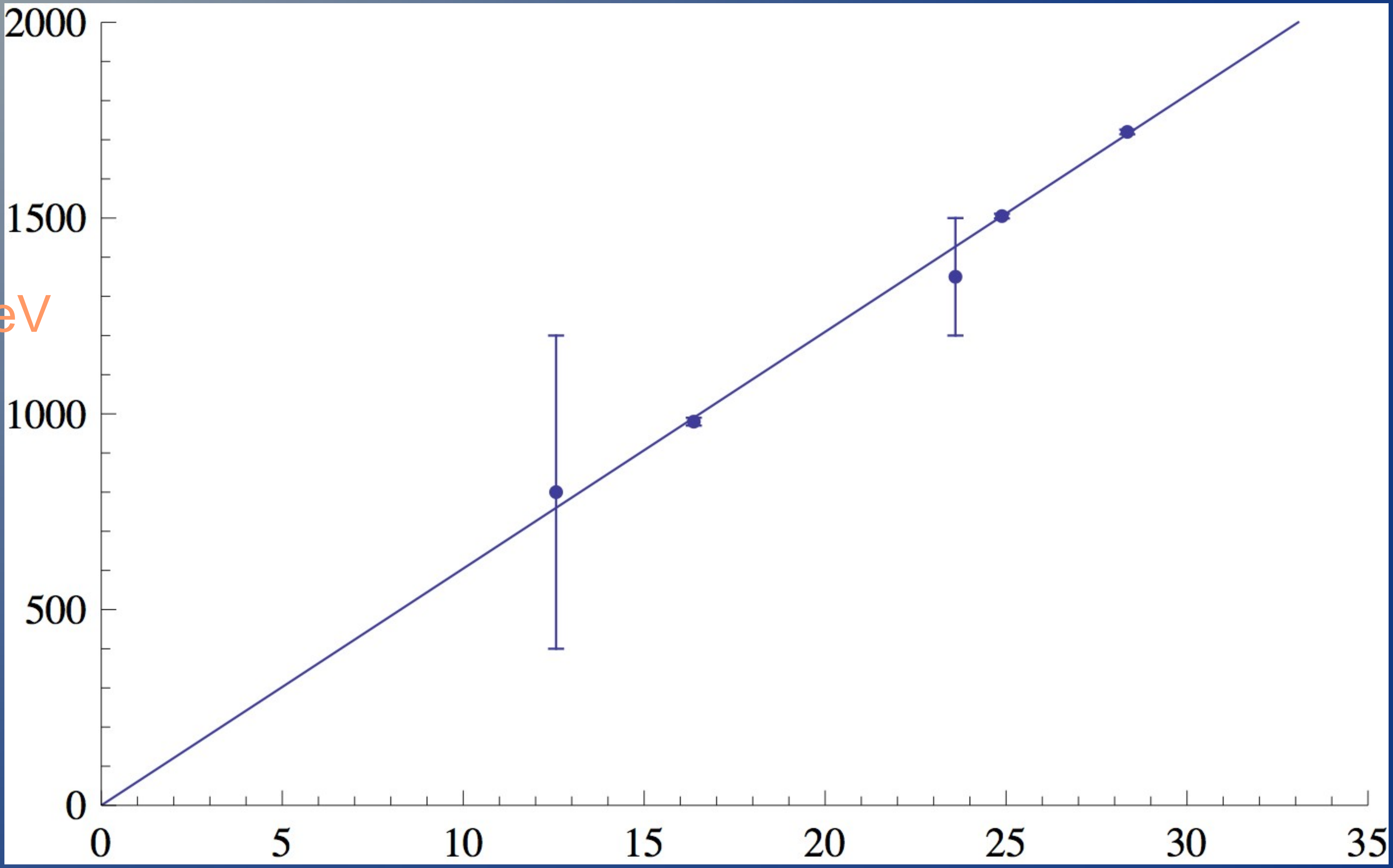
Data and Predictions for f_J States



Model Predictions



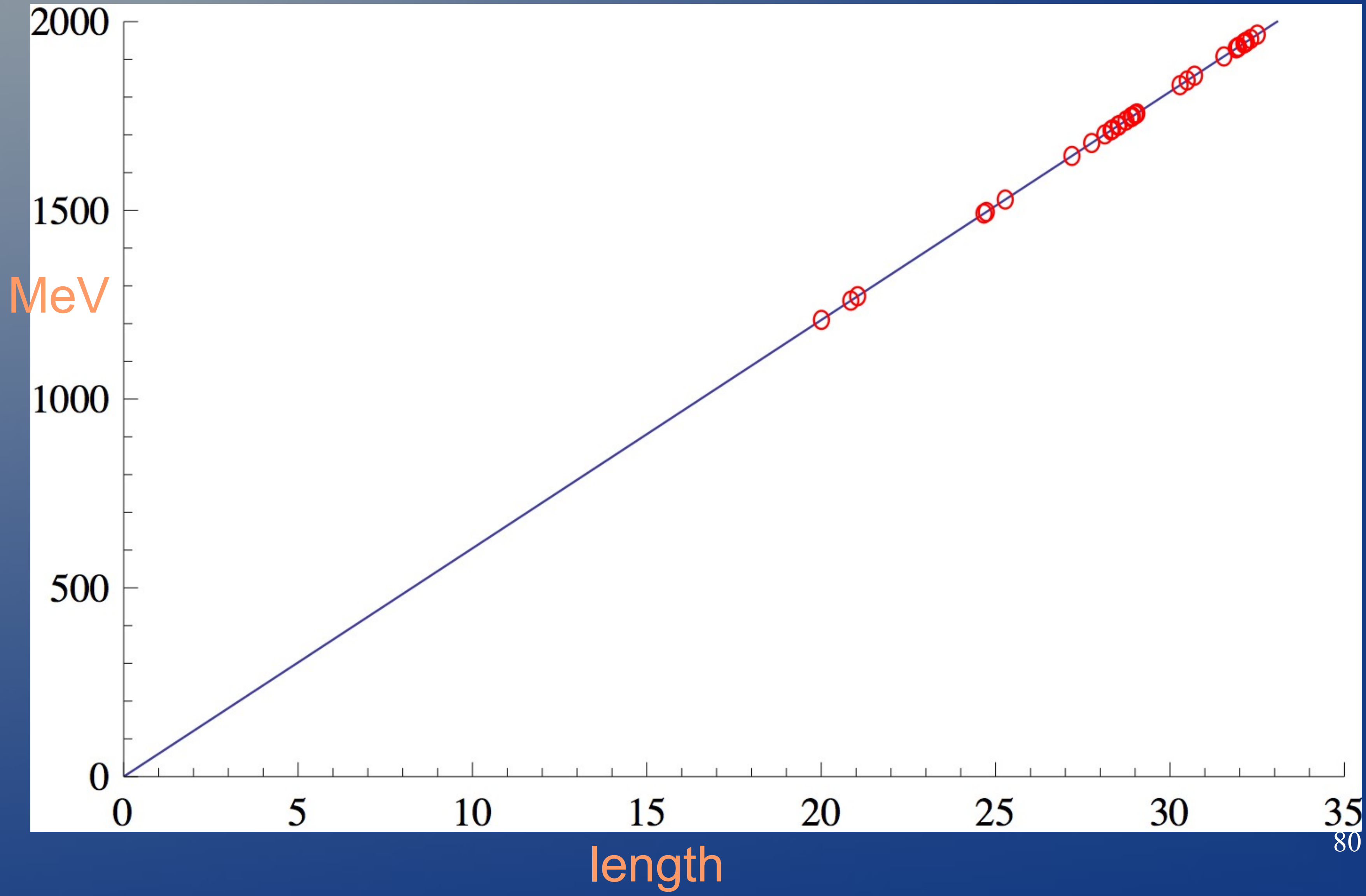
Fit of f_0 data only



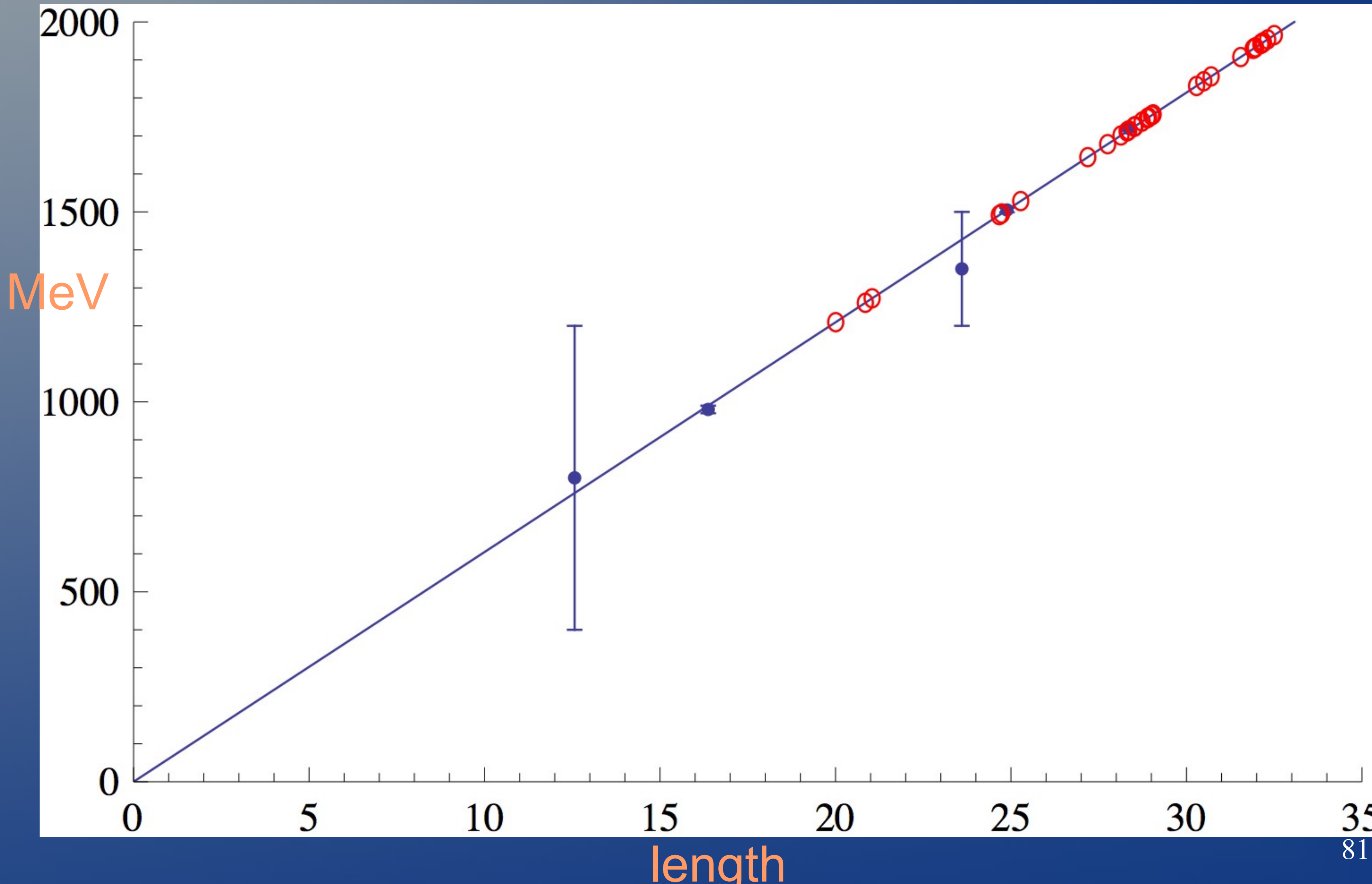
MeV

length

