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Models of Neutrino Masses & Mixings

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Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103
G.A, R. Franceschini, hep-ph/0512202.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];
G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

Lecture 3

Survey of basic ideas on model building
Part 2: "Exceptional" models

- What if θ_{23} is really ~maximal and θ_{13}~0
- Tri-Bimaximal Mixing (HPS)

Harrison-Perkins-Scott

- HPS from A4

- Still large space for non maximal 23 mixing

2-σ interval $0.32 < \sin^2\theta_{23} < 0.62$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

Very small θ_{13} theoretically hard

"Normal" models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

"Exceptional" models: θ_{23} very close to maximal and/or θ_{13} very small
or: a special value for θ_{12}, or....

What if θ_{23} is really maximal?

Would be challenging!

All existing models invoke peculiar symmetries (typically non abelian, continuous or discrete)

Early models: Barbieri et al, Wetterich....

The most general mass matrix for $\theta_{13}= 0$ and θ_{23} maximal is given by (after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Grimus, Lavoura..., Ma,....

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Note it is symmetric under 2 <-> 3 or μ <-> τ exchange

Suggests a model based on 2 <-> 3 symmetry. Not easy!

Imposing a 2-3 perm. symmetry on $L^T m_\nu L$ does not work, because $\overline{R}$ L then produces a charged lepton mixing that spoils θ_{23} max.

For simplicity, a 2x2 model of μ <-> τ symmetry:

$$L^T L \to \begin{bmatrix} z & w \\ w & z \end{bmatrix} \qquad U_\nu^T \begin{bmatrix} z & w \\ w & z \end{bmatrix} U_\nu = \begin{bmatrix} z-w & 0 \\ 0 & z+w \end{bmatrix} \qquad U_\nu = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$m' = V^\dagger m U_e$$

$$m'^\dagger = U_e^\dagger m^\dagger V$$

$$m'^\dagger m' = U_e^\dagger m^\dagger m U_e$$

Charged lepton mass:

$$\bar{R}L \to \begin{bmatrix} a & a \\ b & b \end{bmatrix} \sim m$$

Note: eignvls 0,2

$$m^\dagger m = \begin{bmatrix} a^* & b^* \\ a^* & b^* \end{bmatrix} \cdot \begin{bmatrix} a & a \\ b & b \end{bmatrix} = (|a|^2 + |b|^2) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad U_e = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Thus $U_{P\text{-}NMS} = U_e^+ U_\nu = 1$ and there is no mixing!!

For example, in this SU(5) model the μ <-> τ symmetry on L is badly broken

Mohapatra, Nasri, Hai-Bo Yu '06

F ~ $\overline{5}$ (contains L)
T ~ 10 h ~ 5
S ~ 15 H ~ 45

$$F_\mu \leftrightarrow F_\tau$$
$$(h, \bar{h}) \leftrightarrow (h, \bar{h})$$
$$(H, \bar{H}) \leftrightarrow (-H, -\bar{H})$$

Superpotential: $W = Y_{15} FFS + Y_5 TTh + Y_5 TF\bar{h} + Y_{45} TFH.$

$$M_\nu = Y_{15} < S > = \begin{bmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{bmatrix}$$

$$M_e = Y_5^T < \bar{h} > -3Y_{45}^T < H > = \begin{bmatrix} A_1 & E_1 & E_1 \\ B_1 & D_1 & D_1 \\ C_1 & F_1 & F_1 \end{bmatrix} - 3 \begin{bmatrix} 0 & E_2 & -E_2 \\ 0 & D_2 & -D_2 \\ 0 & F_2 & -F_2 \end{bmatrix}$$

μ <-> τ symmetry breaking

As a result, for parameter choices that fit masses, θ_{23} is not necessarily too close to maximal

