

High multiplicity QCD at NLO

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5th February 2014

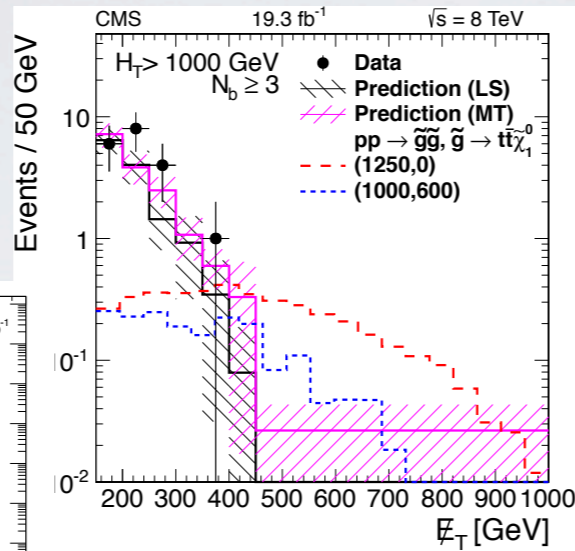
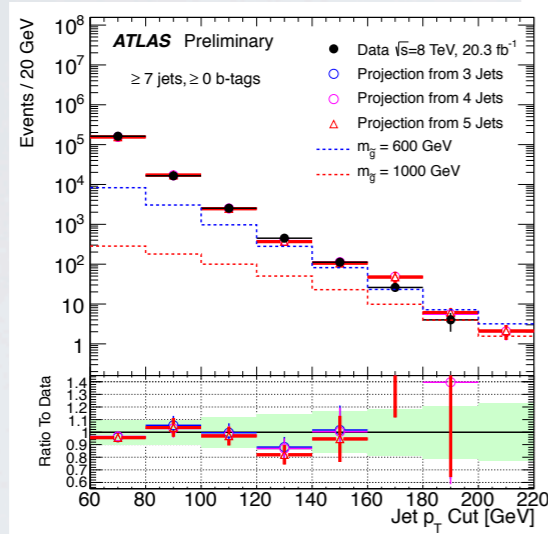
University of Edinburgh

Outline

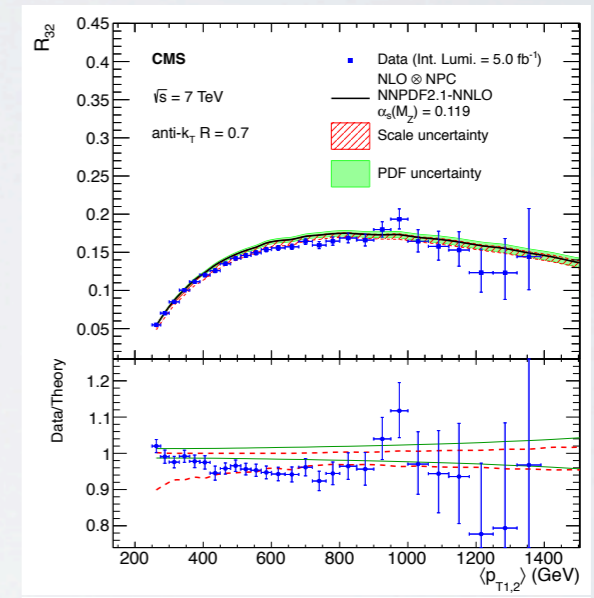
- On-shell methods for loop amplitudes
- NLO computations with NJET + SHERPA
- $pp \rightarrow 5$ jets SB, Biedermann, Uwer, Yundin [arXiv:1309.6585]
- $pp \rightarrow \mathcal{N} + 3$ jets SB, Guffanti, Yundin [arXiv:1312.5927]

Multi-jet production

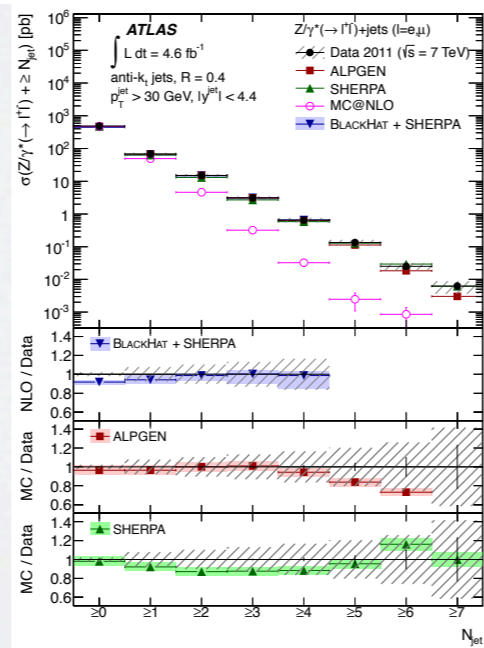
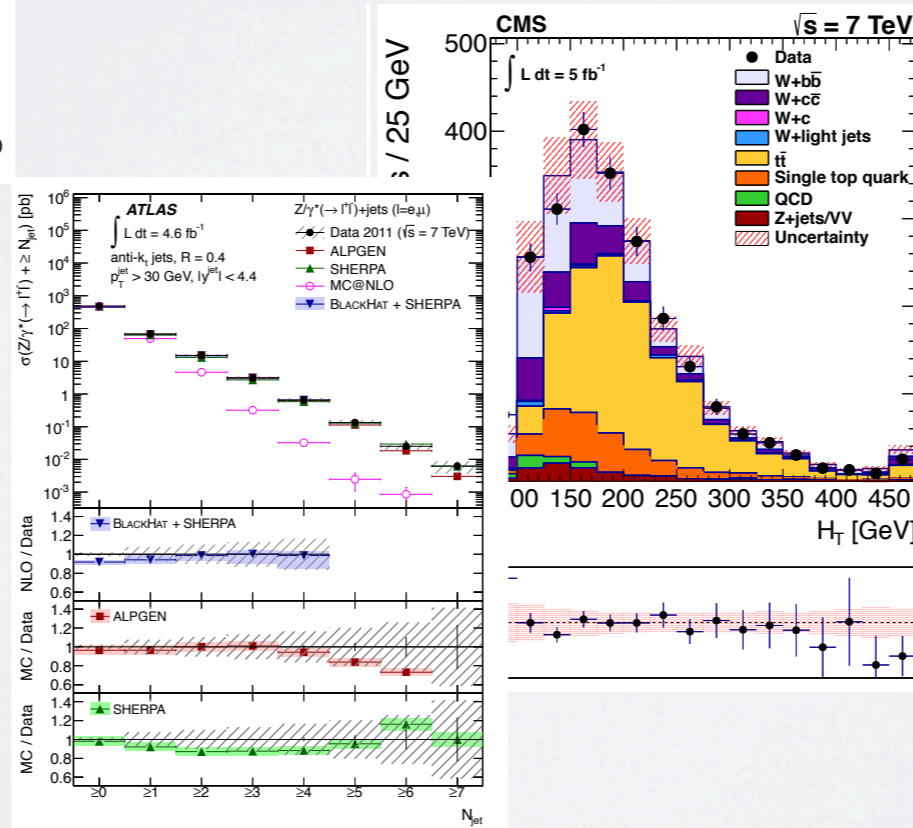
BSM searches



Strong coupling measurements



Precision QCD



Next-to-leading order

everything fixed
order today

LO PDFs

LO α_s

$$\sigma_{hh \rightarrow X}^{\text{LO}} = \sum_{i,j} \int f_{i/h_1}^{\text{LO}} f_{j/h_2}^{\text{LO}} \otimes \int_n \alpha_s^{n-2} d\sigma_{ij \rightarrow X}^{\text{B}}$$

$$\sigma_{hh \rightarrow X}^{\text{NLO}} = \sum_{i,j} \int f_{i/h_1}^{\text{NLO}} f_{j/h_2}^{\text{NLO}} \otimes \left(\int_n \alpha_s^{n-2} (d\sigma_{ij \rightarrow X}^{\text{B}} + \alpha_s d\sigma_{ij \rightarrow X}^{\text{V}}) + \int_{n+1} \alpha_s^{n-1} d\sigma_{ij \rightarrow X}^{\text{R}} \right)$$

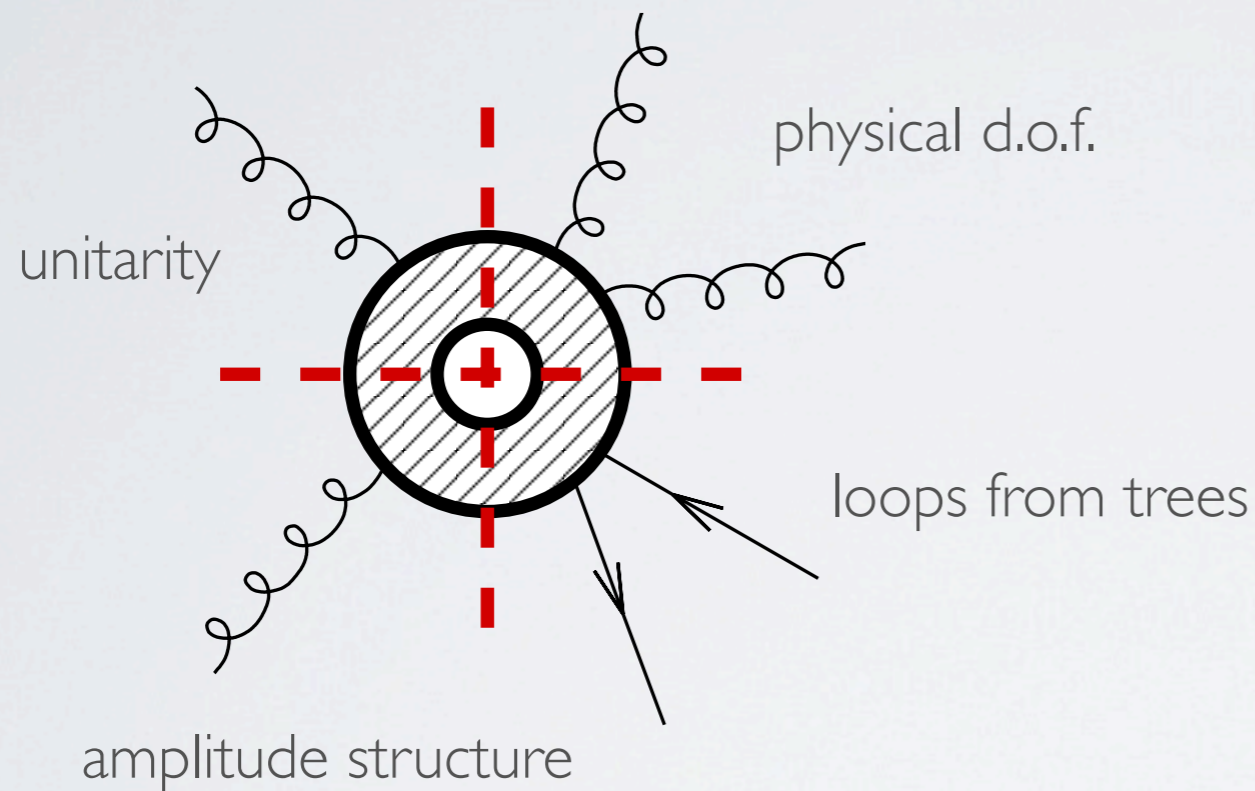
NLO α_s

kinematics

colour

$$d\sigma^{\text{V}} = d\Phi \sum_{\text{colours}, ij} \sum_{\text{spins}, s} 2\text{Re} \left([A_i^{(0)}(s)]^\dagger \cdot C_{i,j} \cdot A_j^{(1)}(s) \right)$$

On-shell amplitudes in gauge theory



simple final expressions

$$\text{MHV}(i^-, j^-) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Parke, Taylor (1986)

colour ordered primitive amplitudes

fixed external legs and propagators

One-loop amplitudes

$$\text{Shaded Circle} = \sum_{t \in \text{boxes}} c_{4;t} \text{Box} + \sum_{t \in \text{triangles}} c_{3;t} \text{Triangle} + \sum_{t \in \text{bubbles}} c_{2;t} \text{Bubble}$$

$$A_n^{(1)} = \sum_{p=2}^4 \sum_{1 \leq i_1 < \dots < i_p \leq n} c_{p;i_1 \dots i_p} \int \frac{d^D k}{(2\pi)^D} \frac{1}{l_{i_1} l_{i_2} \dots l_{i_p}}$$

$$l_i = k - p_{1,i}$$

Unitarity and Discontinuities

$$1 = SS^\dagger = (1 + iT)(1 - iT^\dagger) \Rightarrow TT^\dagger = i(T^\dagger - T)$$

$$A = \langle i|T|f\rangle \quad 1 = \sum \int d\text{LIPS}|k\rangle\langle k|$$

Cutkosky rules:
imaginary part obtained from

$$\frac{1}{k^2 + i0^+} \longrightarrow i\delta^{(+)}(k^2)$$

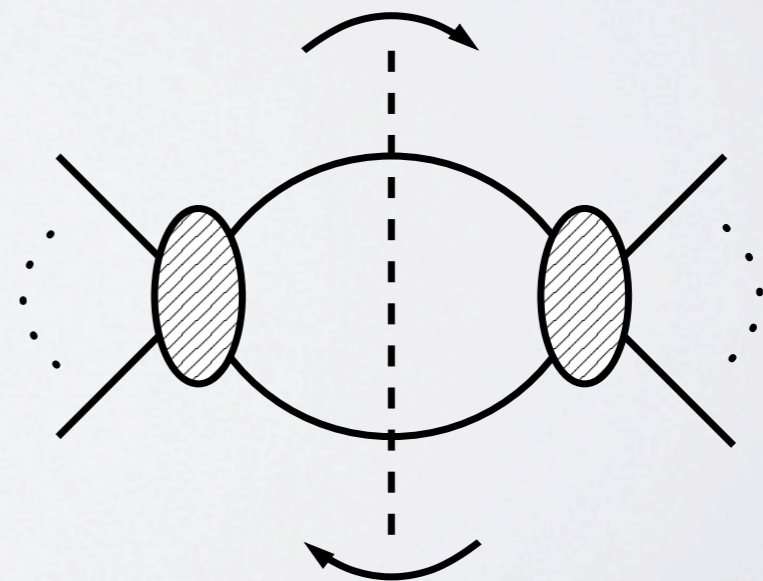
$$\Rightarrow \text{Disc}_{P_{i,j-1}}(A^{(1)}) = \sum \int d\text{LIPS}(k, P_{i,j-1})\delta^{(+)}(k)\delta^{(+)}(k - P_{i,j-1})$$

$$A^{(0)}(k, p_i, \dots, p_{j-1}, -k - P_{i,j-1})A^{(0)}(k + P_{i,j-1}, p_j, \dots, p_{i-1}, -k)$$

Classic S-matrix theory -
perform dispersion integral to
obtain full amplitude

Modern unitarity method -
use cuts to find coefficient
of basis integrals

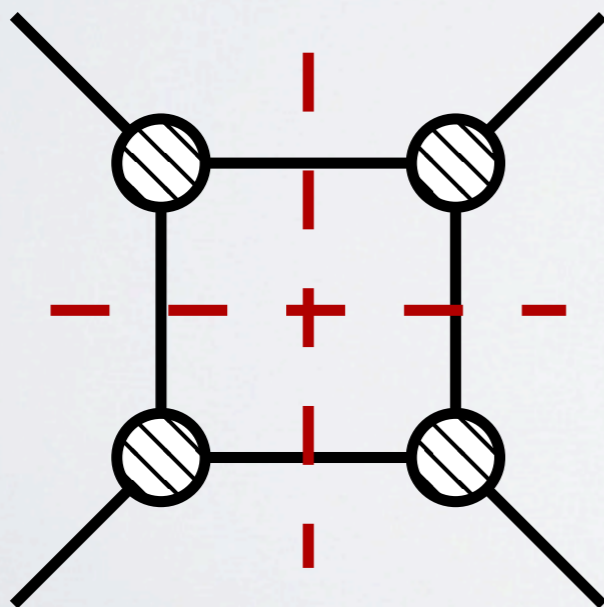
Bern, Dixon, Dunbar, Kosower (1994)



