Standard Model of Particle Physics

Chris Sachrajda

School of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ
UK

SUSSP61, St Andrews
August 8th – 23rd 2006

Contents

I start however by:

- Summarising the main points of Lecture 3.
- Discussing the last four slides of lecture 2.

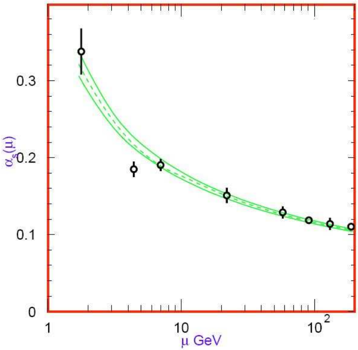
Main Points of Lecture 3

- *Asymptotic Freedom* $\Rightarrow g(\mu)$ decreases (logarithmically) with μ.

Main Points of Lecture 3

- *Asymptotic Freedom* $\Rightarrow g(\mu)$ decreases (logarithmically) with μ.
- If there is a single (hard) scale Q in the process, it is convenient to take $\mu \simeq Q$ and perform perturbation theory in $\alpha_s(Q) = g^2(Q)/(4\pi)$.

The Running Coupling Constant



$\alpha_s(\mu)$ at values of μ where they are measured, (τ-width, Υ-decays, Deep Inelastic Scattering, e^+e^- Event Shapes at 22 and 59 GeV, Z–Width, e^+e^- Event Shapes at 135 and 189 GeV (PDG(2005)).

PDG(2005) result:

$$\alpha_s(M_Z) = 0.1176 \pm 0.002\,.$$

Main Points of Lecture 3

- *Asymptotic Freedom* $\Rightarrow g(\mu)$ decreases (logarithmically) with μ.
- If there is a single (hard) scale Q in the process, it is convenient to take $\mu \simeq Q$ and perform perturbation theory in $\alpha_s(Q) = g^2(Q)/(4\pi)$.

Main Points of Lecture 3

- *Asymptotic Freedom* $\Rightarrow g(\mu)$ decreases (logarithmically) with μ.
- If there is a single (hard) scale Q in the process, it is convenient to take $\mu \simeq Q$ and perform perturbation theory in $\alpha_s(Q) = g^2(Q)/(4\pi)$.
- In general there are contributions from *long-distance* regions of phase-space $\Rightarrow$ there is a danger that we cannot use perturbation theory.

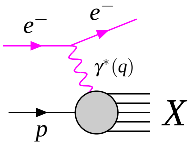
Main Points of Lecture 3

- *Asymptotic Freedom* $\Rightarrow g(\mu)$ decreases (logarithmically) with μ.
- If there is a single (hard) scale Q in the process, it is convenient to take $\mu \simeq Q$ and perform perturbation theory in $\alpha_s(Q) = g^2(Q)/(4\pi)$.
- In general there are contributions from *long-distance* regions of phase-space $\Rightarrow$ there is a danger that we cannot use perturbation theory.
- For sufficiently inclusive processes the Bloch-Nordsieck and Kinoshita-Lee-Naumberg Theorems (generalized to QCD) $\Rightarrow$ these long-distance contributions cancel and we can use perturbation theory, e.g. for $e^+e^- \to$ hadrons

$$\sigma = \sigma_0 \left(3\sum_f Q_f^2\right)\left(1 + \frac{\alpha_s(Q)}{\pi} + 1.411\,\frac{\alpha_s^2(Q)}{(\pi)^2} - 12.8\,\frac{\alpha_s^3(Q)}{(\pi)^3} + \cdots\right)$$

where I have neglected the contribution from the Z_0 intermediate state.

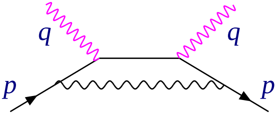
Main Points of Lecture 3 - Cont.



- In Deep Inelastic Scattering (DIS), the kinematical variable $x = -q^2/(2p \cdot q)$ has the physical interpretation of being the fraction of the hadron's momentum carried by the struck quark.
- DIS yields information about the momentum distribution of partons in the target hadron, i.e. the parton distribution functions $f_f(x, q^2)$.

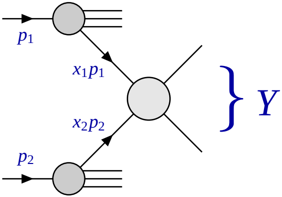
Main Points of Lecture 3 - Cont.

- In DIS the long-distance contributions do not cancel (no sum over degenerate initial states).



- We therefore cannot calculate the parton distribution functions $f_f(x,q^2)$ in perturbation theory.
- We can however, calculate the behaviour with q^2 (at large q^2) - Scaling Violations.

Main Points of Lecture 3 - Cont.



