

Leptons, Loops and Dipoles in Randall-Sundrum Models

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Outline:

1. Brief intro to RS
2. General strategy
3. Calculations in 5D
4. Lepton phenomenology



Minimal RS model: Setup

Slice of AdS_5 in interval ($[1/k, 1/T]$ in conformal coordinates)

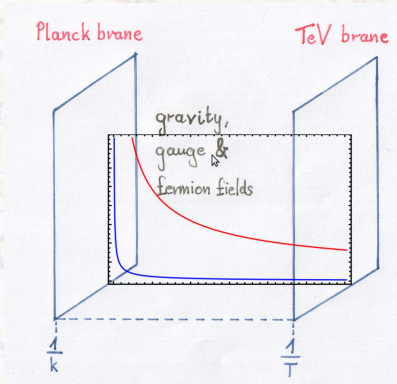
$$ds^2 = \left(\frac{1}{kz}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$M_{Pl}^{4d^2} \approx \frac{M_{Pl}^{5d^3}}{k}$$

$$\varepsilon \equiv \frac{T}{k} \approx 10^{-16} \approx \frac{1 \text{ TeV}}{M_{Pl}^{4d}}$$

proper distance between branes (= boundaries):

$$1/k \times \ln(k/T)$$



Minimal RS model: All SM fields except the Higgs in the bulk; no additional fields

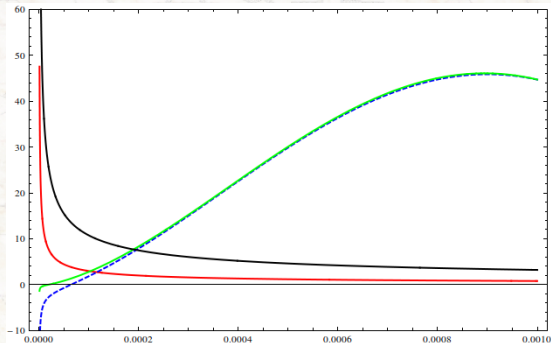
Custodially protected RS: Add $SU(2)_R$ gauge fields and increase fermion content

lowest KK resonance mass $\sim 2.5T$



Minimal RS model: Setup

- in the KK mode language



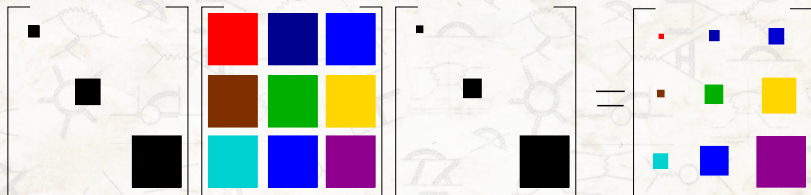
- Note that:

$$Y_{ij}^{4D} \sim f_{L_i}^{(0)}(1/T) y_{ij}^{(5D)} g_{E_i}^{(0)}(1/T)$$



Minimal RS model: Setup

- the equation in a picture



- Note that:

$$Y_{ij}^{AD} \sim f_{L_i}^{(0)}(1/T) y_{ij}^{(5D)} g_{E_i}^{(0)}(1/T)$$



Boundary Conditions and Spectrum

- Boundary conditions (\equiv parities under S^1/Z_2 orbifold Z_2 symmetries)

$$\partial_z A_\mu|_{z=1/T, 1/k} = 0$$

$$A_5|_{z=1/T, 1/k} = 0$$

$$L_R|_{z=1/T, 1/k} = 0$$

$$E_L|_{z=1/T, 1/k} = 0$$

Only L_L, E_R have zero modes ($m_0 = 0$). KK excitations describe massive gauge bosons and vector-like fermions.

- 5D gauge fixed such that A_μ and A_5 decouple $\rightarrow \xi$
- wrong-chirality Higgs couplings (WCHC)

$$\int d^4x [(\bar{L}\Phi)E + h.c.]|_{z=1/T} = \int d^4x [(\bar{L}_L\Phi)E_R + (\bar{L}_R\Phi)E_L + h.c.]|_{z=1/T}$$

The WCHC $(\bar{L}_R\Phi)E_L$ vanishes for a brane-localized Higgs due to the boundary condition. Too naive!



