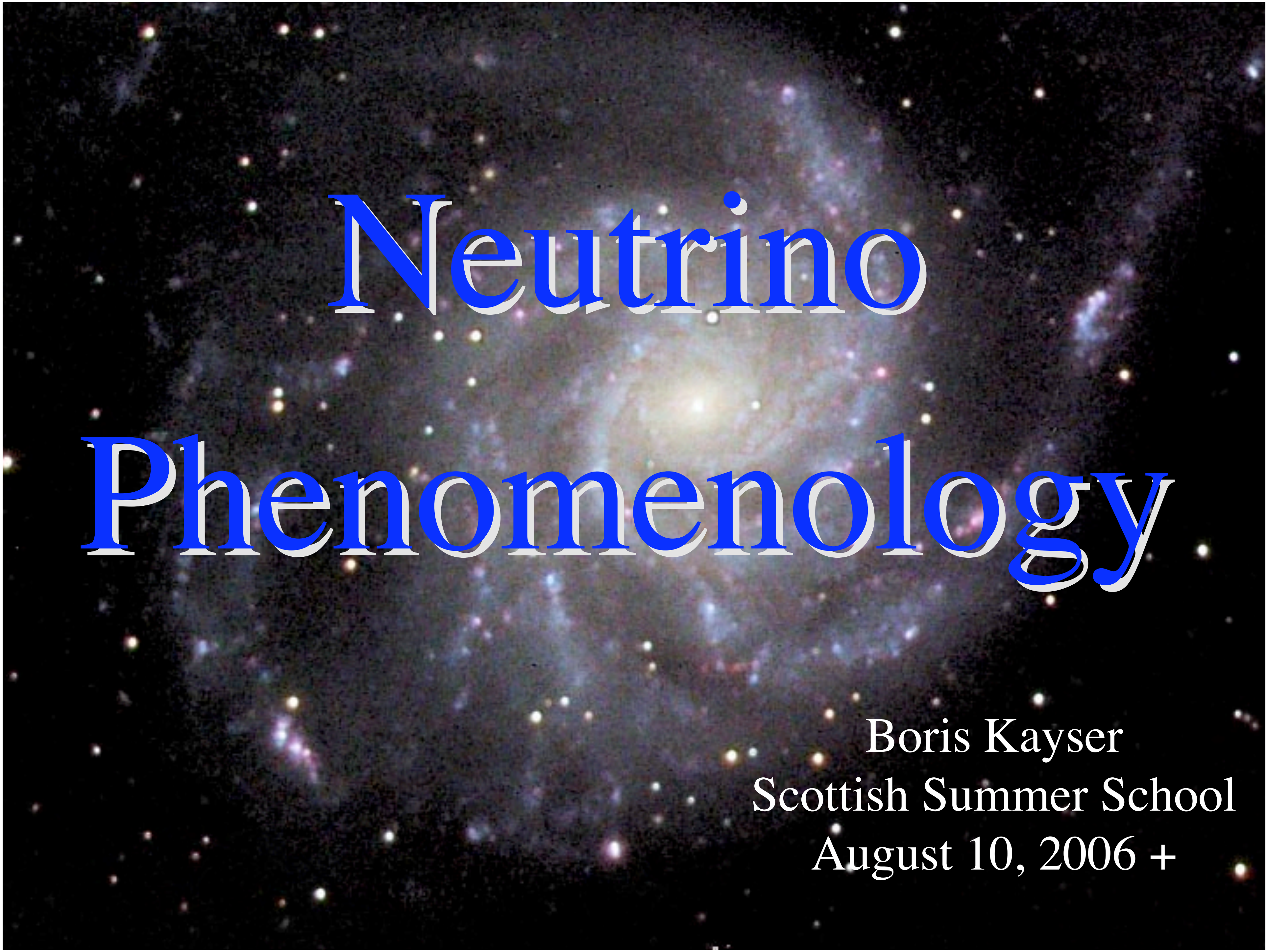
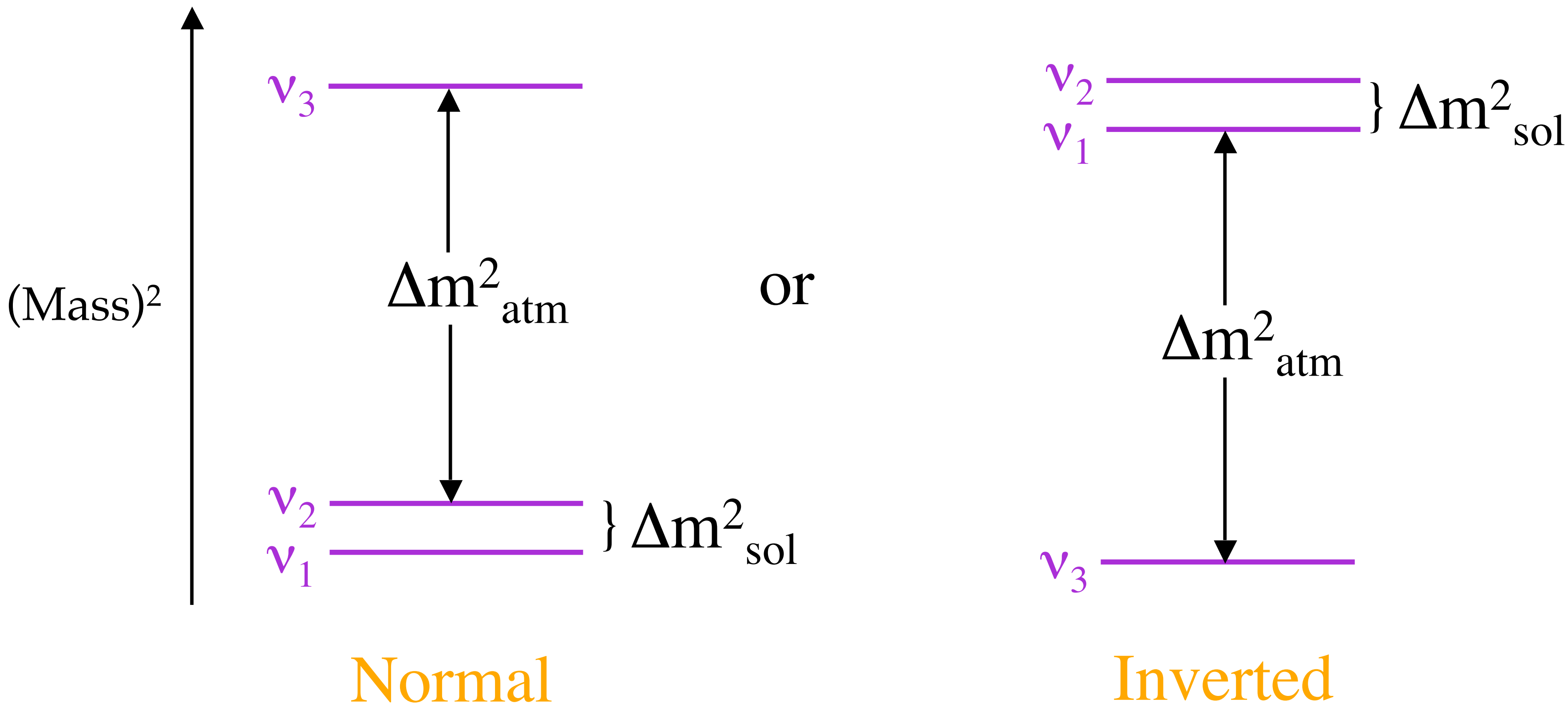