- In hadron-hadron hard-scattering collisions, such as $h_1 + h_2 \to Y + X$, where for example, Y can be a heavy particle (resonance, Higgs, i.e. Drell-Yan Processes) or two (or more) jets at large transverse momentum.

$$\sigma(h_1(p_1) + h_2(p_2) \to Y + X) \quad = $$
$$\int_0^1 dx_1 \int_0^1 dx_2 \sum_{f_1,f_2} f_{f_1}(x_1, Q^2) f_{f_2}(x_2, Q^2)\, \sigma(f_1 + f_2 \to Y + \text{anything})\,.$$

The Discrete Symmetries P, C and CP

Parity

$$(\vec{x}, t) \to (-\vec{x}, t).$$

The vector and axial-vector fields transform as:

$$V_\mu(\vec{x}, t) \to V^\mu(-\vec{x}, t) \quad \text{and} \quad A_\mu(\vec{x}, t) \to -A^\mu(-\vec{x}, t).$$

- The vector and axial-vector currents transform similarly.

Left-handed components of fermions $\psi_L = (\frac{1}{2}(1-\gamma^5)\psi)$ transform into right-handed ones $\psi_R = (\frac{1}{2}(1+\gamma^5)\psi)$, and vice-versa.

- Since CC weak interactions in the SM only involve the left-handed components, parity is not a good symmetry of the weak force.
- QCD and QED are invariant under parity transformations.

Charge Conjugation – Charge conjugation is a transformation which relates each complex field ϕ with $\phi^\dagger$.

Under C the currents transform as follows:

$$\bar{\psi}_1 \gamma_\mu \psi_2 \to -\bar{\psi}_2 \gamma_\mu \psi_1 \quad \text{and} \quad \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \to \bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1,$$

where ψ_i represents a spinor field of type (flavour or lepton species) i.

CP – Under the combined CP-transformation, the currents transform as:

$$\bar{\psi}_1 \gamma_\mu \psi_2 \to -\bar{\psi}_2 \gamma^\mu \psi_1 \quad \text{and} \quad \bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 \to -\bar{\psi}_2 \gamma^\mu \gamma_5 \psi_1.$$

The fields on the left (right) hand side are evaluated at $(\vec{x},t)$ ($(-\vec{x},t)$).

CP Cont.

Consider now a charged current interaction:

$$(W^1_\mu - iW^2_\mu)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V_{ij}D^j + (W^1_\mu + iW^2_\mu)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V^*_{ij}U^i,$$

U^i and D^j are up and down type quarks of flavours i and j respectively.

Under a CP transformation, the interaction term transforms to:

$$(W^1_\mu + iW^2_\mu)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V_{ij}U^i + (W^1_\mu - iW^2_\mu)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V^*_{ij}D^j$$

- CP-invariance requires V to be real
  (or more strictly that any phases must be able to be absorbed into the definition of the quark fields).
- For CP-violation in the quark sector we therefore require 3 generations.

Higgs Mass and Interactions

Imagine that the Higgs potential is

$$V = -\mu^2\,(\phi^\dagger\phi) + \lambda\,(\phi^\dagger\phi)^2 \quad \text{and write} \quad \phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix} \quad \text{where} \quad v^2 = \frac{\mu^2}{\lambda}\,.$$

In terms of $h(x)$:

$$V = \mu^2\,h^2 + \sqrt{\lambda}\mu\,h^3 + \frac{\lambda}{4}\,h^4\,.$$

- We know $v = \mu/\sqrt{\lambda} = 250\,$GeV from M_W and other quantities.
- The mass of the Higgs is $\sqrt{2}\mu$. Today, we have no direct way of knowing this.
- The larger that m_h is, the stronger are the Higgs self interactions.
- Finally, I stress that even if the overall picture is correct, the Higgs sector may be more complicated than the simplest picture presented here.

Lecture 4 — Flavourdynamics and Non-Perturbative QCD

1. Introduction
2. Leptonic Decays
3. Introduction to Lattice Phenomenology
4. HQET (Heavy Quark Effective Theory)
5. Semileptonic Decays

The CKM Matrix

- The charged-current interactions are of the form

$$J_\mu^+ = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

- 2005 Particle Data Group summary for the magnitudes of the entries:

$$\begin{pmatrix} 0.9739 - 0.9751 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.014 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix} .$$

- The Wolfenstein parametrization is

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar\rho - i\bar\eta) & -A\lambda^2 & 1 \end{pmatrix} .$$

Exploring the Limits of the Standard Model

- A central approach to seeking evidence for physics *Beyond the Standard Model* is to over-determine $\bar{\rho}$ and $\bar{\eta}$ and check for consistency.
- For processes dominated by loop-effects (penguins, boxes etc) in particular, new BSM particles would contribute to the amplitudes
  $\Rightarrow$ SM predictions would be *wrong*
  $\Rightarrow$ inconsistencies. *T*he major difficulty in this approach is our inability to control non-perturbative QCD effects to sufficient precision.

