

Rare $b \rightarrow s \ell^+ \ell^-$ decays

– getting ready –

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Outline

I) Introduction: EFT of $|\Delta B| = |\Delta S| = 1$ decays

II) Experimental status & measurements

$$B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-, \quad B \rightarrow K \ell^+ \ell^-, \quad B_s \rightarrow \mu^+ \mu^-$$

III) Exclusive decays $B \rightarrow K^{(*)} \ell^+ \ell^-$

A) Low- & high- q^2 regions

B) Form factor relations

C) Optimized observables in $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

IV) Fits and implications

Effective Theory of $|\Delta B| = |\Delta S| = 1$ decays

Flavour changes in SM – only via W^\pm exchange

$U_i = \{u, c, t\}$:

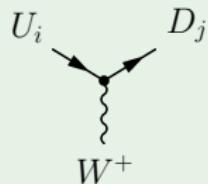
$$Q_U = +2/3$$

$D_j = \{d, s, b\}$:

$$Q_D = -1/3$$

$$\mathcal{L}_{CC} = \frac{g_2}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^+$$

~ Cabibbo-Kobayashi-Maskawa (CKM) matrix



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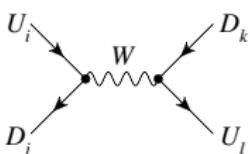
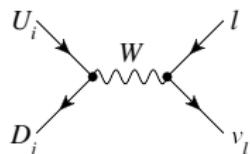
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W^+

Tree: only $U_i \rightarrow D_j$ & $D_i \rightarrow U_j$

$Q_i \neq Q_j \Rightarrow$ charged current (CC)



$$H \rightarrow \ell \nu_\ell$$

$$H_1 \rightarrow H_2 + \ell \nu_\ell$$

$$H_1 \rightarrow H_2 H_3$$

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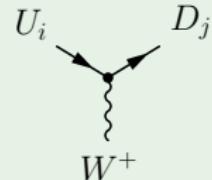
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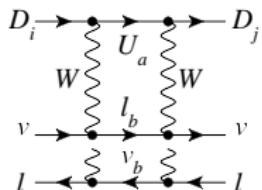
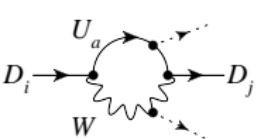
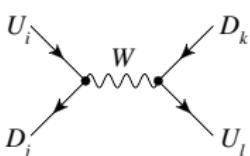
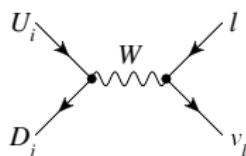


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Loop: $D_i \rightarrow D_j$ (& $U_i \rightarrow U_j$)

$Q_i = Q_j \Rightarrow$ neutral current (FCNC)



$$H \rightarrow \ell \nu_\ell$$

$$H_1 \rightarrow H_2 + \ell \nu_\ell$$

$$H_1 \rightarrow H_2 + \{\gamma, Z, g\}$$

$$\{\gamma, Z, g\} \rightarrow \{\gamma, \bar{\ell} \ell, H_3\}$$

$$H_1 \rightarrow \bar{\ell} \ell$$

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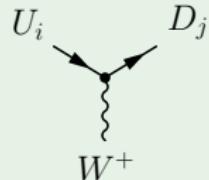
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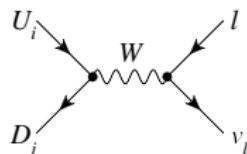


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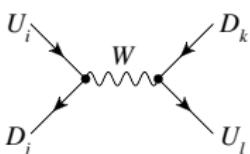
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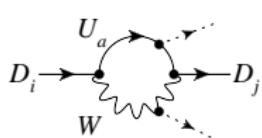


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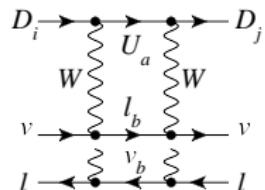


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$$H_1 \rightarrow \bar{\ell}\ell$$

$$H_1 \rightarrow H_2 + \{\bar{\ell}\ell, \bar{\nu}\nu\}$$

$$\mathcal{A} \sim G_F V_{ij}$$

$$\sim G_F V_{ij} V_{lk}^*$$

$$\sim G_F g \sum_a V_{ai} V_{aj}^* f(m_a)$$

$$\sim G_F g^2 \sum_{a,b} V_{ai} V_{aj}^* f(m_{a,b})$$

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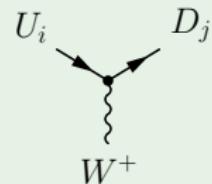
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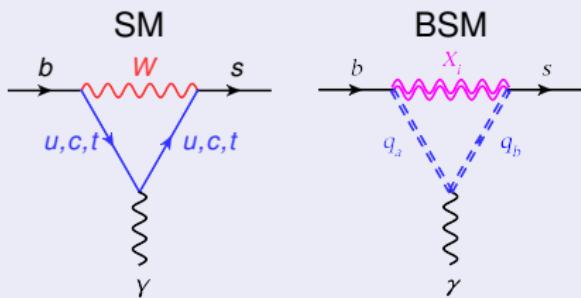


In the SM: FCNC-decays w.r.t. tree-decays are ...

quantum fluctuations = loop-suppressed

- ⇒ no suppression of contributions beyond SM (BSM) wrt SM itself
- ⇒ indirect search for BSM signals

BUT requires high precision,
experimentally and theoretically !!!



B -Hadron decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA

\gg

ext. mom'a in B restframe

\gg

QCD-bound state effects

$$M_W \approx 80 \text{ GeV}$$

$$M_Z \approx 91 \text{ GeV}$$

$$M_B \approx 5 \text{ GeV}$$

$$\Lambda_{\text{QCD}} \approx 0.5 \text{ GeV}$$

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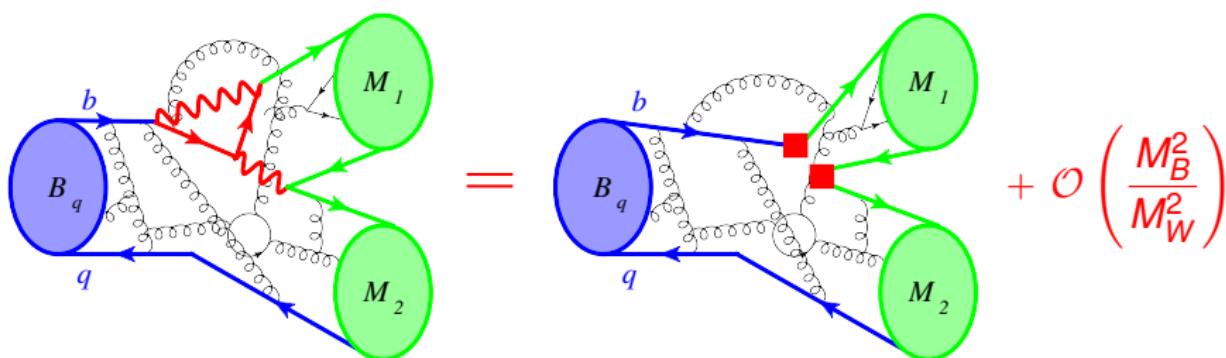
electroweak IA \gg ext. mom'a in *B* restframe

$$\begin{aligned} M_W &\approx 80 \text{ GeV} \\ M_Z &\approx 91 \text{ GeV} \end{aligned}$$

$$M_B \approx 5 \text{ GeV}$$

electroweak scale is “short-distance = local” compared to external momenta

⇒ Effective theory of $|\Delta B| = |\Delta S| = 1$ decays ⇒ separation of scales



renormalization group (RG) resums large QCD log's due to gluons
with virtuality $> M_B$ to all orders in α_s

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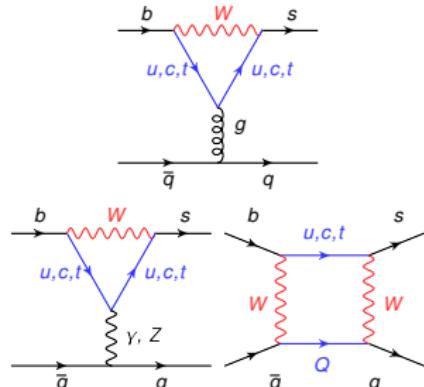
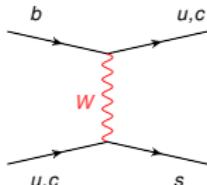
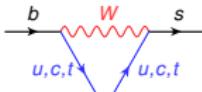
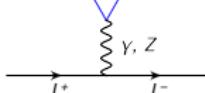
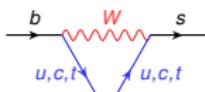
$$\mathcal{L}_{\text{eff}} \sim G_F V_{\text{CKM}} \times \left[\sum_{9,10} C_i^{\ell\bar{\ell}} \mathcal{O}_i^{\ell\bar{\ell}} + \sum_{7\gamma, 8g} C_i \mathcal{O}_i + \text{CC} + (\text{QCD \& QED-peng}) \right]$$

semi-leptonic

electro- & chromo-mgn

charged current

QCD & QED -penguin



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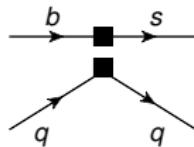
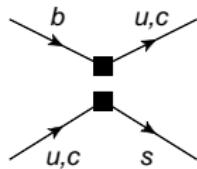
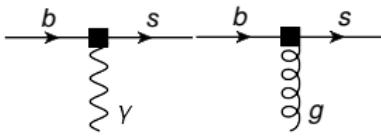
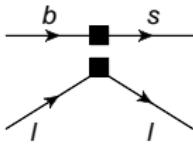
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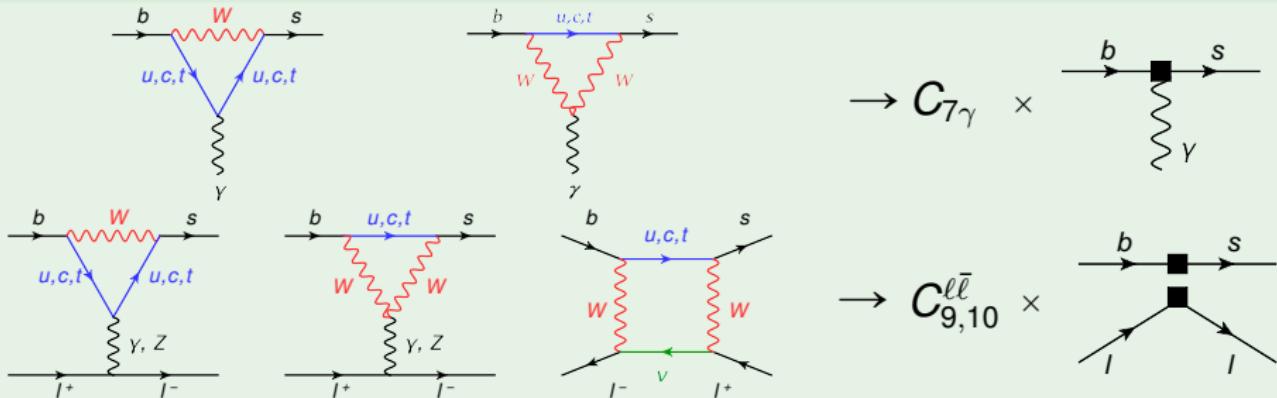
C_i = **Wilson coefficients:** contains short-dist. pmr's (heavy masses M_t, \dots – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

⇒ in SM known up to next-to-next-to-leading order

\mathcal{O}_i = **higher-dim. operators:** flavour-changing coupling of light quarks

EFT (Effective Field Theory) in the SM (Standard Model) for ...

$b \rightarrow s + \gamma$ and $b \rightarrow s + \ell^+ \ell^-$

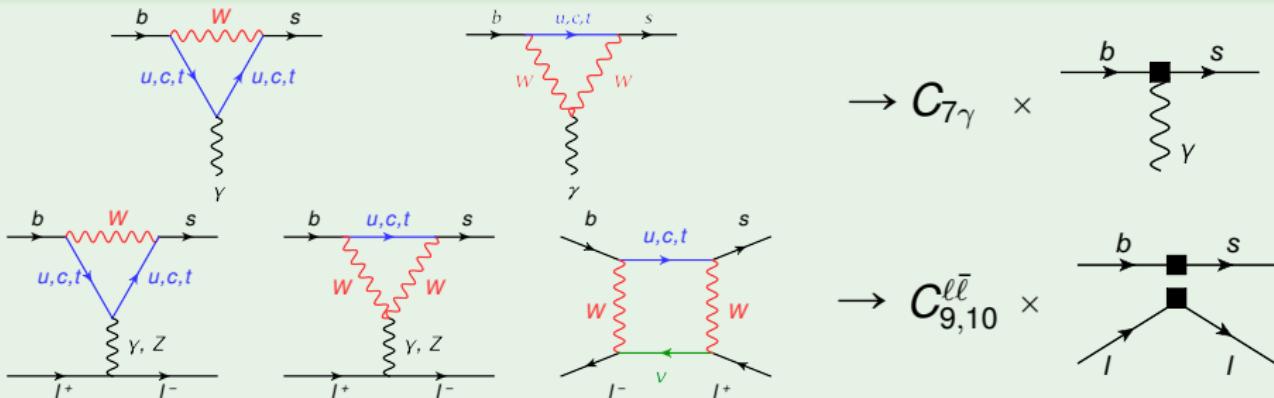


$$\mathcal{O}_{7\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s} \sigma^{\mu\nu} P_R b] F_{\mu\nu},$$

$$\mathcal{O}_{9,10}^{\ell\bar{\ell}} = \frac{\alpha_e}{4\pi} [\bar{s} \gamma^\mu P_L b] [\bar{\ell} \gamma_\mu (1, \gamma_5) \ell]$$

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and

- current-current op's $b \rightarrow s + Q\bar{Q}$, ($Q = u, c$)
- QCD penguin op's $b \rightarrow s + q\bar{q}$, ($q = u, d, s, c, b$)
- chromo-magnetic dipole $b \rightarrow s + \text{gluon}$

\Rightarrow induce backgrounds

$$b \rightarrow s + (q\bar{q}) \rightarrow s + \ell^+ \ell^-$$

vetoed in exp's for $q = c$: J/ψ and ψ'

More $b \rightarrow s + (\gamma, \ell^+ \ell^-)$ operators beyond the SM ...

... frequently considered in model-(in)dependent searches

SM' = χ -flipped SM analogues

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S + P = scalar + pseudoscalar

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T + T5 = tensor

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new Dirac-structures beyond SM:

- **SM'** : right-handed currents
- S + P : higgs-exchange & box-type diagrams
- T + T5 : box-type diagrams, Fierzed scalar tree exchange

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Extension of EFT beyond the SM ...

$$\mathcal{L}_{\text{eff}}(\mu_b) = \mathcal{L}_{\text{QED} \times \text{QCD}}(u, d, s, c, b, e, \mu, \tau, ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{\text{CKM}} \sum_{\text{SM}} (C_i + \Delta C_i) \mathcal{O}_i + \sum_{\text{NP}} C_j \mathcal{O}_j(???)$$

- ⇒ ΔC_i ... NP contributions to SM C_i
- ⇒ $\sum_{\text{NP}} C_j \mathcal{O}_j$... NP operators (e.g. $C'_{7,9,10}$, $C^{(')}_{S,P}$, ...)
- ⇒ ??? ... additional light degrees of freedom (\Leftarrow not pursued in the following)

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model-dep.