Neutrino Phenomenology

Boris Kayser
Scottish Summer School
August 10, 2006 +

What We Have Learned

The $(\text{Mass})^2$ Spectrum



$\Delta m^2_{sol} \cong 8.0 \times 10^{-5}\ \text{eV}^2$, $\quad \Delta m^2_{atm} \cong 2.7 \times 10^{-3}\ \text{eV}^2$

Are there *more* mass eigenstates, as LSND suggests?

Leptonic Mixing

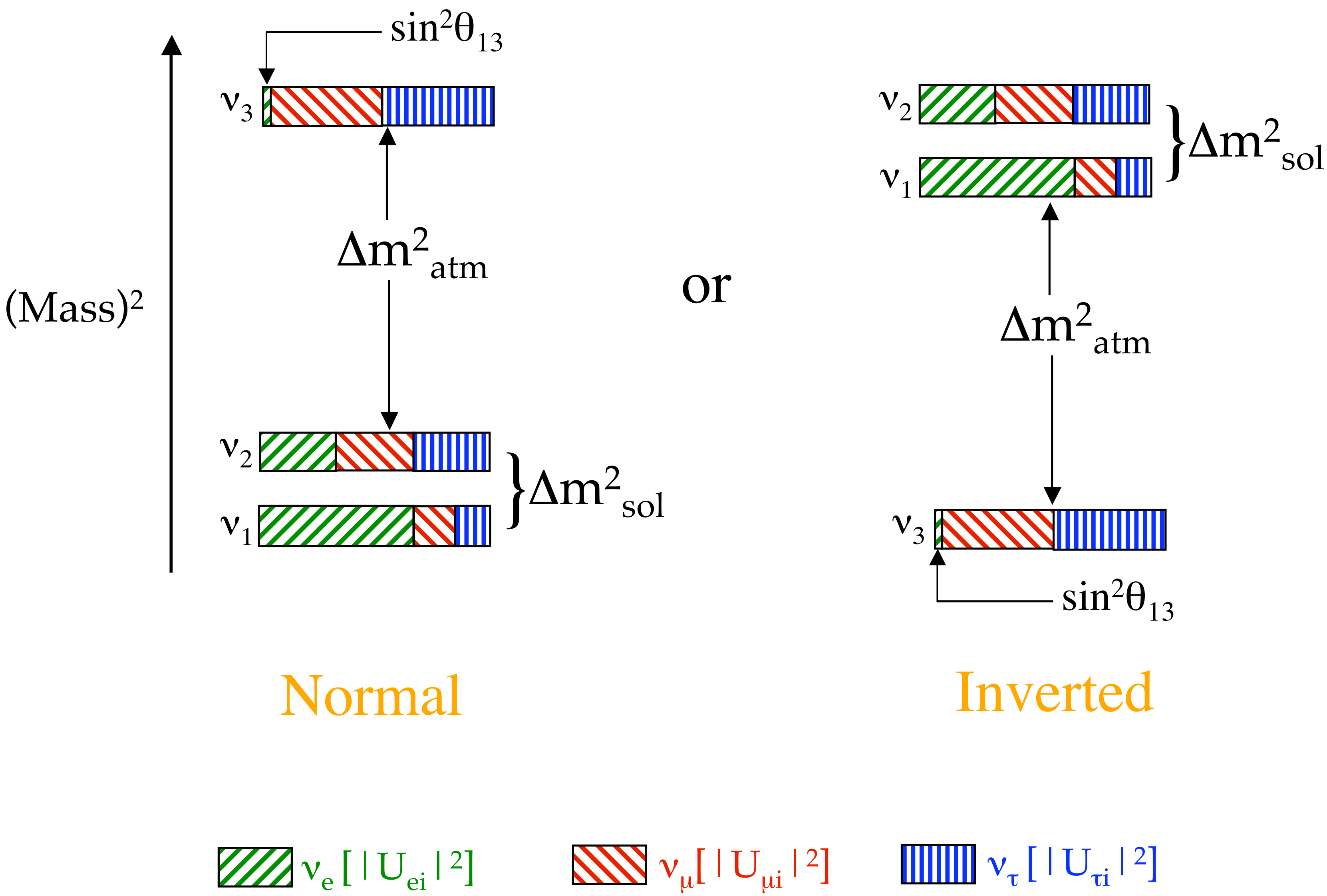
This has the consequence that —

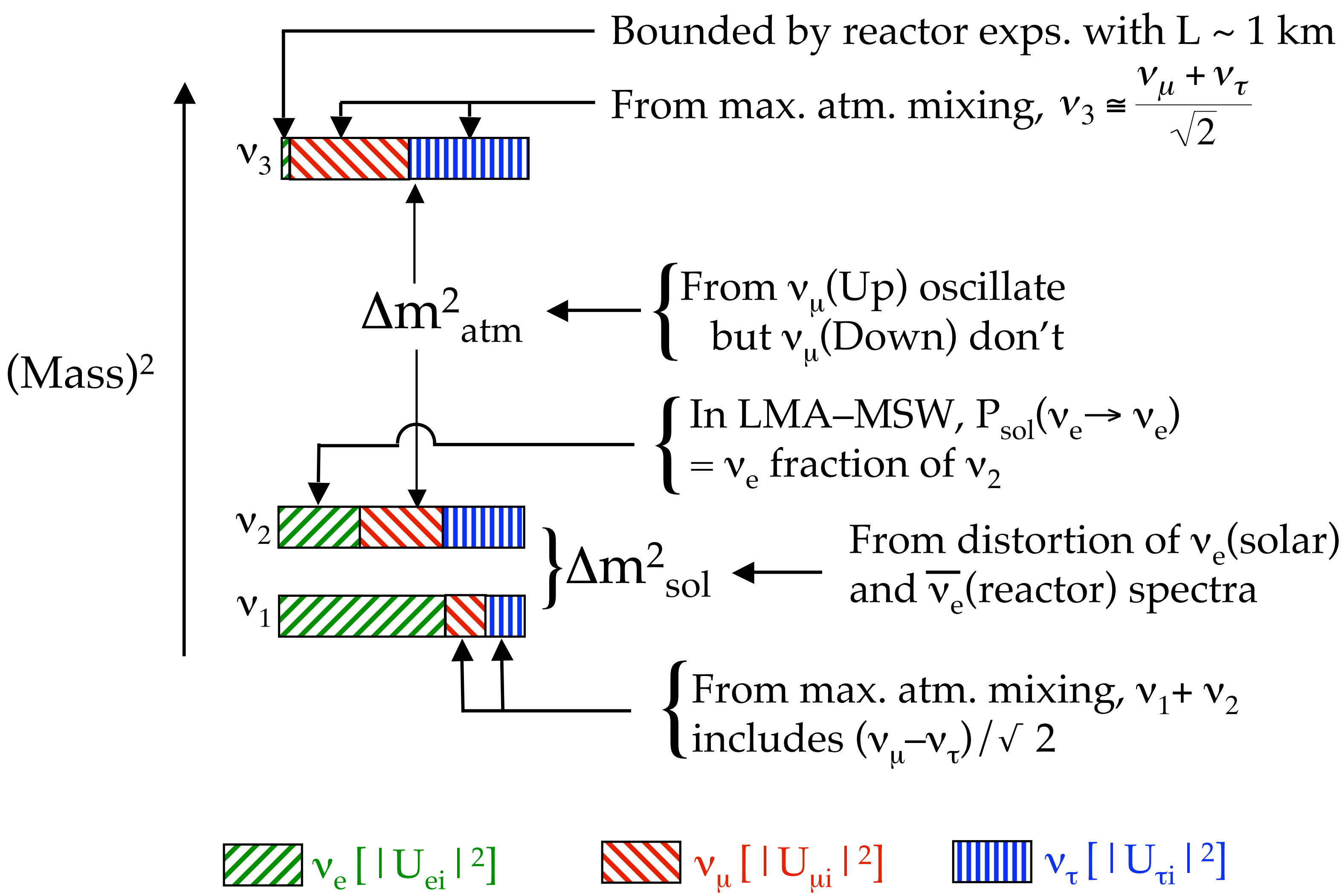
$$|\nu_i> = \sum_\alpha U_{\alpha i} \, |\nu_\alpha> .$$

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

The spectrum, showing its approximate flavor content, is




Bounded by reactor exps. with L ~ 1 km
From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$
ν_3
(Mass)^2
Δm^2_{atm}
From ν_μ(Up) oscillate
but ν_μ(Down) don't
In LMA–MSW, $P_{sol}(\nu_e \rightarrow \nu_e)$
= ν_e fraction of ν_2
ν_2
ν_1
Δm^2_{sol}
From distortion of ν_e(solar)
and $\overline{\nu}_e$(reactor) spectra
From max. atm. mixing, $\nu_1 + \nu_2$
includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$
ν_e [$|U_{ei}|^2$]
ν_μ [$|U_{\mu i}|^2$]