For most quantities, the uncertainties are dominated by theory.

Exploring the Limits of the Standard Model

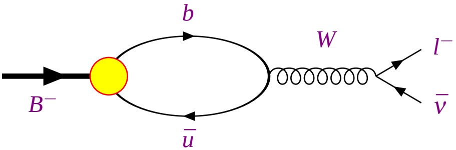
- A central approach to seeking evidence for physics *Beyond the Standard Model* is to over-determine $\bar{\rho}$ and $\bar{\eta}$ and check for consistency.
- For processes dominated by loop-effects (penguins, boxes etc) in particular, new BSM particles would contribute to the amplitudes
$\Rightarrow$ SM predictions would be *wrong*
$\Rightarrow$ inconsistencies. The major difficulty in this approach is our inability to control non-perturbative QCD effects to sufficient precision.

For most quantities, the uncertainties are dominated by theory.

- For the remainder of these lectures I will illustrate the determination of the CKM matrix elements.

Leptonic Decays of Mesons

- The difficulty in making predictions for weak decays of hadrons is in controlling the non-perturbative strong interaction effects.
- As a particularly simple example consider the leptonic decays of pseudoscalar mesons in general and of the B-meson in particular.



- Non-perturbative QCD effects are contained in the matrix element

$$\langle 0|\,\bar{b}\gamma^\mu(1-\gamma^5)u\,|B(p)\rangle\ .$$

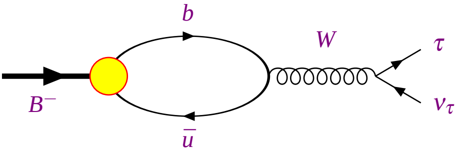
- Lorentz Inv. + Parity $\Rightarrow$ $\langle 0|\,\bar{b}\gamma^\mu u\,|B(p)\rangle = 0.$
- Similarly $\langle 0|\,\bar{b}\gamma^\mu\gamma^5 u\,|B(p)\rangle = if_B p^\mu\ .$

All QCD effects are contained in a single constant, f_B, the B-meson's *(leptonic) decay constant.* ($f_\pi \simeq 132\,$MeV)

Leptonic Decays of Mesons Cont.

2006 Result from Belle:

$$B(B \to \tau\nu_\tau) = \left(1.06^{+0.34}_{-0.28}\,(\text{stat.})^{+0.18}_{-0.16}\,(\text{syst.})\right) \times 10^{-4}\,. \qquad (\texttt{hep-ex/0604018})$$



$$B(B \to \tau\nu_\tau) = f_B^2\,|V_{ub}|^2\,\frac{G_F^2 m_B m_\tau^2}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_B\,.$$

Thus the measurement of the branching ratio gives us information about $f_B|V_{ub}| \Rightarrow$ in order to determine V_{ub} we need to know $f_B \Rightarrow$ requires non-perturbative QCD.

Introduction to Lattice Phenomenology

- Lattice phenomenology starts with the evaluation of correlation functions of the form:

  $$\langle 0|\,O(x_1,x_2,\cdots,x_n)\,|0\rangle = \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\overline{\psi}]\,e^{iS}\,O(x_1,x_2,\cdots,x_n)\ ,$$

  where $O(x_1,x_2,\cdots,x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function:

  $$Z = \int [dA_\mu]\,[d\psi]\,[d\overline{\psi}]\,e^{iS}\ .$$

- These formulae are written in Minkowski space, whereas Lattice calculations are performed in Euclidean space ($\exp(iS) \to \exp(-S)$ etc.).
- The physics which can be studied depends on the choice of the multilocal operator O.
- The functional integral is performed by discretising space-time and using Monte-Carlo Integration.

Two-Point Correlation Functions

Consider two-point correlation functions of the form:

$$C_2(t) = \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, J(\vec{x},t)\, J^\dagger(\vec{0},0)\, |0\rangle \ ,$$

where J and $J^\dagger$ are any interpolating operators for the hadron H which we wish to study and the time t is taken to be positive.

- We assume that H is the lightest hadron which can be created by $J^\dagger$.
- We take $t > 0$, but it should be remembered that lattice simulations are frequently performed on periodic lattices, so that both time-orderings contribute.

Two-Point Correlation Functions (Cont.)

$$C_2(t) = \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, J(\vec{x},t)\, J^\dagger(\vec{0},0)\, |0\rangle \;,$$

Inserting a complete set of states $\{|n\rangle\}$:

$$\begin{aligned} C_2(t) &= \sum_n \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, J(\vec{x},t)\, |n\rangle\, \langle n| J^\dagger(\vec{0},0)\, |0\rangle \\ &= \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \langle 0|\, J(\vec{x},t)\, |H\rangle\, \langle H| J^\dagger(\vec{0},0)\, |0\rangle + \cdots \end{aligned}$$

where the $\cdots$ represent contributions from heavier states with the same quantum numbers as H.

