# Neutrino-Nucleon Deep Inelastic Scattering

# Neutrino-Nucleon 'n a Nutshell

- Charged Current: W<sup>±</sup> exchange
  - Quasi-elastic Scattering: (Target changes but no break up) v<sub>µ</sub> + n → µ<sup>-</sup> + p
  - Nuclear Resonance Production: (Target goes to excited state)  $\nu_{\mu} + n \rightarrow \mu^{-} + p + \pi^{0}$  (N<sup>\*</sup> or  $\Delta$ )  $n + \pi^{+}$
  - Deep-Inelastic Scattering: (Nucleon broken up)

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v_{\mu} + quark \rightarrow \mu^{-} + quark'
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- Neutral Current: Z<sup>0</sup> exchange
  - Elastic Scattering: (Target unchanged)  $v_{\mu} + N \rightarrow v_{\mu} + N$
  - Nuclear Resonance Production: (Target goes to excited state)  $v_{\mu} + N \rightarrow v_{\mu} + N + \pi$  (N<sup>\*</sup> or  $\Delta$ )
  - Deep-Inelastic Scattering (Nucleon broken up)  $v_{\mu}$  + quark  $\rightarrow v_{\mu}$  + quark



# **Scattering Variables**



Scattering variables given in terms of invariants

•More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.



Measured quantities:  $E_h$ , E',  $\theta$ 

4-momentum Transfer<sup>2</sup>: 
$$Q^2 = -q^2 = -\left(p'-p\right)^2 \approx \left(4EE'\sin^2(\theta/2)\right)_{Lab}$$
  
Energy Transfer:  $v = (q \cdot P)/M_T = \left(E - E'\right)_{Lab} = (E_h - M_T)_{Lab}$   
Inelasticity:  $y = (q \cdot P)/(p \cdot P) = (E_h - M_T)/(E_h + E')_{Lab}$   
Fractional Momentum of Struck Quark:  $x = -q^2/2(p \cdot q) = Q^2/2M_T v$   
Recoil Mass<sup>2</sup>:  $W^2 = (q + P)^2 = M_T^2 + 2M_T v - Q^2$   
CM Energy<sup>2</sup>:  $s = (p + P)^2 = M_T^2 + \frac{Q^2}{xy}$ 



Deep Inelastic Scattering Cont.



Much intuition was gained from the Feynman-Bjorken parton picture. Noting that the typical scale of strong-interactions is 1 fm or 200 MeV, consider a frame in which  $|\vec{p}|$  is large

$$x \equiv \frac{-q^2}{2p \cdot q} \qquad (\xi p + q)^2 \simeq 0 \implies 2\xi p \cdot q + q^2 \simeq 0 \implies \xi = x.$$

The experimentally measurable quantity *x* gives the fraction of the proton's momentum carried by the struck quark (in the *infinite momentum* frame).

#### Parton Interpretation of DIS

Mass of target quark  $m_q^2 = x^2 P^2 = x^2 M_T^2$ ν μ  $q = p^{\nu} - p^{\mu}$ хP (1 - x)P

Neutrino scatters off a parton inside the nucleon

Mass of final state quark

$$m_{q^2}^2 = (xP + q)^2$$

In "infinite momentum frame", x is momentum of partons inside the nucleon

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

# So why is cross-section so large?

- (at least compared to ve<sup>-</sup> scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_{0}^{Q_{\text{max}}^2 \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$
$$s = m_e^2 + 2m_e E_v$$

- But we just learned for DIS that effective mass of each target quark is  $m_q = xm_{nucleon}$
- So much larger target mass means larger  $\sigma_{TOT}$

# Chirality, Charge in CC v-q Scattering

- Total spin determines inelasticity distribution
  - Familiar from neutrinoelectron scattering

pount-tike scattering mplies linear with energy

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi} \left( x d(x) + x u(x)(1-y)^2 \right)$$
$$\frac{d\sigma^{\overline{\nu}p}}{dxdy} = \frac{G_F^2 s}{\pi} \left( x d(x) + x u(x)(1-y)^2 \right)$$
but what is this "q(x)"?