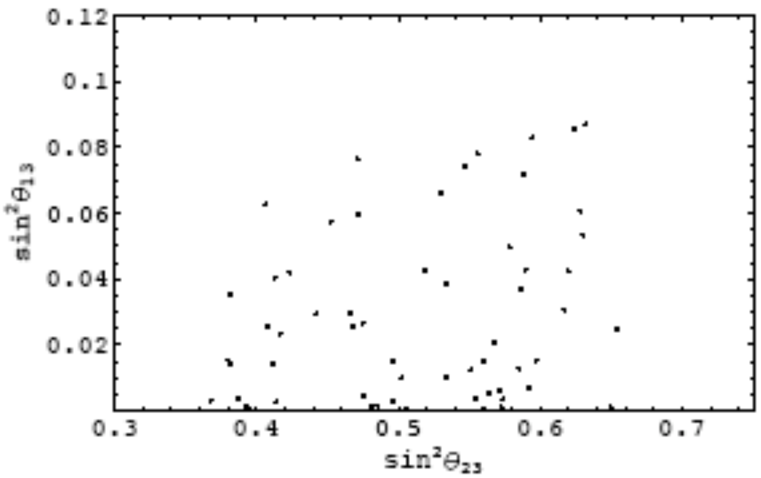
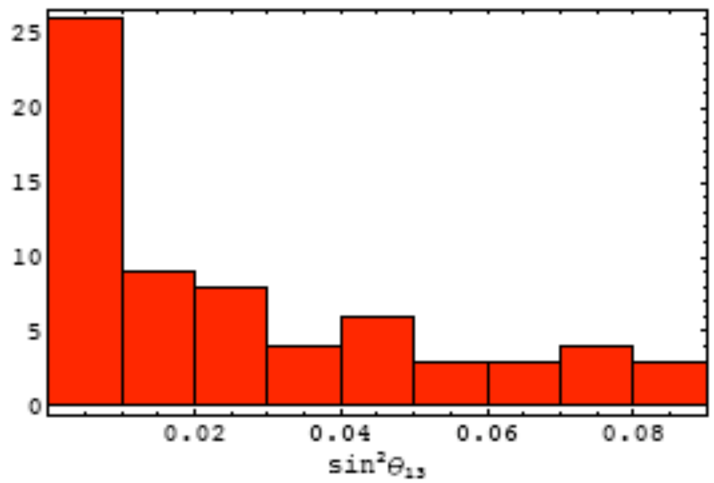


FIG. 1: Scatter plot in the $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ plane.

and $\theta_{13} \sim o(\lambda_C^2)$



Finally, it looks like a "normal" model!

Similarly a symmetry $\nu_{\mu R}$ <-> $\nu_{\tau R}$ is not sufficient:

Majorana: $M_{RR}^{-1} = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$

Dirac: $m_\nu^D \sim \bar{R}L \to \begin{bmatrix} a & b \\ a & b \end{bmatrix}$

See-saw: $m_\nu = m_\nu^{DT} M_{RR}^{-1} m_\nu^D = \begin{bmatrix} a & a \\ b & b \end{bmatrix} \begin{bmatrix} x & y \\ y & x \end{bmatrix} \begin{bmatrix} a & b \\ a & b \end{bmatrix} = (x+y) \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$

Not μ <-> τ symmetric
Mixing not maximal

Clearly a more elaborate broken symmetry is needed:
In some models, discrete broken symmetries are used to make charged leptons diagonal and Dirac ν masses diagonal and μ <-> τ symmetric, while the perm. symmetry is in the Majorana RR matrix

e.g. Grimus, Lavoura

An interesting particular case

Harrison, Perkins, Scott

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

$$U=\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

Fogli et al '05

$\sin^2\theta_{12} = 1/3$: 0.290-0.342

$\sin^2\theta_{23} = 1/2$: 0.39-0.53

$\sin^2\theta_{13} = 0$: < 0.02

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

Two extremes:

- In anarchy all ν mixing angles and mass ratios are random. Apparent hierarchies are from fluctuations

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

It is interesting to construct models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) are very interesting

Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103

Alternative models based on $SU(3)_F$ or $SO(3)_F$

Verzielas, G. Ross King

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has 4!/2 = 12 elements.