- 1) decoupling of new heavy particles @ NP scale: $\mu_{\text{NP}} \gtrsim M_W$
- 2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)
 C_i are correlated, depend on fundamental parameters

model-indep.

extending SM EFT-Lagrangian → new C_j
 C_j are UN-correlated free parameters

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Experimental results

$$B \rightarrow K^* \ell^+ \ell^-$$

$$B \rightarrow K \ell^+ \ell^-$$

$$B_s \rightarrow \mu^+ \mu^-$$

$\Delta B = 1$ FCNC's: Rich phenomenology . . .

$b \rightarrow s + \gamma$

$B \rightarrow K^* \gamma \quad (B_s \rightarrow \phi \gamma)$

- Br
- time-dep. CP asym's: S, C, H
- iso-spin asymmetry Δ_0

$B \rightarrow X_s \gamma$

- $Br, dBr/dE_\gamma$
- A_{CP} in $B \rightarrow X_s \gamma$ and $B \rightarrow X_{s+d} \gamma$

$B_s \rightarrow \gamma \gamma$

- $Br (A_{CP})$

$b \rightarrow s + \ell^+ \ell^-$

$B_s \rightarrow \ell^+ \ell^- : Br$

$B \rightarrow K \ell^+ \ell^- : dBr/dq^2, A_{FB}(q^2), F_H(q^2)$

$B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^- \quad (B_s \rightarrow \phi(\rightarrow K\bar{K}) \ell^+ \ell^-)$

- $dBr/dq^2, A_{FB}(q^2), F_{L,T}(q^2), \dots$

- $d^4 Br/dq^2 d\cos\theta_\ell d\cos\theta_K d\phi \rightarrow 12$ angular obsv's $J_{1,\dots,9}^{(s,c)}$
→ optimized obsv's $A_T^{(2,3,4,\text{re,im})}, P_{1,\dots,6}, H_T^{(1,\dots,5)}$

$B \rightarrow X_s \ell^+ \ell^- : dBr/dq^2, A_{FB}(q^2), H_{T,L}(q^2)$

$\Delta B = 1$ FCNC's: Rich phenomenology . . .

$b \rightarrow s + \gamma$

$B \rightarrow K^* \gamma \quad (B_s \rightarrow \phi \gamma)$

- Br
- time-dep. CP asym's: S, C, H
- iso-spin asymmetry Δ_0

$B \rightarrow X_s \gamma$

- Br, dBr/dE_γ
- A_{CP} in $B \rightarrow X_s \gamma$ and $B \rightarrow X_{s+d} \gamma$

$B_s \rightarrow \gamma \gamma$

- Br (A_{CP})

$b \rightarrow s + \ell^+ \ell^-$

$B_s \rightarrow \ell^+ \ell^- : Br$

$B \rightarrow K \ell^+ \ell^- : dBr/dq^2, A_{FB}(q^2), F_H(q^2)$

$B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^- \quad (B_s \rightarrow \phi(\rightarrow K\bar{K}) \ell^+ \ell^-)$

- $dBr/dq^2, A_{FB}(q^2), F_{L,T}(q^2), \dots$

- $d^4 Br/dq^2 d\cos\theta_\ell d\cos\theta_K d\phi \rightarrow 12$ angular obsv's $J_{1,\dots,9}^{(s,c)}$
 → optimized obsv's $A_T^{(2,3,4,\text{re,im})}, P_{1,\dots,6}, H_T^{(1,\dots,5)}$

$B \rightarrow X_s \ell^+ \ell^- : dBr/dq^2, A_{FB}(q^2), H_{T,L}(q^2)$

. . . to test short-distance flavor couplings C_i :

$i = 7, 7'$

$i = 7^{(')}, 9^{(')}, 10^{(')}, S^{(')}, P^{(')}, T(5), \dots$

BUT need non-perturbative hadronic input:

Form factors: $(B \rightarrow K) \rightarrow f_{+,T,0}$ and $(B \rightarrow K^*, B_s \rightarrow \phi) \rightarrow V, A_{0,1,2}, T_{1,2,3}$

Decay constants and LCDA's : $B_{d,s}, K, K^*, \phi, \dots$

Heavy quark expansion parameters : $\lambda_{1,2}, \dots$, Shape-functions . . .

Experimental data: $b \rightarrow s \ell^+ \ell^-$ – number of events

# of evts	BaBar 2012 471 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 9.6 fb^{-1}	LHCb 2011/12 1 fb^{-1}	
$B^0 \rightarrow K^{*0} \ell \bar{\ell}$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	288 ± 20	900 ± 34	
$B^+ \rightarrow K^{*+} \ell \bar{\ell}$			24 ± 6	76 ± 16	
$B^+ \rightarrow K^+ \ell \bar{\ell}$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	319 ± 23	1232 ± 40	
$B^0 \rightarrow K_S^0 \ell \bar{\ell}$			32 ± 8	60 ± 19	
$B_s \rightarrow \phi \ell \bar{\ell}$			62 ± 9	77 ± 10	
$B_s \rightarrow \mu \bar{\mu}$				emerging	
$\Lambda_b \rightarrow \Lambda \ell \bar{\ell}$			51 ± 7		
$B^+ \rightarrow \pi^+ \ell \bar{\ell}$		limit		25 ± 7	

- CP-averaged results
- vetoed q^2 region around J/ψ and ψ' resonances
- † unknown mixture of B^0 and B^\pm

Babar arXiv:1204.3933

Belle arXiv:0904.0770

CDF arXiv:1107.3753 + 1108.0695
+ ICHEP 2012

LHCb LHCb-CONF-2012-008
(-003, -006),
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Outlook / Prospects

Belle reprocessed all data $711 \text{ fb}^{-1} \rightarrow$ final analysis ?

LHCb end of 2012 additional $\gtrsim 2 \text{ fb}^{-1}$ and $(5 - 7) \text{ fb}^{-1}$ by the end of 2017

ATLAS / CMS pursue also analysis of $B \rightarrow K^* \mu \bar{\mu}$ and $B \rightarrow K \mu \bar{\mu}$

Belle II expects about (10-15) K events $B \rightarrow K^* \ell\bar{\ell}$ ($\gtrsim 2020$)

[A.J.Bevan arXiv:1110.3901]

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Angular analysis of $\bar{B} \rightarrow \bar{K}^* [\rightarrow \bar{K}\pi] + \ell^+ \ell^-$

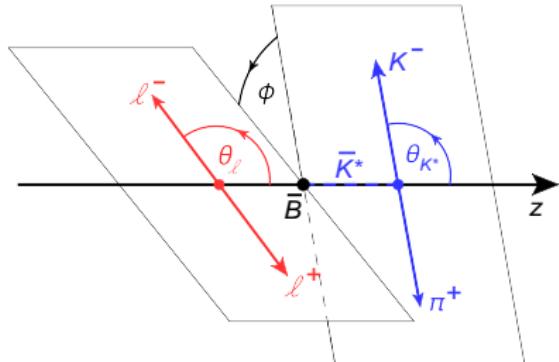
4-body decay with on-shell \bar{K}^* (vector)

1) $q^2 = m_{\ell\bar{\ell}}^2 = (\vec{p}_\ell + \vec{p}_{\bar{\ell}})^2 = (\vec{p}_{\bar{B}} - \vec{p}_{\bar{K}^*})^2$

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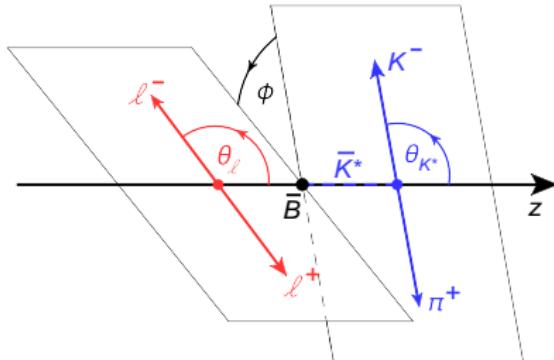
4) $\phi \angle (\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B -RF



Angular analysis of $\bar{B} \rightarrow \bar{K}^* \rightarrow \bar{K}\pi$ + $\ell^+\ell^-$

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 - 4) $\phi \angle (\vec{p}_{\bar{K}} \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B -RF



$J_i(q^2)$ = “Angular Observables”

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} = J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_\ell \\ + J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi \\ + (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi \\ + J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

Angular analysis of $\bar{B} \rightarrow \bar{K}^* [\rightarrow \bar{K}\pi] + \ell^+ \ell^-$

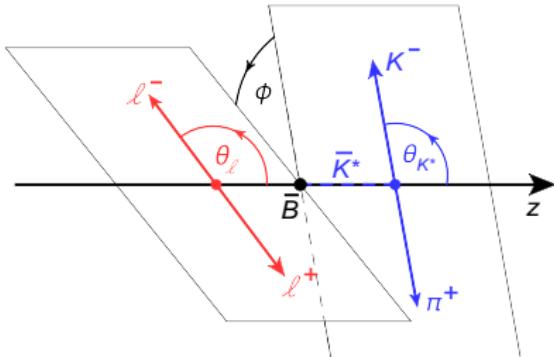
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$\Rightarrow 2 \times (12 + 12) = 48$ if measured separately: A) decay + CP-conj and B) for $\ell = e, \mu$

Angular analysis of $\bar{B} \rightarrow \bar{K}^* [\rightarrow \bar{K}\pi] + \ell^+ \ell^-$

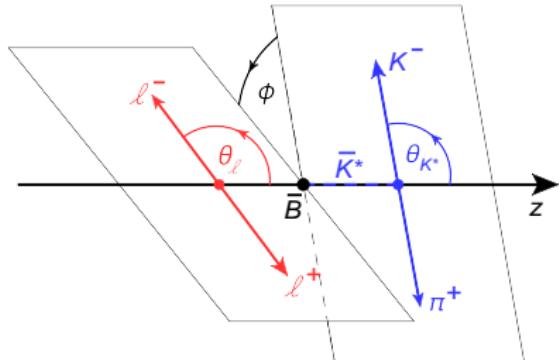
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CP-conj. decay $B^0 \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \ell^+ \ell^-$: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

$$\text{CP-even : } J_{1,2,3,4,7} \longrightarrow + \bar{J}_{1,2,3,4,7} [\delta_W \rightarrow -\delta_W]$$

$$\text{CP-odd : } J_{5,6,8,9} \longrightarrow - \bar{J}_{5,6,8,9} [\delta_W \rightarrow -\delta_W]$$

with weak phases δ_W conjugated

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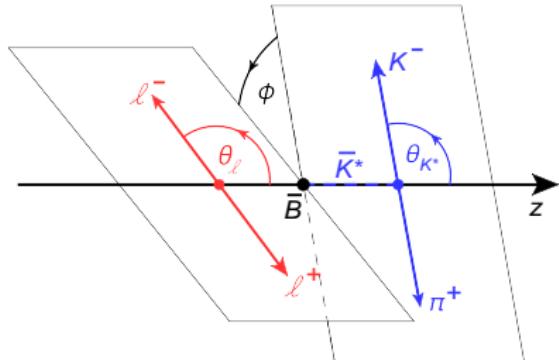
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with weak phases δ_W conjugated

1) CP-odd : $A_{\text{CP}} \sim (J_i - \bar{J}_i) \sim d^4(\Gamma + \bar{\Gamma})$ = flavour-untagged B samples

2) (naive) T-odd $J_{7,8,9}$: $A_{\text{CP}} \sim \cos\delta_s \sin\delta_W \rightarrow$ not suppressed by small strong phases δ_s

Which operators contribute to which J_i ?

J_i	$\text{SM}^{(\prime)}, \text{SM} \times \text{SM}'$	S	P	$\text{SM}^{(\prime)} \times (\text{S}, \text{P})$	T, T5	$\text{SM}^{(\prime)} \times (\text{T}, \text{T5})$	$(\text{S}, \text{P}) \times (\text{T}, \text{T5})$
1s	1	—	—	—	1	m_ℓ	—
1c	1	1	1	m_ℓ/m_b	1	$m_\ell (\text{SM}^{(\prime)} \times \text{T5})$	—
2s	1	—	—	—	1	—	—
2c	1	—	—	—	1	—	—
3	1	—	—	—	1	—	—
4	1	—	—	—	1	—	—
5	1	—	—	m_ℓ	—	m_ℓ	$\text{S} \times \text{T5}, \text{P} \times \text{T}$
6s	1	—	—	—	—	m_ℓ	—
6c	—	—	—	m_ℓ	—	$m_\ell (10^{(\prime)} \times \text{T})$	$\text{S} \times \text{T5}, \text{P} \times \text{T}$
7	1	—	—	m_ℓ	—	m_ℓ	$\text{P} \times \text{T5}, \text{S} \times \text{T}$
8	1	—	—	—	$\text{T} \times \text{T5}$	—	—
9	1	—	—	—	$\text{T} \times \text{T5}$	—	—

[Krüger/Matias hep-ph/0502060], [Altmannshofer et al. arXiv:0811.1214v5], [Alok et al. 1008.2367, CB/Hiller/van Dyk 1212.2321]

— = no contribution

1 = order one contribution

m_ℓ = kinematic suppression by lepton mass: $m_\ell/\sqrt{q^2}$

Naive factorization &
narrow width approximation
of $K^* \rightarrow K\pi$

Data for $B \rightarrow K^* + \ell^+ \ell^-$: Br , A_{FB} , F_L

angular analysis in each q^2 -bin in θ_ℓ , θ_K

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_K} = \frac{3}{2} F_L \cos^2 \theta_K + \frac{3}{4} (1 - F_L) \sin^2 \theta_K$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_\ell} = \frac{3}{4} F_L \sin^2 \theta_\ell + \frac{3}{8} (1 - F_L)(1 + \cos^2 \theta_\ell) + A_{FB} \cos \theta_\ell$$

