Non-perturbative Gauge-Higgs Unification

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Outline

- Introduction
- Non-perturbative GHU
- Lattice results

Based on work in collaboration with
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Outline

- no fermions, but fundamental properties of gauge theory
- what is the origin of the Higgs mechanism?
- are (non-renormalizable) 5d gauge theories viable?
- is the Higgs mass stable under quantum corrections?
- what is the mass hierarchy in the gauge-Higgs sector?

Gauge-Higgs Unification

Gauge field in 5 dimensions

$$A_M^C \xrightarrow[SU(2)]{\mathcal{M}_5=E_4\times S^1/\mathbb{Z}_2} \left\{A_\mu^3\,,\,A_5^{1,2}\right\} \text{“even” fields}$$

5d $SU(2)$ gauge field decomposes into 4d $U(1)$ gauge field and a 4d scalar (Higgs) h

Higgs field

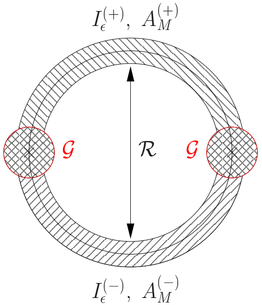
Higgs is identified with 5d components of the gauge field
Physical degrees of freedom are Polyakov loops in the 5th dimension
Higgs potential (zero at tree level)?
Non-renormalizability?

Gauge field on a circle

Spurion field $\mathcal{G}$

$SU(N)$ gauge fields on S^1 require two open charts and a transition function $\mathcal{G} \in SU(N)$ on the overlaps (size ϵ)

$$A_M^{(-)} = \mathcal{G} A_M^{(+)} \mathcal{G}^{-1} + \mathcal{G} \partial_M \mathcal{G}^{-1}$$



$S^1 : x_5 \in (-\pi R, \pi R]$; Reflection $\mathcal{R}$

$$\begin{aligned} z = (x_\mu, x_5) \quad &\rightarrow \quad \bar{z} = (x_\mu, -x_5) \\ A_M(z) \quad &\rightarrow \quad \alpha_M A_M(\bar{z})\,, \quad \alpha_\mu = 1\,,\ \alpha_5 = -1 \end{aligned}$$

Gauge field on the $S^1/\mathbb{Z}_2$ orbifold

Orbifold projection

$\mathbb{Z}_2$ projection: $\mathcal{R}\, A_M^{(+)} = A_M^{(-)}$

Only $A_M = A_M^{(+)}$ and $\mathcal{G}$ on $I_\epsilon = \{x_\mu, x_5 \in (-\epsilon, \pi R + \epsilon)\}$

$$\mathcal{R}\, A_M = \mathcal{G}\, A_M\, \mathcal{G}^{-1} + \mathcal{G}\, \partial_M \mathcal{G}^{-1}$$

Gauge-covariance under gauge transformation Ω requires

$$\mathcal{G} \to (\mathcal{R}\,\Omega)\, \mathcal{G}\, \Omega^{-1} \quad \text{and} \quad D_M \mathcal{G} \equiv 0$$

Gauge field on the interval I_0

$\epsilon \to 0$, at the boundaries $\mathcal{G}|_{x_5=0,\pi R} = g$ constant

$$\alpha_M\, A_M = g\, A_M\, g^{-1} \qquad \text{Dirichlet boundary conditions}$$
$$-\alpha_M\, \partial_5 A_M = g\, \partial_5 A_M\, g^{-1} \qquad \text{Neumann boundary conditions}$$

Gauge symmetry on the orbifold

Breaking at the boundaries

$\mathcal{G} = g$ constant implies for gauge transformations Ω

$$[g, \Omega] = 0 \quad \text{on the boundaries}$$

Choice of g defines a inner automorphism which assigns parities to group generators T^A [Hebecker and March-Russell, 2002]

$$g\,T^a\,g^{-1} \;=\; T^a \;\text{(unbroken)}\,, \quad g\,T^{\hat{a}}\,g^{-1} \;=\; -T^{\hat{a}} \;\text{(broken)}$$

Gauge symmetry is $G = SU(N)$ in the bulk and

$$G = SU(p+q) \;\xrightarrow{g}\; H = SU(p) \times SU(q) \times U(1)$$

on the boundaries

Gauge symmetry on the orbifold

Example 1. $SU(2) \xrightarrow{g} U(1)$

$g = \mathrm{diag}(-i, i)$: even fields (i.e. non-zero on the boundaries)