RS & Lepton-flavour observables

- Exhaustive phenomenology of tree-level processes (electroweak, flavour)
from gluon FCNCs KK gluon mass ~ 20 TeV w/o extra flavour structure [e.g. Csaki, Falkowski, Weiler, 2008]
- Higgs production in gluon-gluon fusion [Casagrande et al., 2010; Azatov et al., 2010; Carena et al., 2012; Malm et al., 2013, Archer et al. 2014]
- quark FCNCs: $b \rightarrow s\gamma$, $b \rightarrow sg$ [Gedelia, Isidori, Perez 2009; Blanke et al., 2012], and $c \rightarrow ug$ [Delaunay et al., 2012]
- lepton observables
 - ▶ $a_\mu - a^{SM} = 239(63)(48) \times 10^{-11}$
 - ▶ $\text{Br}(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13} (10^{-14})$ MEG (upgraded)
 - ▶ $\text{Br}(\mu \rightarrow 3e) < 1.0 \times 10^{-12}$ SINDRUM
 - ▶ $\text{Br}^{\text{Au(Al)}}(\mu N \rightarrow eN) < 7 \times 10^{-13} (10^{-16})$ SINDRUM II (DeeMe, Mu2E, COMET)



Lepton Flavour Observables

- $g - 2$
pure loop-induced effect in RS; flavour diagonal; [Davoudiasl et al. 2000]
- d_e
pure loop-induced effect in RS; flavour diagonal, CP violating;
- $\mu \rightarrow e\gamma$
pure loop-induced effect in RS; LFV [Agashe et al., 2006, Csaki et al., 2010]
- $\mu \rightarrow 3e$
tree-dominated effect (?) [Agashe et al., 2006, Grojean et al., 2003; Csaki et al., 2010]
- $\mu \rightarrow e$ in Au
tree-dominated effect (?); provides “orthogonal” information [Agashe et al., 2006; Chang & Ng, 2005; Csaki et al., 2010]

This talk:

- a general strategy
- How do you calculate loops in 5D?



Lagrangian (minimal model)

$$\begin{aligned} S_{5D} = & \int d^4x \int_{1/k}^{1/T} dz \sqrt{G} \left\{ -\frac{1}{4} F^{MN} F_{MN} - \frac{1}{4} W^{a,MN} W_{MN}^a \right. \\ & + \sum_{\psi=E,L} \left(e_m^M \left[\frac{i}{2} \bar{\psi}_i \Gamma^m (D_M - \overleftarrow{D}_M) \psi_i \right] - M_{\psi_i} \bar{\psi}_i \psi_i \right) \left. \right\} + S_{\text{GF+ghost}} \\ & + \int d^4x \left\{ (D^\mu \Phi)^\dagger D_\mu \Phi - V(\Phi) - \left(\frac{T}{k} \right)^3 \left[y_{ij}^{(5D)} (\bar{L}_i \Phi) E_j + \text{h.c.} \right] \right\} \end{aligned}$$

$$\Gamma^m = (\gamma^\mu, i\gamma_5)$$

$$e_m^M = (kz) \delta_m^M$$

$$M_{\psi_i} = c_{L/E_i} \cdot k$$

Two scales:

k determines the size of all parameters

T put in by hand to address the quantum corrections to the Higgs mass



Strategy

- distinct scale hierarchy $k \gg T \gg v \gg m_\ell$
 $\underbrace{\hspace{1.5cm}}_{UV} \quad \underbrace{\hspace{1.5cm}}_{IR}$

matching scale μ

- strategy:

1. Step (in symmetric phase \rightarrow no vev):

integrate out the “bulk” \rightarrow match onto an $SU(2)_L \times U(1)_Y$ symmetric effective theory

$$\mathcal{L}_{RS} \rightarrow \mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{T^2} \sum_i C_i \mathcal{O}_i \quad \text{[Buchmüller & Wyler]}$$

2. Step: change into the “broken” phase

3. Step: compute observables in using 4d effective theory



1. Step: EFT before EWSB

- distinct scale hierarchy $k \gg T \gg v \gg m_\ell$
 $\underbrace{\hspace{1.5cm}}_{UV} \gg \underbrace{\hspace{1.5cm}}_{IR}$
- strategy: integrate out the “bulk” by matching onto an $SU(3) \times SU(2) \times U(1)_Y$ invariant Lagrangian at a scale $T \gg \mu \gg v$ in the unbroken theory:

$$\mathcal{L}_{RS} \rightarrow \mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{T^2} \sum_i C_i \mathcal{O}_i$$