⇒ fitted F_L and A_{FB}

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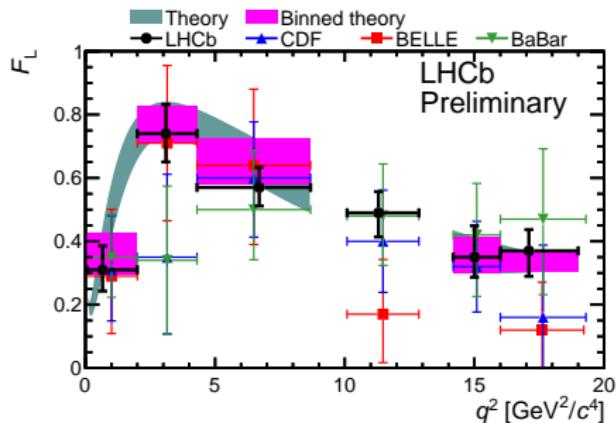
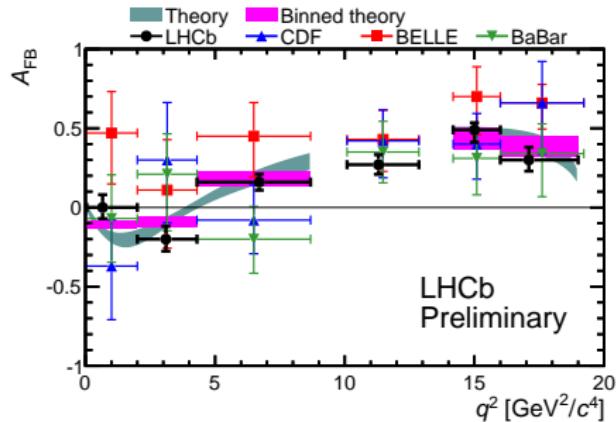
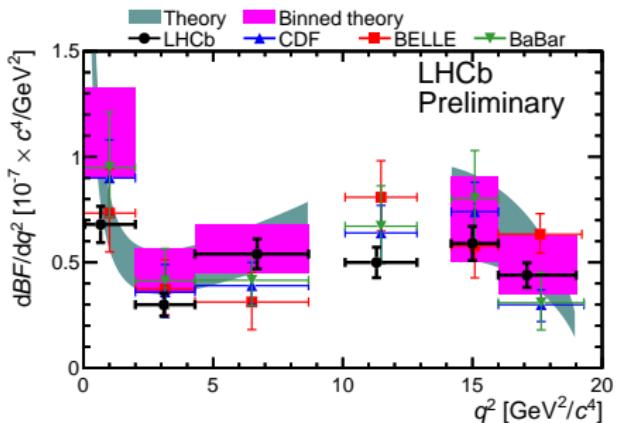
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\Rightarrow fitted F_L and A_{FB}

SM-predictions: CB/Hiller/van Dyk arXiv:1105.0376

form factors Ball/Zwicky hep-ph/0412079



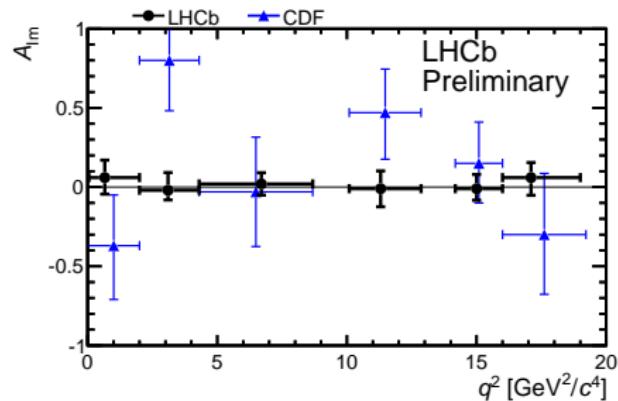
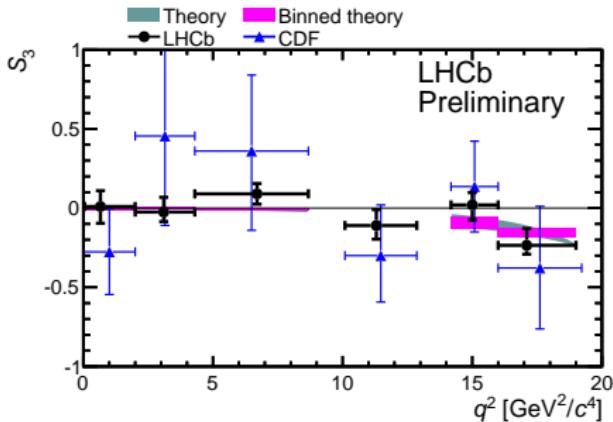
Data for $B \rightarrow K^* + \ell^+ \ell^-$:

measurement of $A_T^{(2)}$, A_{im} from CDF and S_3 , S_9 from LHCb

$$\frac{2\pi}{(\Gamma + \bar{\Gamma})} \frac{d(\Gamma + \bar{\Gamma})}{d\phi} = 1 + S_3 \cos 2\phi + (A_{im} \text{ or } S_9) \sin 2\phi$$

with

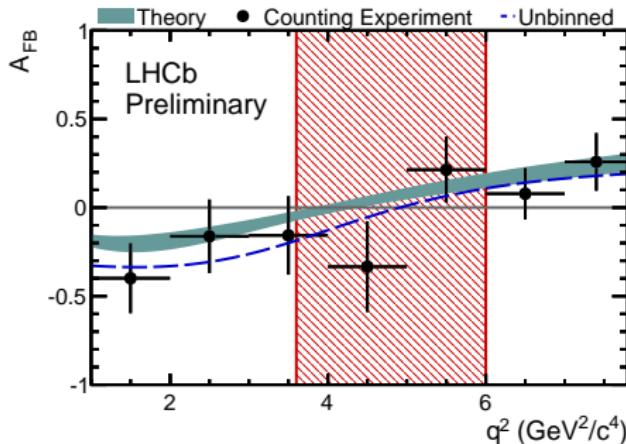
$$S_3 = \frac{J_3 + \bar{J}_3}{\Gamma + \bar{\Gamma}} = \frac{1}{2}(1 - F_L) A_T^{(2)}, \quad A_{im} = A_9 = \frac{J_9 - \bar{J}_9}{\Gamma + \bar{\Gamma}}, \quad S_9 = \frac{J_9 + \bar{J}_9}{\Gamma + \bar{\Gamma}},$$



Data for $B \rightarrow K^* + \ell^+ \ell^-$:

Zero-crossing of A_{FB} in low- q^2 region:

finer q^2 -bins than before: $\Delta q^2 = 1 \text{ GeV}^2$



Measurement: [LHCb Collab. LHCb-CONF-2012-008]

$$q_0^2 = (4.9^{+1.1}_{-1.3}) \text{ GeV}^2$$

Theory (SM): $q_0^2 = (4.0 \dots 4.3 \pm 0.3) \text{ GeV}^2$

[Beneke/Feldmann/Seidel hep-ph/0412400]

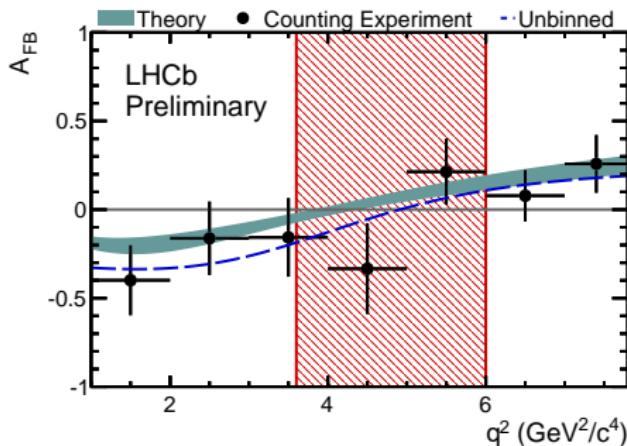
[Ali/Kramer/Zhu hep-ph/0601034]

[CB/Hiller/van Dyk/Wacker arXiv:1111.2558]

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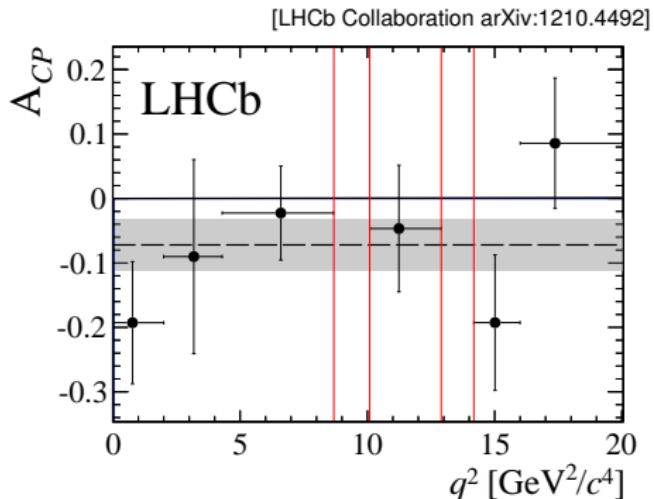
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 [CB/Hiller/van Dyk/Wacker arXiv:1111.2558]

Rate CP asymmetry A_{CP}



$B \rightarrow K^*(\rightarrow K\pi) + \ell^+\ell^-$ and S -wave:

Theorists assume P -wave K^{*0} (+ narrow-width approx.) decaying in $(K\pi)$ -final state . . .

. . . BUT in reality: resonant and non-resonant production of $(K\pi)$ in S -wave config for $(K\pi)$ -inv. mass $\sqrt{p^2}$ around $M_{K^*} \approx 892$ MeV (D -wave contr. from $K^{*0}(1430)$ negligible for $\sqrt{p^2} < 1.2$ GeV)

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Modification of angular observables J_i due to S-wave

$$i = 3, 6, 9 \quad \text{not affected}$$

$$i = 4, 5, 7, 8 \quad J_i \sin 2\theta_K \rightarrow J_i \sin 2\theta_K + \mathcal{J}_i \sin \theta_K$$

$$i = 1s, 1c, 2s, 2c \quad J_{is} \sin^2 \theta_K + J_{ic} \cos^2 \theta_K \rightarrow (J_{is} + \mathcal{J}_{is}) \sin^2 \theta_K + (J_{ic} + \mathcal{J}_{ic}) \cos^2 \theta_K \\ + \mathcal{J}_{isc} \cos \theta_K$$

[Lu/Wang arXiv:1111.1513, Becirevic/Tayduganov 1207.4004, Blake/Egede/Shires 1210.5279, Matias 1209.1525]

⇒ $\mathcal{J}_{4,5,7,8,1sc,2sc}$: interference of S- and P-wave, can be separated by angular analysis

⇒ $\mathcal{J}_{1s,1c,2s,2c}$: pure S-wave, must be measured in $\sqrt{p^2}$ sidebands around M_{K^*}

S-wave contribution in $B^0 \rightarrow J/\psi K^+ \pi^- \approx 7\%$ for $0.8 \text{ GeV} < \sqrt{p^2} < 1.0 \text{ GeV}$ [BaBar hep-ex:0411016]

⇒ no information yet on $B^0 \rightarrow K^+ \pi^- \ell^+ \ell^-$

$B \rightarrow K^*(\rightarrow K\pi) + \ell^+\ell^-$ and S-wave:

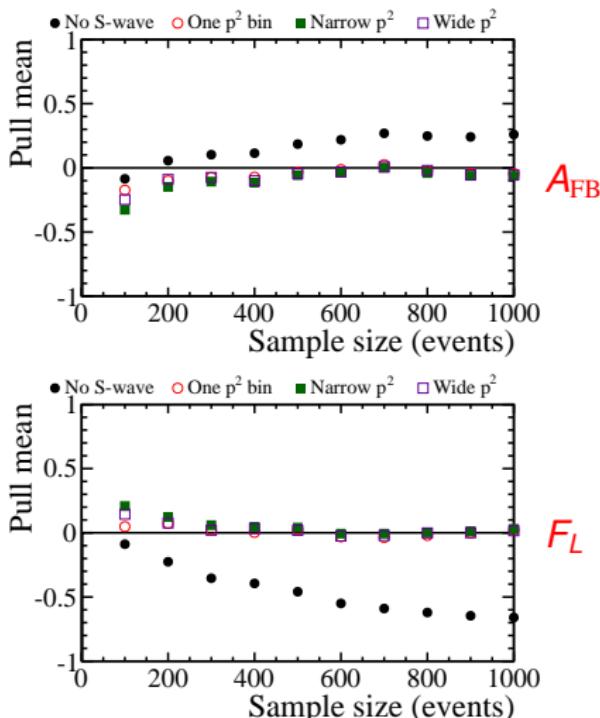
Theorists assume P-wave K^{*0} (+ narrow-width approx.) decaying in $(K\pi)$ -final state . . .

. . . BUT in reality: resonant and non-resonant production of $(K\pi)$ in S-wave config for $(K\pi)$ -inv. mass $\sqrt{p^2}$ around $M_{K^*} \approx 892$ MeV (D -wave contr. from $K^{*0}(1430)$ negligible for $\sqrt{p^2} < 1.2$ GeV)

Inclusion of S-wave in angular analysis

[Blake/Egede/Shires arXiv:1210.5279]

- ⇒ based on toy samples, modeling S-wave $\sqrt{p^2}$ -dep. using LASS-parametrisation
[LASS Collab. NPB296 (1987) 493]
- ⇒ Pull mean after refitting observable from toy sample depending on sample size (in given q^2 -bin) when
 - ignoring S-wave
 - for different treatment of $\sqrt{p^2}$ -dependence
- ⇒ “Inclusion of S-wave component will be mandatory in future experiments”

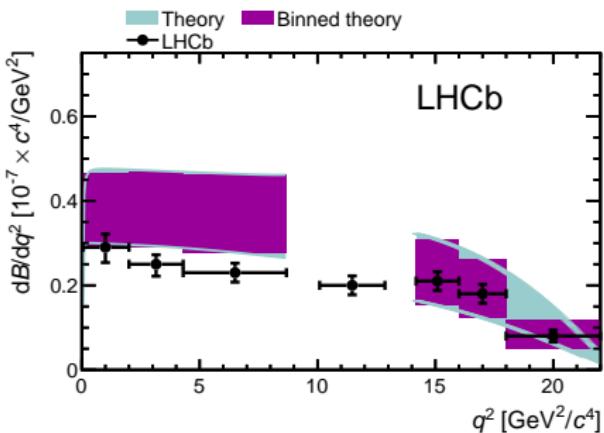


$B \rightarrow K + \ell^+ \ell^-$: 3-body decay \rightarrow 2 kinematic variables: q^2, θ_ℓ

$$\frac{1}{(d\Gamma/dq^2)} \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \frac{3}{4} [1 - F_H] \sin^2\theta_\ell + \frac{1}{2} F_H + A_{FB} \cos\theta_\ell$$