ν_τ [$|U_{\tau i}|^2$]

The 3 X 3 Unitary Mixing Matrix

Caution: We are *assuming* the mixing matrix U to be 3 x 3 and unitary.

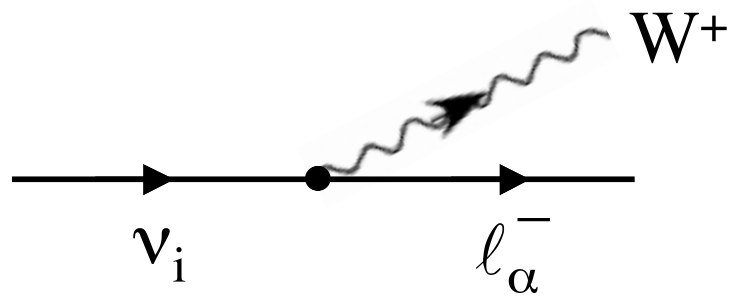
$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \overline{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} \right)(CP)^{-1} = \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}^{+}$$

Phases in U will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

When $|\nu_i\rangle \rightarrow |e^{i\varphi}\nu_i\rangle$, $U_{\alpha i} \rightarrow e^{i\varphi}\, U_{\alpha i}$

When $|\ell_\alpha^-\rangle \rightarrow |e^{i\varphi}\,\ell_\alpha^-\rangle$, $U_{\alpha i} \rightarrow e^{-i\varphi}\, U_{\alpha i}$

Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

Exception: If the neutrino mass eigenstates are their own antiparticles, then —

Charge conjugate →

$$\nu_i = \nu_i^{c} = C\overline{\nu_i}^{T}$$

One is no longer free to phase-redefine ν_i without consequences.

U can contain additional CP-violating phases.

How Many Mixing Angles and $\not{CP}$ Phases Does U Contain?