Predictions from fit to f_0 data



Data and Predictions for f_0 States



The Tight Knot Spectrum in QCD

Tom Kephart
Vanderbilt University

Presented at the
Isaac Newton Institute
TOD Programme
17 November 2012

The Tight Knot Spectrum in QCD

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International Centre for Mathematical Sciences Edinburgh

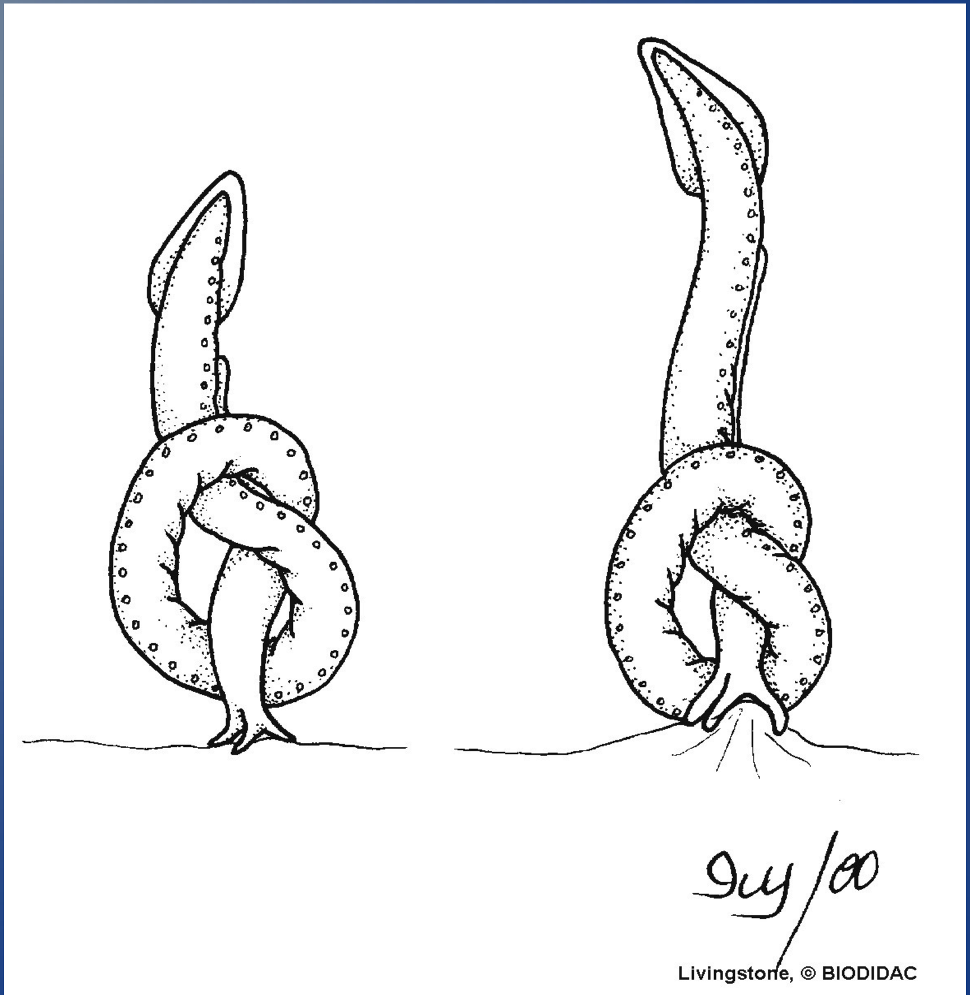
3rd TOD workshop   Isaac Newton Institute