- A_μ^3: "photon/Z"
- $A_5^{1,2}$: complex "Higgs"

Example 2. $SU(3) \xrightarrow{g} SU(2) \times U(1)$

$g = \mathrm{diag}(-1, -1, 1)$: even fields

- $A_\mu^{1,2,3,8}$: "photon,Z and $W^\pm$"
- $A_5^{4,5,6,7}$: doublet of complex "Higgs" in the fundamental representation of $SU(2)$

Minimal model to reproduce the Standard Model Higgs sector

Effective Higgs potential on $S^1/\mathbb{Z}_2$

Kaluza–Klein (KK) expansion

$$E(x,x_5)=\frac{1}{\sqrt{2\pi R}}E^{(0)}(x)+\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}E^{(n)}(x)\cos(nx_5/R) \quad \text{even fields}$$

$$O(x,x_5)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}O^{(n)}(x)\sin(nx_5/R) \quad \text{odd fields}$$

4d KK masses

$$(m_nR)^2 = n^2$$

For energies $E \ll 1/R$, 4d effective theory of $E^{(0)}(x)$

Coleman–Weinberg potential

For a real scalar field in D-dimensional Euclidean space

$$\int[\mathrm{D}\phi]\mathrm{e}^{-S_{\mathrm{E}}} \sim \mathrm{e}^{-V} \equiv \frac{1}{\sqrt{\det[-\partial_\mu\partial_\mu + M^2]}}$$

Take $D=4$ and insert KK masses m_n (only $n \neq 0$)

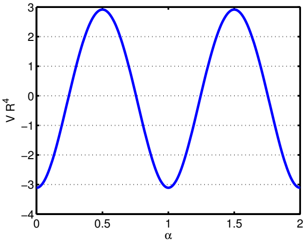
Hosotani mechanism

Example: SU(2) [Kubo, Lim and Yamashita, 2002]

A vev $\langle A_5^1 \rangle$ shifts KK masses $(m_n R)^2 = n^2\,,\ (n \pm \alpha)^2$ $(n \neq 0)$

$$V(\alpha) = -\frac{3 \cdot 2 \cdot P}{64\pi^6 R^4} \sum_{m=1}^{\infty} \frac{\cos(2\pi m \alpha)}{m^5}\,, \quad \alpha \;=\; g_5 \,\langle A_5^1 \rangle\, R$$

- pure gauge: $P = 3$, minimum at $\alpha = 0, 1, \ldots$, lowest $H = U(1)$ KK gauge boson is massless
- $+\ N_{\mathrm{f}}$ adjoint fermions: $P = 3 - 4N_{\mathrm{f}}$, minimum is at $\alpha = 1/2, 3/2, \ldots$ for $N_{\mathrm{f}} > 3/4$, gauge boson has $m_Z = 1/(2R)$



Hosotani mechanism

Hosotani mechanism [Hosotani, 1983, 1989]

- gauge theory + massless fermions, vev $\alpha = g_5 \left\langle A_5^{\hat{a}} \right\rangle R$
- minimization of $V(\alpha)$ yields $\alpha_{\rm min}$
- dynamical fermion masses $m_{\rm f} = \alpha_{\rm min}/(2R)$
- Hosotani breaking $H \rightarrow ?$, dynamical gauge boson masses $m_Z = \alpha_{\rm min}/R$?
- if $\alpha_{\rm min} \in \mathbb{Z}$ the rank of H is not broken.
  Polyakov loop $\langle P \rangle = \exp(2\pi i \alpha T^{\hat{a}})$, Cartan generators H_i

$$[\langle P \rangle, H_i] = 0 \quad \text{for integer } \alpha$$

- in order to get $\alpha_{\rm min} \neq 0, 1$ one needs extra matter fields
- it is hard to get a Higgs heavier than the gauge boson, cf. [Antoniadis, Benakli and Quiros, 2001], [Lim and Maru, 2007]