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LHCb arXiv:1209.4284 : $\langle Br \rangle, \langle A_{FB} \rangle, \langle F_H \rangle$

and previous results for $\langle Br \rangle$ from

Belle arXiv:0904.0770

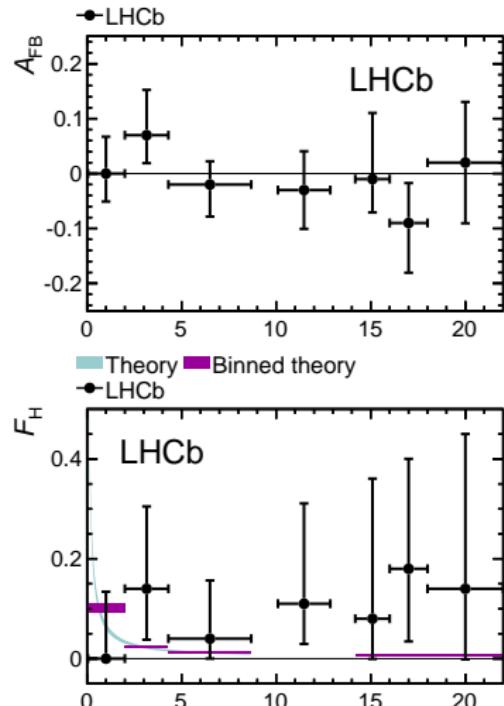
CDF arXiv:1107.3753

BaBar arXiv:1204.3933

SM prediction:

CB/Hiller/van Dyk/Wacker arXiv:1111.2558

form factors from Khodjamirian et al. arXiv:1006.4945



$B_s \rightarrow \mu^+ \mu^-$

SM prediction: $Br[B_s \rightarrow \mu^+ \mu^-] \approx 3.5 \times 10^{-9}$ [De Bruyn et al. arXiv:1204.1737]

time-integrated accounting for B_s -mixing

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time-integrated accounting for B_s -mixing

Beyond SM:

$$Br \sim |C_S - C'_S|^2 + \left| (C_P - C'_P) + \frac{2m_\ell}{m_{B_s}} (C_{10} - C'_{10}) \right|^2$$

$$B_s \rightarrow \mu^+ \mu^-$$

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Beyond SM: $Br \sim |C_S - C'_S|^2 + \left| (C_P - C'_P) + \frac{2m_\ell}{m_{B_s}} (C_{10} - C'_{10}) \right|^2$

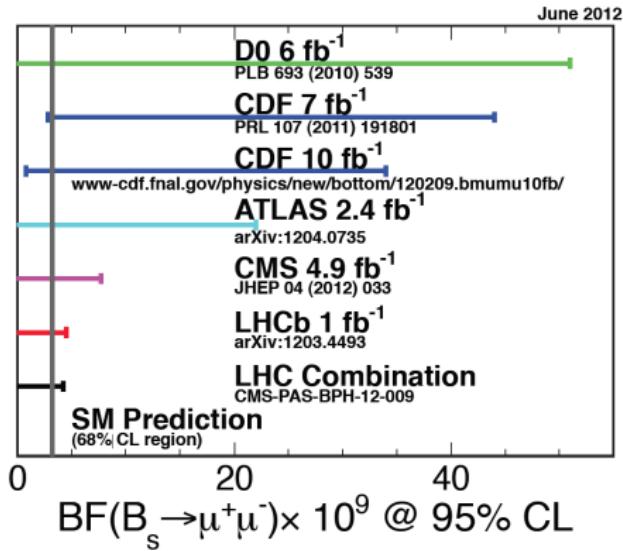
- since ~ 10 years CDF and DØ lowered upper bound from:

$$\mathcal{O}(10^{-6}) \rightarrow \mathcal{O}(10^{-8})$$

- nowadays measurements from:
CDF, DØ, LHCb, ATLAS and CMS
 \Rightarrow LHCb finds signal with 3.5σ

$$Br = (3.2^{+1.5}_{-1.2}) \times 10^{-9}$$

based on 2.1 fb^{-1}
[LHCb Collaboration arXiv:1211.2674]



Exclusive decays

$- B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^- -$

Exclusive $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

Hadronic amplitude $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

neglecting 4-quark operators

$$\mathcal{M} = \langle K\pi | C_7 \times \text{Feynman diagram } + C_{9,10} \times \text{Feynman diagram } | B \rangle$$

The equation shows the hadronic amplitude \mathcal{M} as a sum of two terms. The first term is $\langle K\pi | C_7 \times$ followed by a Feynman diagram. The diagram consists of a horizontal line with a black square vertex labeled b , a vertical wavy line labeled γ , and another black square vertex labeled s . The second term is $C_{9,10} \times$ followed by another Feynman diagram. This diagram has a horizontal line with a black square vertex labeled b , a vertical line with a black square vertex labeled s , and two diagonal lines meeting at a central point, each labeled ℓ .

Exclusive $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

Hadronic amplitude $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

neglecting 4-quark operators

$$\mathcal{M} = \langle K\pi | C_7 \times \text{Feynman diagram } + C_{9,10} \times \text{Feynman diagram } | B \rangle$$

\mathcal{M} may be expressed in terms of transversity amplitudes of K^* ($m_\ell = 0$)

... using narrow width approximation & intermediate K^* on-shell

⇒ "just" requires $B \rightarrow K^*$ form factors $V, A_{1,2}, T_{1,2,3}$:

$$A_\perp^{L,R} \sim \sqrt{2\lambda} \left[(C_9 \mp C_{10}) \frac{V}{M_B + M_{K^*}} + \frac{2m_b}{q^2} C_7 T_1 \right],$$

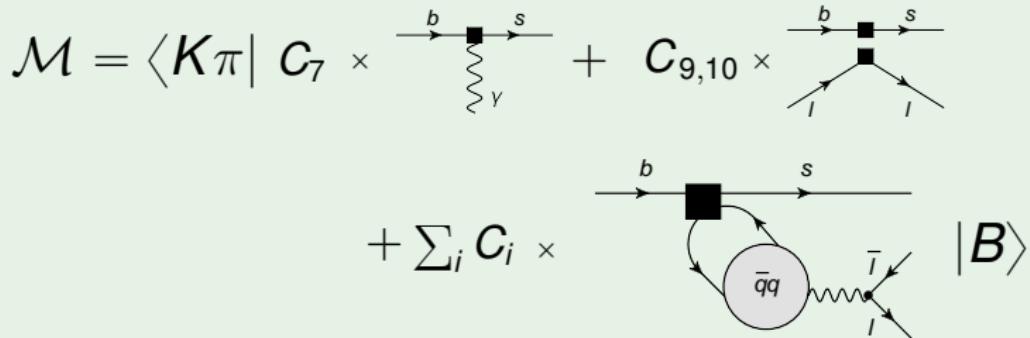
$$A_\parallel^{L,R} \sim -\sqrt{2} (M_B^2 - M_{K^*}^2) \left[(C_9 \mp C_{10}) \frac{A_1}{M_B - M_{K^*}} + \frac{2m_b}{q^2} C_7 T_2 \right],$$

$$A_0^{L,R} \sim -\frac{1}{2M_{K^*}\sqrt{q^2}} \left\{ (C_9 \mp C_{10}) [\dots A_1 + \dots A_2] + 2m_b C_7 [\dots T_2 + \dots T_3] \right\}$$

Exclusive $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

Hadronic amplitude $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

including 4-quark operators



... but 4-Quark operators and \mathcal{O}_{8g} have to be included

- current-current $b \rightarrow s + (\bar{u}u, \bar{c}c)$
- QCD-penguin operators $b \rightarrow s + \bar{q}q$ ($q = u, d, s, c, b$)

\Rightarrow large peaking background around certain $q^2 = (M_{J/\psi})^2, (M_{\psi'})^2$:

$$B \rightarrow K^{(*)}(\bar{q}q) \rightarrow K^{(*)}\bar{\ell}\ell$$

q^2 - regions in $b \rightarrow s \ell^+ \ell^-$

$$K^{(*)}\text{-energy in } B\text{-rest frame: } E_{K^{(*)}} = (M_B^2 + M_{K^{(*)}}^2 - q^2)/(2 M_B)$$

⇒ Two regions in q^2 where theory can give reliable predictions beyond naive factorization

q^2 -region	low- q^2 : $q^2 \ll M_B^2$	high- q^2 : $q^2 \sim M_B^2$
$K^{(*)}$ -recoil	large recoil: $E_{K^{(*)}} \sim M_B/2$	low recoil: $E_{K^{(*)}} \sim M_{K^{(*)}} + \Lambda_{\text{QCD}}$
theory method	QCDF, nl OPE: $q^2 \in [1, 6] \text{ GeV}^2$	OPE + HQET: $q^2 \geq (14 \dots 15) \text{ GeV}^2$

[QCDF: Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

[non-local OPE: Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945 & 1211.0234]

[local OPE: Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

⇒ $\bar{c}c$ vetoed in experiment

$$dBr[B \rightarrow K^* \ell^+ \ell^-]/dq^2$$

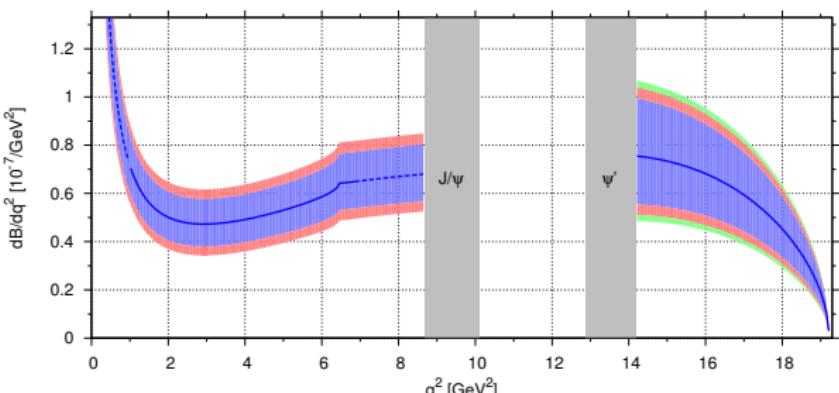
⇒ light resonances

$$q^2 \lesssim 1 \text{ GeV}^2 \text{ not vetoed}$$

small for CP-aver. obs's
relevant for CP-asy's

[Jäger/Martin-Camalich 1212.2263]

[Khodjamirian/Mannel/Wang 1211.0234]



Low- q^2 = Large Recoil

QCD Factorisation (QCDF)

[Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

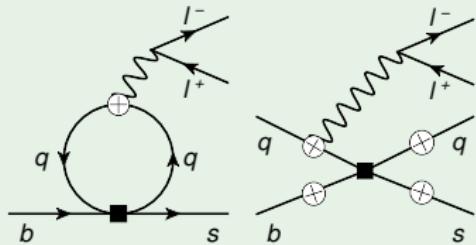
= (large recoil + heavy quark) limit [also Soft Collinear ET (SCET)]

$$\left\langle \bar{\ell} \ell K_a^* \left| H_{\text{eff}}^{(i)} \right| B \right\rangle \sim$$

$$C_a^{(i)} \times \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K*} + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$$

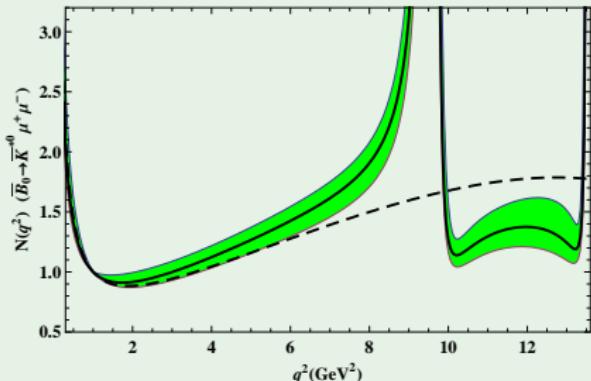
$C_a^{(i)}$, $T_a^{(i)}$: perturbative kernels in α_s ($a = \perp, \parallel$, $i = u, t$)

ϕ_B , $\phi_{a,K*}$: B - and K_a^* -distribution amplitudes



$c\bar{c}$ -contributions

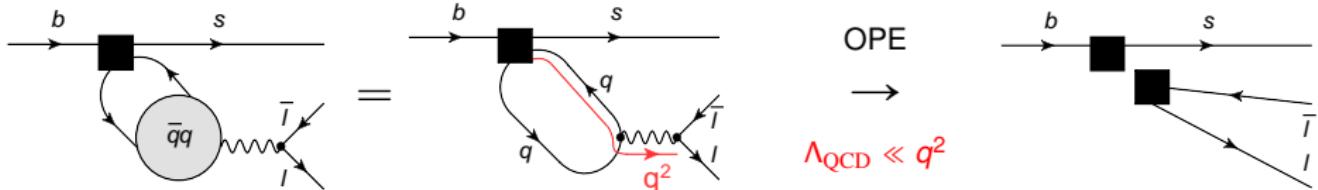
[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]



- OPE near light-cone incl. soft-gluon emission (non-local operator) for $q^2 \leq 4 \text{ GeV}^2 \ll 4m_c^2$
- hadronic dispersion relation using measured $B \rightarrow K^{(*)}(\bar{c}c)$ amplitudes at $q^2 \geq 4 \text{ GeV}^2$
- $B \rightarrow K^{(*)}$ form factors from LCSR
- up to (15-20) % in rate for $1 < q^2 < 6 \text{ GeV}^2$

High- q^2 = Low Recoil

Hard momentum transfer ($q^2 \sim M_B^2$) through $(\bar{q}q) \rightarrow \bar{\ell}\ell$ allows local OPE



$$\begin{aligned} \mathcal{M}[\bar{B} \rightarrow \bar{K}^* + \bar{\ell}\ell] &\sim \frac{8\pi^2}{q^2} i \int d^4x e^{iq \cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\text{eff}}(0), j_\mu^{\text{em}}(x)\} | \bar{B} \rangle [\bar{\ell} \gamma^\mu \ell] \\ &= \left(\sum_a C_{3a} Q_{3a}^\mu + \sum_b C_{5b} Q_{5b}^\mu + \sum_c C_{6c} Q_{6c}^\mu + \mathcal{O}(\text{dim} > 6) \right) [\bar{\ell} \gamma_\mu \ell] \end{aligned}$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $\text{dim} = 3$ operators: $\langle \bar{K}^* | Q_{3,a} | \bar{B} \rangle \sim$ usual $B \rightarrow K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$Q_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) [\bar{s} \gamma_\nu (1 - \gamma_5) b] \quad \rightarrow \quad C_9 \rightarrow C_9^{\text{eff}}, \quad (V, A_{1,2})$$

$$Q_{3,2}^\mu = \frac{im_b}{q^2} q_\nu [\bar{s} \sigma_{\nu\mu} (1 + \gamma_5) b] \quad \rightarrow \quad C_7 \rightarrow C_7^{\text{eff}}, \quad (T_{1,2,3})$$

$\text{dim} = 3$ α_s matching corrections are also known

$m_s \neq 0$ 2 additional $\text{dim} = 3$ operators, suppressed with $\alpha_s m_s / m_b \sim 0.5\%$,
NO new form factors

$\text{dim} = 4$ absent

$\text{dim} = 5$ suppressed by $(\Lambda_{\text{QCD}}/m_b)^2 \sim 2\%$,
explicite estimate @ $q^2 = 15 \text{ GeV}^2$: < 1%

$\text{dim} = 6$ suppressed by $(\Lambda_{\text{QCD}}/m_b)^3 \sim 0.2\%$ and small QCD-penguin's: $C_{3,4,5,6}$
spectator quark effects: from weak annihilation

beyond OPE duality violating effects

- based on Shifman model for c -quark correlator + fit to recent BES data
- $\pm 2\%$ for integrated rate $q^2 > 15 \text{ GeV}^2$

\Rightarrow OPE of exclusive $B \rightarrow K^{(*)}\ell^+\ell^-$ predicts small sub-leading contributions !!!