- relevant operators include

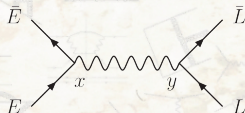
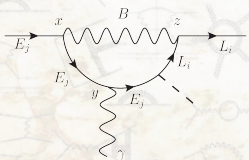
$$\begin{aligned} \sum_i C_i \mathcal{O}_i \supset & a_{B,ij} \bar{L}_i \Phi \sigma_{\mu\nu} E_j B^{\mu\nu} + a_{W,ij} \bar{L}_i \tau^a \Phi \sigma_{\mu\nu} E_j W^{a,\mu\nu} + \text{h.c.} \\ & + c_{1,i} (\bar{E}_i \gamma_\mu E_i) (\Phi^\dagger i D^\mu \Phi) + c_{2,i} (\bar{L}_i \gamma_\mu L_i) (\Phi^\dagger i D^\mu \Phi) \\ & + c_{3,i} (\bar{L}_i \gamma^\mu \tau^a L_i) (\Phi^\dagger \overleftrightarrow{i\tau^a} D_\mu \Phi) \\ & + b_{1,ijkl} (\bar{L}_i \gamma_\mu L_i) (\bar{L}_k \gamma^\mu L_l) + b_{2,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{E}_j \gamma^\mu E_j) \\ & + b_{3,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{E}_j \gamma^\mu E_j) + \dots \\ & + b_{L\tau Q,ij} (\bar{L}_i \gamma_\mu \tau^a L_i) (\bar{Q}_j \gamma^\mu \tau^a Q_j) + b_{LQ,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{Q}_j \gamma^\mu Q_j) \\ & + b_{EQ,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{Q}_j \gamma^\mu Q_j) + b_{Lu,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{u}_j \gamma^\mu u_j) \\ & + b_{Ld,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{d}_j \gamma^\mu d_j) + b_{Eu,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{u}_j \gamma^\mu u_j) \\ & + b_{Ed,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{d}_j \gamma^\mu d_j) + \dots \end{aligned}$$



1. Step: EFT before EWSB

- relevant operators include

$$\begin{aligned}
 \sum_i C_i \mathcal{O}_i \supset & a_{B,ij} \bar{L}_i \Phi \sigma_{\mu\nu} E_j B^{\mu\nu} + a_{W,ij} \bar{L}_i \tau^a \Phi \sigma_{\mu\nu} E_j W^{a,\mu\nu} + \text{h.c.} \\
 & + c_{1,i} (\bar{E}_i \gamma_\mu E_i) (\Phi^\dagger i D^\mu \Phi) + c_{2,i} (\bar{L}_i \gamma_\mu L_i) (\Phi^\dagger i D^\mu \Phi) \\
 & + c_{3,i} (\bar{L}_i \gamma^\mu \tau^a L_i) (\Phi^\dagger \overleftrightarrow{i\tau^a D}_\mu \Phi) \\
 & + b_{1,ijkl} (\bar{L}_i \gamma_\mu L_i) (\bar{L}_k \gamma^\mu L_l) + b_{2,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{E}_j \gamma^\mu E_j) \\
 & + b_{3,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{E}_j \gamma^\mu E_j) + \dots \\
 & + b_{L\tau Q,ij} (\bar{L}_i \gamma_\mu \tau^a L_i) (\bar{Q}_j \gamma^\mu \tau^a Q_j) + b_{LQ,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{Q}_j \gamma^\mu Q_j) \\
 & + b_{EQ,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{Q}_j \gamma^\mu Q_j) + b_{Lu,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{u}_j \gamma^\mu u_j) \\
 & + b_{Ld,ij} (\bar{L}_i \gamma_\mu L_i) (\bar{d}_j \gamma^\mu d_j) + b_{Eu,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{u}_j \gamma^\mu u_j) \\
 & + b_{Ed,ij} (\bar{E}_i \gamma_\mu E_i) (\bar{d}_j \gamma^\mu d_j) + \dots
 \end{aligned}$$



2. Step: After EWSB

Changing to the 'broken' phase

$$\Phi \rightarrow \left(\begin{array}{c} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + iG) \end{array} \right) \quad E_i \rightarrow V_{ij} P_R \psi_j, \quad L_i \rightarrow U_{ij} P_L \left(\begin{array}{c} \nu_j \\ \psi_j \end{array} \right)$$

gives

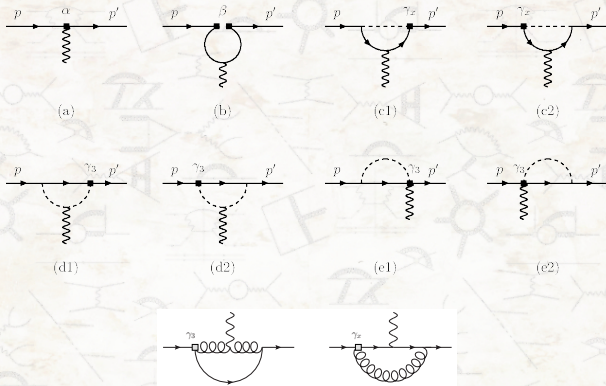
$$\begin{aligned} \sum_i C_i \mathcal{O}_i \rightarrow & \frac{\alpha_{ij} + \alpha_{ij}^*}{2} \frac{v}{\sqrt{2}} \bar{\psi}_i \sigma_{\mu\nu} \psi_j F^{\mu\nu} + \frac{\alpha_{ij} - \alpha_{ij}^*}{2i} \frac{v}{\sqrt{2}} \bar{\psi}_i \sigma_{\mu\nu} i\gamma_5 \psi_j F^{\mu\nu} \\ & + \beta_{ijkl} (\bar{\psi}_i \gamma^\mu P_L \psi_j) (\bar{\psi}_k \gamma_\mu P_R \psi_l) \\ & + \gamma_{1,ij} \frac{v}{2} (\bar{\psi}_i P_L \gamma_\mu \psi_j) (i\partial^\mu H) + [\gamma_{2,ij} + \gamma_{3,ij}] \frac{v}{2} (\bar{\psi}_i P_R \gamma_\mu \psi_j) (i\partial^\mu H) \\ & + \gamma_{3,ij} \frac{v}{\sqrt{2}} (\bar{\psi}_i P_R \gamma^\mu \nu_j) (-i\partial_\mu \phi^-) + \gamma_{3,ij} \frac{v}{\sqrt{2}} (\bar{\psi}_i P_R \gamma^\mu \nu_j) (eA_\mu \phi^-) \\ & + \text{h.c. of previous line} + \dots \end{aligned}$$