Higgs mass

Higgs mass

Zero at tree level (5d gauge invariance). From the potential

$$(m_H R)^2 = R^4 g_4^2 \frac{\mathrm{d}^2 V}{\mathrm{d}\alpha^2}\bigg|_{\alpha=\alpha_{\min}} , \quad g_4^2 = \frac{g_5^2}{2\pi R}$$

same as from Feynman diagram perturbation theory [Gersdorff, Irges and Quiros (2002); Cheng, Matchev and Schmalz (2002)]

Solution to hierarchy problem

- finite (bulk) mass
- insensitive to the UV cut-off Λ (at 1-loop)
- boundary mass term $\mathrm{tr}\,\{[A_5, g][A_5, g^{-1}]\} \sim \Lambda^2$: invariant under H but is excluded because $\mathrm{tr}\,\{D_5\mathcal{G}\, D_5\mathcal{G}\} \equiv 0$ on I_ϵ [Irges and FK, 2005]

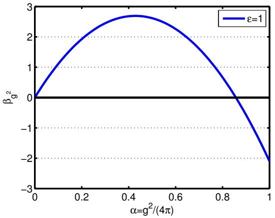
Renormalization

Renormalizability of $SU(N)$ gauge theory in $D > 4$ dimensions?

- bare coupling has negative dimension $[g_D] = (4 - D)/2$
- perturbative limit $g_D \to 0$ is trivial
- for $\epsilon = D - 4 \ll 1$ and $g^2 \sim \mu^\epsilon g_D^2$ [Peskin, 1980]

$$\mu \frac{\mathrm{d}g^2}{\mathrm{d}\mu} = \beta_{g^2} = \epsilon g^2 - \frac{22N}{3} \frac{g^4}{16\pi^2} + \ldots$$

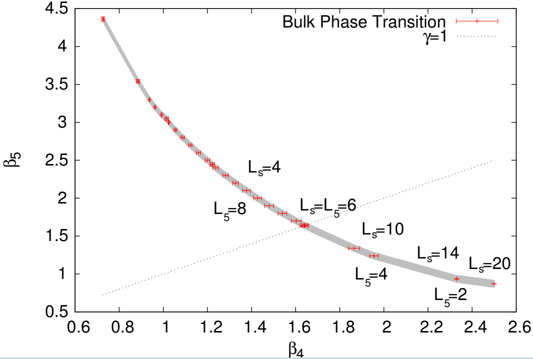
- $g_\star^2 = 24\pi^2\epsilon/(11N)$ non-trivial UV fixed point
- (truncated) RG flow supports existence for $D = 5$ [Gies, 2003]
- what are the relevant operators?



Non-perturbative phase diagram

Monte Carlo simulations on the lattice [FK, Luz and Rago, 2011]

- anisotropic Wilson plaquette gauge action, S^1
- non-trivial UV fixed point ↔ 2nd order phase transition
- in "infinite" volume only a 1st order phase transition, cf. [Del Debbio, Kenway, Lambrou and Rinaldi, 2013]



Lattice action

Orbifold action

Euclidean $T \times L^3 \times (N_5 + 1)$ lattice, spacings a_4 and a_5
gauge links $U_M(n) \in SU(2)$ connect $n + \hat{M}$ with n
anisotropic Wilson plaquette action

$$S_W^{\text{orb.}} = \frac{\beta}{2}\Big[\frac{1}{\gamma}\sum_{\text{4d p}} w\,\text{tr}\,\{1-P_{\mu,\nu}(n)\} + \gamma\sum_{\text{5d p}}\text{tr}\,\{1-P_{\mu,5}(n)\}\Big]$$

$$w = \begin{cases} \frac{1}{2} & \text{boundary plaquette} \\ 1 & \text{in all other cases.} \end{cases}, \qquad \pi R = N_5 a_5\,,$$