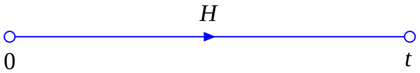
Finally using translational invariance:

$$C_2(t) = \frac{1}{2E}\, e^{-iEt} \left| \langle 0| J(\vec{0},0) |H(p)\rangle \right|^2 + \cdots \;,$$

where $E = \sqrt{m_H^2 + \vec{p}^{\,2}}$.

Two-Point Correlation Functions (Cont.)

$$C_2(t) = \frac{1}{2E}\, e^{-iEt} \left| \langle 0 | J(\vec{0},0) | H(p) \rangle \right|^2 + \cdots \,.$$



- In Euclidean space $\exp(-iEt) \to \exp(-Et)$.
- By fitting $C(t)$ to the form above, both the energy (or, if $\vec{p}=0$, the mass) and the modulus of the matrix element

  $$\left| \langle 0 | J(\vec{0},0) | H(p) \rangle \right|$$

  can be evaluated.
- Example: if $J = \bar{u}\gamma^\mu\gamma^5 d$ then the decay constant of the π-meson can be evaluated,

  $$\left| \langle 0 | \, \bar{u}\gamma^\mu\gamma^5 d \, | \pi^+(p) \rangle \right| = f_\pi \, p^\mu \ ,$$

  (the physical value of $f_\pi \simeq$ is 132 MeV).

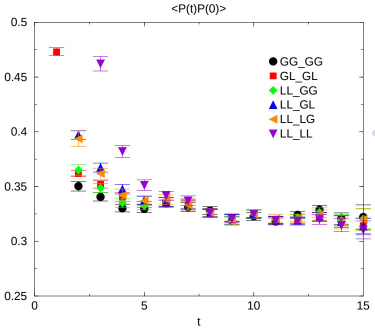
Effective Masses

At zero momentum

$$C_2(t) = \text{Constant} \times e^{-mt}$$

so that it is sensible to define the *effective mass*

$$m_{\text{eff}}(t) = \log\left(\frac{C(t)}{C(t+1)}\right) .$$



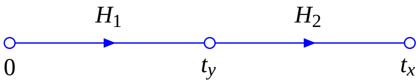
Effective Mass Plot for a Pseudoscalar Meson. UKQCD Collaboration.

Three-Point Correlation Functions

Consider now a three-point correlation function of the form:

$$C_3(t_x,t_y) = \int d^3x\,d^3y\; e^{i\vec{p}\cdot\vec{x}}\; e^{i\vec{q}\cdot\vec{y}}\; \langle 0|\, J_2(\vec{x},t_x)\, O(\vec{y},t_y)\, J_1^\dagger(\vec{0},0)\,|0\rangle\ ,$$

where $J_{1,2}$ may be interpolating operators for different particles and we assume that $t_x > t_y > 0$.

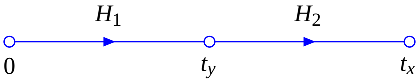


For sufficiently large times t_y and $t_x - t_y$

$$\begin{aligned} C_3(t_x,t_y) \;&\simeq\; \frac{e^{-E_1 t_y}}{2E_1}\,\frac{e^{-E_2(t_x-t_y)}}{2E_2}\,\langle 0|J_2(0)|H_2(\vec{p})\rangle \\ &\quad\times\langle H_2(\vec{p})|O(0)|H_1(\vec{p}+\vec{q})\rangle\;\langle H_1(\vec{p}+\vec{q})|J_1^\dagger(0)|0\rangle\ , \end{aligned}$$

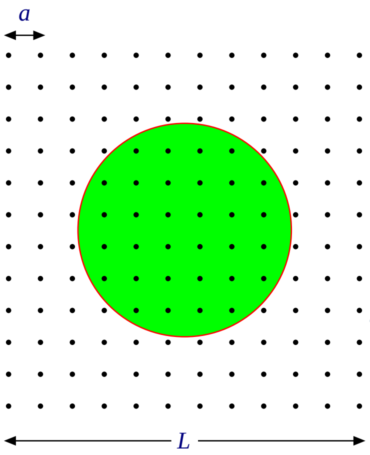
where $E_1^2 = m_1^2 + (\vec{p}+\vec{q})^2$ and $E_2^2 = m_1^2 + \vec{p}^{\,2}$.

Three-Point Correlation Functions



- From the evaluation of two-point functions we have the masses and the matrix elements of the form $|\langle 0|J|H(\vec{p})\rangle|$. Thus, from the evaluation of three-point functions we obtain matrix elements of the form $|\langle H_2|O|H_1\rangle|$.
- Important examples include:
  - $K^0 - \bar{K}^0$ ($B^0 - \bar{B}^0$) mixing. In this case
  $$O = \bar{s}\gamma^\mu(1-\gamma^5)d\,\bar{s}\gamma_\mu(1-\gamma^5)d\,.$$
  - Semileptonic and rare radiative decays of hadrons of the form $B \to \pi, \rho$ + leptons or $B \to K^*\gamma$. Now O is a quark bilinear operator such as $\bar{b}\gamma^\mu(1-\gamma^5)u$ or an *electroweak penguin* operator.