An element is abcd which means 1234 --> abcd

C_1:	1 = 1234			
C_2:	T = 2314	ST = 4132	TS = 3241	STS = 1423
C_3:	T^2 = 3124	ST^2 = 4213	T^2S = 2431	TST = 1342
C_4:	S = 4321	T^2ST = 3412	TST^2 = 2143	

Thus A4 transf.s can be written as:

1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

C_1, C_2, C_3, C_4 are equivalence classes

x, x' in same class if [$x' \sim gxg^{-1}$] g: group element

Three singlet inequivalent represent'ns:

Recall:

$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: \; S=1,\, T=1 \\ 1': \; S=1,\, T=\omega \\ 1'': \; S=1,\, T=\omega^2 \end{cases}$$

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\omega^3 = 1$$

$$1 + \omega + \omega^2 = 0$$

$$\omega^2 = \omega^*$$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1', 1"

In the (S-diag basis) consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

(a_1,a_2,a_3) $\xrightarrow{T}$ (a_3,a_1,a_2)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3 \qquad 3 \sim (a_2 b_3, a_3 b_1, a_1 b_2)$$

$$1' = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \qquad 3 \sim (a_3 b_2, a_1 b_3, a_2 b_1)$$

$$1'' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

e.g. $1' = a_1 b_1+\omega a_2 b_2+\omega^2 a_3 b_3 \xrightarrow{T} a_3 b_3+\omega a_1 b_1+\omega^2 a_2 b_2 =$
$= \omega\,[a_1 b_1+\omega a_2 b_2+\omega^2 a_3 b_3]$

while 1' is inv. under S

Under A4

lepton doublets l ~ 3
e^c, μ^c, τ^c ~ 1, 1′, 1″ respectively
gauge singlet flavons ϕ, ϕ', ξ, (ξ') ~ 3, 3, 1,(1) respectively
driving fields (for SUSY version) ϕ_0, ϕ'_0, ξ_0 ~ 3, 3, 1

Additional symmetries: broken $U(1)_F$ symmetry (ch. lepton masses) with e^c, μ^c, τ^c charges (3 or 4,2,0)
and a discrete symmetry (dep. on versions) : for example
Z: (e^c, μ^c, τ^c)-> -i (e^c, μ^c, τ^c), l -> il, ϕ -> ϕ, (ξ,ϕ') -> - (ξ,ϕ')

The Yukawa interactions in the lepton sector are:

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + ...$$

⊕ Here is without see-saw (with see-saw is also OK: wait!)

Structure of the model

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

$$\langle\varphi'\rangle = (v', 0, 0)$$
$$\langle\varphi\rangle = (v, v, v)$$
$$\langle\xi\rangle = u$$

$$m_l = v_d \frac{v}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu \omega^2 & y_\mu \omega \\ y_\tau & y_\tau \omega & y_\tau \omega^2 \end{pmatrix}$$

Spectrum free.
Diagonalized by U_e:

the big plus of A4

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

$$l \rightarrow \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} l = Vl = l_{diag}$$

⊕ From here it follows that U_{HPS} is the mixing matrix

m_ν in the basis of diagonal charged leptons is:

$$m_\nu|_{ldiag} \sim V^* \begin{bmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{bmatrix} V^* = \begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

which in turn can be written as:

$$m_\nu|_{ldiag} \sim U^T \begin{bmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{bmatrix} U$$

with:

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The crucial issue is to guarantee the strict alignment

$$\begin{aligned}\langle\varphi'\rangle &= (v',0,0)\\ \langle\varphi\rangle &= (v,v,v)\\ \langle\xi\rangle &= u\end{aligned}$$

We have constructed two completely natural versions of the model:

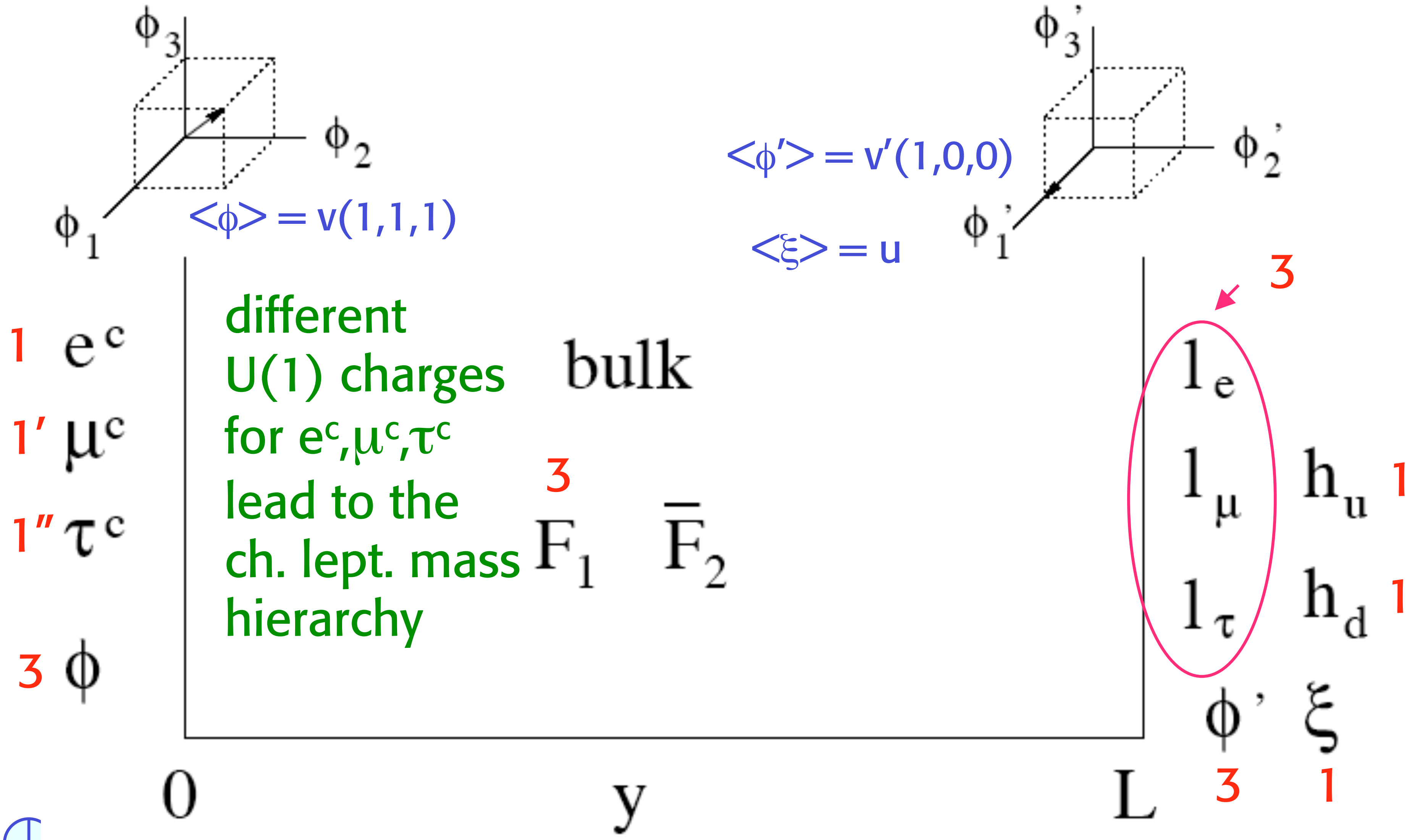
- a version in 5 dimensions
- a SUSY version in 4-dim (with more fields)

We first briefly discuss the 5-dim version

GA, Feruglio hep-ph/0504165

The model has 1 compactified extra dim. and 2 branes

(crucial issue: guarantee and protect the vev alignment)