BUT, still missing $B \rightarrow K^{(*)}$ form factors @ high- q^2
for predictions of angular observables J_i

Main theory uncertainty: form factors (FF)

Currently, FF only known from LCSR @ low q^2

⇒ @ high q^2 only extrapolations based on some q^2 -dependence

- pole approximations

[Ball/Zwicky hep-ph/0406232 + 0412079]

- series expansion (z -expansion)

[Bharucha/Feldmann/Wick arXiv:1004.3249]

[Khodjamirian/Mannel/Pivovavrov/Wang arXiv:1006.4945]

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@ high q^2 : Lattice QCD required to use observables

like Br , A_{FB} , F_L , F_H , ...

⇒ work in progress

- $B \rightarrow K$

[Zhou et al. arXiv:1111.0981, Bouchard et al. arXiv:1210.6992]

- $B \rightarrow K^*$

[Liu et al. arXiv:1101.2726]

($B \rightarrow K$ technically easier on lattice,

$B \rightarrow K^*$ systematically limited → need to solve problem of unstable K^* on lattice)

Optimised observables in $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$

Angular observables

$$J_i(q^2) \sim \{\text{Re}, \text{Im}\} \left[A_m^{L,R} \left(A_n^{L,R} \right)^* \right] \\ \sim \sum_a (C_a F_a) \sum_b (C_b F_b)^*$$

$A_m^{L,R} \dots K^*$ -transversity amplitudes $m = \perp, \parallel, 0$

$C_a \dots$ short-distance coefficients

$F_a \dots$ form factors

Angular observables

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$A_m^{L,R} \dots K^*$ -transversity amplitudes $m = \perp, \parallel, 0$

$C_a \dots$ short-distance coefficients

$F_a \dots$ form factors

simplify when using form factor relations:

low K^* recoil limit: $E_{K^*} \sim M_{K^*} \sim \Lambda_{\text{QCD}}$

[Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 \approx V, \quad T_2 \approx A_1, \quad T_3 \approx A_2 \frac{M_B^2}{q^2}$$

large K^* recoil limit: $E_{K^*} \sim M_B$

[Charles et al. hep-ph/9812358, Beneke/Feldmann hep-ph/0008255]

$$\xi_{\perp} \equiv \frac{M_B}{M_B + M_{K^*}} V \approx \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 \approx T_1 \approx \frac{M_B}{2E_{K^*}} T_2$$

$$\xi_{\parallel} \equiv \frac{M_B + M_{K^*}}{2E_{K^*}} A_1 - \frac{M_B - M_{K^*}}{M_{K^*}} A_2 \approx \frac{M_B}{2E_{K^*}} T_2 - T_3$$

Low hadronic recoil

$$A_i^{L,R} \sim C^{L,R} \times f_i$$

$$C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1 + \hat{M}_{K^*}} V, \quad f_\parallel = \sqrt{2} (1 + \hat{M}_{K^*}) A_1, \quad f_0 = \frac{(1 - \hat{s} - \hat{M}_{K^*}^2)(1 + \hat{M}_{K^*})^2 A_1 - \hat{\lambda} A_2}{2 \hat{M}_{K^*} (1 + \hat{M}_{K^*}) \sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Low hadronic recoil

FF symmetry breaking

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s)$$

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$$C_7^{\text{SM}} \approx -0.3, C_9^{\text{SM}} \approx 4.2, C_{10}^{\text{SM}} \approx -4.2$$

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(“helicity FF’s” [Bharucha/Feldmann/Wick arXiv:1004.3249])

Low hadronic recoil

FF symmetry breaking OPE

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda, \alpha_s) + \mathcal{O}(\lambda^2), \quad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

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Low hadronic recoil

⇒ small, apart from possible duality violations

FF symmetry breaking

OPE

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Large hadronic recoil

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_{\perp,\parallel} + \mathcal{O}(\alpha_s, \lambda),$$

$$A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

$$C_\perp^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b M_B}{q^2} C_7,$$

$$C_\parallel^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b}{M_B} C_7,$$

Low hadronic recoil

 \Rightarrow small, apart from possible duality violations

FF symmetry breaking

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(“helicity FF’s” [Bharucha/Feldmann/Wick arXiv:1004.3249])

Large hadronic recoil

 \Rightarrow limited, end-point-divergences at $\mathcal{O}(\lambda)$

$$A_{\perp,\parallel}^{L,R} \sim \pm C_\perp^{L,R} \times \xi_{\perp,\parallel} + \mathcal{O}(\alpha_s, \lambda), \quad A_0^{L,R} \sim C_\parallel^{L,R} \times \xi_\parallel + \mathcal{O}(\alpha_s, \lambda)$$

2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

$$C_\perp^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b M_B}{q^2} C_7, \quad C_\parallel^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b}{M_B} C_7,$$

“Optimized observables” in $B \rightarrow K^* \ell^+ \ell^-$ Not yet measured (except $A_T^{(2)}$) !!!

Idea: reduce form factor (FF) sensitivity by combination (usually ratios) of angular obs's J_i
⇒ guided by large energy limit @ low- q^2 and Isgur-Wise @ high- q^2 FF-relations

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 ⇒ guided by large energy limit @ low- q^2 and Isgur-Wise @ high- q^2 FF-relations

@ low- q^2 = large recoil

$$A_T^{(2)} = P_1 = \frac{J_3}{2 J_{2s}}, \quad A_T^{(\text{re})} = 2 P_2 = \frac{J_{6s}}{4 J_{2s}}, \quad A_T^{(\text{im})} = -2 P_3 = \frac{J_9}{2 J_{2s}},$$

$$H_T^{(1)} = P_4 = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}}, \quad H_T^{(2)} = P_5 = \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}},$$

$$P_6 = \frac{-J_7/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} - J_3)}}, \quad A_T^{(3)} = \sqrt{\frac{(2 J_4)^2 + J_7^2}{-2 J_{2c} (2 J_{2s} + J_3)}}, \quad A_T^{(4)} = \sqrt{\frac{J_5^2 + (2 J_8)^2}{(2 J_4)^2 + J_7^2}}$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]

[CB/Hiller/van Dyk arXiv:1006.5013]

[Becirevic/Schneider arXiv:1106.3283]

[Matias/Mescia/Ramon/Virto arXiv:1202.4266]

“Optimized observables” in $B \rightarrow K^* \ell^+ \ell^-$ Not yet measured (except $A_T^{(2)}$) !!!

Idea: reduce form factor (FF) sensitivity by combination (usually ratios) of angular obs's J_i
⇒ guided by large energy limit @ low- q^2 and Isgur-Wise @ high- q^2 FF-relations

@ high- q^2 = low recoil

$$H_T^{(1)} = P_4 = \frac{\sqrt{2}J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}},$$

$$H_T^{(2)} = P_5 = \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}},$$

$$H_T^{(3)} = \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$H_T^{(4)} = Q = \frac{\sqrt{2}J_8}{\sqrt{-J_{2c}(2J_{2s} + J_3)}},$$

$$H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - (J_3)^2}},$$

$$\frac{A_{im}}{A_{FB}} = \frac{J_9}{J_{6s}}, \quad \text{and} \quad \frac{J_8}{J_5}$$

(A_{im} already measured by CDF, but large uncertainty)

[CB/Hiller/van Dyk arXiv:1006.5013]

[Matias/Mescia/Ramon/Virto arXiv:1202.4266 + 1207.2753]

[CB/Hiller/van Dyk arXiv:1212.2321]

Optimized observables $B \rightarrow K^* \ell^+ \ell^-$ @ low $q^2 \dots$

... experiments provide only A_{FB} , F_L , S_3 , however optimized observables related as:

$$A_T^{(2)} = P_1 = \frac{2 S_3}{1 - F_L}, \quad A_T^{(re)} = 2P_2 = -\frac{4}{3} \frac{A_{FB}}{(1 - F_L)}$$

convert A_{FB} , F_L , $S_3 \rightarrow P_1$, P_2

[Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753]

in q^2 -bins: [2, 4.3] and [4.3, 8.68] GeV 2 (naive theorist conversion due to lacking correlations)

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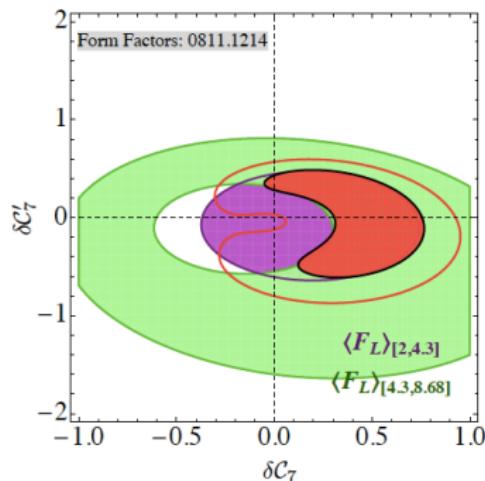
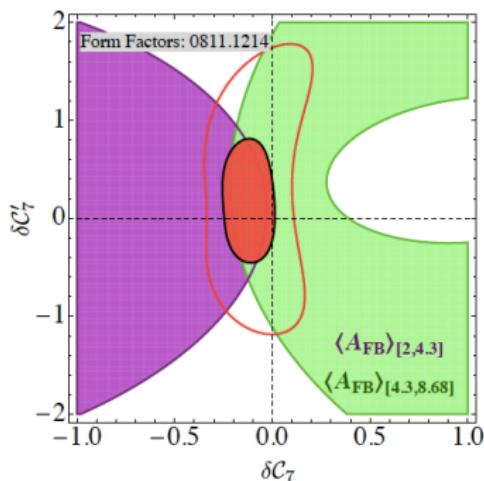
in q^2 -bins: [2, 4.3] and [4.3, 8.68] GeV 2 (naive theorist conversion due to lacking correlations)

in $\delta C_7 - \delta C_7'$ plane

A_{FB}

and

F_L



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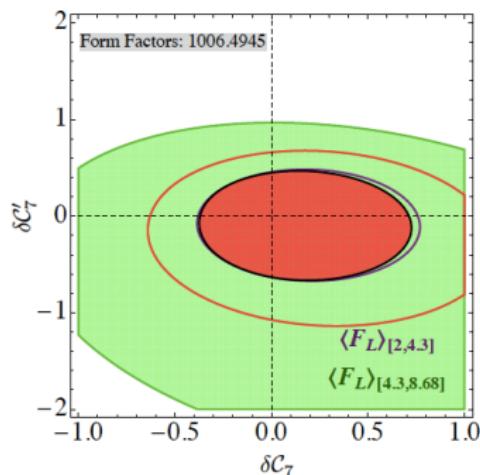
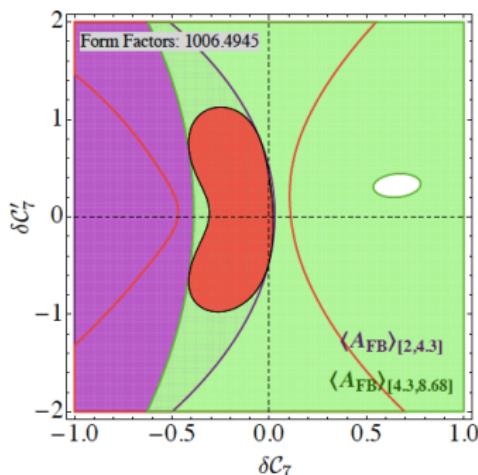
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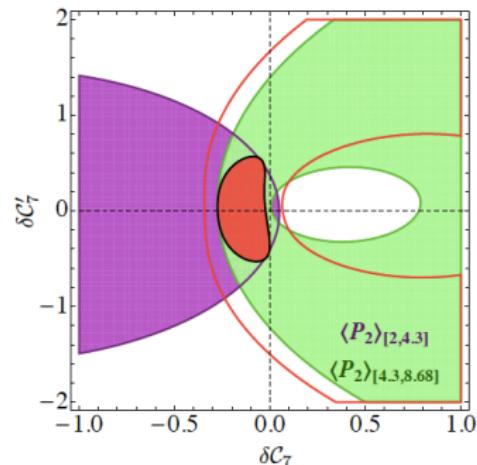
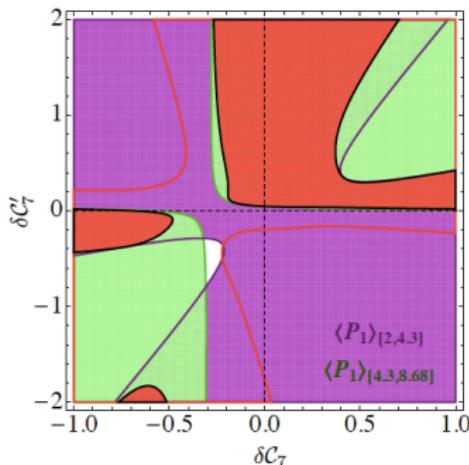
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in $\delta C_7 - \delta C_7'$ plane

P_1

and

P_2



A_{FB} and F_L are sensitive to form factors, P_1 and P_2 not!

OPE predictions @ high- q^2 beyond SM

[CB/Hiller/van Dyk arXiv:1212.2321]

Scenario	$ H_T^{(1)} = 1$	$H_T^{(2)} = H_T^{(3)}$	$H_T^{(4)} = H_T^{(5)}$	$J_{6c} = 0$	$J_7 = 0$	$J_{8,9} = 0$
SM	✓	✓	✓	✓	✓	✓
SM + (S+P)	✓	$\frac{m_\ell}{Q} \Re C_{79} \Delta_S^*$	✓	$\frac{m_\ell}{Q} \Re C_{79} \Delta_S^*$	$\frac{m_\ell}{Q} \Im C_{79} \Delta_S^*$	✓
SM + (T+T5)	$\frac{M_{K^*}^2}{Q^2} \rho_1^T$	$\frac{m_\ell}{Q} \Re C_{10} C_{T(T5)}^*$	$\frac{M_{K^*}}{Q} \Im \rho_2^T$	$\frac{m_\ell}{Q} \Re C_{10} C_T^*$	$\frac{m_\ell}{Q} \Im C_{10} C_{T5}^*$	$\Im \rho_2^T$
SM + SM'	✓	✓	✓	✓	✓	$\Im \rho_2$
all	$\frac{M_{K^*}^2}{Q^2} \rho_1^T$	$\Re C_{T(T5)} \Delta_{P(S)}^*$	$\frac{M_{K^*}}{Q} \Im \rho_2^{(T)}$	$\Re C_{T(T5)} \Delta_{P(S)}^*$	$\Im C_{T(T5)} \Delta_{S(P)}^*$	$\Im \rho_2^{(T)}$

Low recoil relations predicted by OPE

- SM-like models (first row) and the leading terms that break them in SM extensions
- ✓ = at most corrections of order $\alpha_s \Lambda/m_b$ and $C_7/C_9 \Lambda/m_b$
- $Q = \mathcal{O}(m_b, \sqrt{q^2})$ and $\Delta_{S,P} \equiv (C_{S,P} - C_{S',P'})$

⇒ if too large violations were measured this would imply contributions beyond OPE

with current data, tensor operators are constraint such that

$$| |H_T^{(1)}| - 1 | \lesssim 0.08$$

Fits and implications

– Model-independent –

“Global Fit” = combination of $b \rightarrow s + (\gamma, \ell^+ \ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

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Nuisance parameters

1) process-specific

FF's, decay const's,
LCDA pmr's,
 $\vec{\nu}$ sub-leading Λ/m_b ,
renorm. scales: $\mu_{b,0}$

2) general

quark masses, CKM, ...