Real parameters before constraints:	18
Unitarity constraints — $\sum_i U_{\alpha i}^* U_{\beta i} = \delta_{\alpha\beta}$	
Each row is a vector of length unity:	– 3
Each two rows are orthogonal vectors:	– 6
Rephase the three ℓ_α :	– 3
Rephase two ν_i , if $\overline{\nu}_i \neq \nu_i$:	– 2
Total physically-significant parameters:	4
Additional (Majorana) $\not{CP}$ phases if $\overline{\nu}_i = \nu_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\not{CP}$ phases if $\bar{\nu}_i \neq \nu_i$	$\not{CP}$ phases if $\bar{\nu}_i = \nu_i$
3	1	3

The Mixing Matrix

Atmospheric Cross-Mixing Solar

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Majorana $\not{CP}$ phases

$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 37\text{-}53°$, $\theta_{13} \lesssim 10°$

δ would lead to $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$. $\not{CP}$

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

The Majorana $\not{CP}$ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^* \exp(-\, i m_i^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in neutrino oscillation.

The Open Questions

- What is the absolute scale of neutrino mass?

- Are neutrinos their own antiparticles?

- Are there “sterile” neutrinos?

We must be alert to surprises!

• What is the pattern of mixing among the different types of neutrinos?

What is θ_{13}? Is θ_{23} maximal?

• Is the spectrum like  or  ?

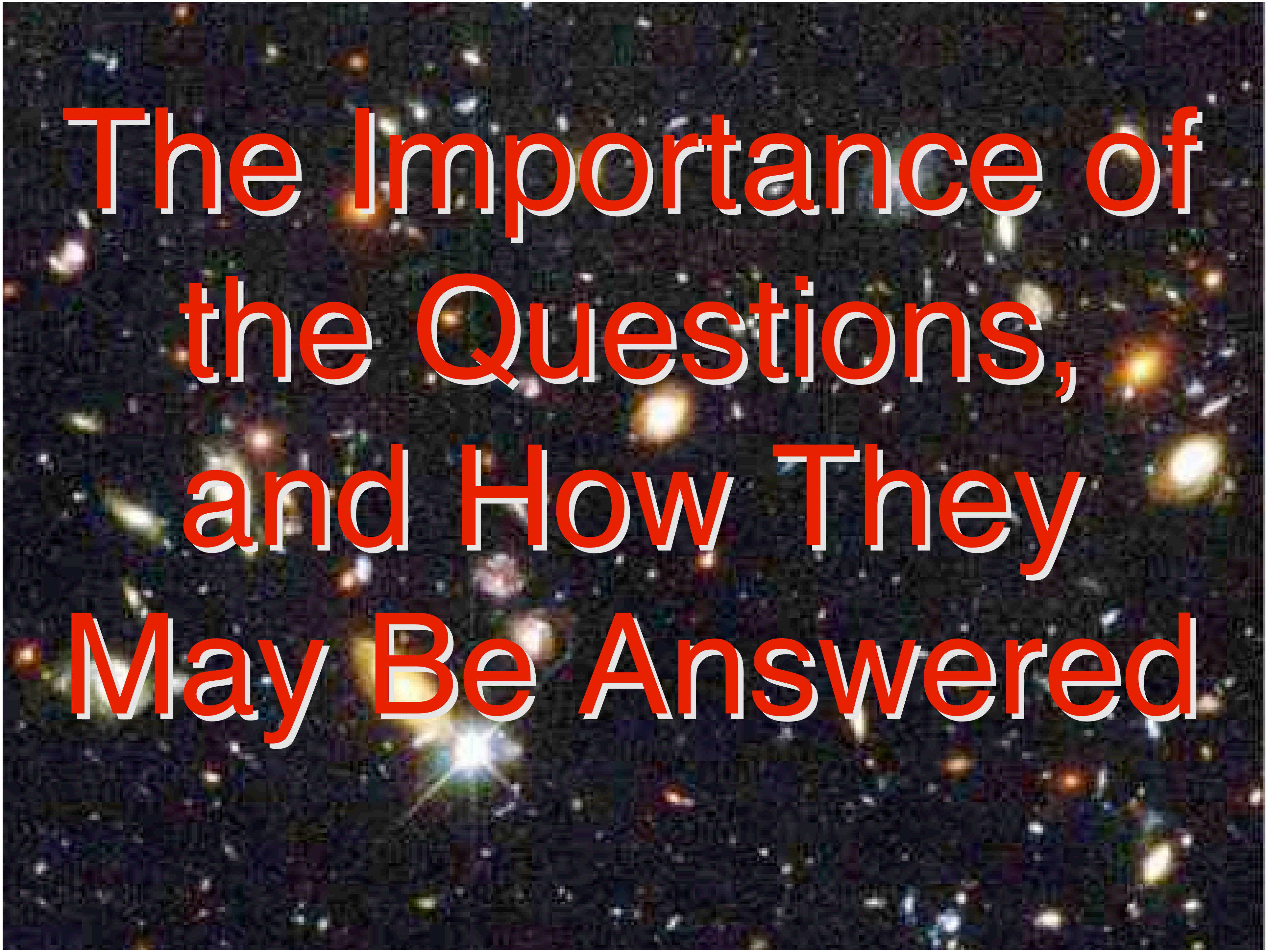
• Do neutrinos violate the symmetry CP?

Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

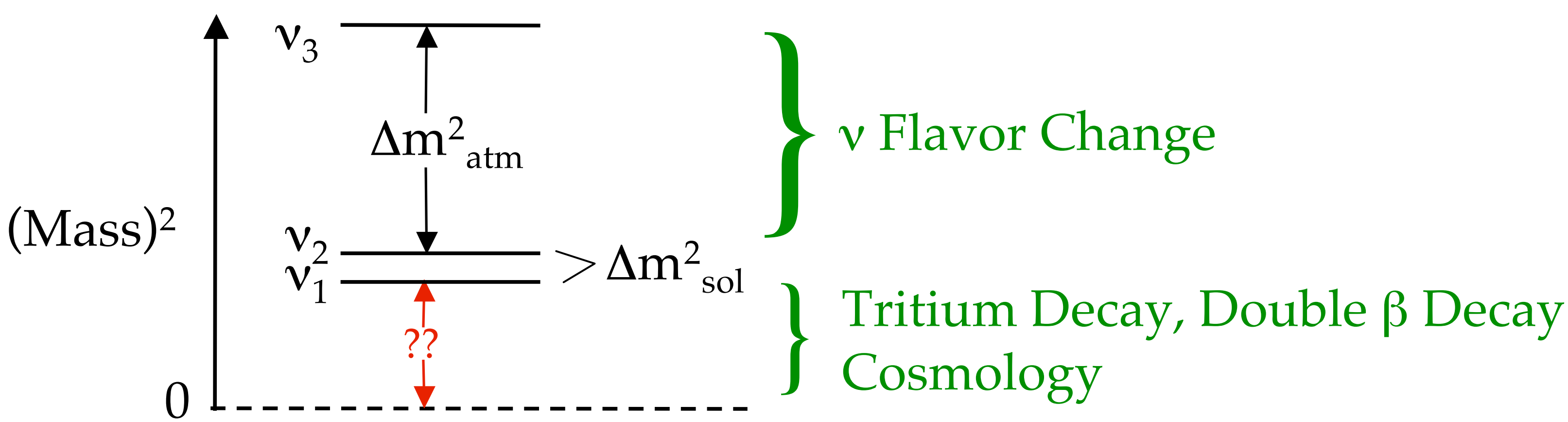
- What can neutrinos and the universe tell us about one another?

- Is CP violation by neutrinos the key to understanding the matter – antimatter asymmetry of the universe?

- What physics is behind neutrino mass?


The Importance of
the Questions,
and How They
May Be Answered

What Is the Absolute Scale of Neutrino Mass?



How far above zero is the whole pattern?

A Cosmic Connection

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

Cosmological Data + Cosmological Assumptions $\Rightarrow$

$$\Sigma\, m_i < (0.17 - 1.0) \text{ eV} .$$

Mass(ν_i) ↑

(Seljak, Slosar, McDonald
Pastor)

If there are only 3 neutrinos,

$$0.04 \text{ eV} \lesssim \text{Mass[Heaviest } \nu_i] < (0.07 - 0.4) \text{ eV}$$

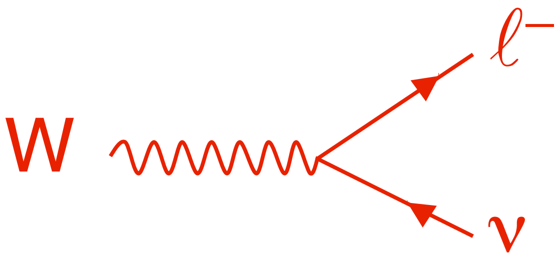
↑ $\sqrt{\Delta m^2_{atm}}$ Cosmology ↑

Are Neutrinos Majorana Particles?

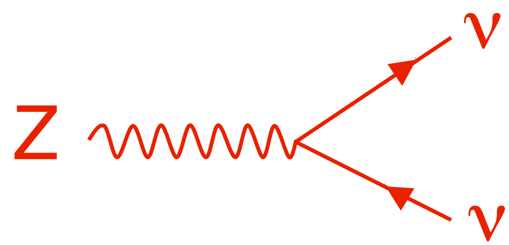
How Can the Standard Model be Modified to Include Neutrino Masses?

Majorana Neutrinos or Dirac Neutrinos?

The S(tandard) M(odel)



and

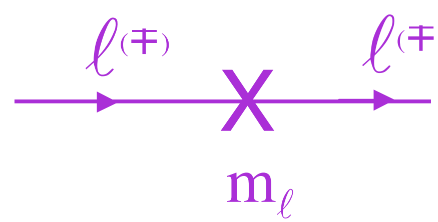


couplings conserve the Lepton Number L
defined by —

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

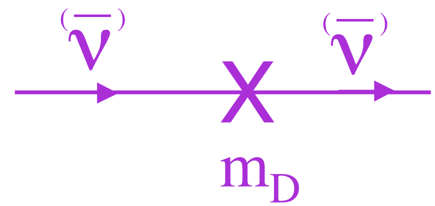
So do the Dirac charged-lepton mass terms

$$m_\ell \overline{\ell_L} \ell_R$$



- Original SM: $m_\nu = 0$.
- Why not add a Dirac mass term,

$$m_D \overline{\nu}_L \nu_R$$



Then everything conserves L, so for each mass eigenstate ν_i,

$$\overline{\nu}_i \neq \nu_i \qquad \text{(Dirac neutrinos)}$$

$$[L(\overline{\nu}_i) = -L(\nu_i)]$$

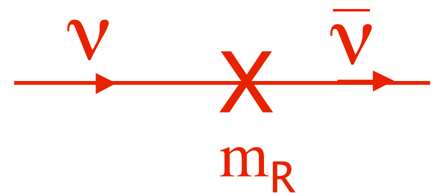
- The SM contains no ν_R field, only ν_L.

To add the Dirac mass term, we had to add ν_R to the SM.

Unlike ν_L, ν_R carries no Electroweak Isospin.

Thus, no SM principle prevents the occurrence of the Majorana mass term

$$m_R \overline{\nu_R^{\,c}} \nu_R$$



But this does not conserve L, so now

$$\bar{\nu}_i = \nu_i \qquad \text{(Majorana neutrinos)}$$

[No conserved L to distinguish $\bar{\nu}_i$ from ν_i]

We note that $\bar{\nu}_i = \nu_i$ means —

$$\bar{\nu}_i(h) = \nu_i(h)$$

helicity

The objects ν_R and $\nu_R{}^c$ in $m_R \overline{\nu_R{}^c}\, \nu_R$ are not the mass eigenstates, but just the neutrinos in terms of which the model is constructed.

$m_R \overline{\nu_R{}^c}\, \nu_R$ induces $\nu_R \longleftrightarrow \nu_R{}^c$ mixing.