Generalized Unitarity

generalized discontinuities: put more propagators on-shell

$$\begin{aligned}
 A_n^{(1)}|_{\text{cut}(1234)} &= \sum_{p=2}^4 \sum_{1 \leq i_1 < \dots < i_p \leq n} c_{p;i_1 \dots i_p} \int \frac{d^D k}{(2\pi)^D} \frac{l_1^2 l_2^2 l_3^2 l_4^2 \delta^{(+)}(l_1^2) \delta^{(+)}(l_2^2) \delta^{(+)}(l_3^2) \delta^{(+)}(l_4^2)}{l_{i_1} l_{i_2} \dots l_{i_p}} \\
 &= c_{4;1234} I_{4;1234}|_{\text{cut}(1234)} \\
 &= \frac{I_{4;1234}|_{\text{cut}(1234)}}{n_s} \sum_{s \in \mathcal{Z}(l_i^2)} A^{(0)}(-l_1^{(s)}, 1, l_2^{(s)}) A^{(0)}(-l_2^{(s)}, 2, l_3^{(s)}) A^{(0)}(-l_3^{(s)}, 3, l_4^{(s)}) A^{(0)}(-l_4^{(s)}, 4, l_1^{(s)})
 \end{aligned}$$

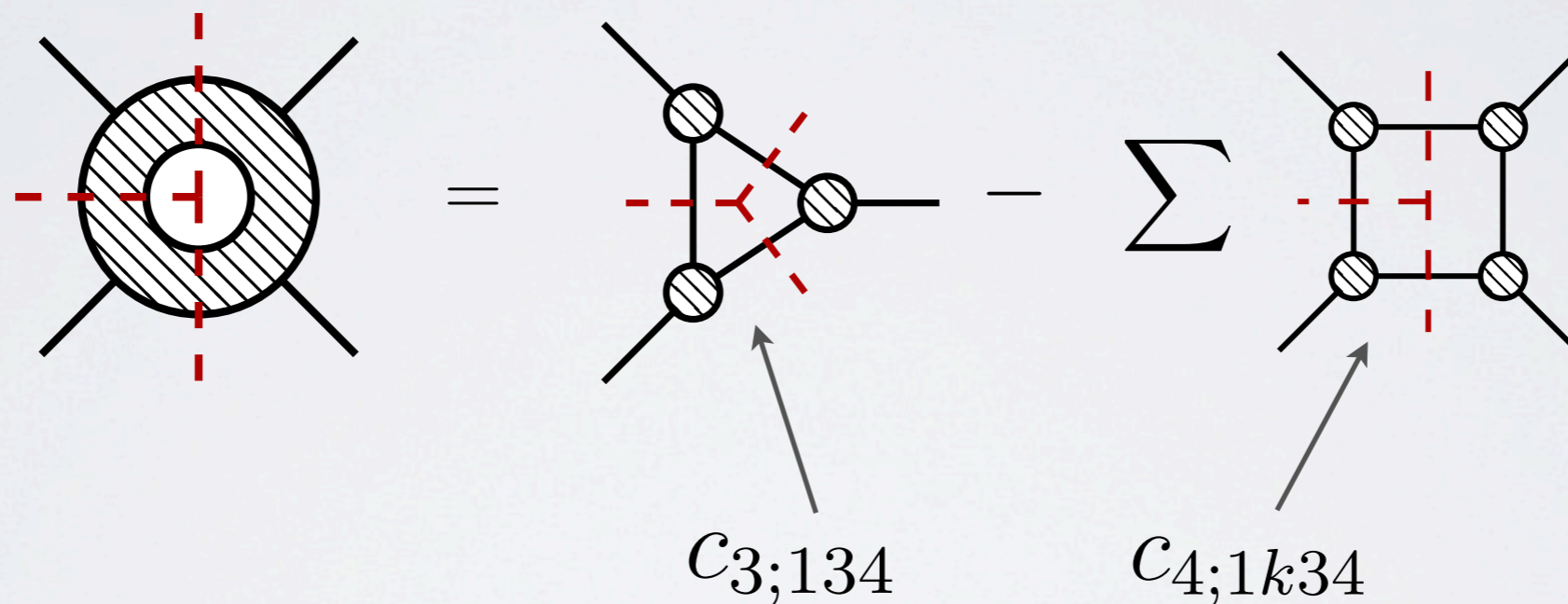


isolate integral coefficients

complex momentum solutions

Generalized Unitarity

top down: subtract leading singularities
and perform further multiple cuts



1-d integral: drop box contribution and find direct
formula for triangle coefficient from product of 3 trees

Forde (2007)

$C_{p;i_1 \dots i_p}$ rational functions - algebraic procedure

Integrand reduction

Ossola, Papadopoulos, Pittau (2006)

Ellis, Giele, Kunszt, (2007)

$$A_n^{(1)} = \sum_{p=2}^4 \sum_{1 \leq i_1 < \dots < i_p \leq n} c_{p;i_1 \dots i_p} \int \frac{d^D k}{(2\pi)^D} \frac{1}{l_{i_1} l_{i_2} \dots l_{i_p}} \quad \rightarrow \quad A_n^{(1)} = \sum_{p=2}^4 \sum_{1 \leq i_1 < \dots < i_p \leq n} \int \frac{d^D k}{(2\pi)^D} \frac{\Delta_{p;i_1 \dots i_p}(k)}{l_{i_1} l_{i_2} \dots l_{i_p}}$$

$$\Delta_{4;1234}(k) = c_{4;1234} + c'_{4;1234} k \cdot \omega_{1234}$$

introduce spurious vectors
to span loop space

$$\begin{aligned} \Delta_{4;1234}(\bar{k}, \mu_{11}) &= c_{4;1234}^{[0]} + c_{4;1234}^{[1]} \bar{k} \cdot \omega_{1234} \\ &+ \mu_{11} \left(c_{4;1234}^{[2]} + c_{4;1234}^{[3]} \bar{k} \cdot \omega_{1234} \right) + \mu_{11}^2 c_{4;1234}^{[4]} \end{aligned}$$

dimensional regulated (with mass shift)

Giele, Kunszt, Melnikov (2008)

SB (2009)

stable numerical algorithm - purely algebraic

Loop integrals: QCDFLoop/FF (Ellis, Zanderighi),
OneLoop (Van Hameren), ...

Dealing with colour

$$\mathcal{A}_n^{(L)} = \sum_j c_j^{(L)} A_{n;j}^{(L)}$$

$SU(N_c)$ colour matrices partial amplitude

$$\sum_{\text{colours}} \left(c_i^{(L_1)} \right)^T c_j^{(L_2)} = \mathcal{C}_{ij}^{(L_1, L_2)}(N_c)$$

$$d\sigma^{\text{B}}(s) = d\Phi \sum_{i,j} [A_{n;j}^{(0)}(s)]^\dagger \mathcal{C}_{ij}^{(0,0)} A_{n;j}^{(0)}(s) \quad d\sigma^{\text{V}}(s) = d\Phi \sum_{i,j} [A_{n;j}^{(0)}(s)]^\dagger \mathcal{C}_{ij}^{(0,1)} A_{n;j}^{(1)}(s)$$

partial amplitudes are a linear combination of primitive amplitudes

$$A_{n;j}^{(L)} = \sum_k a_{k,j} A_n^{[m]} + b_{k,j} A_n^{[f]}$$

Dealing with colour

Feynman diagram
matching algorithm

Ellis, Kunszt, Melnikov, Zanderighi [1105.4319]

Ita, Ozeren [1111.4193]

SB, Biedermann, Uwer, Yundin [1209.0100]

diagrams

$$A_{n;j}^{(L)} = \sum_{i=1}^N D_i = \sum_{i=1}^{\hat{N}} C_i K_i$$

invert to find independent
set of primitives

$$P_i = \sum_{j=1}^{\hat{N}} M_{ij} K_j$$

matching matrix $\{0, +1, -1\}$

match topology only
4-gluon vertex not needed
diagrams symmetries reduce independent set of
primitives (e.g. Furry's theorem)

combinatorial approaches: Melia [1304.7809, 1312.0599];

Schuster [1311.6296]; Weinzierl, Reuschle [1310.0413]

Dealing with colour

Process	$N_{\text{pri}}^{[0]}$	$N_{\text{pri}}^{[m]}$	$N_{\text{pri}}^{[f]}$
$8g$	720	2520	2520
$\bar{u}u + 6g$	720	5040	1800
$\bar{u}u\bar{d}d + 4g$	360	3360	671
$\bar{u}u\bar{d}\bar{s}s + 2g$	120	1344	194
$\bar{u}u\bar{d}\bar{s}s\bar{c}c$	30	384	65

large numbers of primitive amplitudes for high multiplicity

use phase space symmetry to reduce computational cost

$$\begin{aligned} \sigma_{gg \rightarrow n(g)}^V &= \int dPS_n \boxed{A^{(0)\dagger} \cdot \mathcal{C}_{n! \times (n+1)!/2} \cdot A^{(1)}} \\ &= (n-2)! \int dPS_n \boxed{A^{(0)\dagger} \cdot \mathcal{C}_{n! \times (n+1)}^{\text{dsym}} \cdot A^{(1), \text{dsym}}} \end{aligned}$$

factor of (final state gluons)!/2

retain full colour information

Automation with NJET

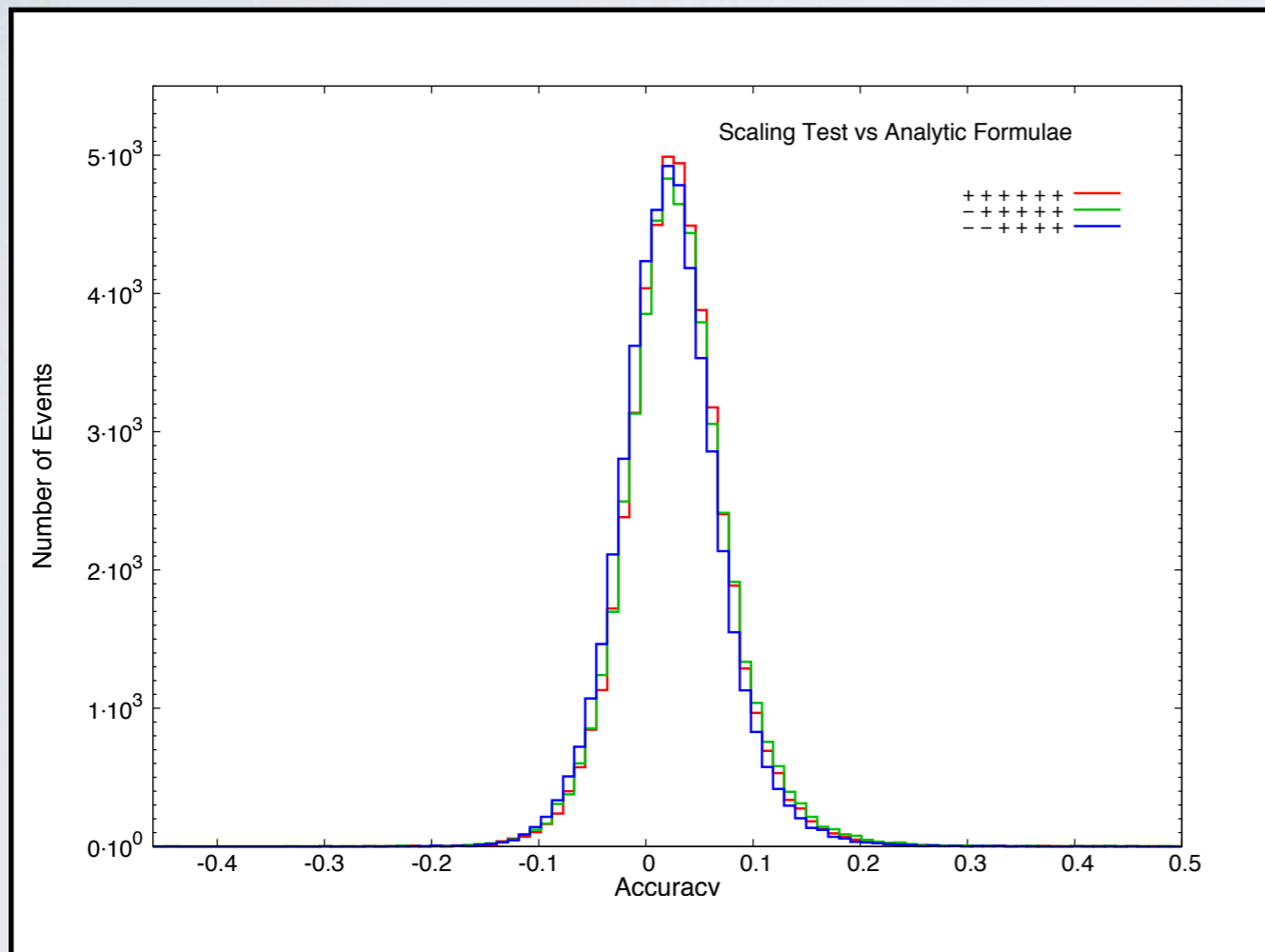
Numerical implementation in C++

Trees	off-shell recursion (Berends-Giele)
Loops	generalized unitarity
Colour	full (via primitive matching), de-symmetrized, leading/sub-leading
Interface	Binoth Les Houches Accord (python)

building on NGLUON [1011.2900]

SB, Biedermann, Uwer, Yundin [1209.0100]

Accuracy



dimension scaling test

$$A(p_i, m_i, \mu_R) = x^{4-n} A(xp_i, xm_i, x\mu_R) := A_{\text{NJET}}(x)$$

$$\# \text{digits} = \log_{10} \left(\frac{A_{\text{NJET}}(s_1) + A_{\text{NJET}}(s_2)}{2(A_{\text{NJET}}(s_1) - A_{\text{NJET}}(s_2))} \right)$$