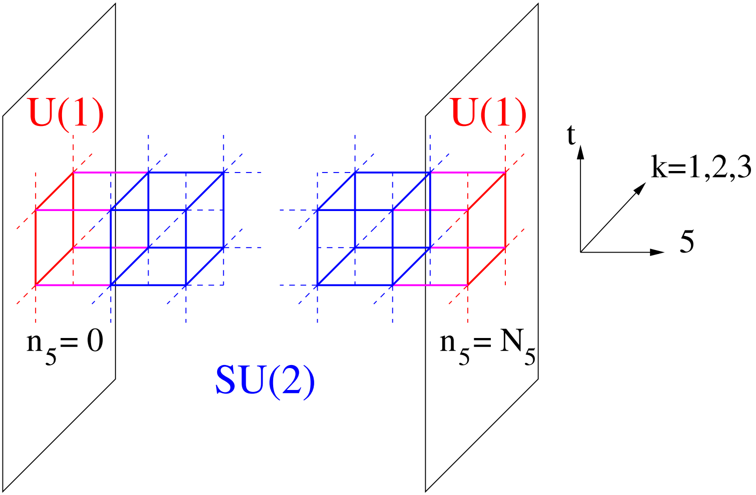
$$\beta_4 = \beta/\gamma\,, \quad \beta_5 = \beta\,\gamma\,, \quad \gamma = a_4/a_5 \text{ (classical level)}$$

Extra dimension $n_5 \in [0, N_5]$ with Dirichlet boundary conditions

$$U_\mu(n) = g\,U_\mu(n)\,g^{-1} \Rightarrow U_\mu(n) = \mathrm{e}^{\phi(n)\,g} \in U(1)$$

at $n_5 = 0$ and $n_5 = N_5$ with $g = -i\sigma^3$ [Irges and FK, 2005]

Lattice fields



Lattice symmetries

Global symmetries

$$Z \otimes F \otimes \mathcal{F}$$

- Z center transformation on 4d hyperplanes by center of G
- F reflection with respect to $n_5 = N_5/2$
- Fixed point symmetry $\mathcal{F} = \mathcal{F}_L \oplus \mathcal{F}_R$

$$\begin{aligned} \mathcal{F}_L : \quad U_5(n_5 = 0) &\rightarrow g_F^{-1}\, U_5(n_5 = 0) \\ U_\nu(n_5 = 0) &\rightarrow g_F^{-1}\, U_\nu(n_5 = 0)\, g_F \end{aligned}$$

$\{g, g_F\} = 0$: $g_F = \mathrm{e}^{i\theta}(-i\sigma^2)$ defines $U(1)$ stick symmetry $\mathcal{S}_{L(R)}$ [Ishiyama, Murata, So and Takenaga, 2010]
$[g, g_F] = 0$: defines a global gauge transfomation

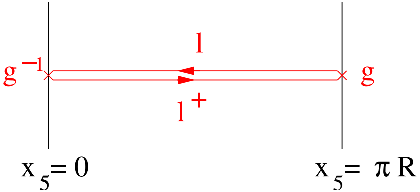
Lattice Higgs operators

Polyakov line P on $S^1/\mathbb{Z}_2$

Orbifolded Polyakov loop

$$P_L = l g l^\dagger g^{-1}$$

where $g = -i\sigma^3$



Lattice Higgs operators:

- $J=0$, $C=1$, $P=1$, $U(1)$ charge is 2
- $\mathrm{tr}\, P_L$: $\not{\mathcal{S}}_L$, $\not{\mathcal{S}}_R$
- $\mathrm{tr}\, \Phi_L \Phi_L^\dagger$: $\mathcal{S}_L$, $\mathcal{S}_R$
  with $\Phi_L = 1/(4N_5)\,[P_L - P_L^\dagger, g] = i\phi^1\sigma^1 + i\phi^2\sigma^2$
- mass(es)

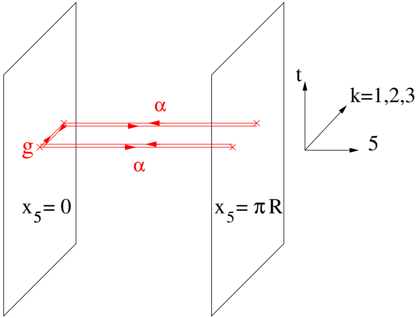
$$C_{ij}(t) = \langle O_i(t) O_j(0)^* \rangle - \langle O_i(t) \rangle \langle O_j(0)^* \rangle \overset{t\to\infty}{\sim} \mathrm{const} \times \exp(-m_O t)$$