As a result of $K^0 \longleftrightarrow \overline{K^0}$ mixing, the neutral K mass eigenstates are —

$$K_{S,L} \cong (K^0 \pm \overline{K^0})/\sqrt{2}\ .$$

As a result of $\nu_R \longleftrightarrow \nu_R{}^c$ mixing, the neutrino mass eigenstate is —

$$\nu_i = \nu_R + \nu_R{}^c = \text{“}\ \nu + \overline{\nu}\ \text{”}\ .$$

Many Theorists Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably Electroweak Isospin Invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

If this is also true for neutrino masses, then neutrinos have Majorana masses.

- The presence of Majorana masses
- $\overline{\nu_i} = \nu_i$ (Majorana neutrinos)
- L not conserved

— are all equivalent

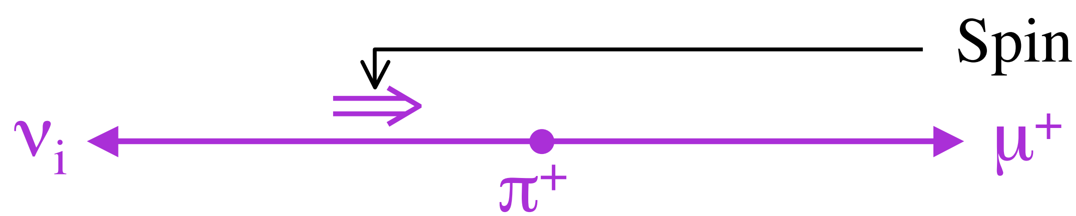
Any one implies the other two.

How Can We Demonstrate That $\overline{\nu}_i = \nu_i$?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve L ($\nu \to \ell^-$; $\overline{\nu} \to \ell^+$).

An Idea that Does Not Work
[and illustrates why most ideas do not work]

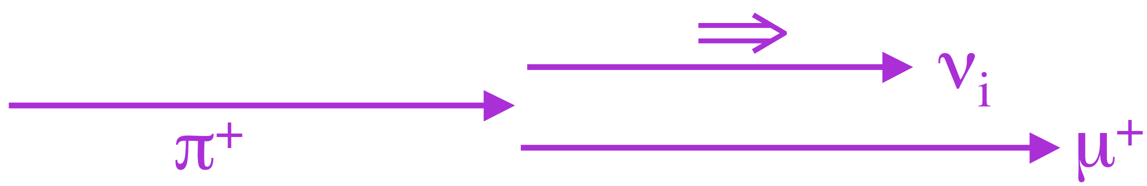
Produce a ν_i via—



Pion Rest Frame

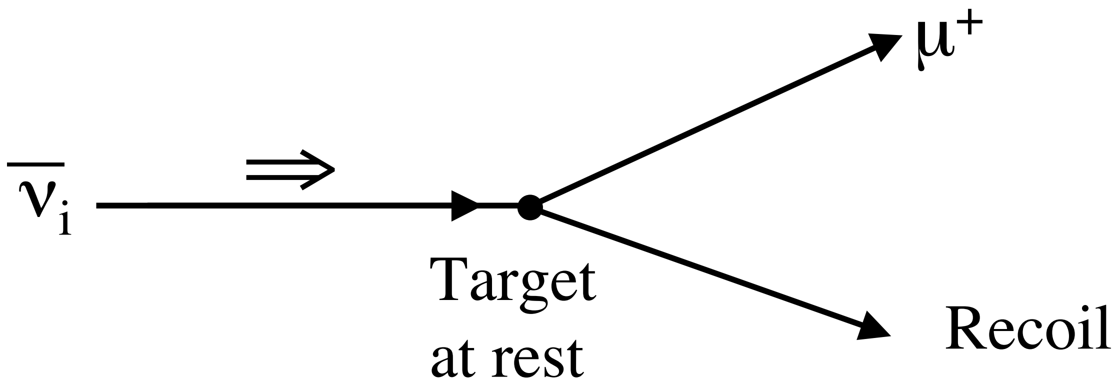
Give the neutrino a Boost:

β_π(Lab) > β_ν(π Rest Frame)



Lab. Frame

The SM weak interaction causes—



$\nu_i = \overline{\nu}_i$ means that $\nu_i(h) = \overline{\nu}_i(h)$.

helicity

If $\nu_i \xrightarrow{\Rightarrow} \; = \; \overline{\nu}_i \xrightarrow{\Rightarrow}$,

our $\nu_i \xrightarrow{\Rightarrow}$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_{\nu_i}}$$

$$\Rightarrow E_\pi(\text{Lab}) \gtrsim 10^5 \text{ TeV if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Fraction of all π – decay ν_i that get helicity flipped

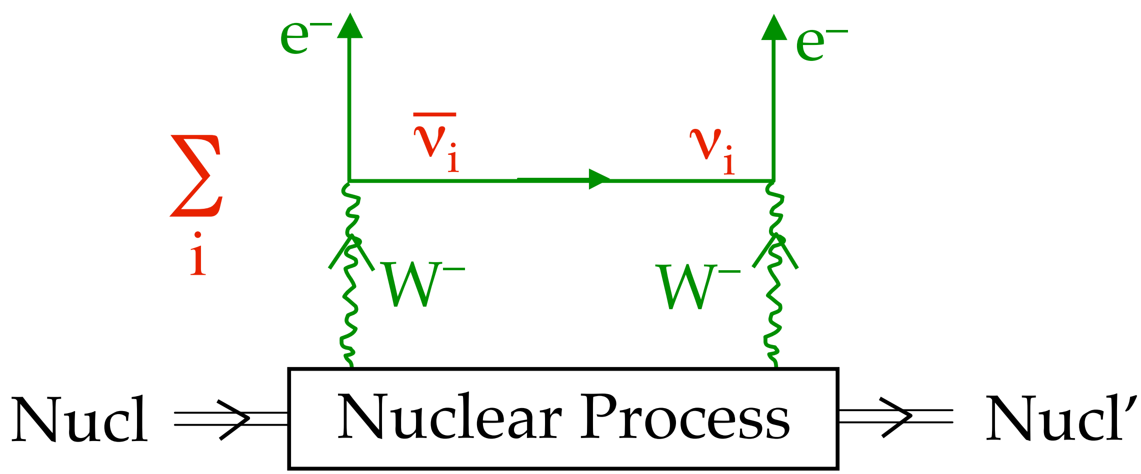
$$\approx \left(\frac{m_{\nu_i}}{E_\nu(\pi \text{ Rest Frame})}\right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Since L-violation comes only from Majorana neutrino *masses*, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & Stodolsky)

