Dynamics and Relativity
Problem Sheet 10: Relativistic collisions (Elastic and Inelastic)

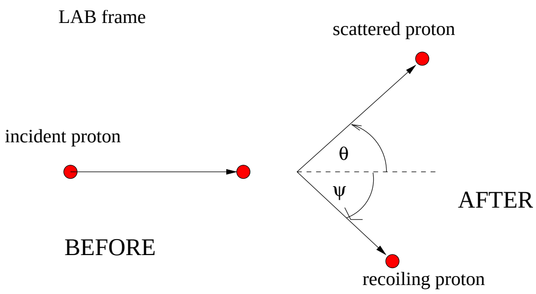
1. Using the CM frame, consider the elastic scattering (in the absence of external forces) of two particles of mass m_1 and m_2.

   (a) Show that mass is conserved in elastic relativistic scattering.

   The incident particles emerge intact after scattering, and we denote their initial and final 3-momenta as $\pm\underline{p}^*$ and $\pm\underline{q}^*$.

   (b) Show that $q^* = p^*$. If $m_1 = m_2 (= m)$, show that the energies of the particles are $E^* = \sqrt{(mc^2)^2 + (p^*c)^2}$ both before and after the collision.

2. After an elastic collision between two protons, the LAB frame scattering and recoil angles are θ and ψ (see diagram).



   (a) Write down the 4-momenta of the two protons before and after the collision using (i) the LAB frame, and (ii) the CM frame.

   (b) Show that θ and ψ satisfy

$$\tan\theta\,\tan\psi = \frac{1}{\gamma(v_{cm})^2}$$

   where v_{cm} is the speed of the CM frame in the lab. Compare this with the non-relativistic result.

3. Consider an elastic collision between two electrons in the CM frame. If the scattering angle is $\theta^* = 45°$ and the electron energies are $E^* = 1$ MeV, determine the LAB frame energy and direction of the electrons after scattering.
   (Assume the rest energy of the electron is 0.5 MeV.)

4. Two particles, with masses m_1 and m_2, move colinearly in an inertial frame S with uniform speeds u_1 and u_2 respectively. On impact they coalesce to form a single particle.

   (a) Show u_3, the speed of the resulting single particle, is given by

$$u_3 = \frac{m_1\gamma(u_1)u_1 + m_2\gamma(u_2)u_2}{m_1\gamma(u_1) + m_2\gamma(u_2)}.$$

   [**Hint:** take the direction of motion to be the x-direction, write down the components of 4-momentum for each particle and use conservation of 4-momentum.]

(b) Show m_3, the mass of the resulting particle, satisfies

$$m_3^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma(u_1)\gamma(u_2)\left[1 - u_1 u_2/c^2\right].$$

[**Hint:** Again, the equations given by 4-momentum conservation are needed. The algebra is greatly simplified by evaluating $\underline{\underline{P}} \cdot \underline{\underline{P}}$, where $\underline{\underline{P}}$ is the total 4-momentum of the system, which is a conserved quantity.]

5. Proton-proton collisions can be used to produce new particles such as pions and antiprotons. For collisions in the LAB frame, calculate the threshold energy (i.e. the energy of the minimum energy of the incident protons) for the reactions:
(a) $p + p \rightarrow p + p + \pi$
(b) $p + p \rightarrow p + p + p + \bar{p}$.
The particle masses are: $m_p = m_{\bar{p}} = 940$ MeV/c^2, and $m_\pi = 135$ MeV/c^2.

6. For the cases listed below, evaluate the ratio λ_c/L where $\lambda_c = h/mc$ and L is the confinement length:

   (a) An electron (mass 0.5 MeV/c^2) in an atom ($L \sim 10^{-10}$ m)

   (b) A proton (mass 935 MeV/c^2) in a nucleus ($L \sim 10^{-14}$ m)

   (c) An "up" quark (mass 2.4 MeV/c^2) in a proton ($L \sim 10^{-15}$ m).

   Comment on your results in the light of Eq. 15.15

$$\frac{T_{min}}{E_{rest}} = \sqrt{1 + \left(\frac{\lambda_c}{L}\right)^2} - 1.$$