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$\vec{\nu}$

Observables

- 1) observables

$$O(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

- 2) experimental data for each observable

$$\text{pdf}(O = o)$$

\Rightarrow probability distribution of values o

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Fit strategies: 1) Put theory uncertainties in likelihood:

- sample $\vec{\theta}$ -space (grid, Markov Chain, importance sampling...)
 - theory uncertainties of O_i at each $(\vec{\theta})_i$: vary $\vec{\nu}$ within some ranges $\Rightarrow \sigma_{\text{th}}(O[(\vec{\theta})_i])$
 - use Frequentist or Bayesian method \Rightarrow 68 & 95 % (CL or probability) regions of $\vec{\theta}$
- $$\chi^2 = \sum \frac{(O_{\text{ex}} - O_{\text{th}})^2}{\sigma_{\text{ex}}^2 + \sigma_{\text{th}}^2}$$

"Global Fit" = combination of $b \rightarrow s + (\gamma, \ell^+ \ell^-)$ observables

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Fit strategies: 2) Fit also nuisance parameters:

- sample $(\vec{\theta} \times \vec{\nu})$ -space (grid, Markov Chain, importance sampling...)
- accounts for theory uncertainties by fitting also $(\vec{\nu})_i$
- use Frequentist or Bayesian method \Rightarrow 68 & 95 % (CL or probability) regions of $\vec{\theta}$ and $\vec{\nu}$

SM basis + real $C_{7,9,10}(4.2 \text{ GeV})$

2D marginalised posterior

[Beaujean/CB/van Dyk/Wacker arXiv:1205.1838]

→ individual constraints at 95 % CR from

$B \rightarrow K^* \gamma$ and

SM basis + real $C_{7,9,10}(4.2 \text{ GeV})$

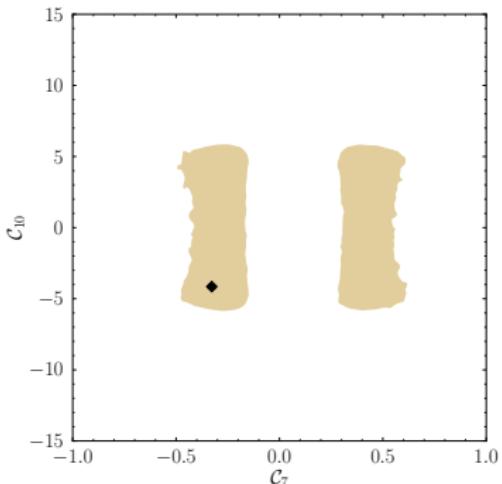
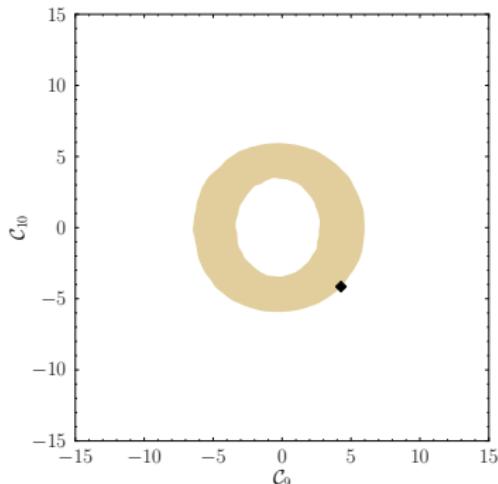
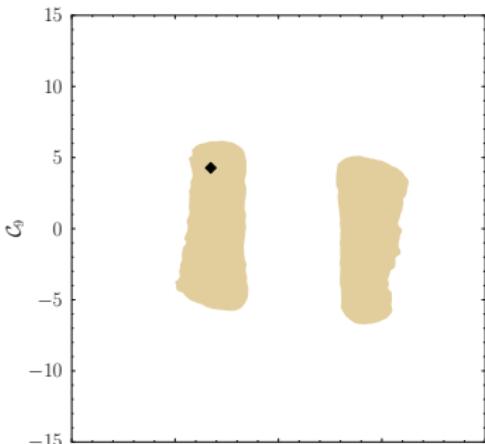
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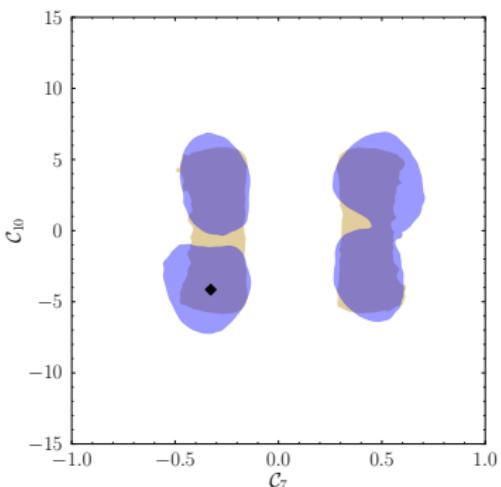
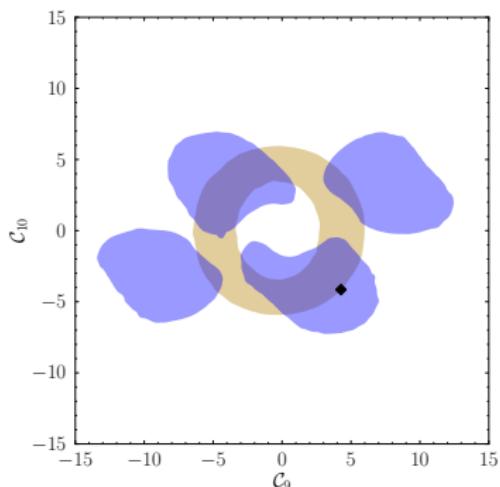
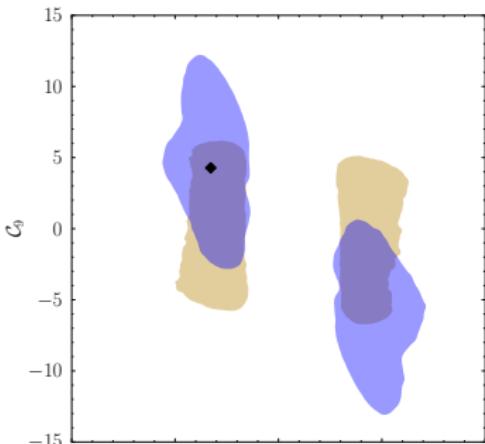
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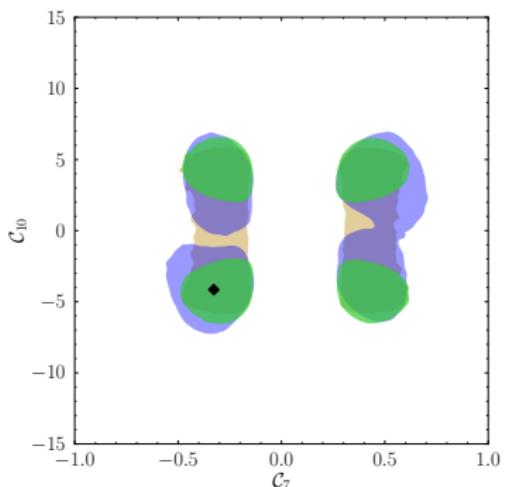
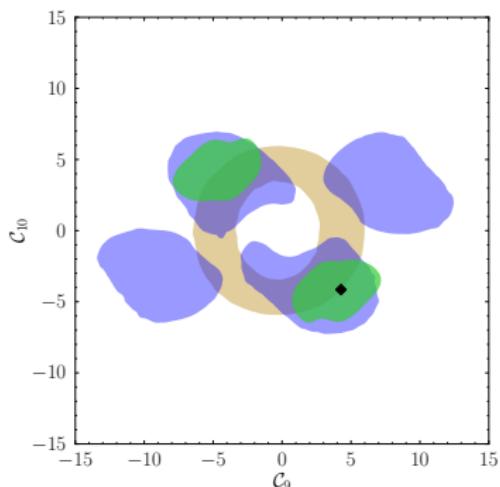
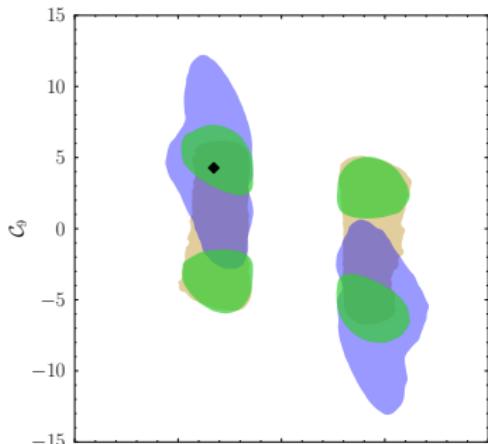
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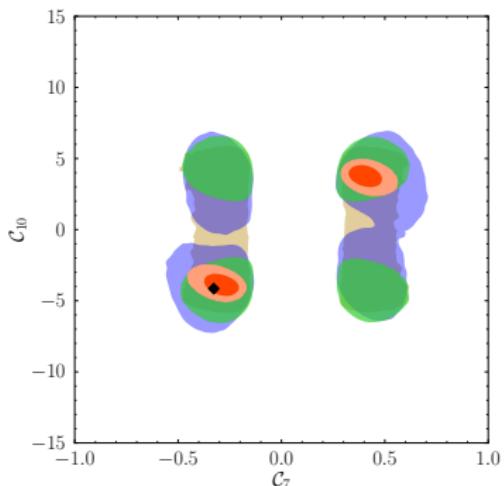
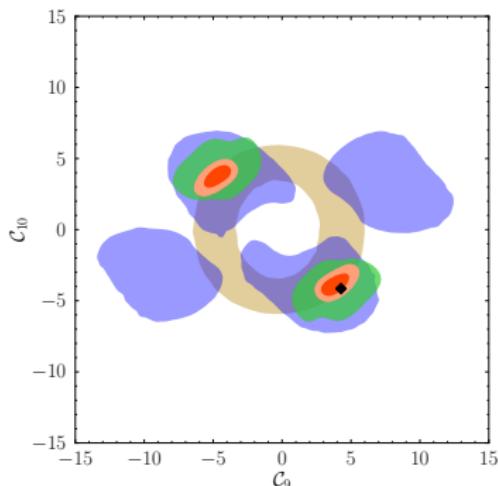
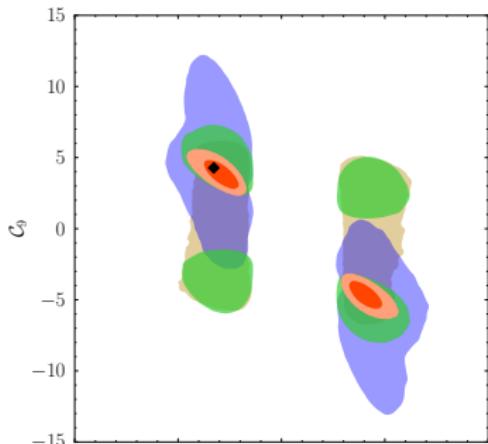
$$B \rightarrow K^* \gamma \quad \text{and} \quad$$

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$$\text{lo-}q^2 B \rightarrow K^* \ell \bar{\ell}$$

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all constraints ($+B_s \rightarrow \mu \bar{\mu}$): **68 % (95 %) CR**



SM basis + real $C_{7,9,10}(4.2 \text{ GeV})$

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[Beaujean/CB/van Dyk/Wacker arXiv:1205.1838]

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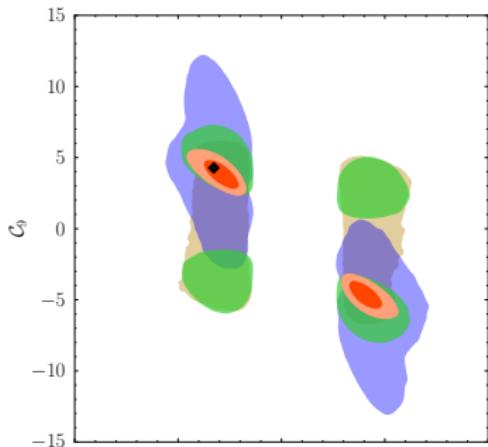
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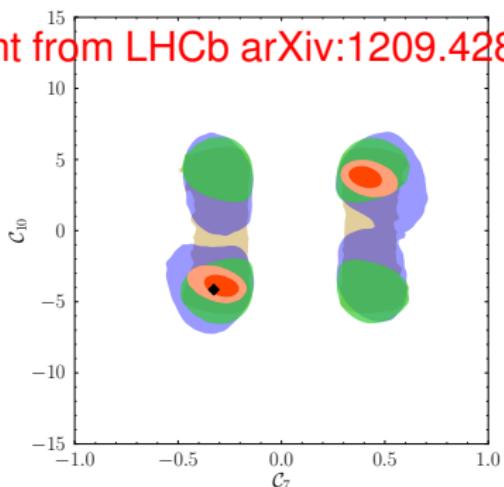
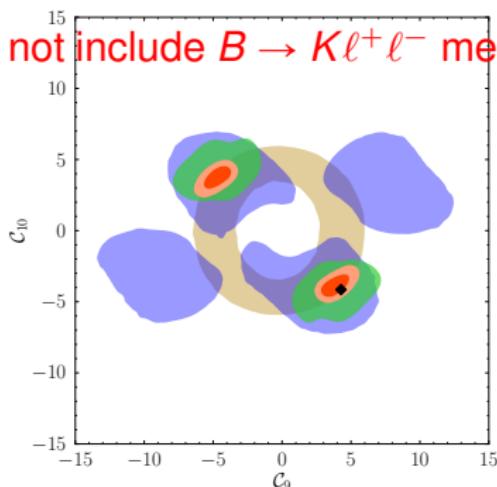
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all constraints (+ $B_s \rightarrow \mu \bar{\mu}$): **68 % (95 %) CR**