the Greek Wilson coefficients are Latin ones dressed with flavour rotation matrices



3.Step: Compute Observables

Example: $g - 2$ & $\mu \rightarrow e\gamma$ correspond to the flavour conserving and violating part of



UV and IR divergences require regularisation

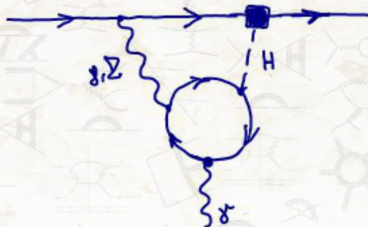
BUT finite due to $\frac{1}{\epsilon} \times \epsilon$

Scheme dependent \rightarrow dependence must cancel with the dependence of the 5D loop in α



3.Step: Compute Observables

Example: $g - 2$ & $\mu \rightarrow e\gamma$ also receive enhanced contributions at two loops



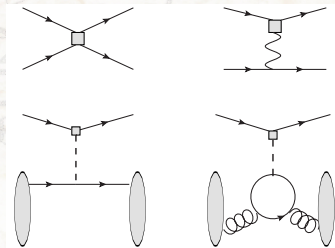
- Barr-Zee type contributions
- arise from FCNC Higgs couplings (effect on $g - 2$ negligible)
- effect on e.g. $\mu \rightarrow e\gamma$ studied

Chang, Hou, Keung '93



3.Step: Matching from effective theory onto effective theory

Example: $\mu \rightarrow e$ & $\mu \rightarrow 3e$

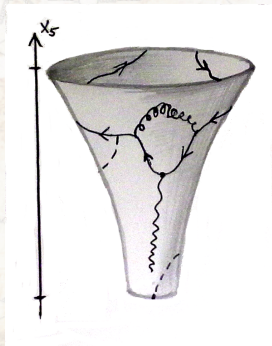


- computation straightforward
- must include insertions of the flavour-changing dipole that also mediates $\mu \rightarrow e\gamma$ (same order in the $1/T$ counting)

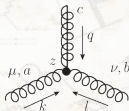


So all we need is ...

Wilson Coefficients



- treat RS as (non-renormalisable) QFT in 5D
derive Feynman rules for 5D theory [Randall & Schwartz 2001]
for the moment we do not consider metric fluctuations
- vertices are simple, e.g.



$$-i \frac{1}{kz} g_5 \epsilon^{abc} \eta_{\mu\nu} (\partial_z|_{\text{on 'a'}} - \partial_z|_{\text{on 'b'}})$$

- propagators are only treated in Fourier space in the flat directions
→ mixed representation
→ each vertex is accompanied by an integral over the fifth dimension



5D Formalism

- work in a 5D QFT
no KK sums; vertices and propagators are five dimensional [Randall, Schwartz, 2001]
- zero-mode (\sim SM fields) must be separated explicitly

$$f_L^{(0)}(z) = \sqrt{\frac{1 - 2c_L}{1 - \epsilon^{1-2c_L}}} \sqrt{T}(kz)^2 (Tz)^{-c_L} \quad g_E^{(0)}(z) = \sqrt{\frac{1 + 2c_E}{1 - \epsilon^{1+2c_E}}} \sqrt{T}(kz)^2 (Tz)^{c_E}$$

- use mixed coordinate-momentum representation for propagators in the unbroken theory [Randall, Schwartz, 2001]
→ propagators depend on 4D momentum and start/end coordinate in the fifth dimension, e.g.

conventions follow Csaki 2010 v1-v6

$$\left[\frac{1}{kz} \right]^4 \mathcal{D} \Delta_L(p, z, z') = i\delta(z - z') 1 \quad \mathcal{D} = \not{p} + i\Gamma^5 \left(\partial_z - \frac{2}{z} \right) - \frac{c_L}{z}$$

$$\Delta_L(p, z, z') = - \underbrace{P_L F_L^+(p, z, z') \not{p} P_R - P_R F_L^-(p, z, z') \not{p} P_L}_{\text{contains to-be SM field}} + \underbrace{P_L d^+ F_L^-(p, z, z') P_L + P_R d^- F_L^+(p, z, z') P_L}_{\sim \text{mass term}}$$