In lowest approximation the action is:

$$
\begin{aligned}
S &= \int d^4x dy \Big\{ \Big[iF_1\sigma^\mu\partial_\mu \overline{F}_1 + iF_2\sigma^\mu\partial_\mu \overline{F}_2 + \frac{1}{2}(F_2\partial_y F_1 - \partial_y F_2 F_1 + h.c.) \Big] \\
&- M(F_1F_2 + \overline{F}_1\overline{F}_2) \\
&+ V_0(\varphi)\delta(y) + V_L(\varphi', \xi)\delta(y-L) \\
&+ [Y_e e^c(\varphi F_1) + Y_\mu \mu^c(\varphi F_1)'' + Y_\tau \tau^c(\varphi F_1)' + h.c.]\,\delta(y) \\
&+ \Big[\frac{x_a}{\Lambda^2}\xi(ll)h_u h_u + \frac{x_d}{\Lambda^2}(\varphi' ll)h_u h_u + Y_L(F_2 l)h_d + h.c. \Big] \delta(y-L) \Big\} + \ldots
\end{aligned}
$$

a Z-parity has also been imposed

$$
(f^c, l, F, \varphi, \varphi', \xi) \xrightarrow{Z} (-if^c, il, iF, \varphi, -\varphi', -\xi)
$$

After integrating out of the F fields one obtains the required effective 4-dim action

$$
\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots
$$

In the flavour basis:

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

$m_\nu = U\,\mathrm{diag}(a+d,a,-a+d)U^T$ (in units of v_u^2/Λ) and $U=U_{HPS}$

In terms of physical param.s (moderate normal hierarchy):

$$|m_1|^2 = \left[-r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_2|^2 = \frac{1}{8\cos^2\Delta(1-2r)}\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_3|^2 = \left[1 - r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.053\ \mathrm{eV})^2$$

- A moderate fine tuning is needed for r

A version with see-saw is also possible

ν_R is a triplet of A4: $\nu^c \sim 3$ No change for ch leptons

Dirac Majorana

$$w_l = \ldots + y(\nu^c l) + x_A \xi(\nu^c \nu^c) + x_B(\varphi_T \nu^c \nu^c)$$

[Discrete parity Z: $\omega, \omega^2, \omega^2, \omega^2$ for l, ν^c, ϕ_T, ξ respectively]

$m_\nu^D \sim 1$ $M_{RR} \sim \begin{bmatrix} A & 0 & 0 \\ 0 & A & D \\ 0 & D & A \end{bmatrix}$ $m_\nu = m_\nu^{DT} M_{RR}^{-1} m_\nu^D \sim M_{RR}^{-1}$

The mass matrix appears just as the inverse of what was before, so that the mixing matrix is the same.

Eigenvalues are the inverse: one can produce inverse hierarchy with realistic θ_{12}, θ_{23} and very small θ_{13}

The model crucially depends on the precise vev alignment

$$\langle \varphi' \rangle = (v', 0, 0)$$
$$\langle \varphi \rangle = (v, v, v)$$
$$\langle \xi \rangle = u$$

The extra dimension with 2 branes allows the decoupling of the ϕ and ξ,ϕ' potentials.

A discrete symmetry is also essential: a separate continuous rotation symmetry on the 2 branes would make any disalignment illusory.

An alternative in 4 dimensions is a SUSY model with driving fields and a superpotential where all terms allowed by symmetry are present (with added fields ξ', ϕ_0, ϕ'_0, ξ_0).

In our models

- all terms allowed by symmetry are present
- all correct'ns are under control and can be made negligible

The 4-dim SUSY version (written in the T-diag basis)

In this basis the ch. leptons are diagonal!