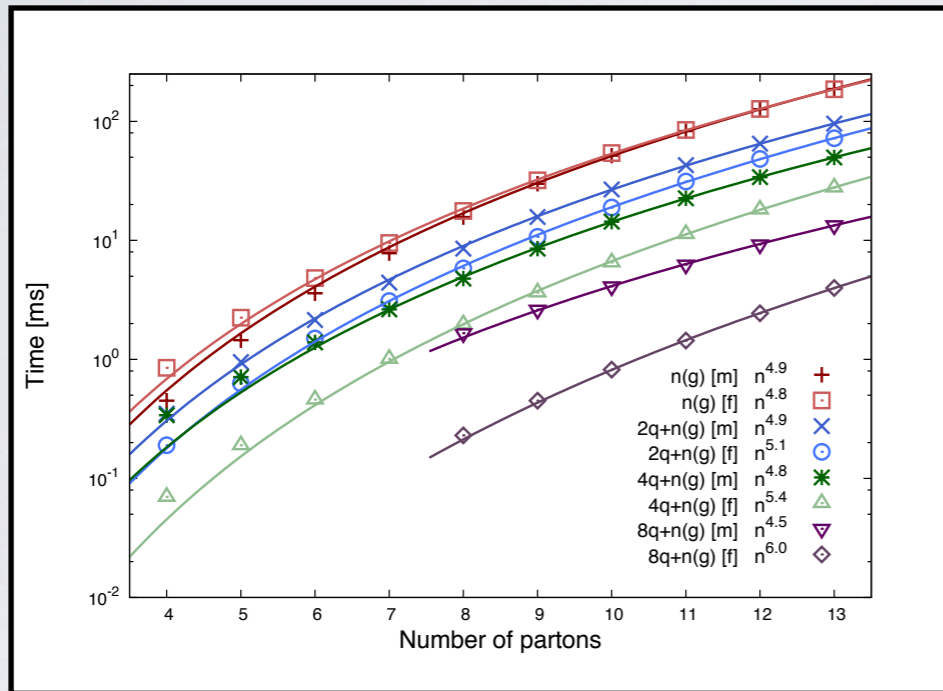
2 calls for the price
of 1 using explicit
vectorization with Vc

$$\text{accuracy} = \log_{10} \left(\frac{A_{\text{NJET}}(s_1) + A_{\text{NJET}}(s_2)}{2(A_{\text{NJET}}(s_1) - A_{\text{NJET}}(s_2))} \right) - \log_{10} \left(\frac{A_{\text{NJET}}(1) + A_{\text{analytic}}}{2(A_{\text{NJET}}(1) + A_{\text{analytic}})} \right)$$

reliable but statistical: add ~ 2 digits on min. accuracy

Performance

primitives scale $\sim n^6$ for $n \lesssim 20$



process	T_{sd} [s]	T_4 digits[s]	(% fixed)	process	T_{sd} [s]	T_4 digits[s]	(% fixed)
4g	0.030	0.030	(0.00)	5g	0.22	0.22	(0.22)
2u2g	0.032	0.032	(0.00)	2u3g	0.34	0.35	(0.06)
2u2d	0.011	0.011	(0.00)	2u2d1g	0.11	0.11	(0.00)
4u	0.022	0.022	(0.00)	4u1g	0.22	0.22	(0.03)

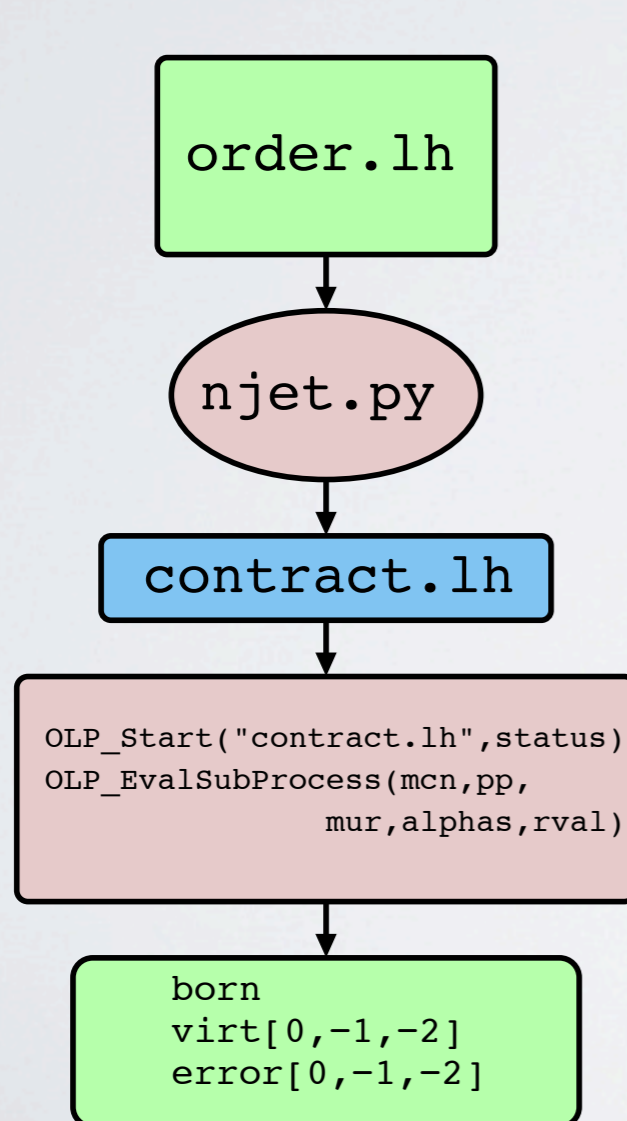
process	T_{sd} [s]	T_4 digits[s]	(% fixed)	process	T_{sd} [s]	T_4 digits[s]	(% fixed)
6g	6.19	6.81	(1.37)	7g	171.3	276.7	(8.63)
2u4g	7.19	7.40	(0.38)	2u5g	195.1	241.2	(3.25)
2u2d2g	2.05	2.06	(0.08)	2u2d3g	45.7	48.8	(0.88)
4u2g	4.08	4.15	(0.21)	4u3g	92.5	101.5	(1.29)
2u2d2s	0.38	0.38	(0.00)	2u2d2s1g	7.9	8.1	(0.23)
2u4d	0.74	0.74	(0.00)	2u4d1g	15.8	16.2	(0.29)
6u	2.16	2.17	(0.02)	6u1g	47.1	48.6	(0.41)

full colour sums a few seconds for $2 \rightarrow 4$

	$gg \rightarrow 2g$	$gg \rightarrow 3g$	$gg \rightarrow 4g$	$gg \rightarrow 5g$
standard sum	0.03	0.22	6.19	171.31
de-symmetrized	0.03	0.07	0.57	3.07

Monte-carlo interface

- BLHA - Binoth Les Houches Accord ([1001.1307], updated [1308.3462])



order file

```
# OLE_order for 5jet production
```

```
MatrixElementSquareType CHsummed  
CorrectionType QCD  
IRregularisation CDR  
AlphasPower 5  
# process list  
21 21 -> 21 21 21 21 21  
1 -1 -> 21 21 21 21 21  
1 -1 -> 21 -2 2 21 21  
1 -1 -> 21 -1 1 21 21  
1 -1 -> 21 -2 2 -3 3  
1 -1 -> 21 -2 2 -2 2  
1 -1 -> 21 -1 1 -1 1
```

contract file

```
# OLE_order for 5jet production  
# Generated file. Do not edit by hand.  
# Signed by NJet 3900867518.  
# 12 1 1e-05 0.01 0 1 1 1 1 0 3 5  
MatrixElementSquareType CHsummed | OK  
CorrectionType QCD | OK  
IRregularisation CDR | OK  
AlphasPower 5 | OK  
# process list  
21 21 -> 21 21 21 21 21 | 1 1 # 70 120 4 64 0 (-2 -1 3 4 5 6 7)  
1 -1 -> 21 21 21 21 21 | 1 2 # 71 120 4 9 0 (-1 -2 3 4 5 6 7)  
1 -1 -> 21 -2 2 21 21 | 1 3 # 72 6 4 9 0 (-1 -2 4 5 3 6 7)  
1 -1 -> 21 -1 1 21 21 | 1 4 # 73 6 4 9 0 (-1 -2 4 5 3 6 7)  
1 -1 -> 21 -2 2 -3 3 | 1 5 # 74 1 4 9 0 (-1 -2 4 5 6 7 3)  
1 -1 -> 21 -2 2 -2 2 | 1 6 # 75 4 4 9 0 (-1 -2 4 5 6 7 3)  
1 -1 -> 21 -1 1 -1 1 | 1 7 # 76 4 4 9 0 (-1 -2 4 5 6 7 3)
```

NJET + Sherpa

$$\sigma_n^{\text{NLO}} = \sigma^{\text{LO}} + \int_n d\sigma_n^{\text{V}} + \int_n d\sigma_n^{\text{I+fac.}} + \int_{n+1} (d\sigma_{n+1}^{\text{R}} - d\sigma_{n+1}^{\text{S}})$$

NJET v2.0

Sherpa MC v2.0.0

leading/sub-leading colour
de-symmetrized colour sums

$$pp \rightarrow \leq 5j$$

$$pp \rightarrow \gamma\gamma + 4j$$

$$pp \rightarrow W^{[\rightarrow l^\pm \nu]} + \leq 5j$$

$$pp \rightarrow Z/\gamma^* [\rightarrow l^+ l^-] + \leq 5j$$

$$pp \rightarrow \gamma + \leq 4j$$

$$pp \rightarrow \gamma\gamma + \leq 4j$$

Comix [Gleisberg, Hoeche (2008)]

CS subtraction [Gleisberg, Krauss (2007)]

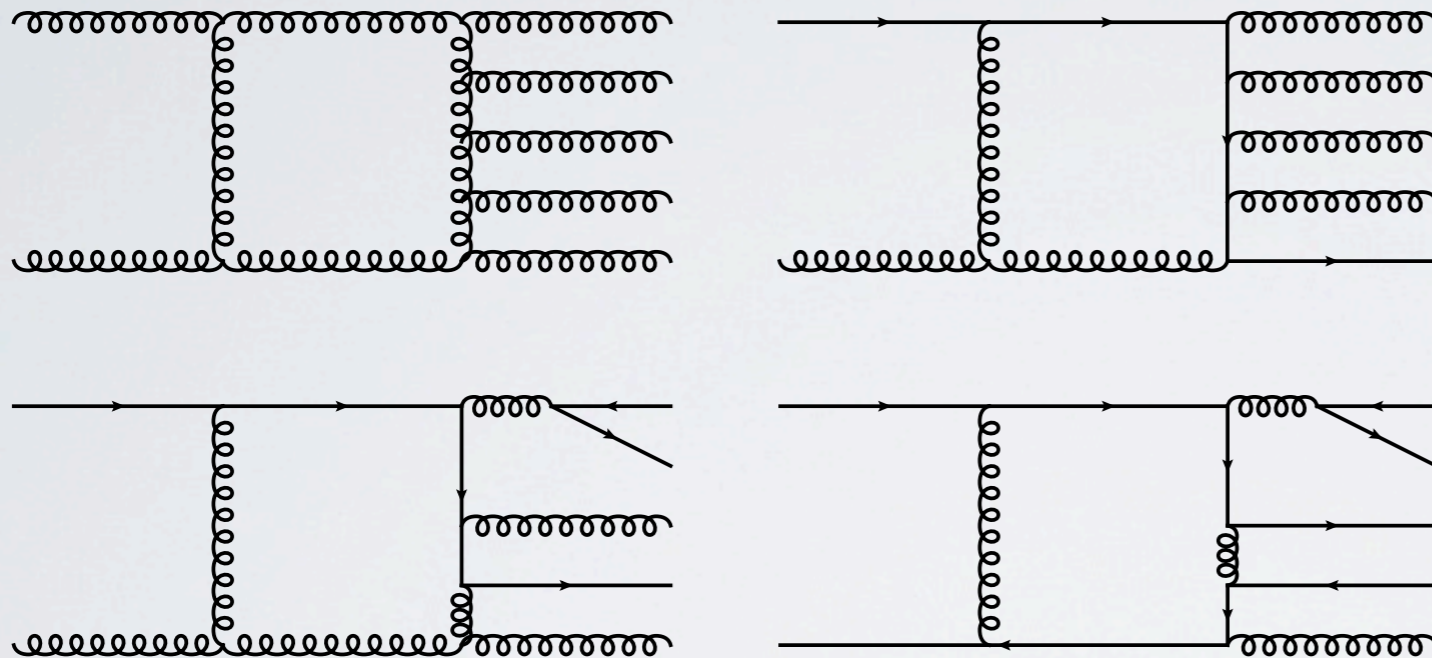
ROOT Ntuple event generation

also:

FastJet [Cacciari, Salam, Soyez (2008)]

LHAPDF [Whalley, Bourilkov, Group (2005)]

Multi-jet production at the LHC



full colour
 $N_f = 5$ flavour scheme
 no top loops

ATLAS cuts [1107.2092]

$p_{T,j_1} > 80$ GeV
 anti-kt $R = 0.4$

$p_{T,j} > 60$ GeV
 $\mu_R = \mu_F = \hat{H}_T/2$
 $|\eta_j| < 2.8$

NLO QCD corrections

$pp \rightarrow \leq 3j$

Nagy (NLOJet++) [hep-ph/0307268]

$pp \rightarrow \leq 4j$

Bern et al. (BlackHat) [1112.3940]

$pp \rightarrow \leq 5j$

SB, Biedermann, Uwer, Yundin [1209.0098]

SB, Biedermann, Uwer, Yundin [1309.6585]

Virtual matrix elements

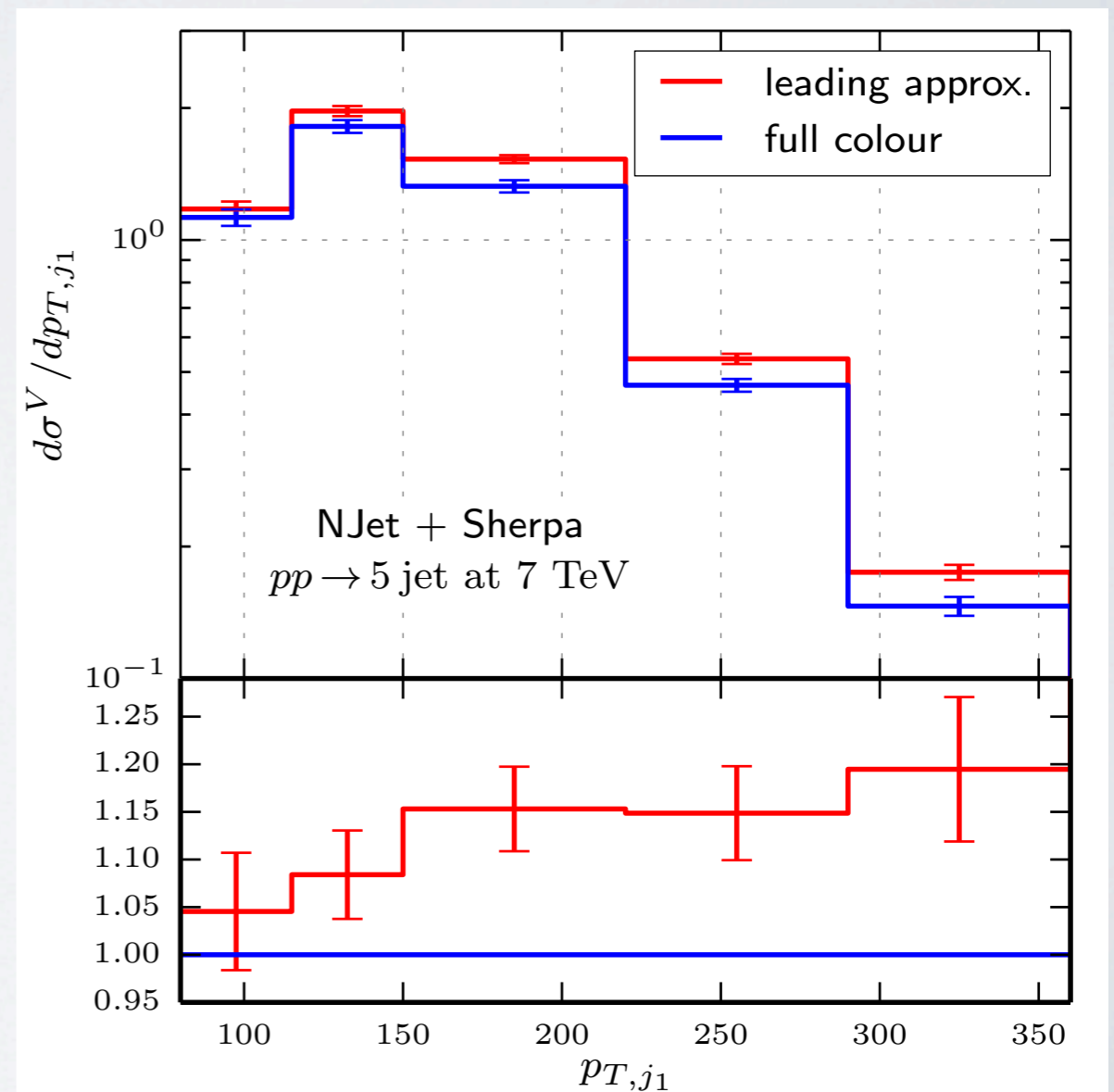
leading part: best case of de-sym.
and leading colour