 $1/4(1+\cos\theta^*)^2 = (1-y)^2$  $\int (1-y)^2 dy = 1/3$ 

 Neutrino/Anti-neutrino CC each produce particular ∆q in scattering

 $vd \rightarrow \mu^{-}u$ 

$$\nu u \rightarrow \mu^+ d$$

# Factorization and Partons

 Factorization Theorem of QCD allows amplitudes for hadronic processes to be written as:

$$A(l+h \to l+X) = \sum_{q} \int dx A(l+q(x) \to l+X) q_h(x)$$

- Parton distribution functions (PDFs) are universal
- Processes well described by single parton interactions
- Parton distribution functions not (yet) calculable from first principles in QCD
- "Scaling": parton distributions are largely independent of Q<sup>2</sup> scale, and depend on fractional momentum, x.



#### Hard Scattering Processes in Hadronic Collisions



$$\sigma(h_1(p_1) + h_2(p_2) \to Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{f_1, f_2} f_{f_1}(x_1) f_{f_2}(x_2) \sigma(f_1 + f_2 \to Y).$$

- The f<sub>fi</sub> s are "known" from Deep Inelastic Scattering.
- It is in this way (modified to take QCD corrections into account) that we were able to make predictions for the cross sections for W and Z production at the SPS or are able to make predictions for Higgs Boson production at the LHC.

Standard Model

### Momentum of Quarks & Antiquarks



Momentum carried by quarks *much greater* than anti-quarks in nucleon

#### y distribution in Neutrino CC DIS



#### Touchstone Question #4: Neutrino and Anti-Neutrino σ<sup>vN</sup>

• Given:  $\sigma_{CC}^{\ \nu} \approx \frac{1}{2} \sigma_{CC}^{\ \nu}$  in the DIS regime (CC) and  $\frac{d\sigma(vq)}{dx} = \frac{d\sigma(\overline{vq})}{dx} = 3\frac{d\sigma(v\overline{q})}{dx} = 3\frac{d\sigma(\overline{vq})}{dx}$ for CC scattering from quarks or anti-quarks of a given momentum,

and that cross-section is proportional to parton momentum, what is the approximate ratio of antiquark to quark momentum in the nucleon?

(a)  $\bar{q}/q \sim 1/3$  (b)  $\bar{q}/q \sim 1/5$  (c)  $\bar{q}/q \sim 1/8$ 

## Momentum of Quarks & Antiquarks



#### Strong Interactions among Partons

Q<sup>2</sup> Scaling fails due to these interactions



 $F_2(x,Q^2)$ 

scatters off "sea" quark

CD scale violations

scaling

 $\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y}$  $\left[ P_{qq}\left(\frac{x}{y}\right) q(y,Q^2) + P_{qg}\left(\frac{x}{y}\right) g(y,Q^2) \right]$ 

•Pqq(x/y) = probability of finding a quark with momentum x within a quark with momentum y

•Pqq(x/y) = probability of finding a q with momentum x within a gluon with momentum y

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{(1-z)} + 2\delta(1-z)$$
$$P_{gq}(z) = \frac{1}{2} \left[ z^2 + (1-z)^2 \right]$$

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#### Deep Inelastic Scattering and QCD Cont.

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- I refer to the standard textbook for the use of the Operator Product Expansion (OPE) to determine the q<sup>2</sup> behaviour of the structure functions.
- The same results can be obtained from the DGLAP equations (let t = log(q<sup>2</sup>/q<sub>0</sub><sup>2</sup>)):

$$\frac{dq^{\mathrm{NS}}(x,t)}{dt} = \frac{\alpha_{s}(t)}{2\pi} \int_{x}^{1} \frac{dy}{y} q^{\mathrm{NS}}(y,t) P_{q \to q}\left(\frac{x}{y}\right)$$

$$\frac{dq^{\mathrm{S}}(x,t)}{dt} = \frac{\alpha_{s}(t)}{2\pi} \int_{x}^{1} \frac{dy}{y} \left\{ q^{\mathrm{S}}(y,t) P_{q \to q}\left(\frac{x}{y}\right) + g(y,t) P_{g \to q}\left(\frac{x}{y}\right) \right\}$$

$$\frac{dg(x,t)}{dt} = \frac{\alpha_{s}(t)}{2\pi} \int_{x}^{1} \frac{dy}{y} \left\{ q^{\mathrm{S}}(y,t) P_{q \to g}\left(\frac{x}{y}\right) + g(y,t) P_{g \to g}\left(\frac{x}{y}\right) \right\}$$



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#### **Scaling from QCD**



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# If you find this difficult to remember...

- It may help you to imagine <u>scaling up</u> a mountain
- Perhaps after yesterday it is more intuitive that as you go up in scale
  - the average momentum of each hiking group decreases
  - and the number of hiking groups increases...