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + ...$$

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

$$\begin{aligned} w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\ &+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2 \end{aligned}$$

and the potential $$V = \sum_i \left|\frac{\partial w}{\partial \phi_i}\right|^2 + m_i^2 |\phi_i|^2 + ...$$

The assumed simmetries are summarised here

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2
$U(1)_F$		2q	q	1								

The driving field have zero vev. So the minimization is:

$$\frac{\partial w}{\partial \varphi_{01}^T} = M\varphi_{T1} + \frac{2g}{3}(\varphi_{T1}^2 - \varphi_{T2}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{01}^S} = g_2\tilde{\xi}\varphi_{S1} + \frac{2g_1}{3}(\varphi_{S1}^2 - \varphi_{S2}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{02}^T} = M\varphi_{T3} + \frac{2g}{3}(\varphi_{T2}^2 - \varphi_{T1}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{02}^S} = g_2\tilde{\xi}\varphi_{S3} + \frac{2g_1}{3}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{03}^T} = M\varphi_{T2} + \frac{2g}{3}(\varphi_{T3}^2 - \varphi_{T1}\varphi_{T2}) = 0 \qquad \frac{\partial w}{\partial \varphi_{03}^S} = g_2\tilde{\xi}\varphi_{S2} + \frac{2g_1}{3}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\tilde{\xi} = 0$$

$$\xi = u$$

$$\varphi_S = (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2$$

In the paper w at NLO is also studied

NLO corrections studied in detail

to m_l — 1st non trivial correction at $o(1/\Lambda^3)$

LO is $1/\Lambda$

to m_ν

LO is $1/\Lambda^2$

$$\frac{x_c}{\Lambda^3}(\varphi_T\varphi_S)'(ll)''h_uh_u \qquad \frac{x_d}{\Lambda^3}(\varphi_T\varphi_S)''(ll)'h_uh_u \qquad \frac{x_e}{\Lambda^3}\xi(\varphi_T ll)h_uh_u$$

to vevs

LO is 1

$$\begin{aligned}
\langle\varphi_T\rangle &\rightarrow (v_T' + \delta v_T, \delta v_T, \delta v_T) \\
\langle\varphi_S\rangle &\rightarrow (v_S + \delta v_1, v_S + \delta v_2, v_S + \delta v_3) \\
\langle\xi\rangle &\rightarrow u \\
\langle\tilde{\xi}\rangle &\rightarrow \delta u'
\end{aligned}$$

δv_T, δv_S, δv_i, $\delta u' \sim o(1/\Lambda)$

All observables get a correction of order $1/\Lambda$

From exp (eg θ_{12}) must be less than 5%

$$0.0022 < \frac{v_S}{\Lambda} \approx \frac{v_T}{\Lambda} \approx \frac{u}{\Lambda} < 0.05$$

In particular $\theta_{13} < \sim 0.05$,
$|tg^2\theta_{23} - 1| < \sim 0.05$

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices are BOTH diagonalised by

$$U_u, U_d \sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

As a result VCKM is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$). But even those are essentially the same for u and d quarks)

Note: it not possible to embed this in a GUT:
with these assignments A4 does not commute with SU(5)

If l ~ 3 then all 5bar ~3, so that d^c_i ~ 3
if e^c, μ^c, τ^c ~ 1, 1', 1" then all 10_i ~ 1, 1', 1"

Realistic quark mass matrices are not easy to obtain from these assigments

For example, for u quarks at leading order:

$$m_u \sim 1.1 + 1'.1'' + 1''.1' \sim a\, u_1 u_1 + b\, (u_2 u_3 + u_3 u_2)$$

or

$$m_u \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$$

Which implies $|m_c| = |m_t|$
and maximal U_{23}

Conclusion

From experiment: a good first approximation for quarks:

$$V_{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

All this is highly non trivial but no real illumination has followed!!

Many models can reproduce the data.

"Normal" models are simplest.

Normal models: θ_{23} large but not maximal, θ_{13} not too small (θ_{13} of order λ_C or λ_C^2 vs θ_{12}, θ_{23} ~o(1))

- Semi anarchy
- Inverse hierarchy

In particular

- Normal hierarchy with suppressed 23 determinant

Well compatible with GUT's (simplest: $SU(5)XU(1)_F$ but SO(10) models are also viable)

Exceptional models: θ_{23} maximal or θ_{13} very small or also: all mixing angles fixed as in HPS (intriguing!!) are peculiar but worth studying (eg A4).