fermion loops included in
sub-leading part

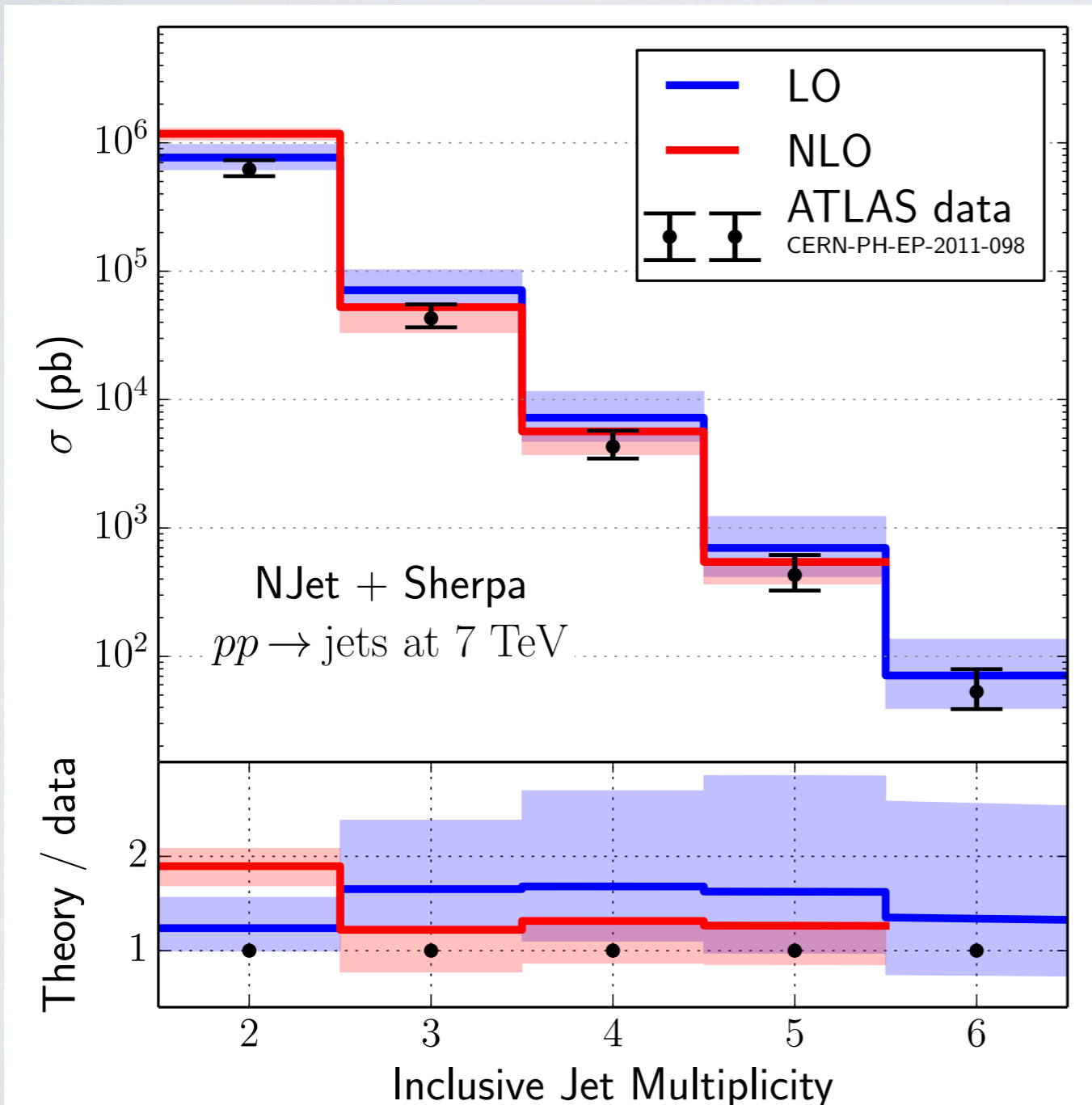
Virtual part	Time per event	QP	QP2	OP
leading	17 s	2%	0.5%	0.01%
subleading	112 s	2.5%	1%	0.05%

average evaluation time on cluster

possibility to switch to octuple
precision - not necessary in practice



Total cross-sections



$$\sigma_5^{7\text{TeV-LO}}(\mu = \hat{H}_T/2) = 0.699(0.004)^{+0.530}_{-0.280} \text{ nb},$$

$$\sigma_5^{7\text{TeV-NLO}}(\mu = \hat{H}_T/2) = 0.544(0.016)^{+0.0}_{-0.177} \text{ nb}.$$

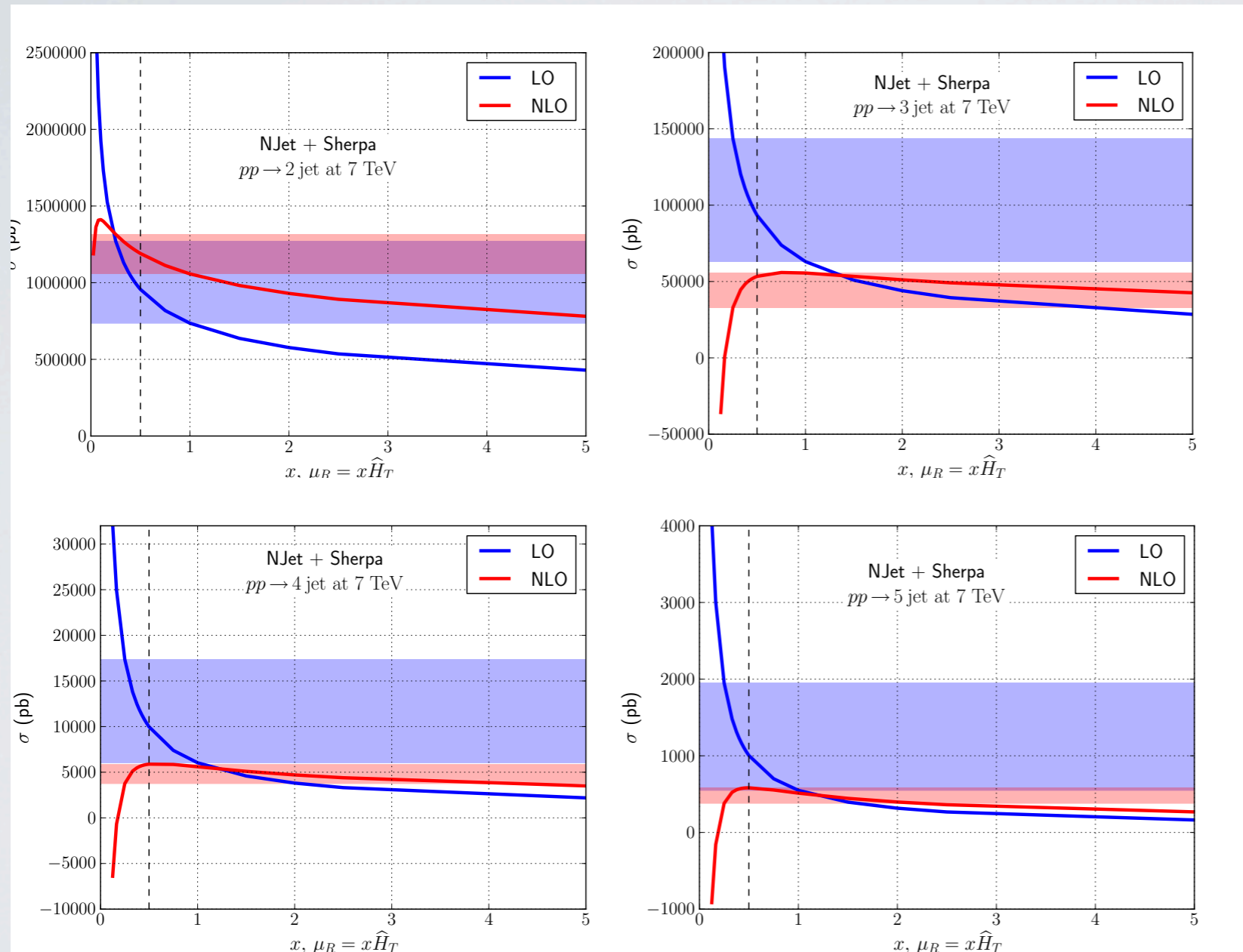
NNPDF2.1 LO

$$\alpha_s(M_Z) = 0.119$$

NNPDF2.3 NLO

$$\alpha_s(M_Z) = 0.118$$

Scale dependence



LO PDF: MSTW2008lo

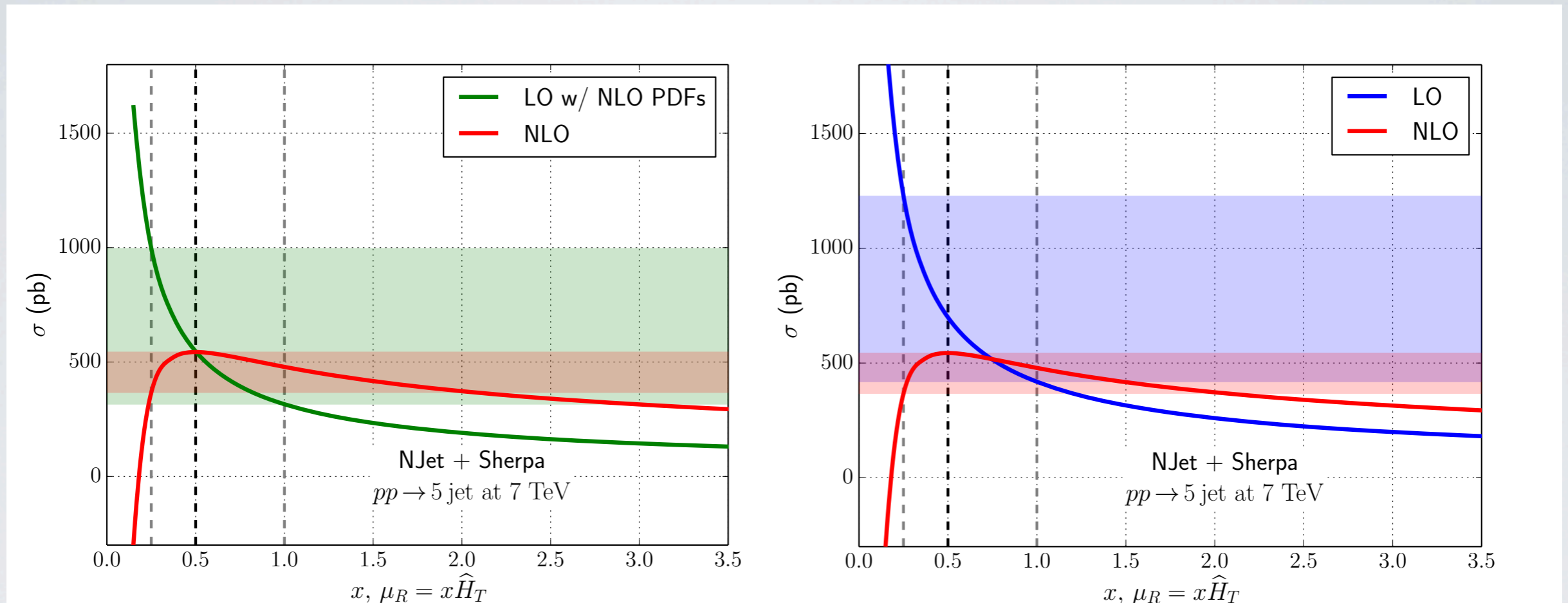
$$\alpha_s(M_Z) = 0.139$$

NLO PDF: MSTW2008nlo

$$\alpha_s(M_Z) = 0.120$$

Dynamical scale attempting include large logarithms

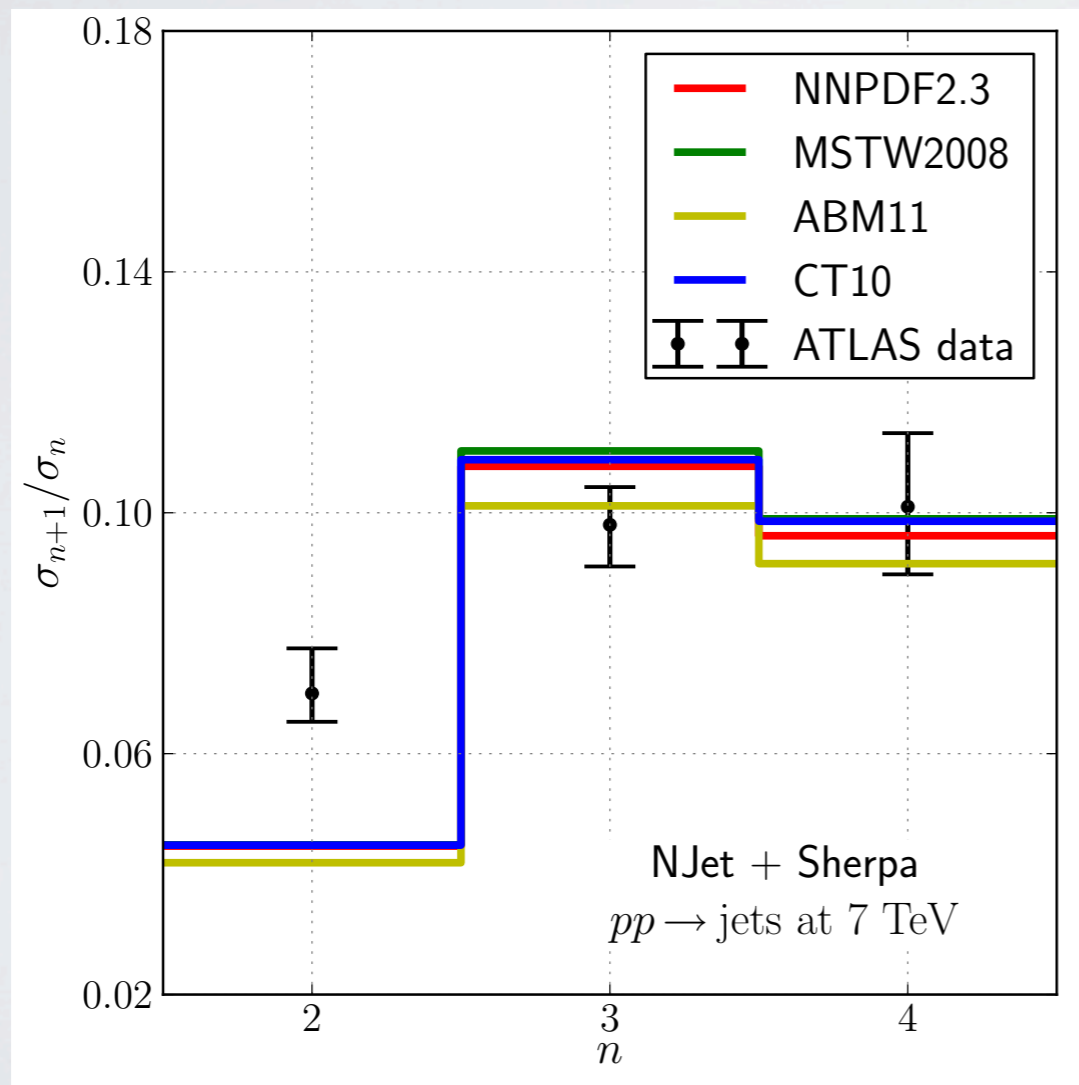
Scale dependence



Majority of NLO corrections are coming from 2-loop running of α_s

Jet Ratios

Reliable quantities for both theory and experiment



fixed order NLO good for di-jets
with asymmetric cuts

c.f. large NLO K-factors Rubin,
Salam, Sapeta [1006.2144]