5D Formalism

- exact solution in the unbroken phase

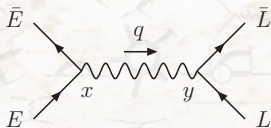
$$F_L^+(p, x, y) = \Theta(x - y) \frac{ik^4 x^{5/2} y^{5/2} \tilde{S}_+(p, x, 1/T, c_L) \tilde{S}_+(p, y, 1/k, c_L)}{S_-(p, 1/T, 1/k, c_L)} \\ + \Theta(y - x) \frac{ik^4 x^{5/2} y^{5/2} \tilde{S}_+(p, y, 1/T, c_L) \tilde{S}_+(p, x, 1/k, c_L)}{S_-(p, 1/T, 1/k, c_L)}$$

$$S_{\pm}(p, x, y, c) = I_{c\pm 1/2}(px) K_{c\pm 1/2}(py) - K_{c\pm 1/2}(px) I_{c\pm 1/2}(py) \\ \tilde{S}_{\pm}(p, x, y, c) = I_{c\pm 1/2}(px) K_{c\mp 1/2}(py) + K_{c\pm 1/2}(px) I_{c\mp 1/2}(py)$$

- similar expressions for the different boson propagators



Tree-level coefficients are 'for free'



$$b_{ij} = -i(-g'_5)^2 \frac{Y_L Y_E}{4} T^2 \int_{1/k}^{1/T} dx dy \frac{f_{L_i}^{(0)2}(y)}{(ky)^4} \frac{g_{E_j}^{(0)2}(x)}{(kx)^4} \Delta_{\perp}(q=0, x, y)$$

the hypercharge boson zero-momentum propagator is

$$\Delta_{\perp}(q, x, y) \stackrel{q \rightarrow 0}{=} \Theta(x-y) \frac{ik}{\ln \frac{k}{T}} \left(-\frac{1}{q^2} + \frac{1}{4} \left\{ \frac{1/T^2 - 1/k^2}{\ln \frac{k}{T}} - x^2 - y^2 + 2x^2 \ln(xT) \right. \right. \\ \left. \left. + 2y^2 \ln(yT) + 2y^2 \ln \frac{k}{T} \right\} + \mathcal{O}(q^2) \right) + (x \leftrightarrow y),$$

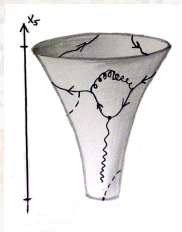
all integrals are elementary

very similar to computation of $\Delta F = 2$ tree-level processes

\leftrightarrow agrees with KK sum calculation [Casagrande et al. 2008]



So all we need is ...

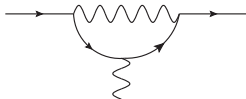


the dipole operator(s) $\cos \Theta_W a^B - \sin \Theta_W a^W$

$$\mathcal{L} \supset a^B \bar{L} \Phi \sigma_{\mu\nu} B^{\mu\nu} E + a^W \bar{L} \Phi \tau^A \sigma_{\mu\nu} W_A^{\mu\nu} E$$

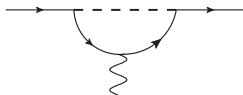
- actual 5D loop with different particle species in the loop
- two (3?) different diagram classes

GAUGE BOSON



one Yukawa interaction
to contribute to $\bar{L} \Phi \sigma \cdot F E$

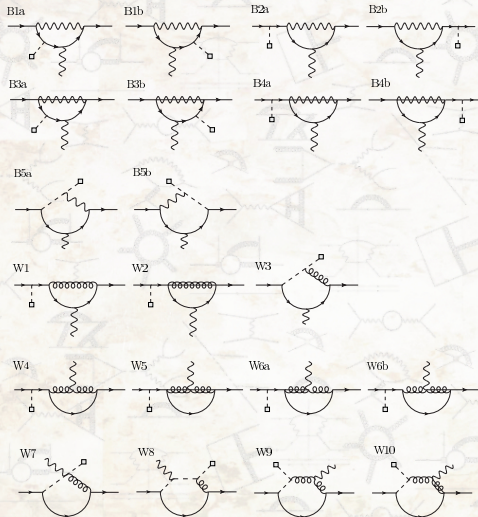
HIGGS BOSON



three Yukawa interactions
to contribute to $\bar{L} \Phi \sigma \cdot F E$



Dipole operator matching—gauge part



- in external line put Φ in vev, and take the superposition of B and W^3 corresponding to the photon

← diagrams in minimal model

- in the custodial model there are 24 non-abelian diagrams and 23 abelian diagrams in 15 distinct topologies