Lattice gauge-boson operators

Vector Polyakov line Z_L on $S^1/\mathbb{Z}_2$ [Irges and FK, 2007]

$$Z_L = g\, U_k\, \alpha_L\, U_k^{-1}\, \alpha_L$$

with $\alpha_L = \Phi_L / \sqrt{\det\, \Phi_L}$



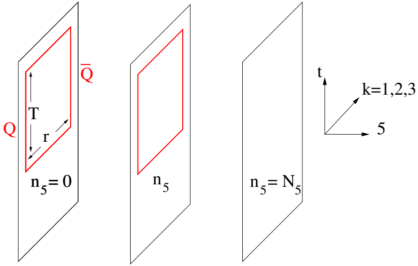
cf. 4d SU(2) Higgs model [Montvay, 1985, 1986], but here Φ_L transforms like a $U(1)$ field strength with charge 2

- $J = 1$, $C = -1$, $P = -1$
- $\mathrm{tr}\,\{Z_L\}$: $\not{\mathcal{S}}_L$, $\mathcal{S}_R$

Wilson loops

4d Wilson loops

$$W(r,T)|_{n_5} = \prod_{\text{rectangle}} U$$



- $\mathrm{tr}\,\{W(r,T)\}$: $\mathcal{S}_L$, $\mathcal{S}_R$
- static potential V between infinitely heavy quark-pair $Q\overline{Q}$:

$$\langle \mathrm{tr}\,\{W(r,T)\}\rangle \overset{T\to\infty}{\sim} \exp(-V(r)T)$$

Spontaneous symmetry breaking (SSB)

SSB in 5d gauge theories

- Higgs mechanism breaks boundary H gauge symmetry?
- perturbatively: SSB can be triggered by a vev $A_5 \to A_5 + v$ ($\langle P \rangle \neq 0$)
- non-perturbatively: the order parameter has to be gauge invariant [Elitzur, 1975]

Order parameters

- $\langle \mathrm{tr}\, P_L \rangle = 0$ defines confined phase, $\langle \mathrm{tr}\, P_L \rangle \neq 0$ defines deconfined phase
- deconfined phase is Higgs phase when $\langle \mathrm{tr}\, Z_L \rangle \neq 0$
- $\mathcal{S}_L$ induces $\mathcal{F}_L$, which contains global gauge transformations
- $\langle \mathrm{tr}\, Z_L \rangle$ is order parameter of SSB

Mean-field expansion

Mean-field for $SU(N)$ [Drouffe and Zuber, 1983]

$SU(N)$ gauge links U are replaced by $N \times N$ complex matrices V and Lagrange multipliers H

$$\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{1}{Z} \int \mathrm{D}U \mathcal{O}[U] \mathrm{e}^{-S_W^{\mathrm{orb.}}[U]} \\
&= \frac{1}{Z} \int \mathrm{D}V \int \mathrm{D}H \, \mathcal{O}[V] \mathrm{e}^{-S_{\mathrm{eff}}[V,H]} \\
S_{\mathrm{eff}} &= S_W^{\mathrm{orb.}}[V] + u(H) + (1/N) \mathrm{Re}\,\mathrm{tr}\,\{HV\} \\
\mathrm{e}^{-u(H)} &= \int \mathrm{D}U \, \mathrm{e}^{(1/N)\mathrm{Re}\,\mathrm{tr}\,\{UH\}}
\end{aligned}$$

Saddle point solution: background (or vev)

$$H \longrightarrow \overline{H} = \overline{h}_0 \, \mathbf{1} \,, \quad V \longrightarrow \overline{V} = \overline{v}_0 \, \mathbf{1} \,, \quad S_{\mathrm{eff}}[\overline{V}, \overline{H}] = \text{minimal}$$

Mean-field expansion

Mean-field background

non-trivial profile [FK, Bunk and Irges, 2005]