# DIS: Relating SFs to Parton Distributions

# **Structure Functions** (SFs)

- A model-independent picture of these interactions can also be formed in terms of nucleon "structure functions"
  - All Lorentz-invariant terms included
  - Approximate zero lepton mass (small correction)

$$\frac{d\sigma^{v,\bar{v}}}{dxdy} \propto \left[ y^2 2xF_1(x,Q^2) + \left(2 - 2y - \frac{M_T xy}{E}\right)F_2(x,Q^2) \pm y(2 - y)xF_3(x,Q^2) \right]$$

- For massless free spin-1/2 partons, one simplification...
  - Callan-Gross relationship, 2xF<sub>1</sub>=F<sub>2</sub>
  - Implies intermediate bosons are completely transverse

Can parameterize transverse cross-section by R<sub>L</sub>. •Callan-Gross violations, M •NLO pQCD,  $g \rightarrow q\overline{q}$ 

 $R_{L} = \frac{\sigma_{L}}{\sigma_{T}} = \frac{F_{2}}{2xF_{1}} \left( 1 + \frac{4M_{T}^{2}x^{2}}{O^{2}} \right)$ 

## SFs to PDFs

- Can relate SFs to PDFs in naïve quark-parton model by matching y dependence
  - Assuming Callan-Gross, massless targets and partons...
  - $F_3: 2y-y^2=(1-y)^2-1$ ,  $2xF_1=F_2: 2-2y+y^2=(1-y)^2+1$

$$2xF_1^{\nu p,CC} = x \left[ d_p(x) + \overline{u_p}(x) + s_p(x) + \overline{c_p}(x) \right]$$
$$xF_3^{\nu p,CC} = x \left[ d_p(x) - \overline{u_p}(x) + s_p(x) - \overline{c_p}(x) \right]$$

- In analogy with neutrino-electron scattering, CC only involves left-handed quarks
- However, NC involves both chiralities (V-A and V+A)
  - Also couplings from EW Unification
  - And no selection by quark charge

$$2xF_{1}^{\nu p,NC} = x \left[ (u_{L}^{2} + u_{R}^{2}) \left( u_{p}(x) + \overline{u_{p}(x)} + c_{p}(x) + \overline{c_{p}(x)} \right) + (d_{L}^{2} + d_{R}^{2}) \left( d_{p}(x) + \overline{d_{p}(x)} + s_{p}(x) + \overline{s_{p}(x)} \right) \right]$$
$$xF_{3}^{\nu p,NC} = x \left[ (u_{L}^{2} - u_{R}^{2}) \left( u_{p}(x) - \overline{u_{p}(x)} + c_{p}(x) - \overline{c_{p}(x)} \right) + (d_{L}^{2} - d_{R}^{2}) \left( d_{p}(x) - \overline{d_{p}(x)} + s_{p}(x) - \overline{s_{p}(x)} \right) \right]$$

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# **Isoscalar Targets**

- Heavy nuclei are roughly neutron-proton isoscalar
- Isospin symmetry implies  $u_p = d_n, d_p = u_n$
- Structure Functions have a particularly simple interpretation in quark-parton model for this case...

$$\frac{d^{2}\sigma^{\nu(\nu)N}}{dxdy} = \frac{G_{F}^{2}s}{2\pi} \left\{ \left( 1 + (1-y)^{2} \right) F_{2}(x) \pm \left( 1 - (1-y)^{2} \right) x F_{3}^{\nu(\bar{\nu})}(x) \right\} \\ 2xF_{1}^{\nu(\bar{\nu})N,CC}(x) = x(u(x) + d(x) + \bar{u}(x) + \bar{d}(x) + s(x) + \bar{s}(x) + c(x) + \bar{c}(x) = xq(x) + x\bar{q}(x) \\ xF_{3}^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x) + xd_{Val}(x) \pm 2x(s(x) - c(x)) \\ \text{where } u_{Val}(x) = u(x) - \bar{u}(x) \right\}$$

# **Nuclear Effects in DIS**

- Well measured effects in charged-lepton DIS
  - Maybe the same for neutrino DIS; maybe not... all precise neutrino data is on Ca or Fe targets!
  - Conjecture: these can be absorbed into effective nucleon PDFs in a nucleus
     Anti-shadowing



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# From SFs to PDFs

- As you all know, there is a large industry in determining Parton Distributions
  - to the point where some of my colleagues on collider experiments might think of parton distributions as an annoying piece of FORTRAN code in their C++ software
- The purpose, of course, exactly related to Chris' point about factorization in his Friday lecture

Calculable in Perturbation Theory



 $\sigma(h_1(p_1) + h_2(p_2) \to Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{f_1, f_2} f_{f_1}(x_1) f_{f_2}(x_2) \sigma(f_1 + f_2 \to Y)$ 

The f<sub>fi</sub> s are "known" from Deep Inelastic Scattering.