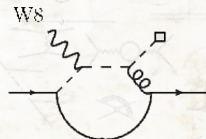
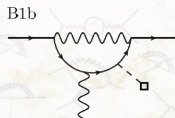
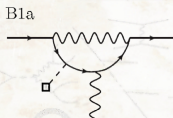


gauge part – extraction the short-distance contribution

There are three different contributions from each diagram

↪ identify à la expansion by regions **Benke, Smirnov '97**

- (1) the SM corrections; all 5D propagators propagate the zero mode. The loop integral does not contain the short-distance scales T, k explicitly, and is **purely** long-distance
needs to be eliminated by subtraction of the zero-mode from **one** gauge boson propagator
- (2) contributions from momentum scales $l \ll T$, at least one of the 5D propagators propagates a KK mode. One-loop long-distance matrix element of tree insertion of four-fermion and fermion-Higgs operator.



- (3) The short-distance contribution: loop momentum of order $l \sim T$ (or k)

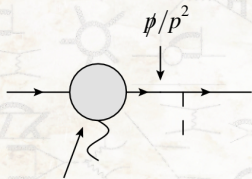
expand in external fermion momenta p, p' to linear/quadratic order eliminates region since the denominator no longer contains long-distance scales



gauge part – remarks

off-shell terms

- non-1PR diagrams are necessary!
- obvious if external leg propagates a massive particle → **short distance effect**
- but even if the particle is on-shell (mass-less mode) the diagram is relevant
- known from B physics → matching of power-suppressed heavy-to-light SCET currents



$$\Lambda^\mu = \Lambda^\mu + \not{p}\Lambda_1^{\text{off},\mu} + \Lambda_2^{\text{off},\mu}\not{p}'$$

- after cancelling the propagator the off-shell vertex-function is part of the short-distance coefficient

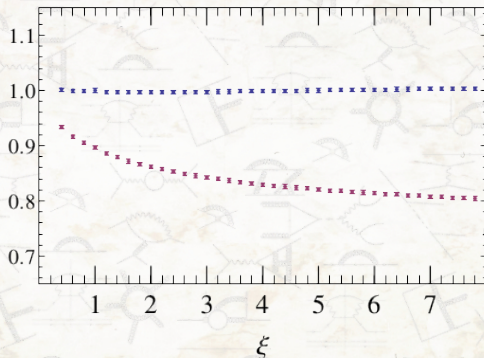


gauge part – remarks

gauge invariance

- calculation done for general 5d gauge parameter ξ
- one can show analytically that the set of 21 1-loop diagrams is gauge invariant (three gauge invariant subsets)

Beneke, Moch, JR 2014



- other checks: scheme independence (naive γ_5 vs general scheme), all integrals can be solved analytically in certain limits (e.g. large loop-momentum)



gauge part – remarks

anapole moment

Most general $U(1)_{em}$ invariant vertex function for on-shell fermions:

$$\Gamma^\mu(p, p') = ieQ_\mu \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m_\ell} q_\nu F_2(q^2) + \frac{i\sigma^{\mu\nu}}{2m_\ell} q_\nu \gamma_5 F_3(q^2) + (q^2 \gamma^\mu - \not{q} q^\mu) \gamma^5 F_4(q^2) \right] u(p, s)$$

- F_1 — charge form factor;
- $a_\mu = (g - 2)_\mu / 2 = F_2(0)$
- F_3 — EDM form factor
- F_4 — anapole moment, $SU(2) \times U(1)_Y$ gauge dependent **Musolf, Holstein, 1991**
 $\not{q} q^\mu \gamma_5 \rightarrow 2m_\mu q^\mu \gamma_5$

The $L[\dots]EF^{\mu\nu}$ vertex contains both $(p + p')^\mu P_R$ and $(p - p')^\mu P_R$. The latter is associated with the anapole moment.

consistency check (analytical even in RS) coefficient of $(p - p')^\mu P_R$ must be proportional to m_ℓ .



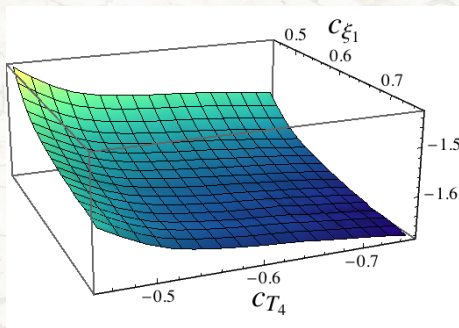
gauge part – result

- general structure of the gauge diagram contribution with full flavour dependence

$$a_{ij}^{\text{gauge}} = ieQ_\mu \frac{\alpha_{em}}{4\pi} \frac{1}{T^2} \times \text{Loop}(c_{L_i}, c_{E_j}) \cdot \log \frac{k}{T} \cdot \underbrace{f_{L_i}^{(0)}(1/T) Y_{ij} g_{E_j}^{(0)}(1/T)}_{=M_{ij}} \frac{T^3}{k^4}$$