The Idea That Can Work —

Neutrinoless Double Beta Decay [$0\nu\beta\beta$]



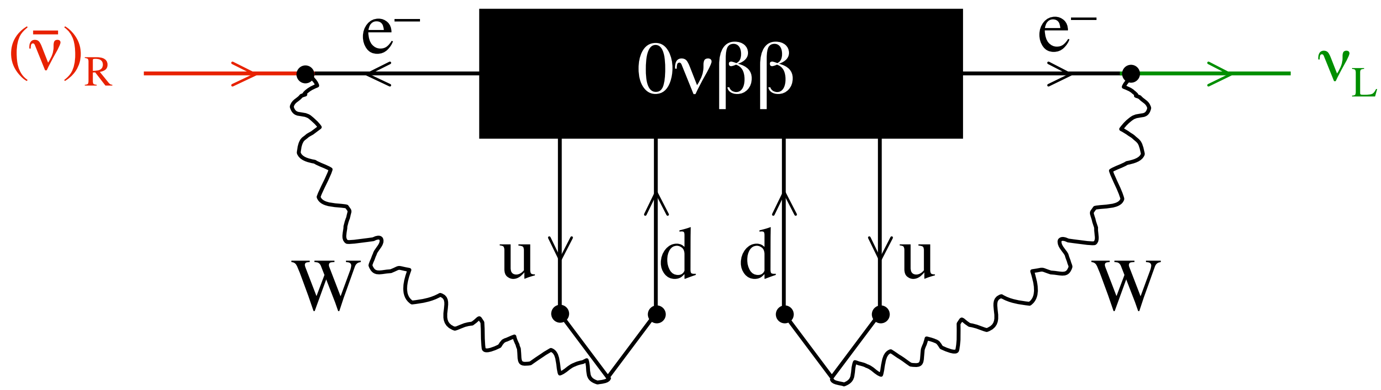
By avoiding competition, this process can cope with the small neutrino masses.

Observation would imply $\not{L}$ and $\bar{\nu}_i = \nu_i$.

Backup Slides

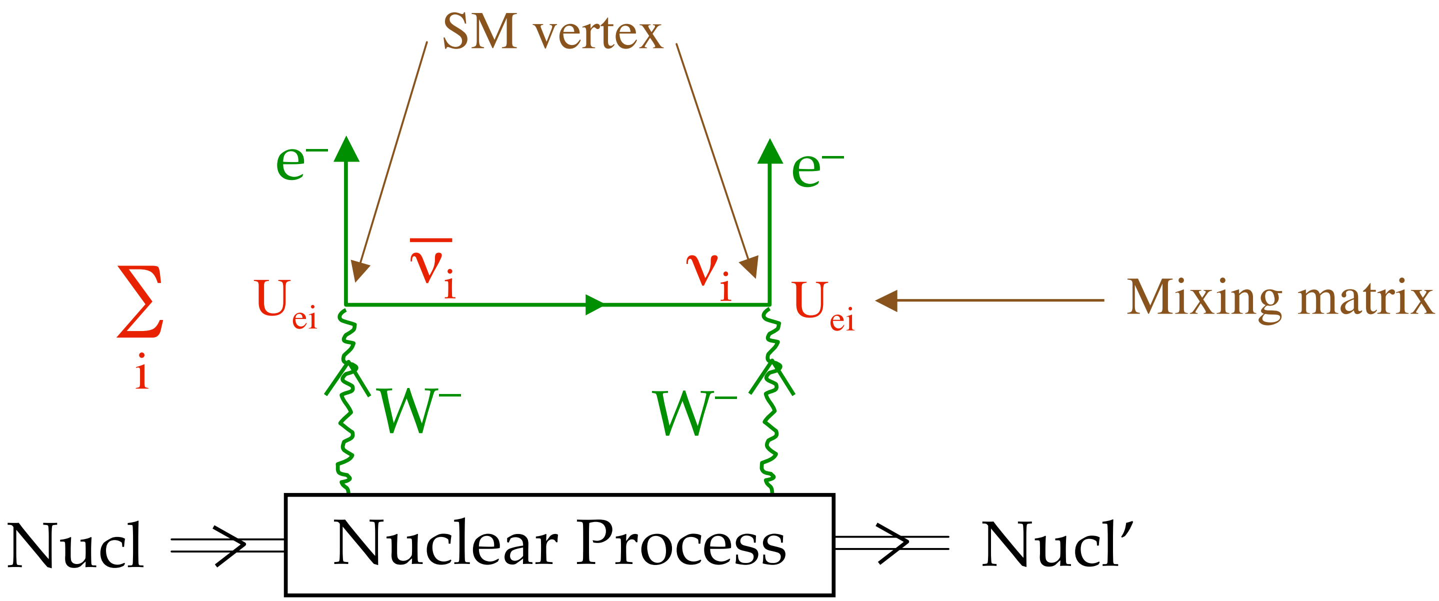
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

Schechter and Valle



$(\bar{\nu})_R \rightarrow \nu_L$: A Majorana mass term

In —



Mass (ν_i)

the $\overline{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] $\propto m_i$

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

The proportionality of 0νββ to mass is no surprise.

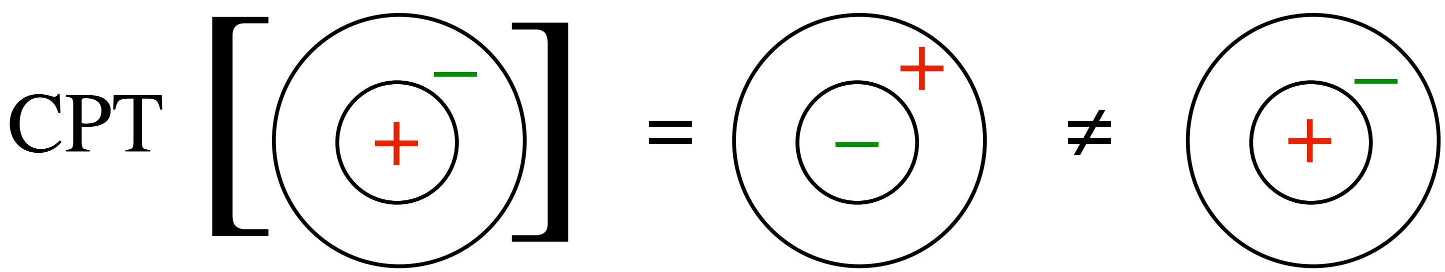
0νββ violates L. But the SM interactions conserve L.