Systematic Uncertainties



We would like

$$L \gg 1\,\mathrm{fm} \quad \text{and} \quad a^{-1} \gg \Lambda_{\mathrm{QCD}}\,.$$

Systematic Uncertainties (Cont.)

- Computing resources limit the number of lattice points which can be included, and hence the precision of the calculation.

  Typically in full QCD we can have about $24 - 32$ points in each spatial direction and so compromises have to be made.
- **Statistical Errors:** The functional integral is evaluated by Monte-Carlo sampling. The statistical error is estimated from the fluctuations of computed quantities within different clusters of *configurations*.
- The different sources of systematic uncertainty are not independent of each other, so the following discussion is oversimplified.

Systematic Uncertainties (Cont.)

- **Discretization Errors (Lattice Artefacts)**: Current simulations are typically performed with

$$a \sim (0.05 - .125)\,\text{fm} \qquad (0.1\,\text{fm} \simeq 2\,\text{GeV})$$

leading to errors of $O(a\Lambda_{\text{QCD}})$ (with Wilson Fermions) or $O(a^2\Lambda^2_{\text{QCD}})$ for *improved* fermion actions.

The errors can be estimated and reduced by:

  - Performing simulations at several values of a and extrapolating to $a = 0$.
  - *Improvement* (Symanzik), i.e. choosing a discretization of QCD so that the errors are formally smaller.

$$f'(x) = \frac{f(x+a) - f(x)}{a} + O(a) \quad \text{or} \quad f'(x) = \frac{f(x+a) - f(x-a)}{2a} + O(a^2)\,.$$

For example, in this way it is possible to reduce the errors from $O(a)$ for Wilson fermions to ones of $O(a^2)$ by the addition of irrelevant operators.

Systematic Uncertainties (Cont.)

- **Chiral Extrapolations:** Simulations are performed with unphysically heavy u and d quarks and the results are then extrapolated to the chiral limit.

  Wherever possible, we use χPT to guide the extrapolation, but it is still very rare to observe chiral logarithms.

  Today, in general, the most significant source of systematic uncertainty is due to the chiral extrapolation.

	m_q/m_s	m_π (MeV)	m_π/m_ρ
$SU(3)$ Limit	1	690	0.68
Currently Typical	1/2	490	0.55
Impressive	1/4	340	0.42
MILC	1/8	240	0.31
Physical	1/25	140	0.18

  For this reason the results obtained using the MILC Collaboration (using *Staggered* lattice fermions) have received considerable attention. Gradually the challenge set by the MILC Collaboration is being taken up by groups using other formulations of lattice fermions (e.g. Improved Wilson, Twisted Mass, Domain Wall, Overlap).
- $\rho \to \pi\pi$ decays have not been achieved on the lattice up to now.

Systematic Uncertainties (Cont.)

- **Finite Volume Effects:** For the quantities described above the finite-volume errors fall exponentially with the volume, e.g.

$$\frac{f_{\pi^\pm}(L) - f_{\pi^\pm}(\infty)}{f_{\pi^\pm}(\infty)} \simeq -\frac{6m_\pi^2}{f_\pi^2}\,\frac{e^{-m_\pi L}}{(2\pi m_\pi L)^{3/2}}\,.$$

  Generally these uncertainties are small at the light-quark masses which can be simulated.

  - For two-particle states (e.g. $K \to \pi\pi$ decays) the finite-volume effects decrease as inverse powers of L, and must be removed.

- **Renormalization of Lattice Operators:** From the matrix elements of the bare operators computed in lattice simulations we need to determine matrix elements of operators renormalized in some standard renormalization scheme (such as $\overline{MS}$).
  - For sufficiently large a^{-1} this can be done in perturbation theory, but lattice perturbation theory frequently has large coefficients $\Rightarrow$ large uncertainties ($O(10\%)$).
  - *Non-perturbative renormalization* is possible and frequently implemented, eliminating the need for lattice perturbation theory.