- $\text{Loop}(c_{L_i}, c_{E_j})$ (here for csRS, RS min is even less sensitive)

Moch, JR 2014



→ expect contribution to $g - 2$ to be independent of the Yukawa structure & FCNCs, are suppressed

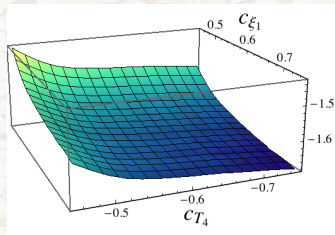


Phenomenology and Results (gauge part)

- a_{ij}^{gauge} relative to the mass matrix

$$a_{ij}^{\text{gauge}} = \text{Const} \times \text{Loop}(c_{L_i}, c_{E_j}) \cdot M_{ij}$$

↪ suppressed FCNCs

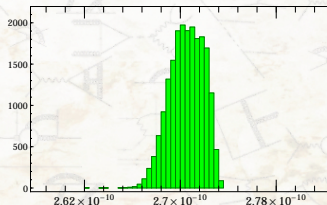


- $g_\mu - 2$

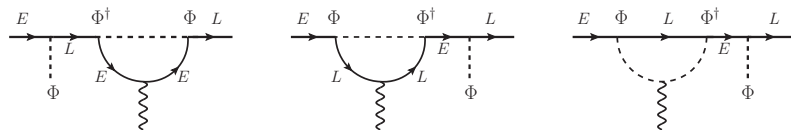
$$\Delta a_\mu^{\text{gauge}} = 27.2(8.8) \cdot 10^{-11} \frac{1 \text{ TeV}^2}{T^2}$$

compared to

$$a_\mu^{\text{exp}} - a_\mu^{\text{the}} = 287(63)(49) \times 10^{-11}$$



Dipole operator matching– Higgs part



- subtlety: a brane localised Higgs should be described as having a delta-function-like profile

$$\delta(z - 1/T) = \lim_{\delta \rightarrow 0} \frac{T}{\delta} \Theta\left(z - \frac{1 - \delta}{T}\right).$$

→ limit of a distribution of width δ/T for $\delta \rightarrow 0$

→ introduces an additional large scale $\frac{T}{\delta}$ for any finite value

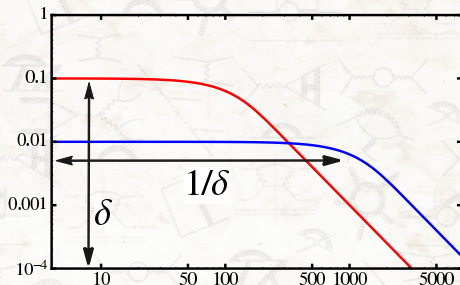
- BUT we also have a dimensional regulator $\rightarrow \epsilon \rightarrow 0$ before or after $\delta \rightarrow 0$?



Dipole operator matching– Higgs part

- answer both options are valid part of the RS model
- the order of limits determines if the Higgs can be resolved or not

Beneke, Moch, JR & Malm et al.



- simple answer (minimal RS):

$$\begin{aligned} \Delta a_{ij} = & (iQ_\mu e) \times \frac{1}{6} \times \frac{1}{16\pi^2} \frac{1}{T^2} \times f_{L_i}^{(0)}(1/T) [YY^\dagger Y]_{ij} g_{E_j}^{(0)}(1/T) \frac{T^3}{k^4} \\ & + (iQ_\mu e) \frac{1}{192\pi^2} \times f_{L_i}^{(0)}(1/T) Y_{ik} iF_{E_k}^-(0, 1/T, 1/T) Y_{kh}^\dagger f_{L_h}^{(0)2}(1/T) Y_{hj} g_{E_j}^{(0)}(1/T) \\ & + (iQ_\mu e) \frac{1}{192\pi^2} \times f_{L_i}^{(0)}(1/T) Y_{ik} g_{E_h}^{(0)2}(1/T) Y_{kh}^\dagger iF_{L_h}^+(0, 1/T, 1/T) Y_{hj} g_{E_j}^{(0)}(1/T) \end{aligned}$$

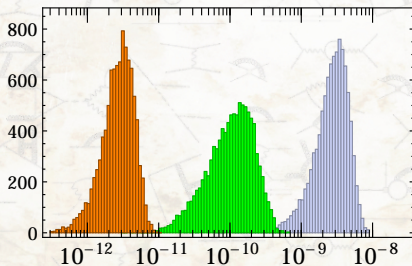


Phenomenology— $g - 2$

- gauge diagram contribution is independent of all model parameters (only overall scales T, k matter)