The L – violation in 0νββ comes from underlying **Majorana** mass terms.

Electromagnetic Properties of Majorana Neutrinos

Majorana neutrinos are very neutral.

No charge distribution:



But for a Majorana neutrino,

$$\mathrm{CPT}\left[\ \nu_i\ \right] = \left[\ \nu_i\ \right]$$

No magnetic or electric dipole moment:

$$\vec{\mu}\left[\;\underset{e^+}{\uparrow}\;\right] = -\,\vec{\mu}\left[\;\underset{e^-}{\uparrow}\;\right]$$

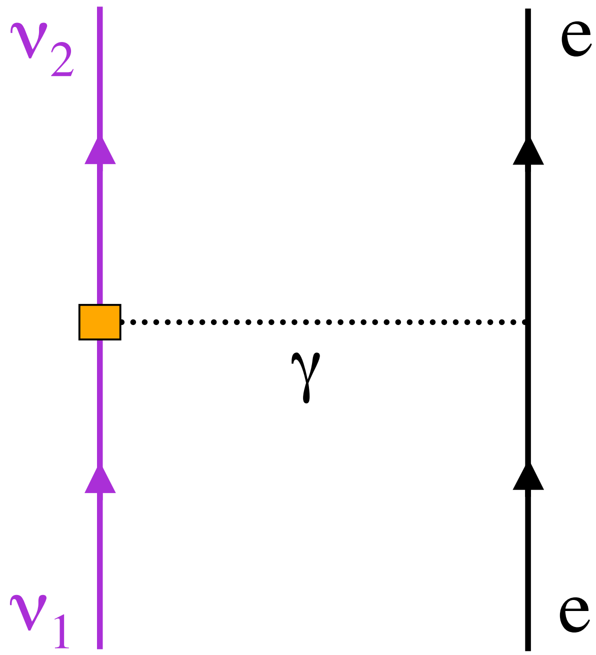
But for a Majorana neutrino,

$$\overline{\nu_i} = \nu_i$$

Therefore,

$$\vec{\mu}\,[\overline{\nu_i}] = \vec{\mu}\,[\nu_i] = 0$$

Transition dipole moments are possible, leading to —



One can look for the dipole moments this way.

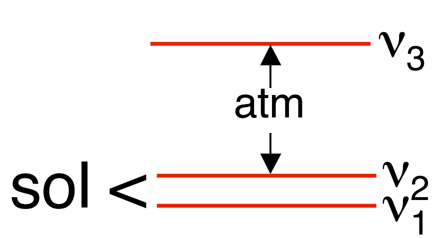
To be visible, they would have to *vastly* exceed Standard Model predictions.

How Large is $m_{\beta\beta}$?

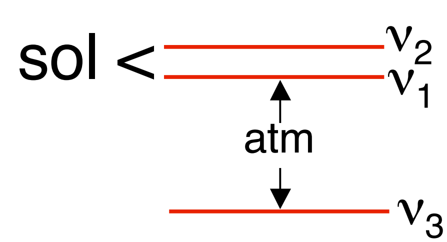
How sensitive need an experiment be?

Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —



or



$$m_{\beta\beta} \equiv \left| \sum m_i U_{ei}^{\,2} \right|$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

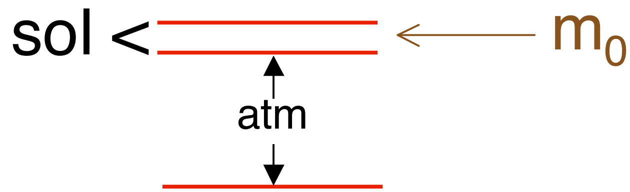
$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The e (top) row of U reads —

$$(U_{e1}, U_{e2}, U_{e3}) = (c_{12}c_{13}e^{i\alpha_1/2}, s_{12}c_{13}e^{i\alpha_2/2}, s_{13}e^{-i\delta})$$

$\theta_{12} \approx \theta_\odot \approx 34°$, but $s_{13}^{\,2} < 0.032$

If the spectrum looks like—



then–

$$m_{\beta\beta} \cong m_0\left[1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right)\right]^{1/2} .$$

$$m_0 \cos 2\theta_\odot \leq m_{\beta\beta} \leq m_0$$

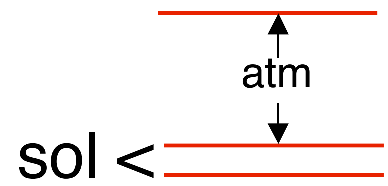
At 90% CL,

$m_0 > 46$ meV (MINOS); $\cos 2\theta_\odot > 0.28$ (SNO),

so

$$m_{\beta\beta} > 13 \text{ meV} .$$

If the spectrum looks like ⟶



then —

$0 < m_{\beta\beta} <$ Present Bound [(0.3–1.0) eV] .

(Petcov *et al.*)

<u>Analyses of $m_{\beta\beta}$ vs. Neutrino Parameters</u>

Barger, Bilenky, Farzan, Giunti, Glashow, Grimus, BK, Kim, Klapdor-Kleingrothaus, Langacker, Marfatia, Monteno, Murayama, Pascoli, Päs, Peña-Garay, Peres, Petcov, Rodejohann, Smirnov, Vissani, Whisnant, Wolfenstein,

Review of $\beta\beta$ Decay: Elliott & Vogel

Evidence for $0\nu\beta\beta$ with $m_{\beta\beta} = (0.05 – 0.84)$ eV?

Klapdor-Kleingrothaus