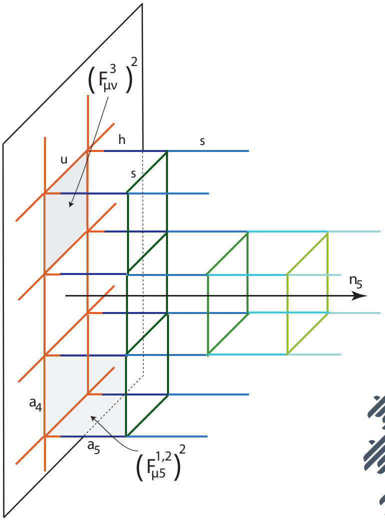
$$\overline{V}(n,\mu) = \overline{v}_0(n_5)\,\mathbf{1}$$
$$\overline{V}(n,5) = \overline{v}_0(n_5+1/2)\,\mathbf{1}$$

vev is not along the algebra
different vev's ↔ colors

Fluctuations

$$v(n) = (\overline{v}_0 + v_0(n))\,\mathbf{1} + i v^A(n)\sigma^A$$

pertubative limit: $\overline{v}_0 \to 1$



Monte Carlo simulations

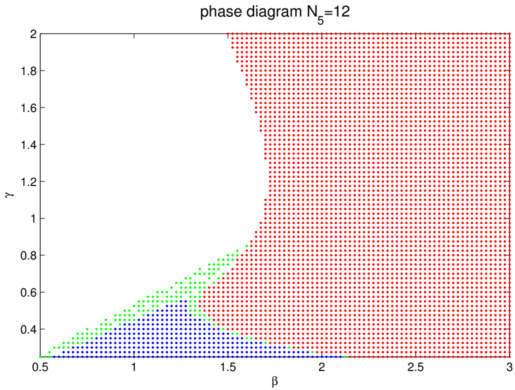
Validity of mean-field expansion

- Propagator $\langle v_i v_j \rangle = (K^{-1})_{ij}$ [Irges, FK and Yoneyama, 2012]
- Observables
  $\langle \mathcal{O}[U] \rangle = \mathcal{O}[\overline{V}] + 1/2(\delta^2 \mathcal{O}/\delta V^2|_{\overline{V}})_{ij}(K^{-1})_{ij} + \ldots$
- Mean-field expansion is expected to be reliable in $D = 5$ dimensions
- Is the mean-field saddle point the dominant one?

Monte Carlo simulations on a (super)computer

- Sample gauge configurations $\{U^{(i)}\}$, $i = 1, \ldots, N$ according to probability $\mathrm{e}^{-S_W^{\mathrm{orb.}}[U]}$
- can be done by local algorithms (heatbath and over-relaxation) for $U(1)$ and $SU(2)$
- Observables $\langle \mathcal{O}(U) \rangle = (1/N) \sum_{i=1}^{N} \mathcal{O}(U^{(i)}) + \mathrm{O}(1/\sqrt{N})$

Mean-field phase diagram



$\overline{v}_0(n_5) = 0$, $\overline{v}_0(n_5 + 1/2) = 0$: confined phase (white)
$\overline{v}_0(n_5) \neq 0$, $\overline{v}_0(n_5 + 1/2) \neq 0$: deconfined phase
$\overline{v}_0(n_5) \neq 0$, $\overline{v}_0(n_5 + 1/2) = 0$: layered phase [Fu and Nielsen, 1984]
$\overline{v}_0(n_5 = 0) \neq 0$, $\overline{v}_0(n_5 = N_5/2) = 0$: hybrid phase

Mean-field phase transitions

Second order phase transition

- the mean-field sees only bulk phase transitions
- critical exponent from $M_H = a_4 m_H(\beta) = (1 - \beta_c/\beta)^\nu$
- for $\gamma > 0.6$ we measure $\nu = 1/4$ in the deconfined phase
- for $\gamma \lesssim 0.6$:
  - layered phase appears
  - approaching the phase boundary deconfined/layered ν is $1/2$
  - and the background goes to zero: second order phase transition

Mean-field dimensional reduction

(Our definition of) Dimensional reduction

- boundary potential can be (locally) fitted by 4d Yukawa

$$V(r) = -b\frac{\exp(-m_Z r)}{r} + \text{const.}$$

with $m_Z \neq 0$:

$$y' \simeq -m_Z \,, \quad y = \ln(r^2 V'(r))$$

- $M_H = a_4 m_H$ and $M_Z = a_4 m_Z$ are < 1
- $m_H R < 1$ and $\rho_{HZ} = m_H / m_Z > 1$

Mean-field continuum limit

Line of constant physics (LCP) [Irges, FK and Yoneyama, 2013]