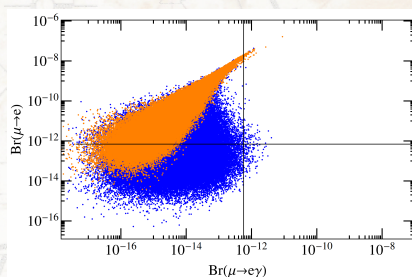
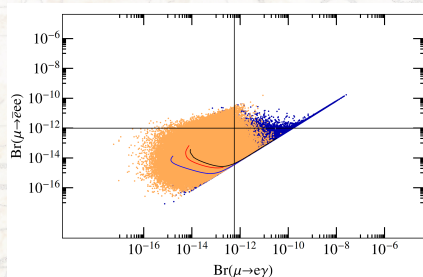
$$\Delta a_\mu \approx 27 \times 10^{-11} \frac{1 \text{ TeV}^2}{T^2}$$

- Higgs contribution depends on the Yukawa size
→ smaller than gauge contribution for Yukawas < 1
→ but can reach $1 \cdot 10^{-9}$ for $Y^> = 3$.



Phenomenology—minimal model

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ ($T = 8$ TeV)

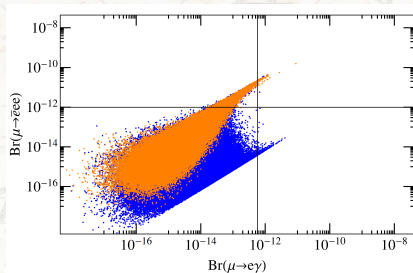
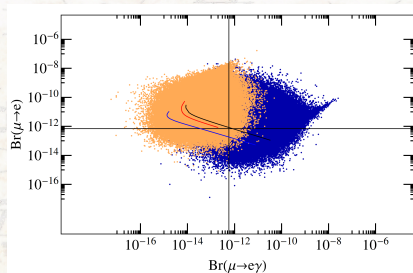


- dipole boundary ($\mu \rightarrow 3e$ dominated by dipole operate)
- in general both tree-level operators (Higgs-fermion and four-fermion) and 5D dipoles are important (esp. in the more natural bulk Higgs case tree-approximation [used in all RS analyses up to now] does not work)
- gauge-contributions prevent reducing $\mu \rightarrow e\gamma$ by just changing the Yukawa magnitude



Phenomenology—minimal model

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ ($T = 8$ TeV)

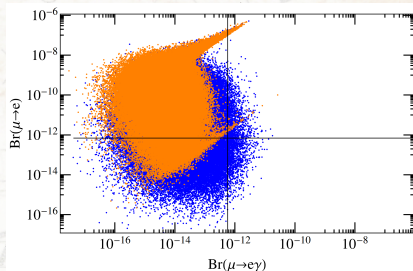
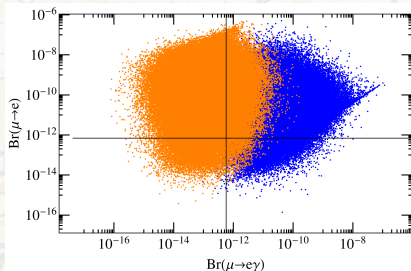


- essentially no correlations \rightarrow genuinely “orthogonal” constraints



Phenomenology—custodial model

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion ($T = 8$ TeV)



- Yukawa size cannot help to escape bounds
- next generation experiments can rule out the parameters space $m_{gluon}^{(1)} < 20$ TeV
- one can invent flavour symmetries to avoid bounds, but it is quite hard to avoid all bounds (different operators contribute differently)

Agashe et al. 2006



Comparisons

- Davoudiasl, Hewett, Rizzo [hep-ph/9911262]
 - + KK sums
 - + dropped external insertion and half the internal ones
 - + Higgs diagrams appear when they should not
 - + subset of abelian diagrams remains
 - + similar order of magnitude but negative sign
- Csaki, Grossman, Tanedo, Tsai [hep-ph/1004.2037]
 - + 5D calculation but no matching onto the effective lagrangian
 - + depending on version without external Higgs insertions
 - + fixed gauge
 - + extracts dominant contributions



Comparisons

- K. Agashe, A. E. Blechman and F. Petriello, [hep-ph/0606021]
 - + studies all LFV muon observables
 - + only Higgs loop-diagrams
 - + consider radiative transition for bulk Higgs
 - + do not include loop contributions to $\mu \rightarrow e$, $\mu \rightarrow 3e$
 - + general conclusions ✓
 - + first to find that Yukawa size cannot be used to evade bounds
 - ▶ first to mention wrong chirality couplings



Summary

- complete computation of the leptonic dimension-six Wilson coefficients including the dipole operators without approximations
- without imposing very specific flavour structures the new experiments will rule out KK modes in excess of 20 TeV
- 'gauge' corrections to $g - 2$ is model independent (like S and T parameter) but too small to help
- Higgs contribution is doubly model-dependent
- unmentioned: dimension-eight contributions, gravity, electric dipole moments, ...