17 October 2012

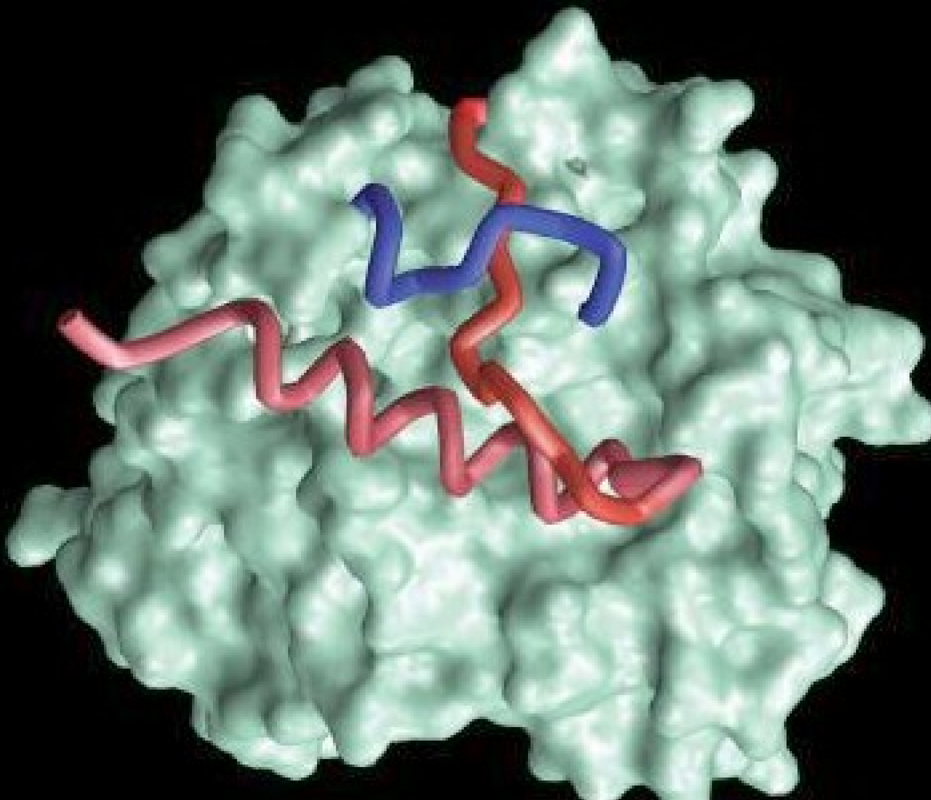
Sea Creatures in Knots

Hagfish – sightless eel

Feeding Hagfish



Proteins can also be Knotted



Computer model of knotted protein in
methanobacterium thermoautotrophicum
Argonne National Laboratory

Results

- Knot Length Errors ~0.1%
- R^2 = .9996 for n = 10 f_0 States
- Slope Parameter

  S = 60.6 +/- 0.91 MeV

Intercept -9.0 +/- 26.1

  $S \sim \Lambda_{QCD}/\pi$

- New States at $E = E_K x S$

E.g.,

4_1^2 at E = 1203 MeV,

7_7^2 at E = 1673 MeV,

etc.

Counting knots

c chiral noninvertible
+ amphichiral noninvertible
- amphichiral invertible
i chiral invertible
a fully amphichiral and invertible

n	c	$+$	$-$	i	a
Sloane	A051766	A051767	A051768	A051769	A052400
3	0	0	0	1	0
4	0	0	0	0	1
5	0	0	0	2	0
6	0	0	0	2	1
7	0	0	0	7	0
8	0	0	1	16	4
9	2	0	0	47	0
10	27	0	6	125	7
11	187	0	0	365	0
12	1103	1	40	1015	17
13	6919	0	0	3069	0
14	37885	6	227	8813	41
15	226580	0	1	26712	0
16	1308449	65	1361	78717	113