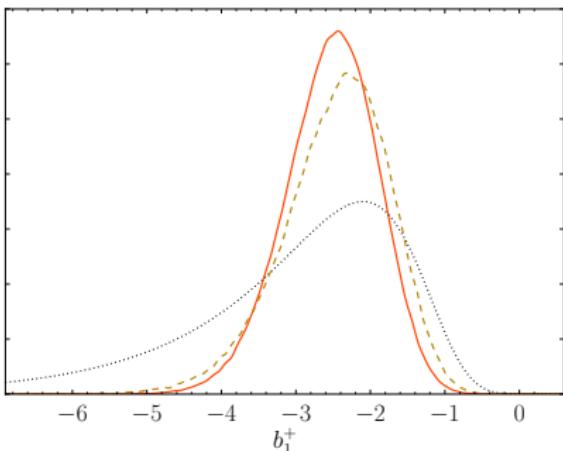
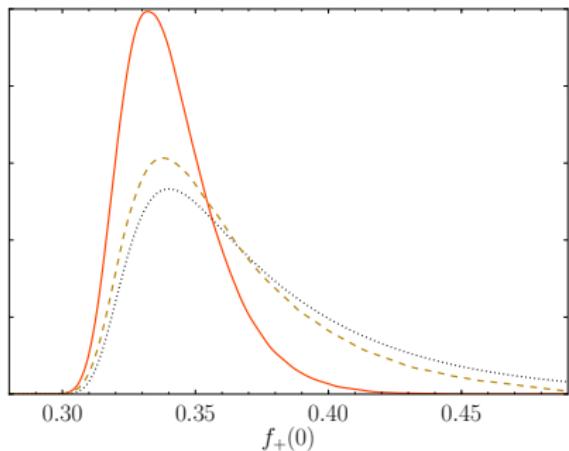
Did not include $B \rightarrow K \ell^+ \ell^-$ measurement from LHCb arXiv:1209.4284



Nuisance parameter – example $B \rightarrow K$ form factor $f_+(q^2)$

$$f_+(q^2) = \frac{f_+(0)}{1 - q^2/M_{\text{res},+}^2} \left[1 + b_1^+ \left(z(q^2) - z(0) + \frac{1}{2} [z(q^2)^2 - z(0)^2] \right) \right],$$

$$z(s) = \frac{\sqrt{\tau_+ - s} - \sqrt{\tau_+ - \tau_0}}{\sqrt{\tau_+ - s} + \sqrt{\tau_+ - \tau_0}}, \quad \tau_0 = \sqrt{\tau_+} \left(\sqrt{\tau_+} - \sqrt{\tau_+ - \tau_-} \right), \quad \tau_{\pm} = (M_B \pm M_K)^2$$



- ⇒ Prior [dotted] from LCSR calculation Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945
- ⇒ Posterior of $f_+(0)$ [left] and b_1^+ [right] using

1) $B \rightarrow K\ell^+\ell^-$ data only [dashed] vs 2) all data [solid, red]

and update in

Altmannshofer/Straub

arXiv:1206.0273

 \Rightarrow based on MCMC + Bayesian inference \Rightarrow included data from

- $B \rightarrow X_s \gamma : Br, A_{CP},$
 $B \rightarrow K^* \gamma : S$
- $B \rightarrow X_s \ell \bar{\ell} : Br,$
 $B \rightarrow K \ell \bar{\ell} : Br,$
 $B \rightarrow K^* \ell \bar{\ell} : Br, A_{FB}, F_L, S_3, A_{im},$
 $B_s \rightarrow \mu \bar{\mu} : Br$

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 $B \rightarrow K^* \ell \bar{\ell} : Br, A_{FB}, F_L, S_3, A_{im},$
 $B_s \rightarrow \mu \bar{\mu} : Br$

⇒ model-indep. NP (real or complex)

- $C_{7,7'}, 9,9', 10,10' \text{ (in varying stages)}$
- $Z\text{-penguin} + C_{7,7'}$
 ⇒ relates $b \rightarrow s \ell \bar{\ell}$ and $b \rightarrow s \nu \bar{\nu}$
- $(C_S - C_{S'}), (C_P - C_{P'})$

and update in

here in 2 parameter scenarios
from arXiv:1206.0273 \Rightarrow

\Rightarrow individual constraints at 95 %

$$S[B \rightarrow K^* \gamma]$$

$$Br[B \rightarrow X_s \gamma], A_{CP}[B \rightarrow X_s \gamma]$$

$$Br[B \rightarrow X_s \ell^+ \ell^-]$$

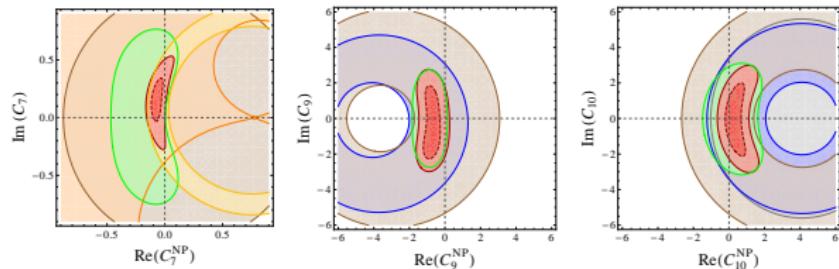
$$B \rightarrow K \ell^+ \ell^-$$

$$B \rightarrow K^* \ell^+ \ell^-$$

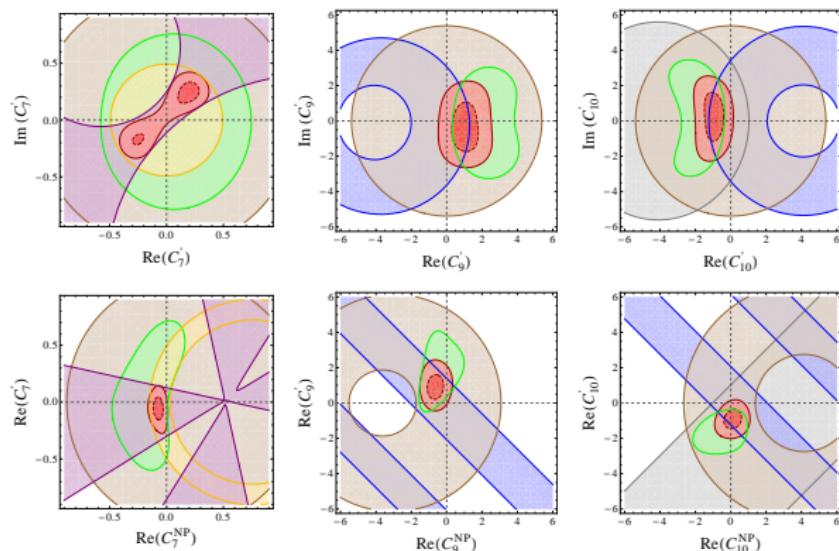
$$B_s \rightarrow \mu^+ \mu^-$$

comb. constraints: 68 % (95 %)

SM operators:



chirality-flipped operators:



and update in

→ predictions of unmeasured observables

- still large T-odd CP-asymmetries

at low- q^2 :

$$|A_7\rangle_{[1,6]} < 35 \%$$

$$|A_8\rangle_{[1,6]} < 21 \%$$

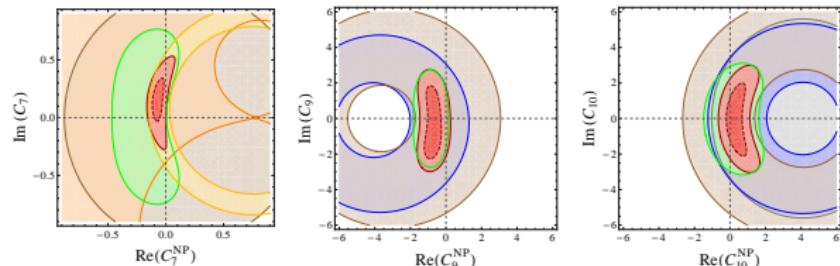
$$|A_9\rangle_{[1,6]} < 13 \%$$

at high- q^2 :

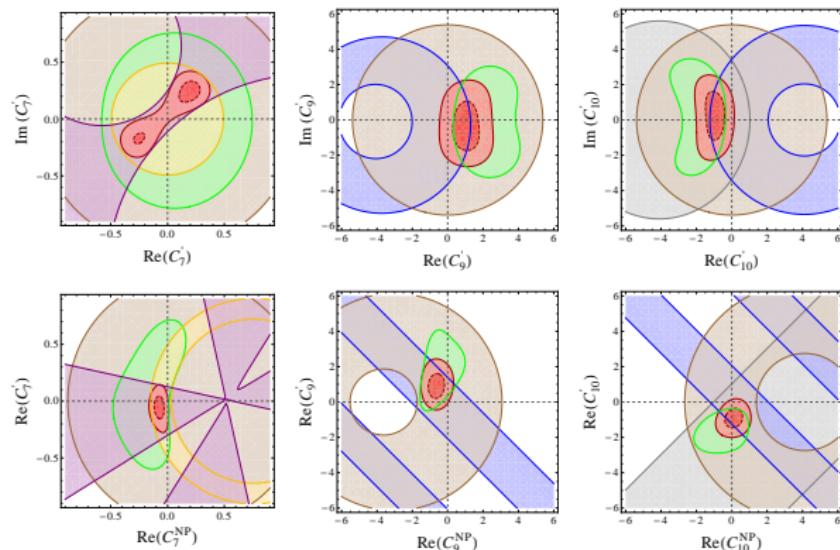
$$|A_8\rangle_{[14,16]} < 12 \%$$

$$|A_9\rangle_{[14,16]} < 20 \%$$

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$$|A_9\rangle_{[14,16]} < 20 \%$$

Lower bounds (at 95% C.L.) on the NP scale Λ of dim-6 op's,
assuming tree-FCNC $c_i = (+1, -1, +i, -i)$ & single operator

$$H_{\text{eff}} = \frac{c_i}{\Lambda^2} \mathcal{O}_i$$

Operator	Λ [TeV] for $ c_i = 1$			
	+	-	+i	-i
$\mathcal{O}_7 = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}$	69	270	43	38
$\mathcal{O}'_7 = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}$	46	70	78	47
$\mathcal{O}_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$	29	64	21	22
$\mathcal{O}'_9 = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$	51	22	21	23
$\mathcal{O}_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$	43	33	23	23
$\mathcal{O}'_{10} = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$	25	89	24	23
$\mathcal{O}_S^{(\prime)} = \frac{m_b}{m_{B_S}} (\bar{s}P_{R(L)} b)(\bar{\ell}\ell)$	93	93	98	98
$\mathcal{O}_P = \frac{m_b}{m_{B_S}} (\bar{s}P_R b)(\bar{\ell}\gamma_5 \ell)$	173	58	93	93
$\mathcal{O}'_P = \frac{m_b}{m_{B_S}} (\bar{s}P_L b)(\bar{\ell}\gamma_5 \ell)$	58	173	93	93

Relation $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow K \ell^+ \ell^-$ interesting because ...

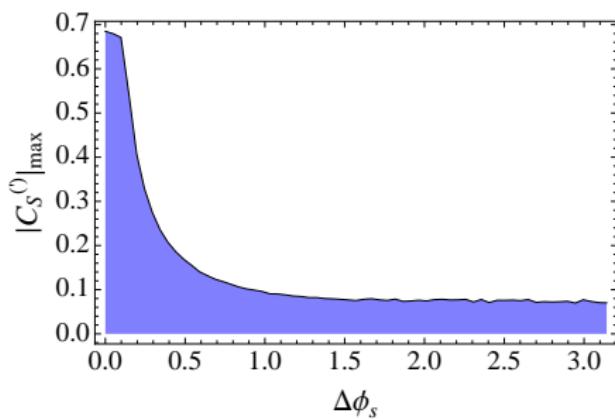
⇒ complementary dependence of

$$B_s \rightarrow \mu^+ \mu^- \rightarrow (C_i - C'_i) \quad \text{for } i = 10, S, P$$

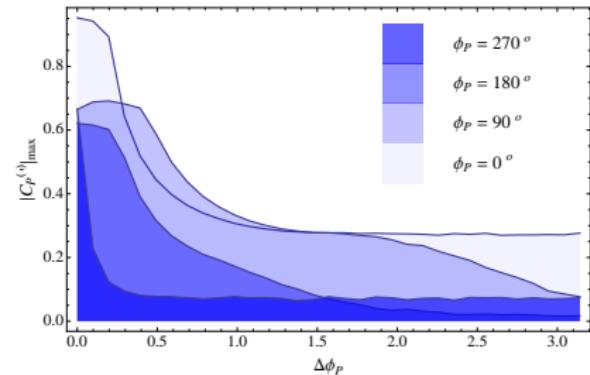
$$B \rightarrow K \ell^+ \ell^- \rightarrow (C_i + C'_i) \quad \text{for } i = 7, 9, 10, S, P$$

⇒ to constrain scalar and pseudo-scalar operators

Only complex $C_{S,S'}$ with relative phase $\Delta\phi_S$



Only complex $C_{P,P'}$ with relative phase $\Delta\phi_P$ and phase ϕ_P of C_P



[Becirevic/Kosnik/Mescia/Schneider arXiv:1205.5811]

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⇒ to constrain scalar and pseudo-scalar operators

⇒ $B \rightarrow K \ell^+ \ell^-$ (A_{FB} , F_H) constrain also T , $T5$

LHCb measurement of F_H arXiv:1209.4284

@ high- q^2 : $q^2 \in [14.2, 16.0], [16.0, 18.0], [18.0, 22.0] \text{ GeV}^2$ implies bound

$$|C_T|^2 + |C_{T5}|^2 \lesssim 0.5$$

assuming single operator dominance

$$F_H \sim \frac{\frac{m_\ell^2/q^2 \times \text{SM}}{\text{SM}} + (|C_T|^2 + |C_{T5}|^2)}{|C_T|^2 + |C_{T5}|^2}$$

!!! form factor f_+ cancels

[CB/Hiller/van Dyk arXiv:1212.2321]

Summary

Implications Summary

- **measurements** (before mid-2012) of Belle, CDF, Babar, LHCb, CMS, ATLAS on rare $B \rightarrow (K, K^*)\ell^+\ell^-$ and $B_s \rightarrow \mu^+\mu^-$ **consistent with SM**:
 - ⇒ two solutions for $C_{7,9,10}$: SM-like sign and sign-flipped
 - $B \rightarrow X_s\gamma$ or other obs. sensitive to eff. part of $C_{7,9}^{\text{eff}}$ might resolve this
 - ⇒ $Br(B \rightarrow K\mu^+\mu^-)$ @ low- q^2 lower than SM
- beyond SM:
 - ⇒ $B_s \rightarrow \mu^+\mu^-$ puts stronger constraints on $C_{S,P,10}^{(')}$
 - ⇒ $B \rightarrow K\mu^+\mu^-$ constrains $(C_{9,10,S,P} + C_{9',10',S',P'})$ and $C_{T,T5}$

!!! Currently measured only obs's with rather large theory uncertainties

EOS = Flavour tool @ TU Dortmund by Danny van Dyk et al.