- bare parameters are β, γ and N_5 (we take $T, L \to \infty$)
- three observables

$$M_H(\beta,\gamma,N_5)\,, \quad M_Z(\beta,\gamma,N_5)\,, \quad M_{Z'}(\beta,\gamma,N_5)$$

at fixed N_5: tune β and γ to fix

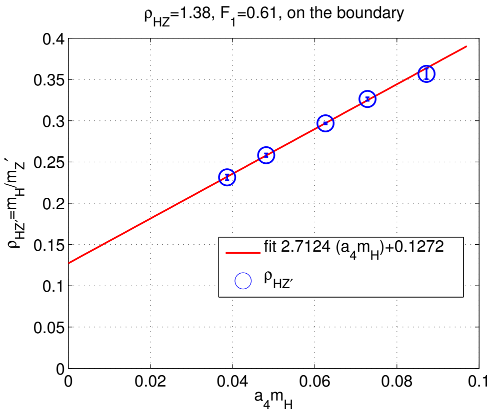
$$F_1 = m_H R = 0.61\,, \quad \rho_{HZ} = 1.38$$

$N_5 \to \infty$: continuum limit of

$$\rho_{HZ'}(N_5) = m_H/m_{Z'}$$

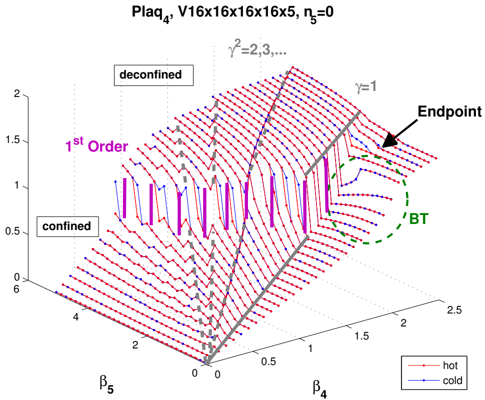
- Symanzik: expect O($a_4 m_H \sim \gamma/N_5$) effects

Mean-field continuum limit



- finiteness ($\neq 0$) of Higgs mass without supersymmetry
- prediction $m_{Z'} \simeq 1\,\mathrm{TeV}$
- a LCP with $\rho_{HZ} = 1.38$ at $\gamma \geq 1$ does not exist

Monte Carlo phase diagram for $N_5 = 4$

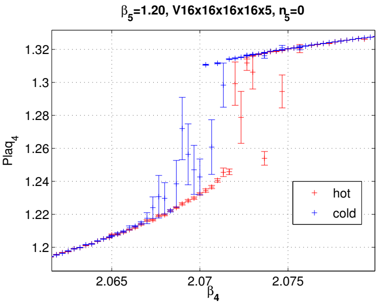


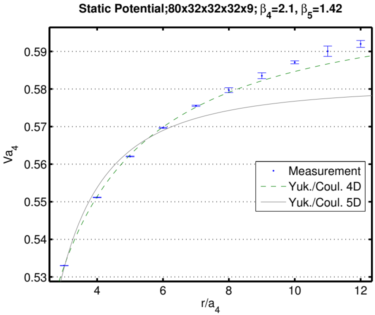
1. "end-point" of the 1st order line emanating from $\gamma = 1$? plaquette susceptibility ≈constant for 12^4 and 16^4
2. transition visible only in the boundary 4d plaquette (not in the bulk) → boundary transition (BT)

Monte Carlo phase diagram

Boundary transition (BT) [Dziennik, 2013, master thesis]

- hysteresis becomes visible on $16^4 \times 5$ volume
- in the vicinity of BT, orbifold with $N_5 = 8$ shows 4d layers
  static potential at $n_5 = 4$ has string tension $a_4^2\sigma \simeq 0.003$





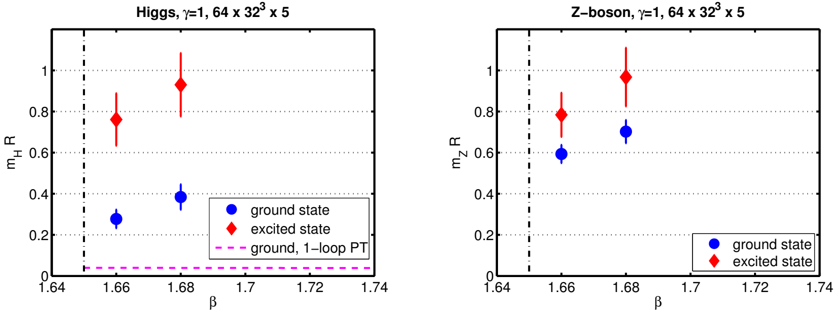
Monte Carlo spectrum

Higgs and Z boson [Dziennik, Irges, FK and Yoneyama, work in progress]