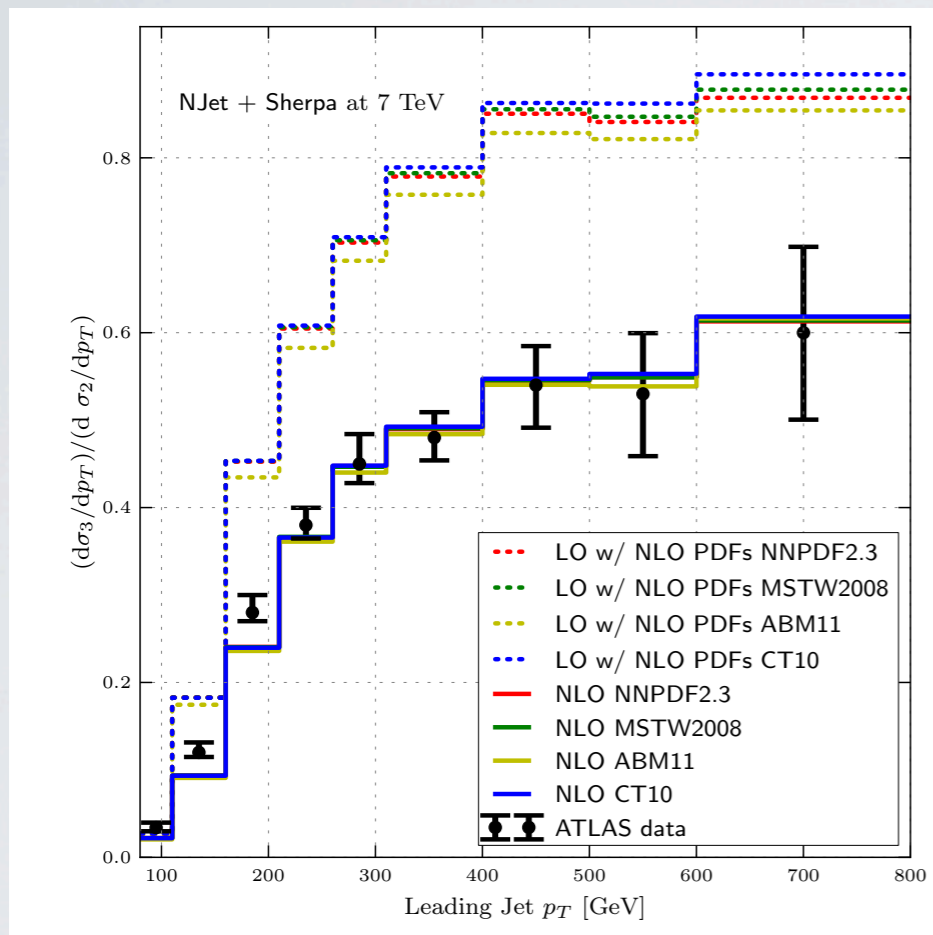
e.g. α_s determination
[1304.7498; CMS-QCD-11-003]

$$\alpha_s(M_Z) = 0.1148 \pm 0.0014 (\text{exp.}) \pm 0.0018 (\text{PDF}) \pm 0.0050 (\text{theory})$$

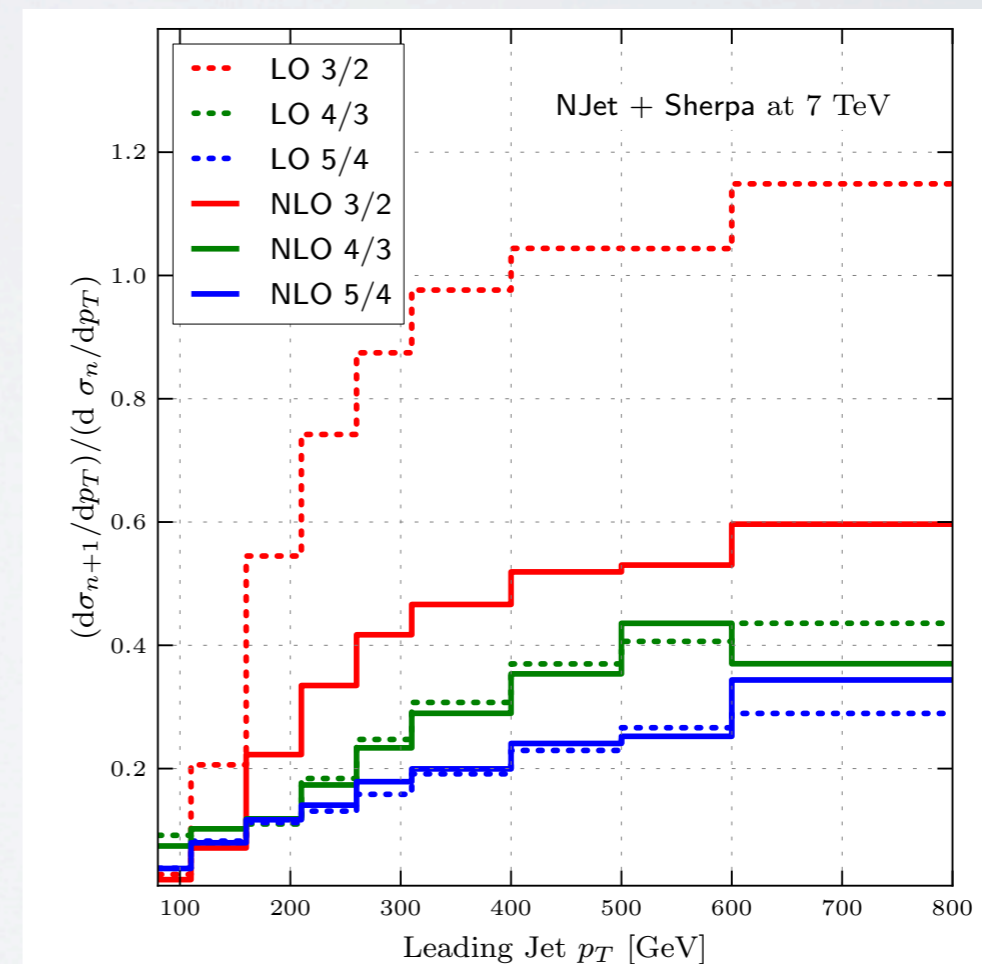
Jet Ratios

large NLO corrections to 3/2 ratio

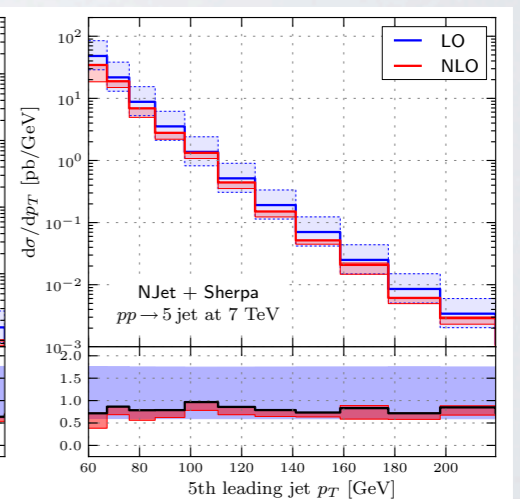
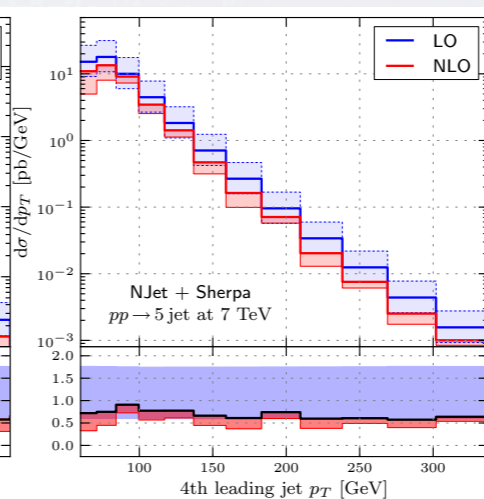
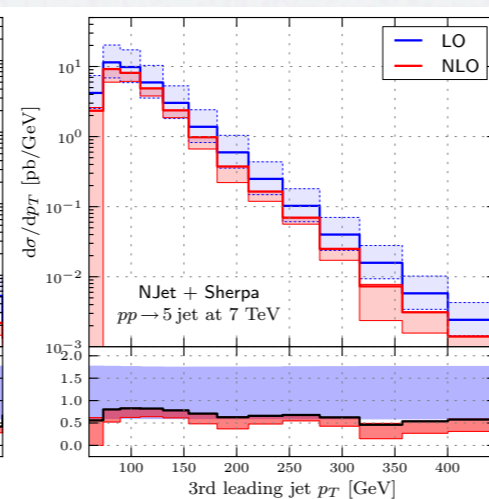
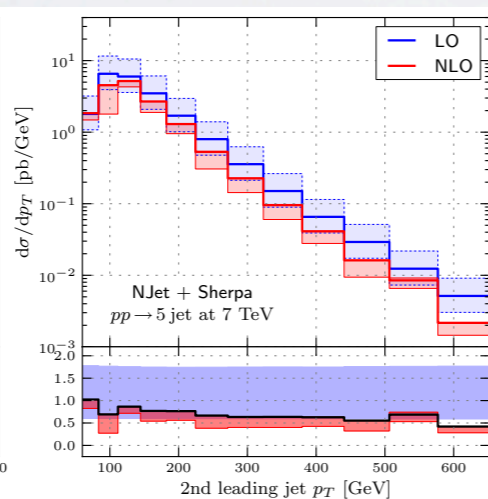
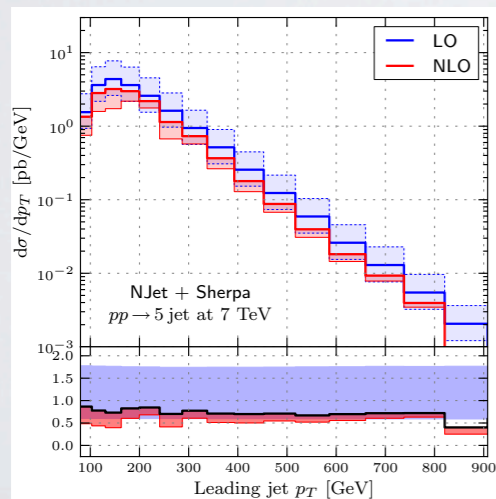
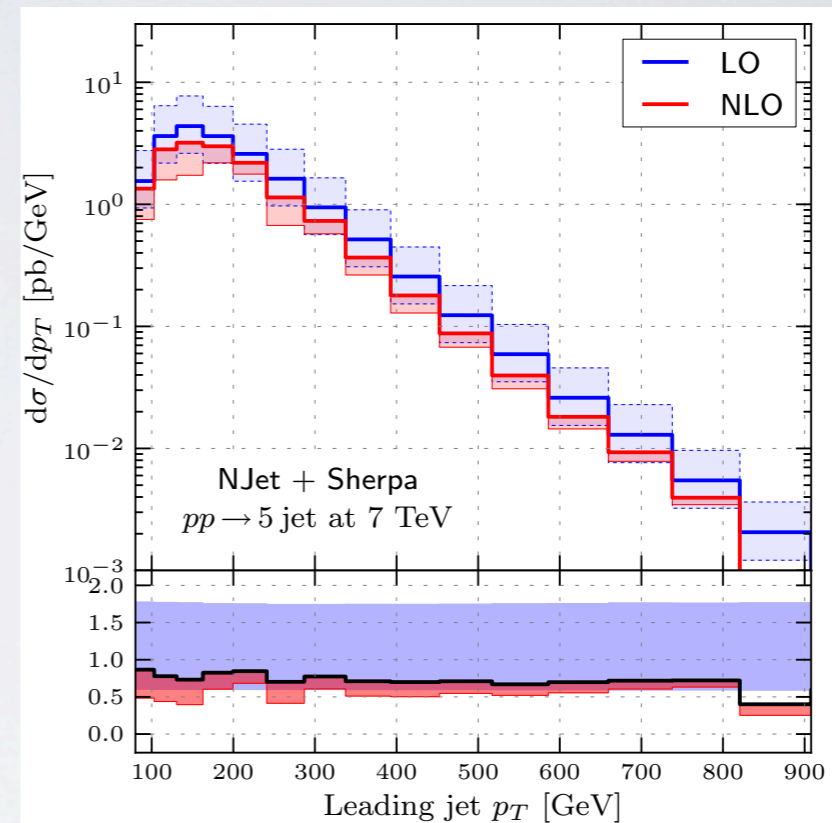
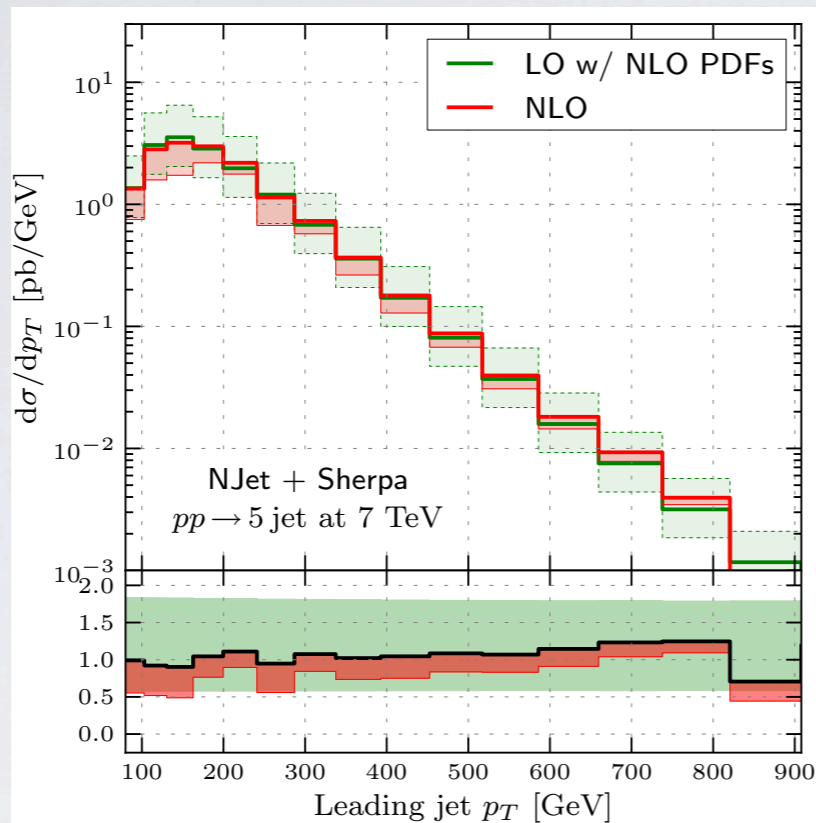
all PDF sets in good agreement with data