Download @ <http://project.het.physik.tu-dortmund.de/eos/>

Outlook

- new $b \rightarrow s + (\gamma, \ell^+ \ell^-)$ data from LHCb, CMS, ATLAS
 - ⇒ LHCb additional 2.2 fb^{-1} to analyze by the end of 2012
 - ⇒ CMS and ATLAS add. $\gtrsim 15 \text{ fb}^{-1}$ in 2012 to search for $B_s \rightarrow \mu^+ \mu^-$ and from 2nd generation Flavor-factory Belle II $\gtrsim 2020$
- However, high exp. statistics → need to account for *S*-wave ($K\pi$)-pairs in $B \rightarrow K^* \ell^+ \ell^-$ [Becirevic/Tayduganov arXiv:1207.4004, Blake/Egede/Shires arXiv:1210.5279]
- first measurements of optimized observables in exclusive $B \rightarrow K^*(\rightarrow K\pi) \ell^+ \ell^-$ @ low- and high- q^2
 - ⇒ combinations with small hadronic uncertainties
- first lattice results of form factors $B \rightarrow K$ and $B \rightarrow K^*$ @ high- q^2 should become available

– Backup Slides –

Remark on $Br[B_s \rightarrow \mu^+ \mu^-]$

So far theorists neglected mixing of $B_s \Rightarrow$ predict Br at $t = 0$: $Br[B_s(t = 0) \rightarrow \bar{\mu}\mu]$

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But with new measurements of $\Delta\Gamma_s$ (incl. sign) from LHCb and CDF, DØ

\Rightarrow experiments actually measure time-integrated Br :

[De Bruyn et al. arXiv:1204.1737]

$$Br[B_s \rightarrow \bar{\mu}\mu] \equiv \frac{1}{2} \int_0^\infty dt \left(\Gamma[B_s(t) \rightarrow \bar{\mu}\mu] + \Gamma[\bar{B}_s(t) \rightarrow \bar{\mu}\mu] \right)$$
$$= \frac{1 + y_s \cdot \mathcal{A}_{\Delta\Gamma}}{1 - y_s^2} Br[B_s(t = 0) \rightarrow \bar{\mu}\mu]$$

with (LHCb '11)

and

$$y_s = \frac{\Delta\Gamma_s}{2\Gamma_s} = 0.088 \pm 0.014$$

\Rightarrow in SM $\mathcal{A}_{\Delta\Gamma}|_{SM} = +1$

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In SM for example

$$Br[B_s \rightarrow \bar{\mu}\mu]_{SM} = (3.53 \pm 0.38) \times 10^{-9}$$

[Mahmoudi/Neshatpour/Orloff arXiv:1205.1845]

largest uncertainties from

$$f_{B_s} = (234 \pm 10) \text{ MeV} \rightarrow 9\% \\ V_{ts} \rightarrow 5\% \\ B_s \text{ lifetime} \rightarrow 2\%$$

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... or using precise ΔM_s measurement to substitute f_{B_s} (and assuming SM) [Buras hep-ph/0303060]

$$Br[B_s \rightarrow \bar{\mu}\mu]_{SM} = \frac{(3.1 \pm 0.2) \times 10^{-9}}{0.91 \pm 0.01} = (3.4 \pm 0.2) \times 10^{-9}$$

[Buras/Girrbach arXiv:1204.5064]

Goodness of fit & Bayes factor

[Beaujean/CB/van Dyk/Wacker arXiv:1205.1838]

sgn(C_7, C_9, C_{10})	best-fit-point	log(MAP)	goodness-of-fit				log(Z)
			T_{like}	p_{like}	T_{pull}	p_{pull}	
(-, +, -)	(-0.295, 3.73, -4.14)	424.31	402.40	59%	48.8	74%	385.1
(+, -, +)	(0.418, -4.64, 3.99)	424.20	402.32	58%	48.9	74%	385.0
(-, -, +)	(-0.392, -3.09, 3.19)	403.72	387.70	0.8%	76.8	3%	363.8
(+, +, -)	(0.557, 2.25, -3.24)	399.70	384.66	0.2%	82.9	1%	360.1
SM: (-, +, -)	(-0.327, 4.28, -4.15)	430.56 [†]	402.30	69%	49.0	82%	392.4

MAP = maximum a posteriori

Z = local evidence = $\int d\vec{\theta} d\vec{\nu} P(D|\theta, \nu) \cdot P(\theta, \nu)$ = “likelihood \times prior”

⇒ 2 methods to derive p -values from 2 statistics T_{like} and T_{pull} :

indicate good fit: $p \sim (60 - 75)\%$

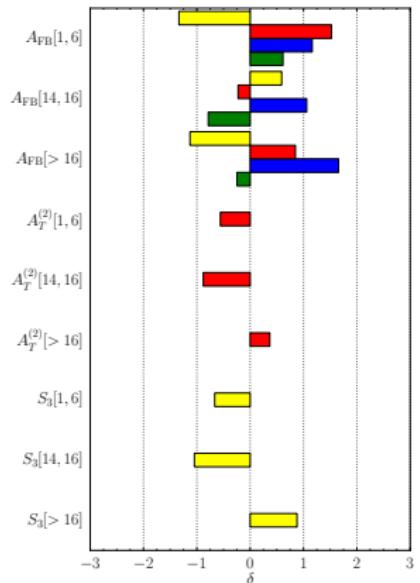
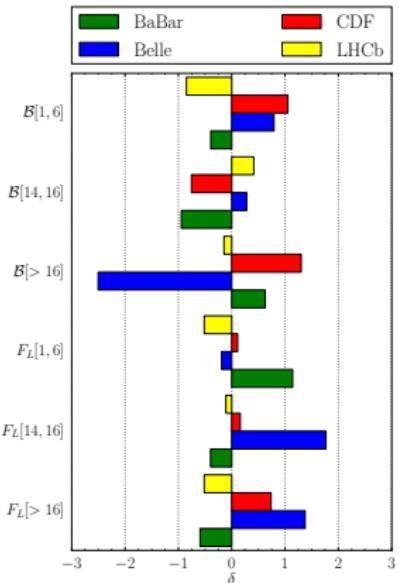
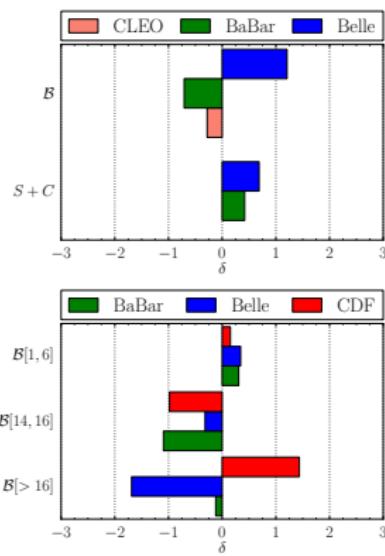
⇒ model comparison: SM = fixed values of Wilson coefficients ⇔ SM-like solution

Bayes factor: $B = \exp(392.4 - 385.1) \approx 1500$ in favor of the simpler model

Pull values of experimental observables

[Beaujean/CB/van Dyk/Wacker arXiv:1205.1838]

22 observables with 59 measurements: $B \rightarrow K^*\gamma$, $B \rightarrow K\ell^+\ell^-$, $B \rightarrow K^*\ell^+\ell^-$



pull definition

$$\delta = \frac{x_{pred}(\vec{\theta}, \vec{\nu}) - x}{\sigma}$$

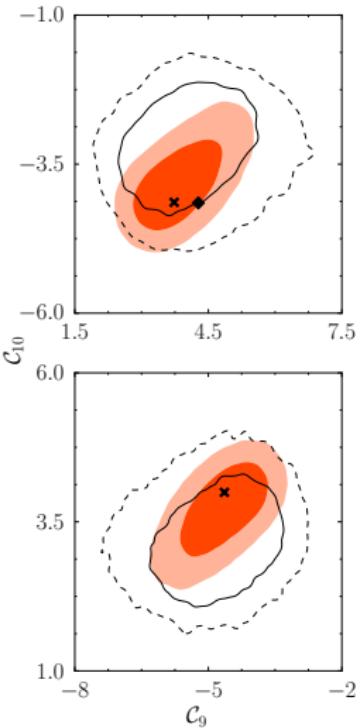
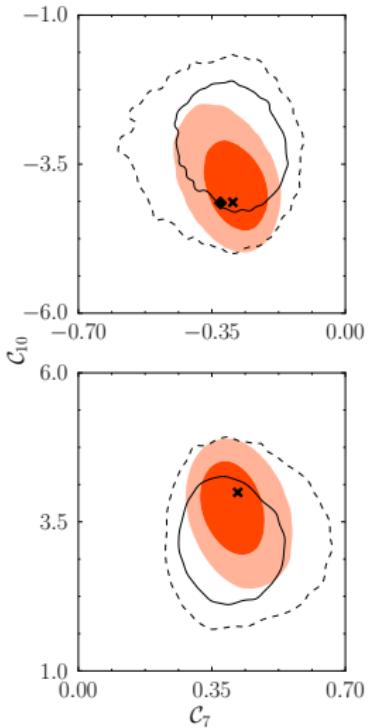
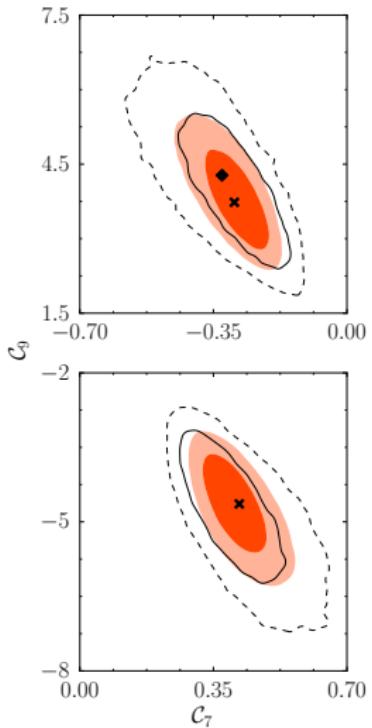
$x_{pred}(\vec{\theta}, \vec{\nu})$ theory prediction at best fit point

x central value of experimental distribution

σ experimental uncertainty

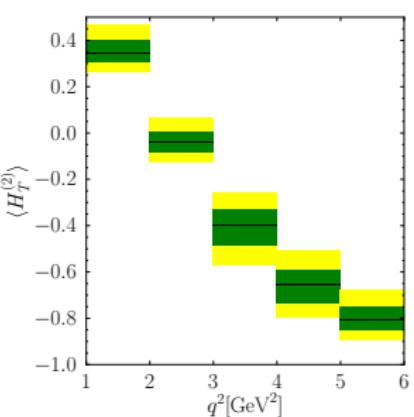
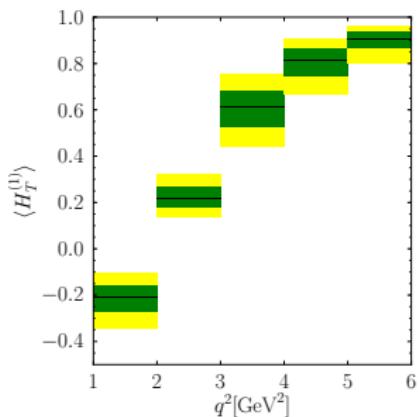
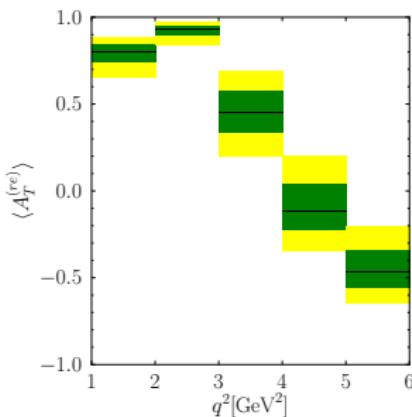
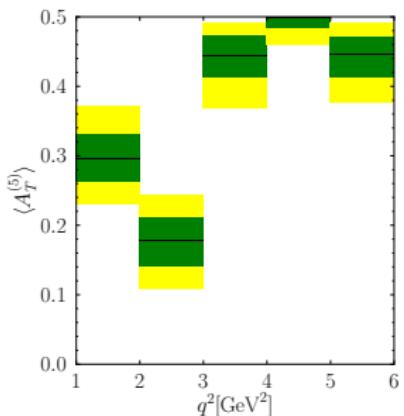
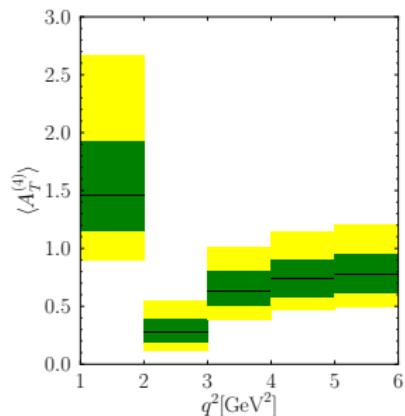
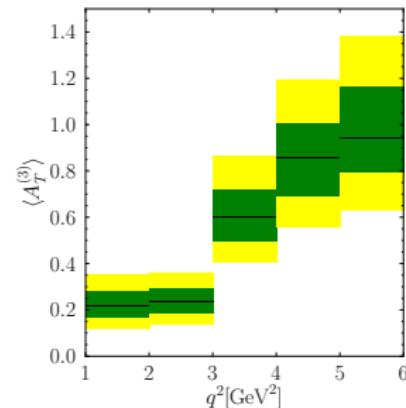
Prior dependence

SM = (\blacklozenge), best fit point = (\times)



95 % (dashed) and 68 % (solid) credibility regions using 3 \times larger prior ranges
⇒ fit still converges

Prediction of yet unmeasured optimized observables @ low- q^2



⇒ Measurements outside these predictions would put simple scenario $C_{7,9,10}$ in trouble

High- q^2 : OPE + HQET

Framework developed by Grinstein/Pirjol hep-ph/0404250

- 1) OPE in Λ_{QCD}/Q with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar{B} \rightarrow \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^6 C_i(\mu) \mathcal{T}_\alpha^{(i)}(q^2, \mu) [\bar{\ell}\gamma^\alpha \ell]$$

$$\begin{aligned} \mathcal{T}_\alpha^{(i)}(q^2, \mu) &= i \int d^4x e^{iq \cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\text{em}}(x)\} | \bar{B} \rangle \\ &= \sum_{k \geq -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle \end{aligned}$$

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	Λ_{QCD}/Q	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	$\Lambda_{\text{QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_j^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

- 2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa V(q^2), \quad T_2(q^2) = \kappa A_1(q^2), \quad T_3(q^2) = \kappa A_2(q^2) \frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)} \right) \frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s \Lambda_{\text{QCD}}/Q)$!!!