Old and New Compilations of f_B and ξ

- “B-Decays from Lattice QCD”, CTS in *B-Decays* (1994, ed. S.Stone):
  $f_B = 180 \pm 40\,\text{MeV} \quad \text{and} \quad f_D = 200 \pm 30\,\text{MeV}\,)\,.$

- ”Heavy Quark Physics from Lattice QCD” J.M.Flynn & CTS in *Heavy Flavours II* (1998, ed. A.J.Buras and M.Lindner):
  - $f_B \qquad (\,f_B = 170 \pm 35\,\text{MeV} \quad \text{and} \quad f_D = 200 \pm 30\,\text{MeV}\,)\,.$
  - $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} = 1.14(8)\,.$

- S.Hashimoto (ICHEP - 2004)

$$f_B = 189 \pm 27\,\text{MeV} \quad \text{and} \quad \xi = 1.23(6).$$

- C.Davies (EPS - 2005)

$$f_B = 216 \pm 22\,\text{MeV} \qquad \frac{f_{B_s}}{f_{B_d}} = 1.20(3)\,.$$

The Davies’ results are from a single calculation .

New Experimental Results and Lattice QCD

G.Martinelli (for UTfit Collaboration) - Ringberg April 2006

- Belle(2006) $\quad B(B^- \to \tau^- \overline{\nu}_\tau) \quad = \quad (1.06^{+0.34}_{-0.28}(\text{syst})^{+0.18}_{-0.16}(\text{stat})) \times 10^{-4}$

$$\Rightarrow f_B = 201 \pm 39\,\text{MeV} \quad (V_{ub} = (38.0 \pm 2.7 \pm 4.7)10^{-4} \text{ from exclusive decays})$$

$$\text{or } \ f_B = 173 \pm 30\,\text{MeV} \quad (V_{ub} = (43.9 \pm 2.0 \pm 2.7)10^{-4} \text{ from inclusive decays})$$

$$\text{or } \ f_B = 180 \pm 31\,\text{MeV} \quad (\text{combined inclusive + exclusive})\,.$$

- CDF (2006) $\quad \Delta m_S \quad = \quad (17.33^{+.42}_{-.21}(\text{stat}) \pm 0.07\text{syst})\,\text{ps}^{-1}$

$$\Rightarrow \quad \xi = 1.15 \pm 0.08 \quad (V_{ub} \text{ exclusive})$$

$$\text{or} \quad \xi = 1.05 \pm 0.10 \quad (V_{ub} \text{ inclusive})$$

$$\text{or} \quad \xi = 1.06 \pm 0.09 \quad (V_{ub} \text{ combined})$$

The Heavy Quark Effective Theory - HQET

- B-physics is playing a central rôle in flavourdynamics and it is useful to exploit the symmetries which arise when $m_Q \gg \Lambda_{\mathrm{QCD}}$.
- The *Heavy Quark Effective Theory* (HQET) is proving invaluable in the study of heavy quark physics.
  - For scales $\ll m_Q$ the physics in HQET is the same as in QCD.
  - For scales $O(m_Q)$ and greater, the physics is different, but can be *matched* onto QCD using perturbation theory.
  - The non-perturbative physics in the same in the HQET as in QCD.

Consider the propagator of a (free) heavy quark:

$$\xrightarrow[\;p\;]{} \quad = i\,\frac{\not p + m}{p^2 - m_Q^2 + i\varepsilon}\ .$$

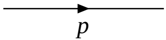
- If the momentum of the quark p is not far from its mass shell,

$$p_\mu = m_Q v_\mu + k_\mu\ ,$$

where $|k_\mu| \ll m_Q$ and v_μ is the (relativistic) four velocity of the hadron containing the heavy quark ($v^2 = 1$), then

$$\xrightarrow[\;p\;]{} \quad = i\,\frac{1+\not v}{2}\,\frac{1}{v\cdot k + i\varepsilon} + O\left(\frac{|k_\mu|}{m_Q}\right).$$

HQET Cont.



$$= i\,\frac{1+\not{v}}{2}\,\frac{1}{v\cdot k+i\varepsilon}+O\left(\frac{|k_\mu|}{m_Q}\right).$$

- $(1+\not{v})/2$ is a projection operator, projecting out the *large* components of the spinors.
- This propagator can be obtained from the gauge-invariant action

$$\mathscr{L}_{HQET}=\bar{h}(iv\cdot D)\frac{1+\not{v}}{2}h$$

  where h is the spinor field of the heavy quark.
- $\mathscr{L}_{HQET}$ is independent of m_Q, which implies the existence of symmetries relating physical quantities corresponding to different heavy quarks (in practice the b and c quarks or Scaling Laws).
- The light degrees of freedom are also not sensitive to the spin of the heavy quark, which leads to a spin-symmetry relating physical properties of heavy hadrons of different spins.