4/3 and 5/4 ratios more stable
QCD corrections



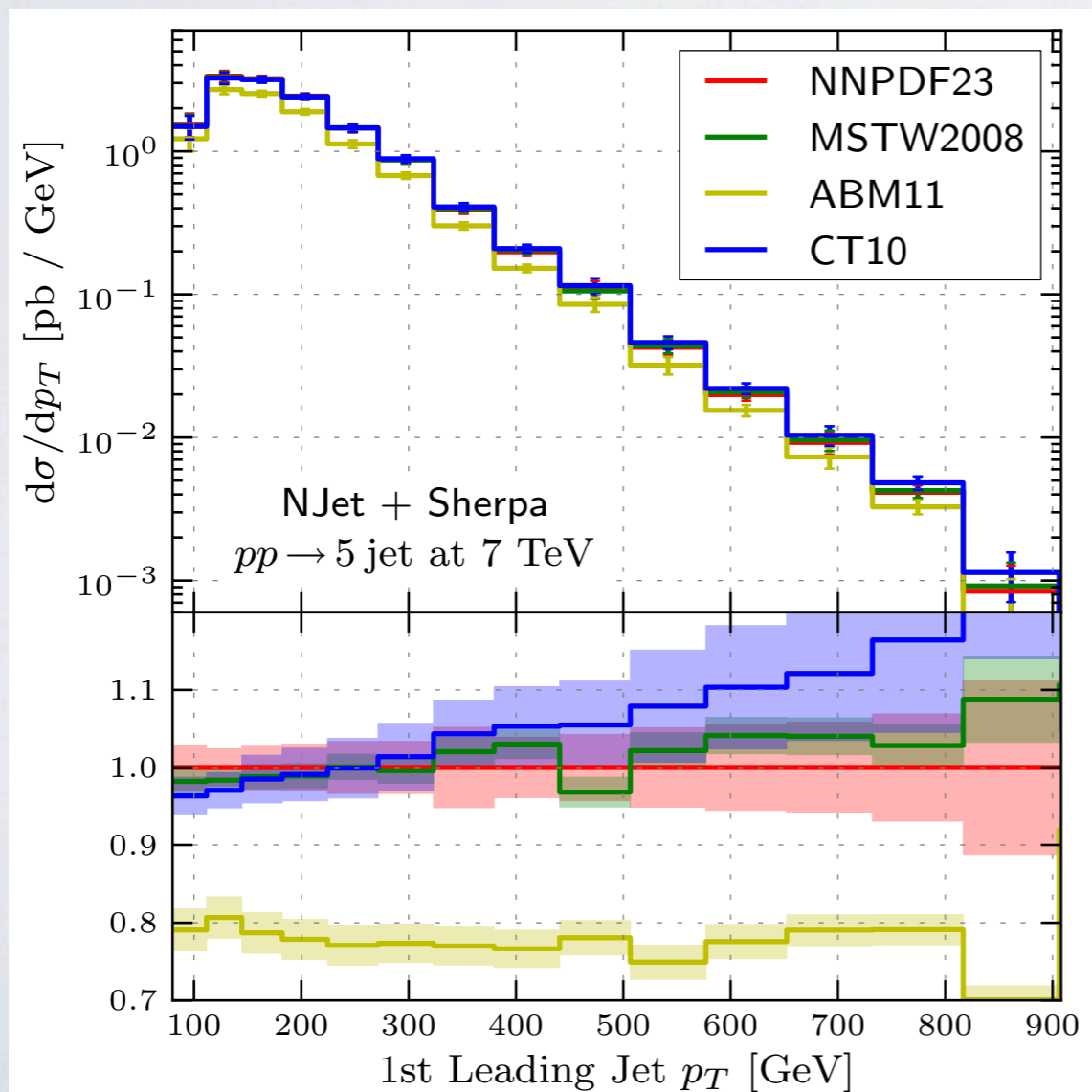
p_T distributions



more distributions at <https://bitbucket.org/njet/njet/wiki/Results/Physics>

PDF dependence

Generally weak dependence
on the choice of PDF fit
(excluding choice for $\alpha_s(M_Z)$)



comparison using all sets with

$$\alpha_s(M_Z) = 0.118$$

some dependence on p_T

significant deviation in
normalization of ABM11

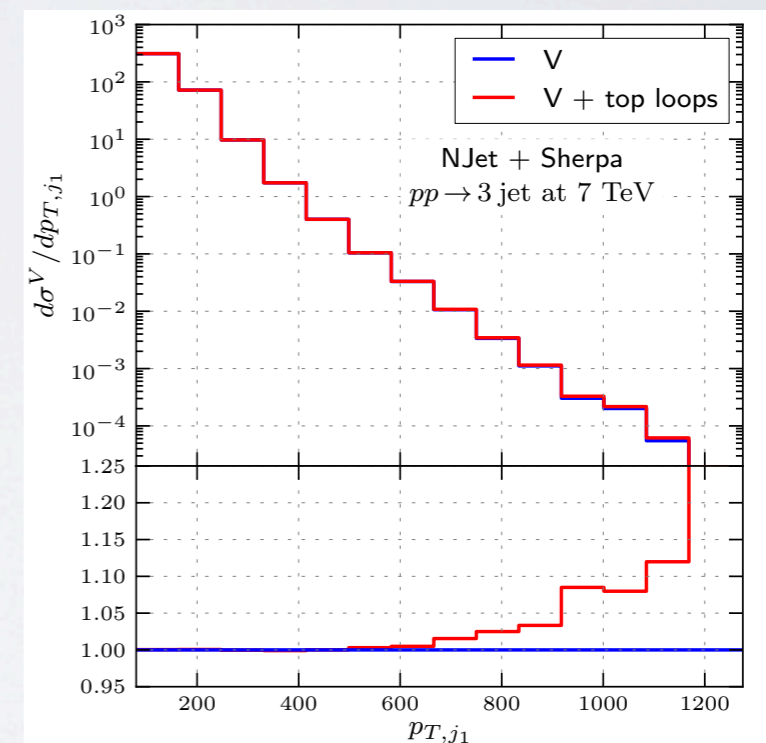
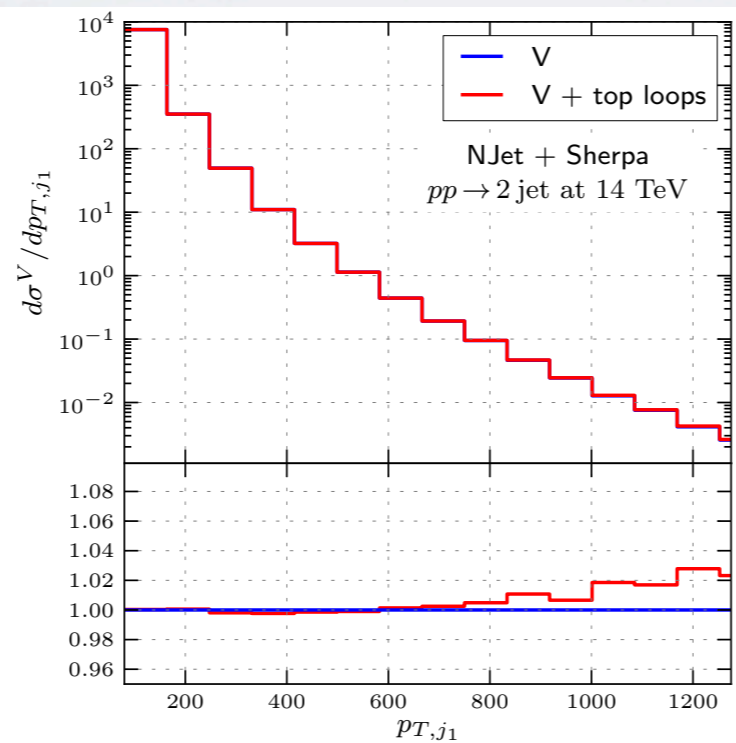
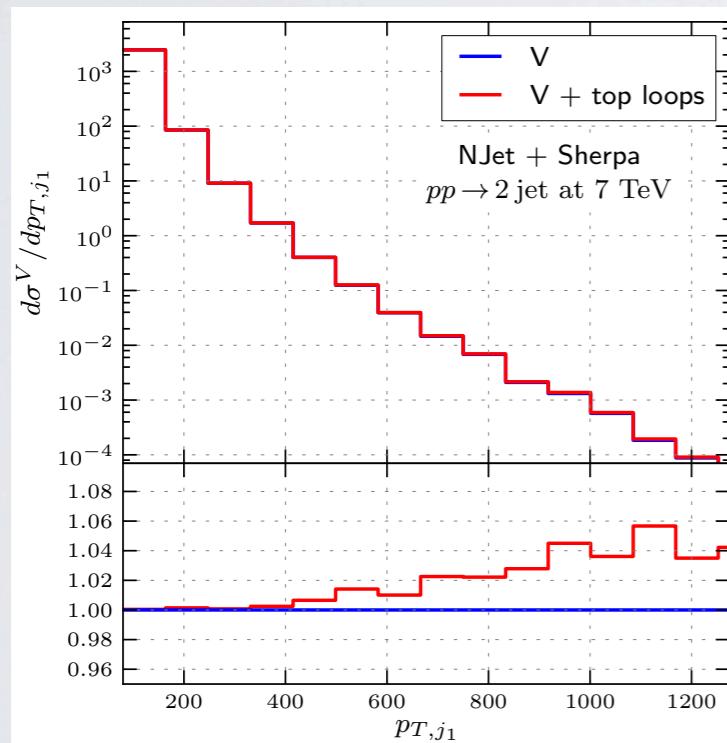
Heavy quark loops

preliminary

top quark loop effects are small ($< 1\%$)

di-jets seem to have additional kinematic suppression

matrix elements checked against MadLoop



corrections grow at very large p_T - still negligible

Efficient Event Generation

- Leading/sub-leading expansion - sample dominant contributions more often
- Separate contributions by number of fermion lines
- ROOT Ntuples - make the most out of the integration run
 - Re-weighting PDFs and renormalization/factorization scales (also jet algorithms with suitable event generation)
- APPLgrid - extremely fast and flexible analysis (very useful for PDF error analyses)

Vector boson production

Validation/feasibility test only

comparison with

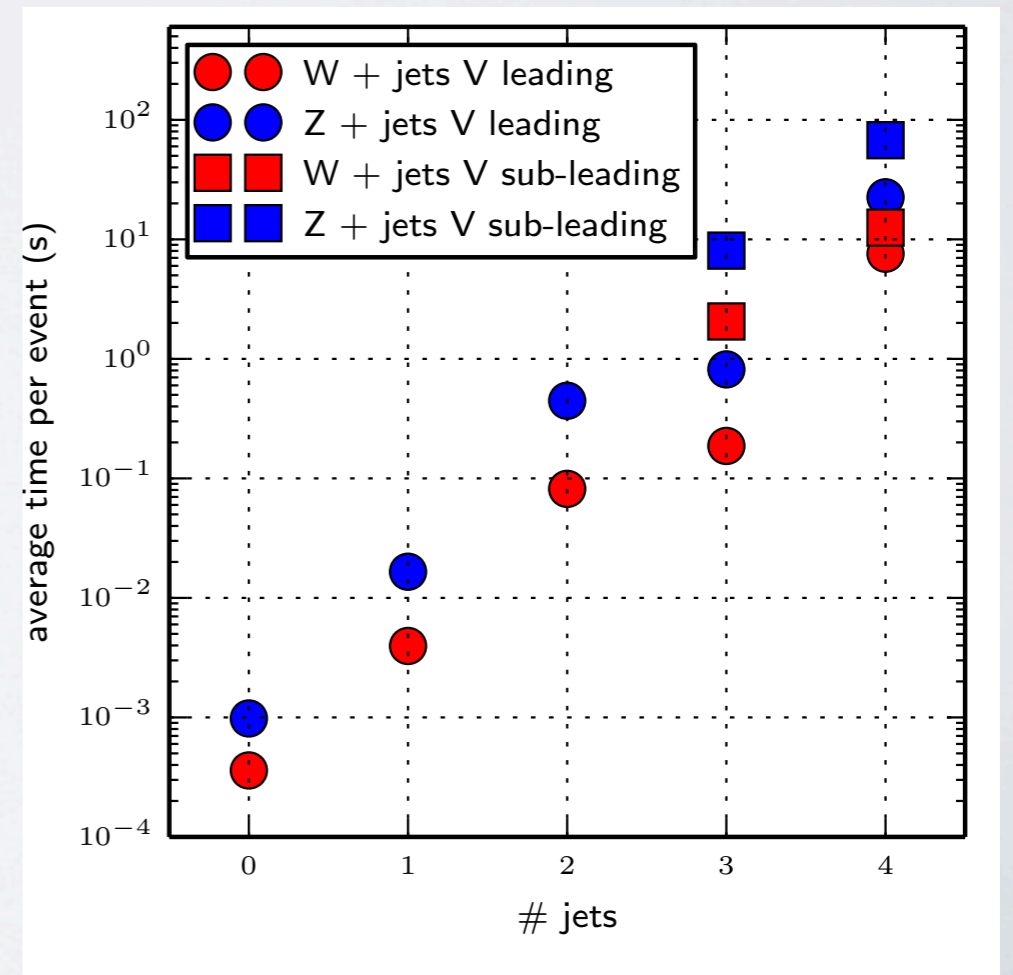
Bern et al. [1304.1253]

$$W^\pm[\rightarrow e^\pm \nu_e] + \leq 4j$$

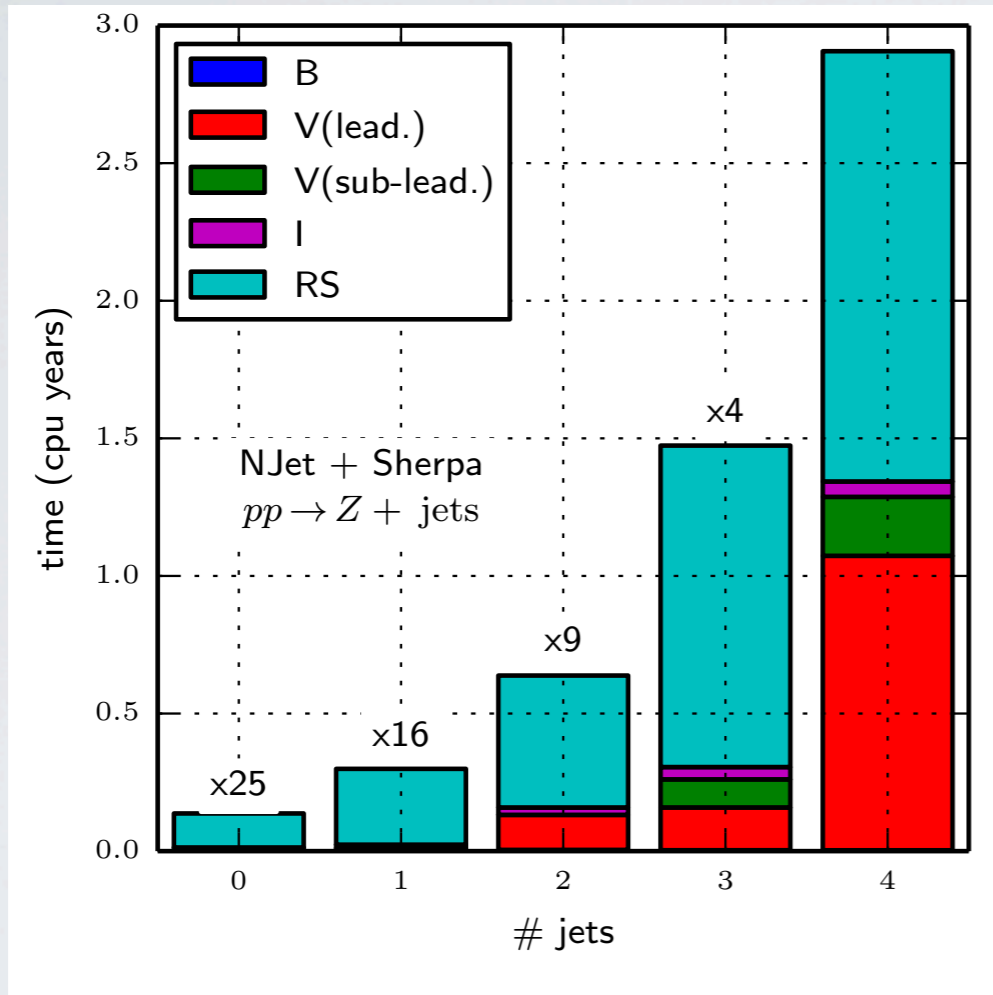
$$Z/\gamma^*[\rightarrow e^+e^-] + \leq 4j$$