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# From SFs to PDFs (cont'd)

We just learned that...

$$2xF_{1}^{\nu(\nu)N,CC}(x) = xq(x) + xq(x)$$
  

$$xF_{3}^{\nu(\nu)N,CC}(x) = xu_{Val}(x) + xd_{Val}(x) \pm 2x(s(x) - c(x))$$
  
where  $u_{Val}(x) = u(x) - u(x)$ 

In charged-lepton DIS

$$2xF_1^{\gamma p}(x) = \left(\frac{2}{3}\right)^2 \sum_{\substack{\text{up type quarks} \\ + \left(\frac{1}{3}\right)^2 \\ \text{down type quarks}}} q(x) + \overline{q}(x)} \overline{q}(x)$$

- So you begin to see how one can combine neutrino and charged lepton DIS and separate
  - the quark sea from valence quarks
  - up quarks from down quarks

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# DIS: Massive Quarks and Leptons

## **Opera at CNGS**



# Lepton Mass Effects in DIS

 Recall that final state mass effects enter as corrections:



- relevant center-of-mass energy is that of the "point-like" neutrinoparton system
- this is high energy approximation
- For  $\nu_\tau$  charged-current, there is a threshold of

$$s_{\min} = (m_{\text{nucleon}} + m_{\tau})^2$$
  
where

$$s_{initial} = m_{\text{nucleon}}^2 + 2E_{\nu}m_{\text{nucleon}}$$
$$\therefore E_{\nu} > \frac{m_{\tau}^2 + 2m_{\tau}m_{\text{nucleon}}}{2m_{\text{nucleon}}} \approx 3.5 \text{ GeV}$$

" $m_{\text{nucleon}}$ " is  $M_T$  elsewhere, but don't want to confuse with  $m_{\tau}$ ..



#### (Kretzer and Reno)

This is threshold for partons with *entire* nucleon momentum
effects big at higher E<sub>v</sub> also

#### **Touchstone Question #5:** What if Taus were Lighter?

- Imagine we lived in a universe where the tau mass was not 1.777 GeV, but was 0.888 GeV
- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?



# **Opera at CNGS**

Goal:  $v_{\tau}$  appearance • 0.15 MWatt source • high energy  $v_{\mu}$  beam 732 km baseline handfuls of events/yr x 10<sup>9</sup>  $P_{osc}^{*} \sigma_{\tau cc}^{*}$  (arbitrary units)  $m^{2})^{l}$ 0.4 GeV  $\Delta m^2 = 3 \ 10^{-3} eV^2$ v<sub>µ</sub> fluence at Gran Sasso (pot 0.35 0.3 0.25 0.2 0.15 v\_fluence 0.1 0.05 0

30 35 40

45 50

E (GeV)

ı mm PER 1.8kTon Pb **Emulsion** layers figures courtesy D. Autero what else is copiously produced in neutrino interactions with  $c\tau \sim 100 \mu m$ and decays to hadrons?

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# **Heavy Quark Production**

- Scattering from heavy quarks is more complicated.
  - Charm is heavier than proton; hints that its mass is not a negligible effect...

$$(q + \zeta p)^2 = p'^2 = m_c^2$$
$$q^2 + 2\zeta p \bullet q + \zeta^2 M^2 = m_c^2$$

Not your father's fractional momentu

(p)

 $\nu_{\mu}$ 

s.d

(q)

(ξp) (p')

Therefore 
$$\zeta \cong \frac{-q^2 + m_c^2}{2p \bullet q}$$

$$\zeta \cong \frac{Q^2 + m_c^2}{2M\upsilon} = \frac{Q^2 + m_c^2}{Q^2 / x}$$

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 $\zeta \cong x \left( 1 + \frac{m_c^2}{Q^2} \right)$ 

"slow rescaling" leads to kinematic suppression of charm production

# Neutrino Dilepton Events

- Neutrino induced charm production has been extensively studied
  - Emulsion/Bubble Chambers (low statistics, 10s of events).
     Reconstruct the charm final state, but limited by target mass.
  - "Dimuon events" (high statistics, 1000s of events)