$64 \times 32^3 \times 5$ lattices at $\gamma = 1$

$m_Z \neq 0$ does not decrease with L (Higgs mechanism!) and $m_Z > m_H$

Excited states for the Higgs and the Z-boson $m_{Z'} \approx m_{H'}$



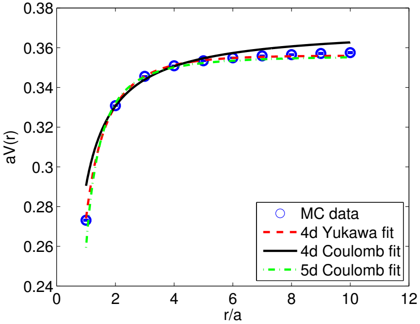
Monte Carlo static potential

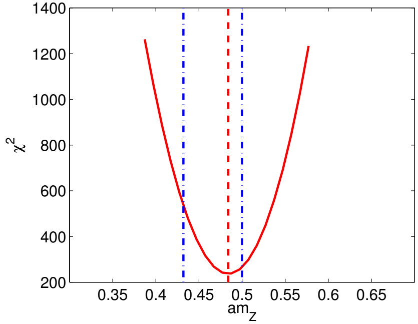
Boundary potential [Dziennik, Irges, FK and Yoneyama, work in progress]

$64 \times 32^3 \times 5$ lattice at $\beta = 1.66$, $\gamma = 1$

4d Yukawa is preferred global fit

fitted Yukawa mass agrees well with directly measured m_Z

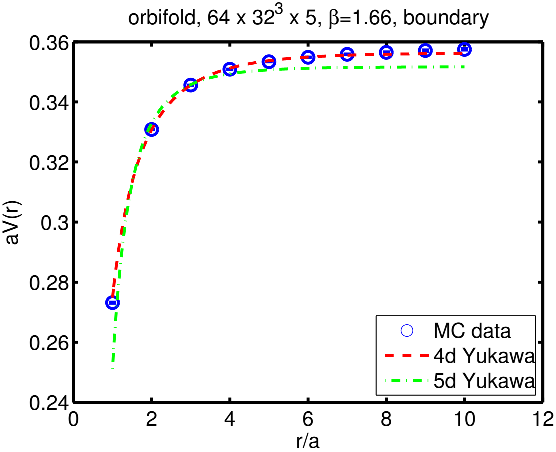




Monte Carlo static potential

Boundary potential [Dziennik, Irges, FK and Yoneyama, work in progress]

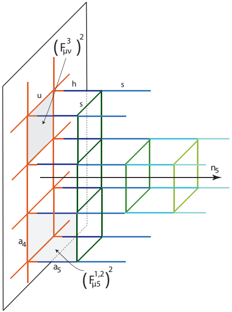
Yukawa fits (SSB) hint at dimensional reduction



Is the orbifold a 5d superconductor slab?

Mean-field background

- orbifold projection induces SSB of translation invariance
- mean-field background defines a "crystal" in coordinate and gauge space



- gauge fluctuations around the mean-field background are like phonons
- Polyakov loop (Higgs) with $U(1)$ charge 2 is like a Cooper pair
- SSB due to gauge-Higgs interaction happens like in a superconductor

Conclusions and outlook

Conclusions

- non-perturbative Gauge-Higgs Unification on orbifold in pure $SU(2)$ gauge theory
- meanfield $(\gamma < 1)$: SSB, 2nd order, LCP with finite m_H and $\rho_{HZ} = m_H/m_Z = 1.38$
- Monte Carlo $(\gamma = 1,\ N_5 = 4)$: SSB, 1st order, $\rho_{HZ} < 1$, static potential fitted by 4d Yukawa with mass m_Z

Outlook

- nature of Monte Carlo phase transition(s) at $\gamma < 1$
- look for $\rho_{HZ} > 1$ and (effective) LCP
- $SU(N > 2)$

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