$$pp \rightarrow \leq W^\pm + 5j$$

good agreement on total
cross sections



Vector boson production



ATLAS JHEP 1307 (2013) 032 [1304.7098]

$$p_{T,j} > 30 \text{ GeV}$$

$$p_{T,e} > 20 \text{ GeV}$$

$$|\eta_e| < 2.47 \quad 1.37 < |\eta_e| < 1.52$$

$$\text{anti-kt } R = 0.4$$

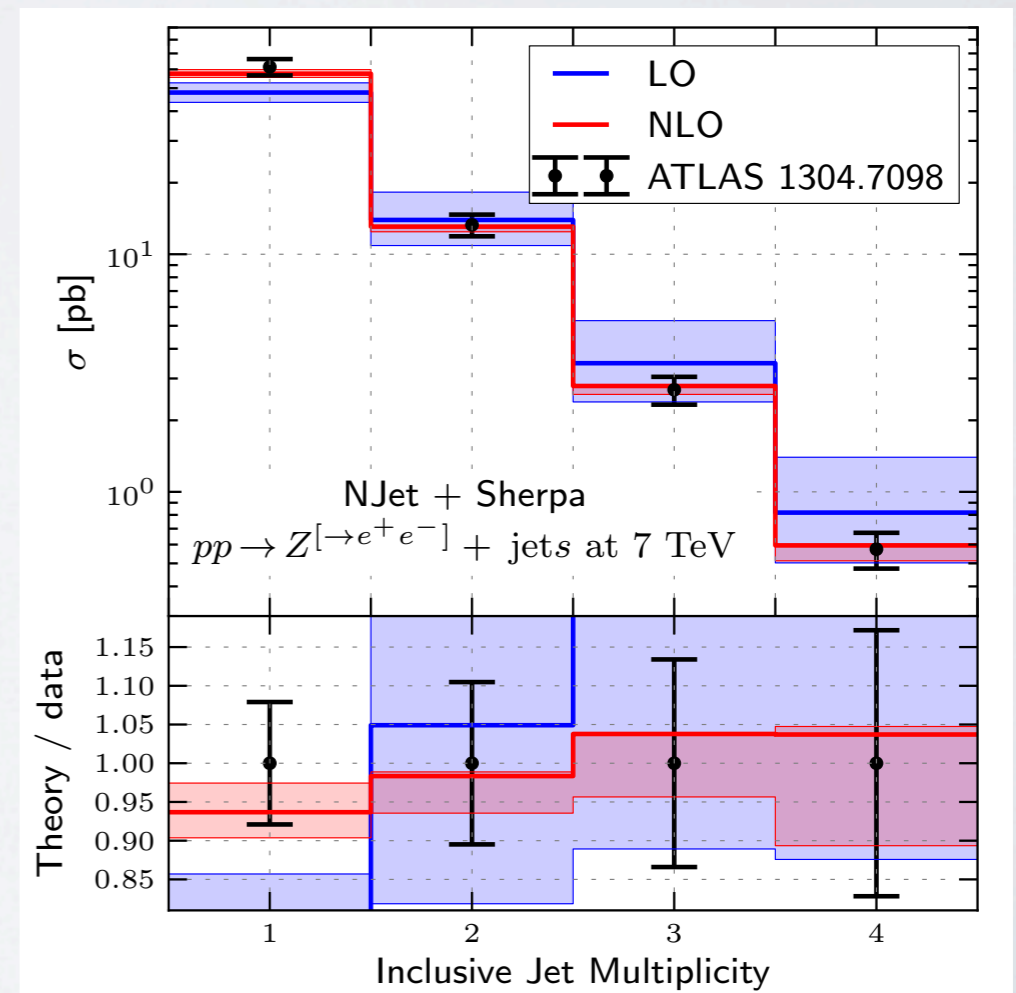
$$|\eta_j| < 4.4$$

$$66 < m_{ee} < 116 \text{ GeV}$$

$$\mu_R = \mu_F = \left(\sqrt{m_{ee}^2 + p_{T,ee}^2} + \hat{H}_T \right) / 2$$

Ita, Bern, Dixon, Febres Cordero,
Kosower, Maitre [1108.2229]

$$Z/\gamma^* [\rightarrow e^+e^-] + \leq 4j$$



Di-photon plus jets

Backgrounds to Higgs measurements $pp \rightarrow H \rightarrow \gamma\gamma$

$pp \rightarrow \gamma\gamma$ NNLO Catani, Cieri, de Florian, Ferrera, Grazzini [1110.2375]

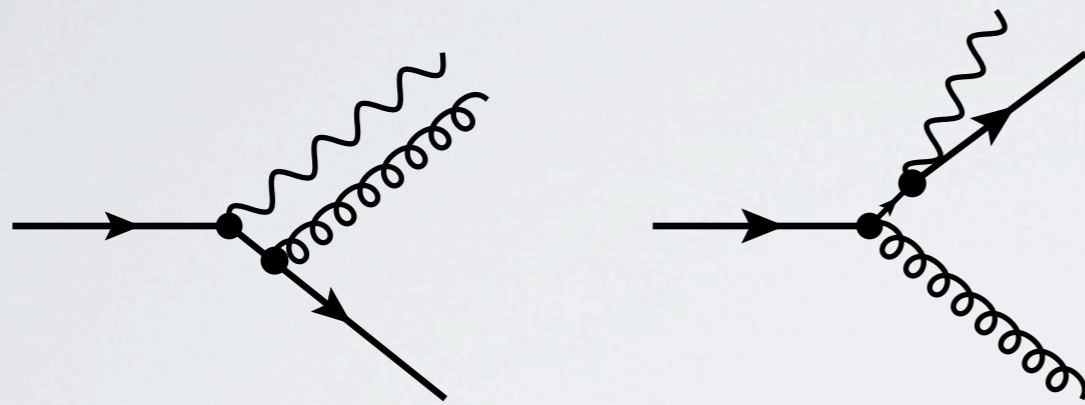
$pp \rightarrow \gamma\gamma + 1j$ NLO Gehrman, Greiner, Heinrich [1303.0824]
Del Duca, Maltoni, Nagy, Trocsanyi [hep-ph/0303012]

$pp \rightarrow \gamma\gamma + 2j$ NLO Gehrman, Greiner, Heinrich [1308.3660]
Bern, Dixon, Febres Cordero, Hoeche, Ita, Kosower, Lo Presti, Maitre [1312.0592]

$pp \rightarrow \gamma\gamma + 3j$ NLO SB, Guffanti, Yundin [1312.5927]

Isolating hard photons

[Frixione (1998)]



Infra-red safe definition of a hard photon must include QCD partons

Smooth cone isolation

- keep soft gluons
- discard partons collinear to photon

$$E_{\text{hadronic}}(r_\gamma) \leq \epsilon p_{T,\gamma} \left(\frac{1 - \cos r_\gamma}{1 - \cos R} \right)^n$$

no need for fragmentation functions

$pp \rightarrow \gamma\gamma + \text{jets}$ at NLO

SB, Guffanti, Yundin [1312.5927]

$$p_{T,j} > 30 \text{ GeV}$$

$$|\eta_j| \leq 4.7$$

$$p_{T,\gamma_1} > 40 \text{ GeV}$$

$$p_{T,\gamma_2} > 25 \text{ GeV}$$

$$|\eta_\gamma| \leq 2.5$$

$$R_{\gamma,j} = 0.5$$

$$R_{\gamma,\gamma} = 0.45$$

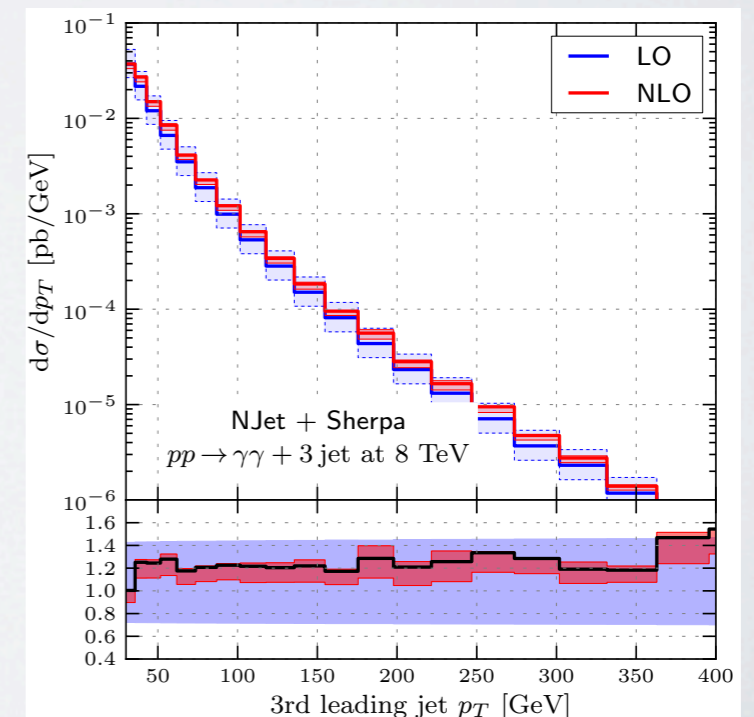
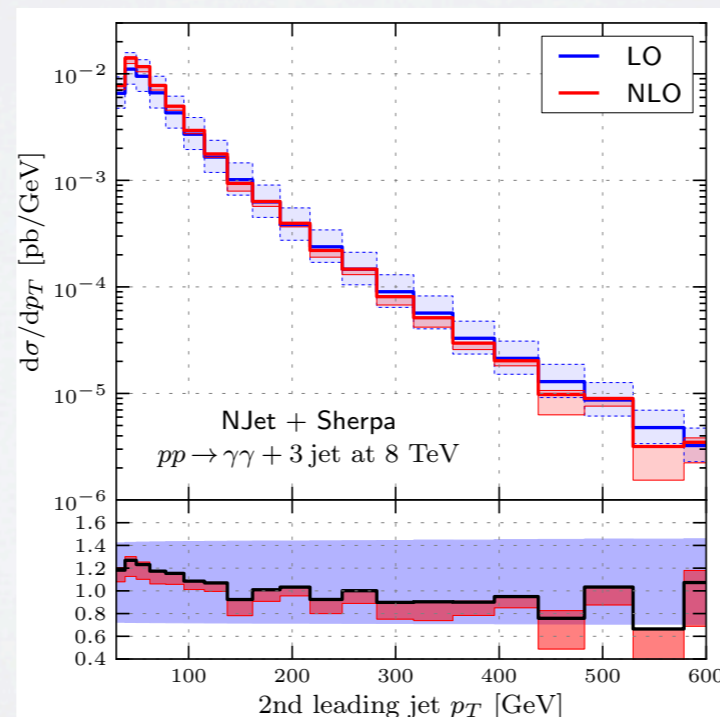
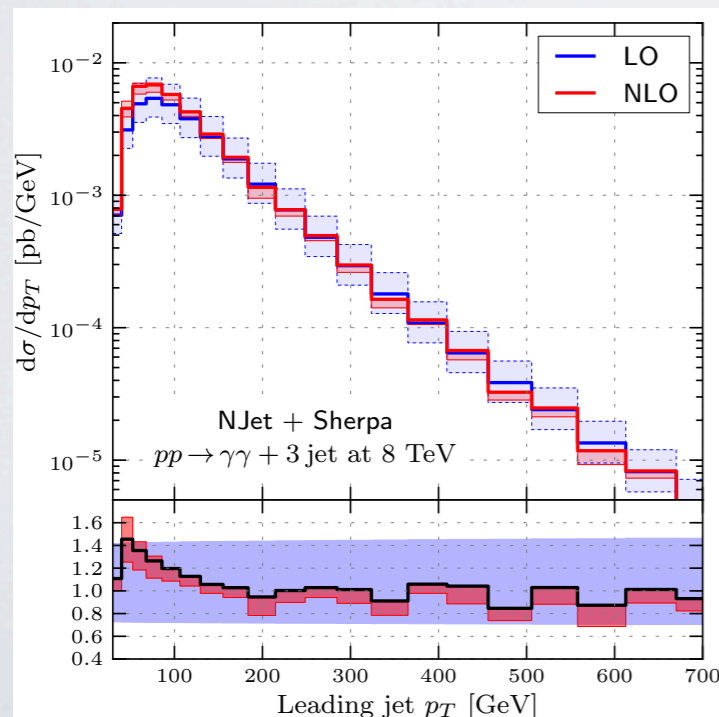
anti- k_T $R = 0.5$ (Fastjet)

Frixione smooth cone
photon isolation

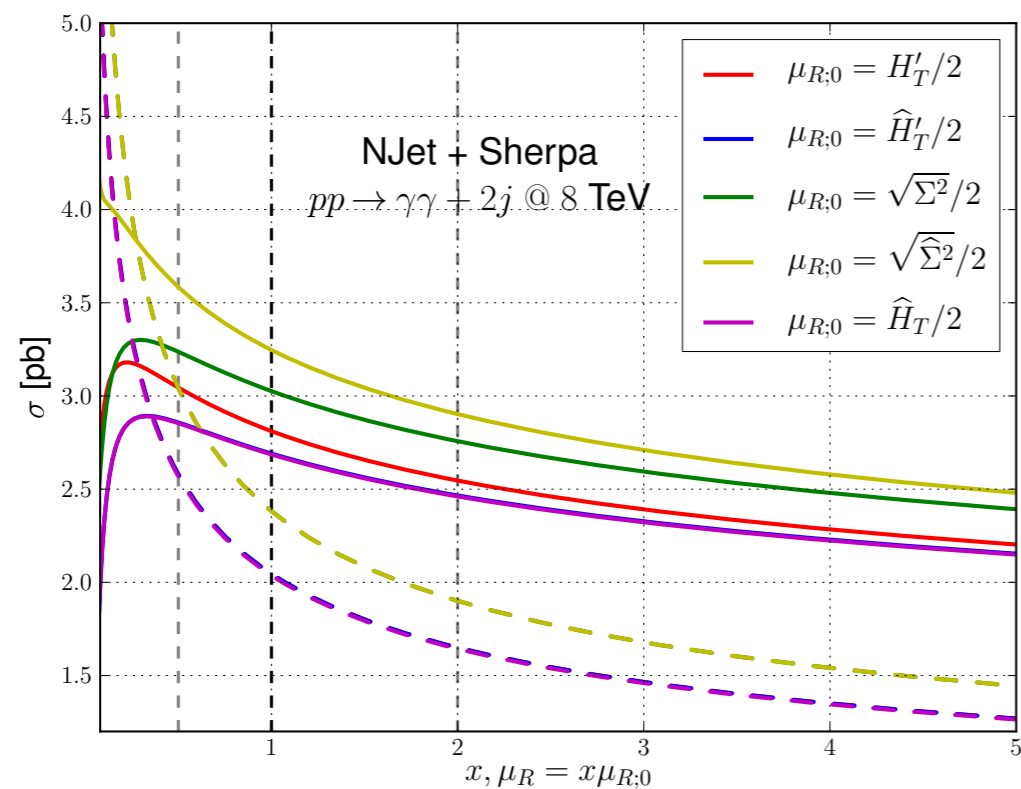
$$\epsilon = 0.05, R = 0.4 \text{ and } n = 1$$