Spin Symmetry in the HQET

- Consider, for example, the correlation function:

$$\int d^3x \, \langle 0| J_H(x) \, J_H^\dagger(0) \, |0\rangle \, ,$$

  - $J_H^\dagger$ and J_H are interpolating operators which can create or annihilate a heavy hadron H.
  - Here I take H to be a pseudoscalar or vector meson.
  - The hadron is produced at rest, with four velocity $v = (1, \vec{0})$.
  - For example take $J_H = \bar{h}\gamma^5 q$ for the pseudoscalar meson and $J_H = \bar{h}\gamma^i q$ ($i = 1,2,3$) for the vector meson. This means that the correlation function will be identical in the two cases except for the factor

$$\gamma^5 \frac{1+\gamma^0}{2} \gamma^5 = \frac{1-\gamma^0}{2}$$

    when H is a pseudoscalar meson, and

$$\gamma^i \frac{1+\gamma^0}{2} \gamma^i = -3 \, \frac{1-\gamma^0}{2}$$

    when it is a vector meson.

Spin Symmetry Cont.

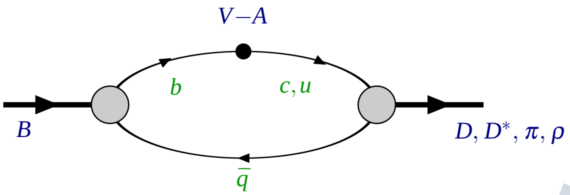
- Correlation functions $\sim \exp(-iM_H t) \Rightarrow$ the pseudoscalar and vector mesons are degenerate (up to relative corrections of $O(\Lambda^2_{QCD}/m_Q)$):

$$M_P = M_V + O(\Lambda^2_{QCD}/m_Q)\,.$$

  (or $M_V^2 - M_P^2 = \text{constant.}$)
- Heavy quark scaling laws (e.g. $f_P \sim 1/\sqrt{M_P}$) can be derived similarly.
- NRQCD is a useful effective theory in studying the physics of heavy quarkonia.

Determination of V_{cb} and V_{ub}

• These can be determined from either inclusive or exclusive decays. I start with a discussion of exclusive decays.

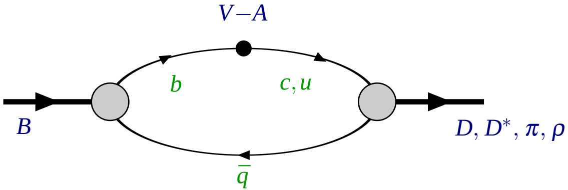


• Space-Time symmetries allow us to parametrise the non-perturbative strong interaction effects in terms of invariant form-factors. For example, for decays into a pseudoscalar meson P $(= \pi, D$ for example)

$$\langle P(k)|V^\mu|B(p)\rangle \;=\; f^+(q^2)\left[(p+k)^\mu - \frac{m_B^2 - m_P^2}{q^2}\,q^\mu\right] + f^0(q^2)\,\frac{m_B^2 - m_P^2}{q^2}\,q^\mu\,,$$

where $q = p - k$.

Determination of V_{cb} and V_{ub} Cont.



- For decays into a vector V $(= \rho, D^*$ for example), a conventional decomposition is

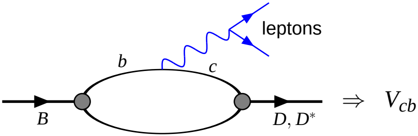
$$\langle V(k,\varepsilon)|V^\mu|B(p)\rangle = \frac{2V(q^2)}{m_B+m_V}\varepsilon^{\mu\gamma\delta\beta}\varepsilon^*_\beta p_\gamma k_\delta$$

$$\langle V(k,\varepsilon)|A^\mu|B(p)\rangle = i(m_B+m_V)A_1(q^2)\varepsilon^{*\mu} - i\frac{A_2(q^2)}{m_B+m_V}\varepsilon^*\!\cdot p\,(p+k)^\mu + i\frac{A(q^2)}{q^2}2m_V\varepsilon^*\!\cdot p\,q^\mu\ ,$$

where ε is the polarization vector of the final-state meson, and $q = p-k$.

$$\{A_3 = \frac{m_B+m_V}{2m_V}A_1 - \frac{m_B-m_V}{2m_V}A_2\ .\}$$

$B \to D^{(*)}$ Semileptonic Decays



- For $B \to D^*$ decays

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{\omega^2-1}\,(\omega+1)^2 \times \left[1 + \frac{4\omega}{\omega+1}\frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right] |V_{cb}|^2\, \mathscr{F}^2(\omega)\,,$$

where $\mathscr{F}(\omega)$ is the IW-function combined with perturbative and power corrections. $(\omega = v_B \cdot v_{D^*})$

- $\mathscr{F}(1) = 1$ up to power corrections and calculable perturbative corrections.

$B \to D^{(*)}$ Semileptonic Decays Cont.

- To determine the difference of $\mathscr{F}(1)$ from 1, the method of double ratios is used. Hashimoto, Kronfeld, Mackenzie, Ryan & Simone (1998)

  For example

$$\mathscr{R}_+ = \frac{\langle D|\bar{c}\gamma^4 b|\bar{B}\rangle \, \langle \bar{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle \, \langle \bar{B}|\bar{b}\gamma^4 b|\bar{B}\rangle} = |h_+(1)|^2$$

  with

$$h_+(1) = \eta_V \left\{ 1 - \ell_P \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 \right\}.$$

  By calculating $\mathscr{R}_+$ and similar ratios of $V \leftrightarrow P$ and $V \leftrightarrow V$ matrix elements all three ℓ's can be determined.