# Neutrino Dilepton Events

- Rate depends on:
  - d, s quark distributions, |V<sub>cd</sub>|
  - Semi-leptonic branching ratios of charm
  - Kinematic suppression and fragmentation



## NuTeV Dimuon Sample



# QCD at Work: Strange Asymmetry?

- An interesting aside...
  - The strange sea can be generated perturbatively from g→s+sbar.
  - BUT, in perturbative generation the momenta of strange and anti strange quarks is equal
    - o well, in the leading order splitting at least. At higher order get a vanishingly small difference.
  - SO s & sbar difference probe non-perturbative ("intrinsic") strangeness
    - o Models: Signal&Thomas, Brodsky&Ma, etc.



(Brodsky & Ma, s-sbar)

0.8

(a)

(b)



# NuTeV's Strange Sea

- NuTeV has tested this
  - NB: very dependent on what is assumed about non-strange sea
  - Why? Recall CKM mixing...

$$\frac{V_{cd}d(x) + V_{cs}s(x) \rightarrow s'(x)}{V_{cd}\overline{d}(x) + V_{cs}\overline{s}(x) \rightarrow \overline{s}'(x)}$$

big

Using CTEQ6 PDFs...

small

 $\int dx \left[ x \left( s - \overline{s} \right) \right] = 0.0019 \pm 0.0005 \pm 0.0014$ c.f.,  $\int dx \left[ x \left( s + \overline{s} \right) \right] \approx 0.02$ 





#### Deep Inelastic Scattering: Conclusions and Summary

- Neutrino-quark scattering is elastic scattering!
  complicated by fact that quarks live in nucleons
- Important lepton and quark mass effects for tau neutrino appearance experiments
- Neutrino DIS important for determining parton distributions
  - particularly valence and strange quarks

# Neutrino-Nucleon Deep Inelastic Scattering Applied...

# DIS NC/CC Ratio

Experimentally, it's "simple" to measure ratios of neutral to charged current cross-sections on an isoscalar target to extract NC couplings



LlewellynSmith Formulae

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \left( \left( u_L^2 + d_L^2 \right) + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\nu)}} \left( u_R^2 + d_R^2 \right) \right)$$



Z-q coupling is  $I_3$ -Qsin<sup>2</sup> $\theta_W$ 

- Holds for isoscalar targets of u and d quarks only
  - Heavy quarks, differences between u and d distributions are corrections
- Isospin symmetry causes PDFs to drop out, even outside of naïve quark-parton model

### Touchstone Question #6: Paschos-Wolfenstein Relation



 If we want to measure electroweak parameters from the ratio of charged to neutral current cross-sections, what problem will we encounter from these processes?

#### Touchstone Question #6: Paschos-Wolfenstein Relation

The NuTeV experiment employed a complicated design to measure
 Paschos - Wolfenstein Relation

How did this help with the heavy quark problem of the previous question?

Hint: what to you know about the relationship of:

 $\sigma(vq)$  and  $\sigma(\overline{vq})$ 

 $R^{-} = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\nu}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\overline{\nu}}} = \rho^{2} \left(\frac{1}{2} - \sin^{2} \theta_{W}\right)$ 

# NuTeV Fit to R<sup>v</sup> and R<sup>vbar</sup>

• NuTeV result:

 $\sin^2 \theta_W^{(on-shell)} = 0.2277 \pm \pm 0.0013(stat.) \pm 0.0009(syst.)$  $= 0.2277 \pm 0.0016$ 

(Previous neutrino measurements gave  $0.2277 \pm 0.0036$ )

Standard model fit (LEPEWWG): 0.2227 ± 0.00037
 A 3<sub>o</sub> discrepancy...



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## NuTeV Electroweak: What does it Mean?



- If I knew, I'd tell you.
- It could be BSM physics. Certainly there are no limits on a Z' that could cause this. But why?
- It could be the asymmetry of the strange sea...
  - it would contribute because the strange sea would not cancel in
  - but it's been measured; not anywhere near big enough
- It could be very large isospin violation
  - if d<sub>p</sub>(x)≠u<sub>n</sub>(x) at the 5% level... it would shift charge current (normalizing) cross-sections enough.
  - no data to forbid it. any reason to expect it?

# Next Lecture: GeV cross-sections, application to $v_{\mu} \rightarrow v_{e}$ , other energy regimes