CT10 NLO PDF set

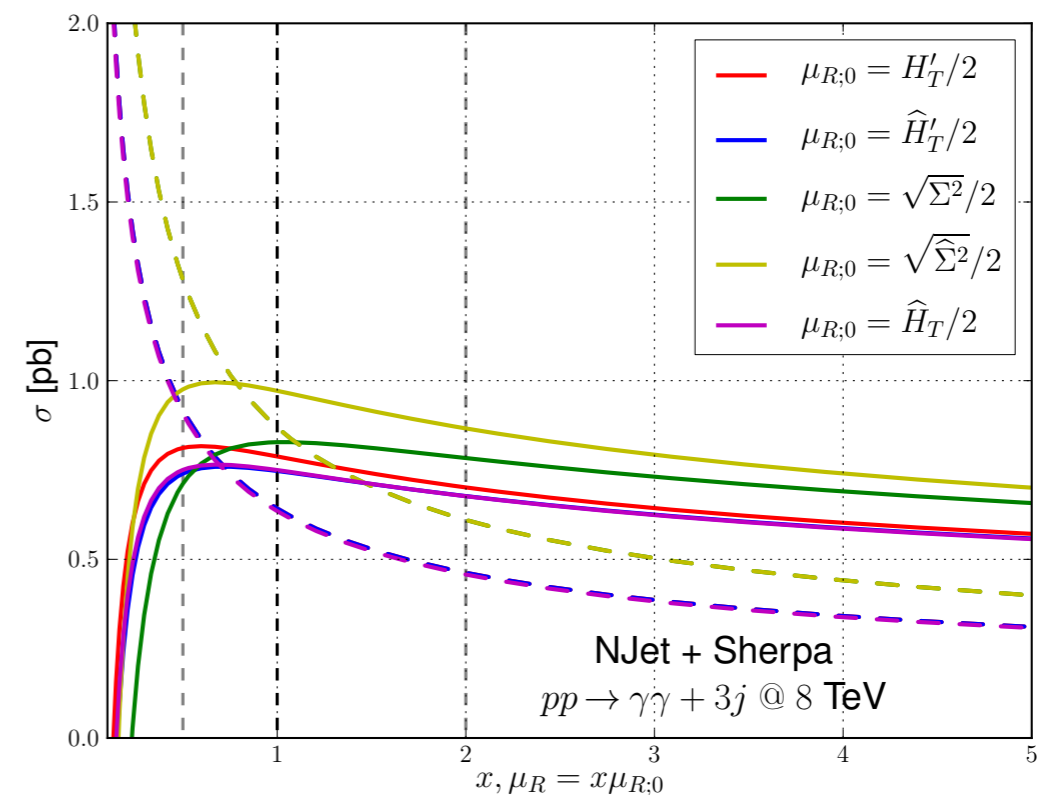
$$\sigma_{\gamma\gamma+3j}^{LO}(\hat{H}'_T/2) = 0.643(0.003)^{+0.278}_{-0.180} \text{ pb} \quad \sigma_{\gamma\gamma+3j}^{NLO}(\hat{H}'_T/2) = 0.785(0.010)^{+0.027}_{-0.085} \text{ pb}$$



Scale dependence



NLO predictions reduce uncertainty from 50% to 20%



Fairly wide range of predictions with different dynamical scales

Scale dependence

$$\hat{H}_T = p_{T,\gamma_1} + p_{T,\gamma_2} + \sum_{i \in \text{partons}} p_{T,i}$$

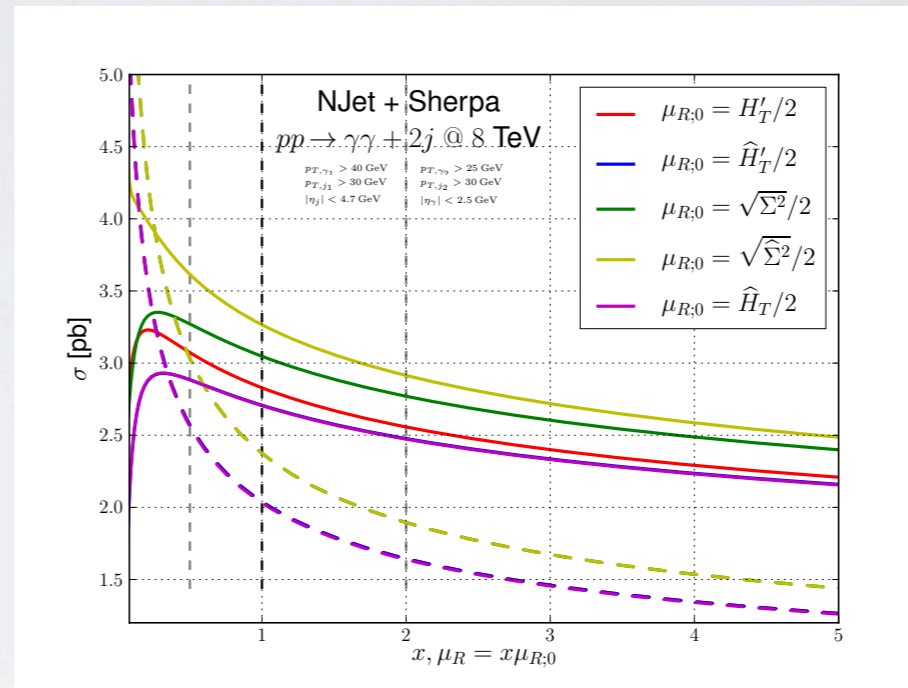
$$\hat{H}'_T = m_{\gamma\gamma} + \sum_{i \in \text{partons}} p_{T,i}$$

$$\hat{\Sigma}^2 = m_{\gamma\gamma}^2 + \sum_{i \in \text{partons}} p_{T,i}^2$$

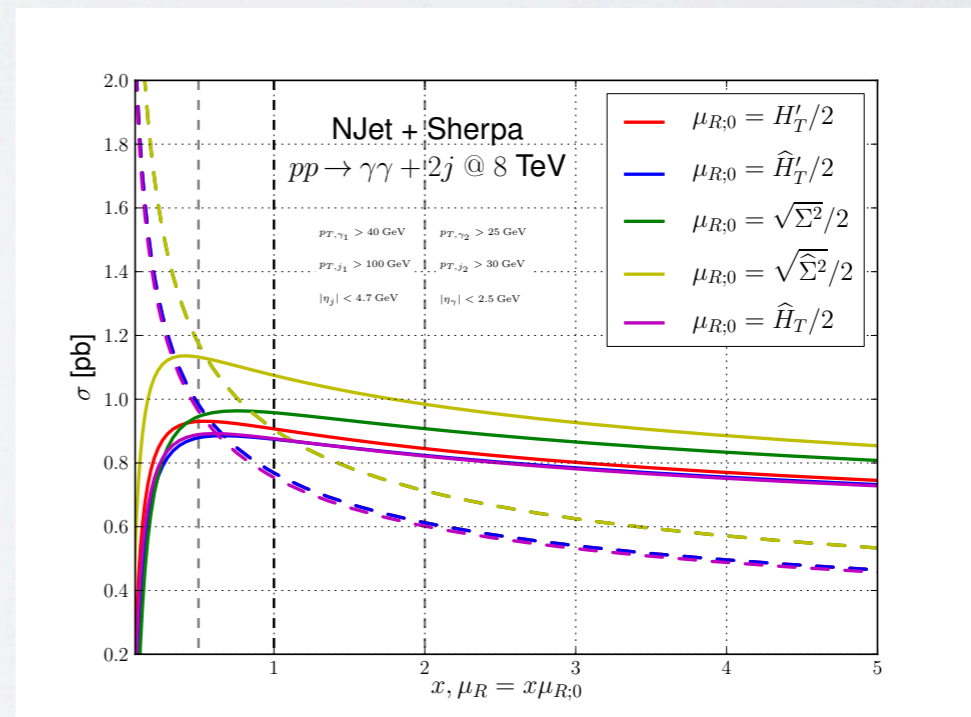
$$H'_T = m_{\gamma\gamma} + \sum_{i \in \text{jets}} p_{T,i}$$

$$\Sigma^2 = m_{\gamma\gamma}^2 + \sum_{i \in \text{jets}} p_{T,i}^2$$

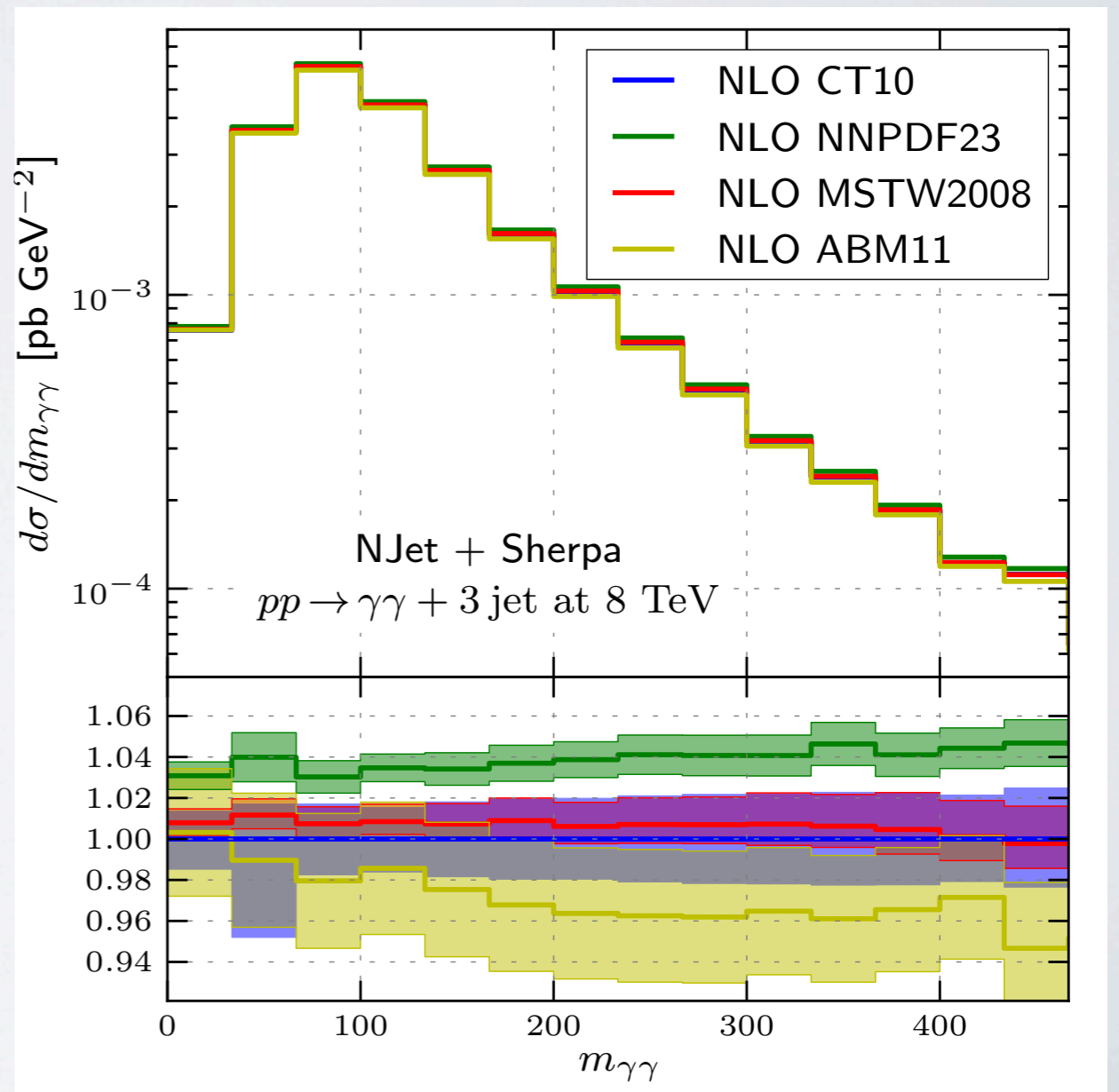
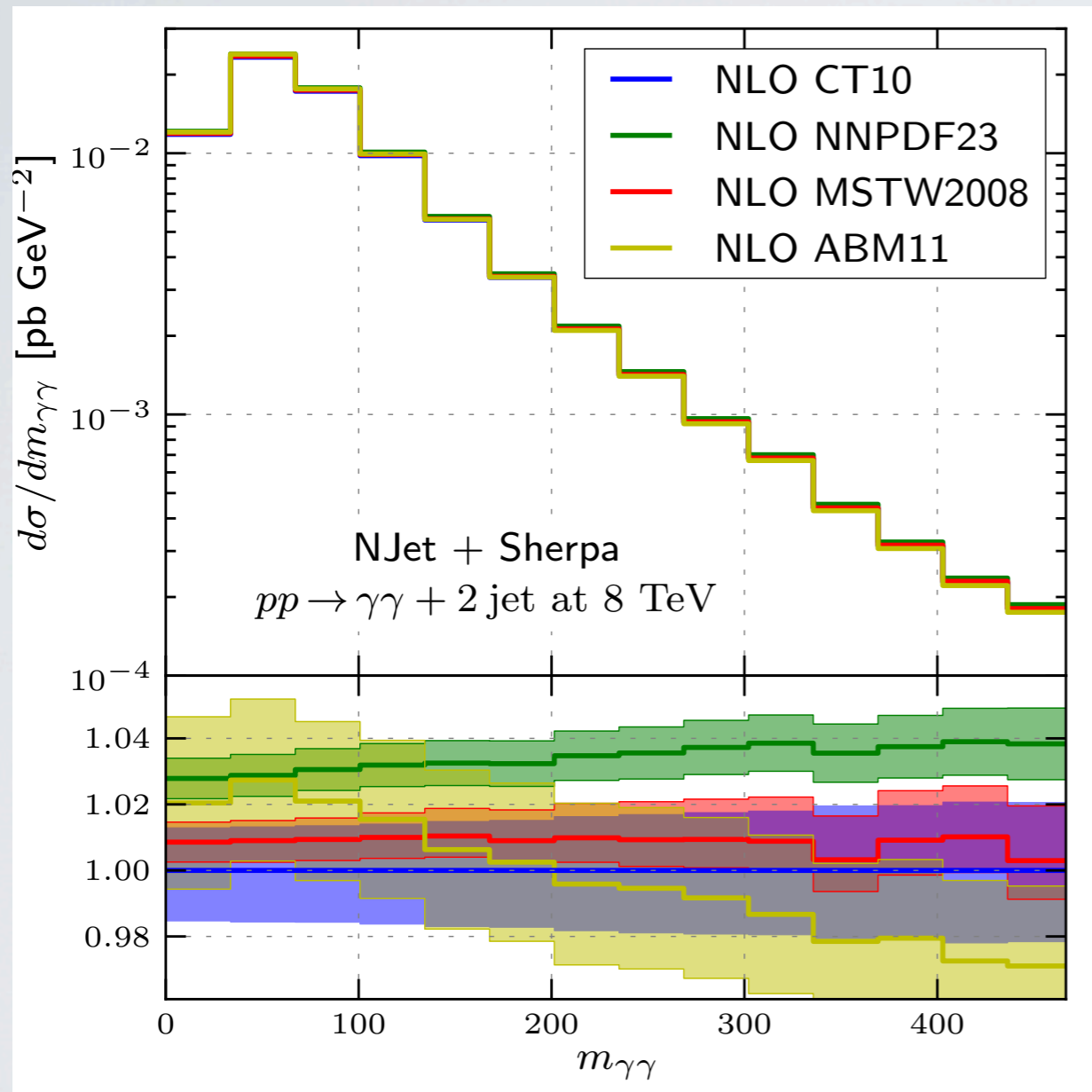
$$p_{T,j_1} > 100 \text{ GeV}$$



$$p_{T,j_1} > 30 \text{ GeV}$$



PDF dependence



Conclusions

- On-shell methods do a good job at keeping theoretical complexity of high-multiplicity amplitudes under control
- NLO computations with NJET + SHERPA <https://bitbucket.org/njet/njet/>
- First computations of NLO QCD corrections to $pp \rightarrow 5j$ and $pp \rightarrow \gamma\gamma + 3j$
- Precision QCD for 13 TeV / 14 TeV LHC runs (as well as current data)
 - matching/merging with parton shower (still hard for pure jets)