- A recent result from the FNAL/MILC/HPQCD Collaborations gives

$$|V_{cb}| = 3.9(1)(3) \times 10^{-2}.$$

  M.Okamoto, `hep-lat/0412044`

- Form Factors for $D \to \pi, K$ semileptonic decays are also being evaluated.

V_{cb} from Inclusive Decays

- V_{cb} is obtained from total semileptonic rate, and from lepton energy and hadronic mass spectra.
- The main tool is the OPE $\Rightarrow$ expansion in inverse powers of m_b, m_c .

$$\Gamma = |V_{cb}|^2\,\hat{\Gamma}_0 m_b^5(\mu)\,(1 + A_{\mathrm{EW}})A^{\mathrm{pert}}(r,\mu)\left\{z_0 + \frac{z_2(r)}{m_b^2} + \frac{z_3(r)}{m_b^3} + \cdots\right\}$$

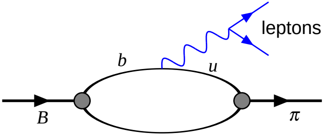
where $r = m_c/m_b$ and the z's are known functions depending on non-perturbative parameters which are determined from the spectra.

- It is difficult to quantify any violations of quark-hadron duality.

PDG(2006) Summary:

$|V_{cb}| = (41.7 \pm 0.7)\,10^{-3}$ (inclusive); $|V_{cb}| = (40.9 \pm 1.8)\,10^{-3}$ (exclusive)

$B \to \pi$ Exclusive Semileptonic Decays from the Lattice



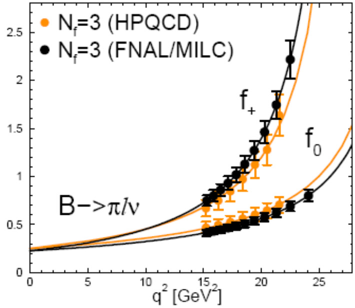
$\Rightarrow \quad V_{ub}$

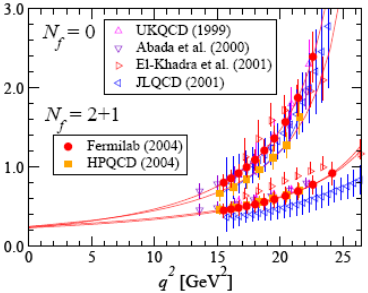
- For exclusive decays we require the form factors and the HQET is significantly less help here. This is the principle uncertainty.
- Small lattice artefacts $\Rightarrow$ momentum of the pion must be small $\Rightarrow$ we obtain form factors at large q^2.

  There is a proposal to eliminate this constraint by using a formulation in which the B-meson is moving. A.Dougall et al., `hep-lat/0509108`
- Experimental results in q^2 bins together with theoretical constraints, helps one use the lattice data to obtain V_{ub} precisely.

  I.Stewart LP2005, T.Becher & R.Hill `hep-lat/0509090`

Recent Results for $B \to \pi$ Form Factors





Courtesy of T.Onogi, Chamonix Flavour Dynamics Workshop, October 2005

HPQCD	Staggered Light & NRQCD Heavy	$\lvert V_{ub}\rvert = 4.04(20)(44)(53) \times 10^{-3}$
FNAL/MILC	Staggered Light & Fermilab Heavy	$\lvert V_{ub}\rvert = 3.48(29)(38)(47) \times 10^{-3}$

V_{ub}

- V_{ub} is also determined from inclusive decays $\bar{B} \to X_u \ell \bar{\nu}_\ell$ using the heavy-quark expansion.
- The difficulty is to remove the backgrounds from the larger $\bar{B} \to X_c \ell \bar{\nu}_\ell$ decays.
- If this is done by going towards the end-point so that $b \to c$ decays are not possible, then we need non-perturbative input (the *shape function*) $\Rightarrow$ limited precision.

PDG(2006) Summary:

$$|V_{ub}| = (4.40 \pm 0.2 \pm 0.27)\, 10^{-3} \quad \text{(inclusive)}; \qquad |V_{ub}| = (3.84^{+0.67}_{-0.49})\, 10^{-3} \quad \text{(exclusive)}$$

- The theoretical uncertainties in the inclusive and exclusive determinations of V_{cb} and V_{ub} are very different and it is reassuring that the results are consistent.
- In terms of the Wolfenstein parameters:

$$|V_{ub}|^2 = A^2 \lambda^6 (\bar{\rho}^2 + \bar{\eta}^2)\,.$$

PDG2006 Unitarity Triangle