Dynamics and Relativity: Hand-in exercise 3

Please hand in your script at the Teaching Office (4315 JCMB) before 12.00 on Thursday 15th December 2011

The Dynamics and Relativity exam will comprise two sections, A and B. There are 4 A questions, and all should be attempted. Section A questions are intended to take about 7.5 minutes each. Section B questions should take approximately 30 minutes each, and the best two from three will count.

Below you will find three A type questions and two B questions, all taken from last year's exam paper. For the 3rd hand-in assessment please answer all questions.

Some Section A Questions

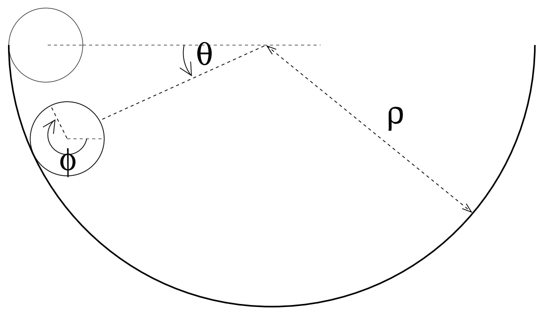
1. At $t = 0$ a projectile is launched from the surface of the earth with speed u at an angle α to the horizontal (which we take as the x-axis of frame S, the rest frame of the Earth). Its S frame position for all subsequent time t is given by:

$$x = ut\cos\alpha, \qquad y = ut\sin\alpha - \frac{1}{2}gt^2.$$

S' is a reference frame which moves with speed u in the initial direction of the particle, while frame S'' falls freely in the Earth's gravitational field. The three frames coincide at $t = 0$ and their Cartesian axes remain parallel throughout.

Find the co-ordinates of the projectile in S' and S'', and describe its motion in these frames.

2. A hoop of radius a and mass m rolls down a surface of semi-circular cross-section with radius ρ, as shown below. θ measures the angle through which hoop moves around the semi-circle, and ϕ measures the rotation of the hoop about its own centre.



(i) Remembering that the hoop rolls, state the relationship between θ and ϕ.

(ii) Given that its moment of inertia is ma^2, express the Lagrangian of the hoop in terms of θ.

(iii) Hence find its equation of motion.

[You may recall that the Euler-Lagrange equation can be written

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = \frac{\partial \mathcal{L}}{\partial q_i}$$

where $\mathcal{L}$ is the Lagrangian and the q_i are generalized co-ordinates.]

3. A planet of mass 1.9×10^{27} kg and radius 7.1×10^{7} m rotates about the axis through its poles. The magnitude of the centrifugal force on an object stationary at the equator of the planet is 0.086 times that of the gravitational force.

Find the period of rotation of the planet.

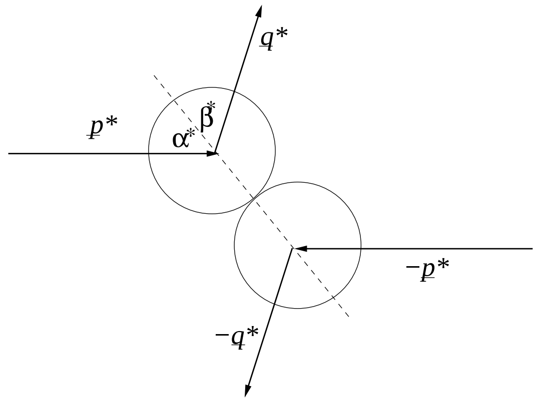
[Newton's gravitational constant is $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}.]

Some B Questions

1. (a) With the aid of a diagram, explain what is meant by the impact parameter, b, the scattering angle, θ, the recoil angle, ψ, and the differential cross-section, $d\sigma/d\Omega$.

(b) What is the relationship between $d\sigma/d\Omega$, θ and b?

(c) Consider a (non-relativistic) collision between two identical smooth hard spheres using the Centre of Mass (CM) frame, as illustrated below.



(i) Using appropriate conservation laws, show that the angle of incidence, α^*, equals the angle of reflection, β^*.

(ii) Find the differential cross-section for this system.

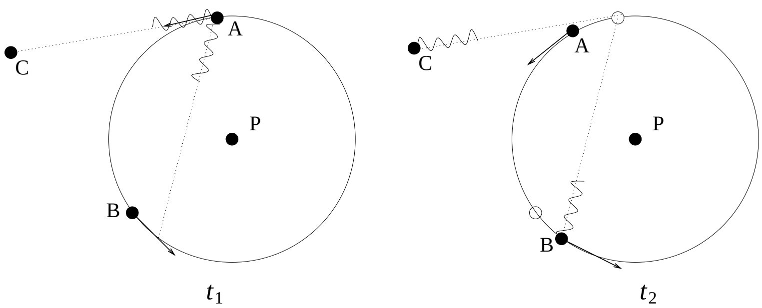
(d) Now consider an elastic collision between two high energy protons. In the CM frame their initial kinetic energy is equal to their rest energy.

(i) Find the initial speed of the protons and their 4-momenta.

(ii) The CM frame scattering angle is $\theta^* = \pi/4$. Find the post-collision 4-momentum of the incident proton in the CM frame.

(iii) Now find the post-collision energy of the incident proton in the *Lab* frame.

2. Two spaceships, A and B, share a circular orbit around a planet, P, as shown below. A third spaceship, C, is at rest in the rest frame of the planet.



At time t_1 light is emitted by A. At time t_2 photons are detected at B and C.

(a) Show that the speeds of A and B must be the same.

(b) State what is meant by the term *4-vector*.

(c) Consider a particle with 4-momentum $\underline{\underline{p}}$ and an observer with 4-velocity $\underline{\underline{U}}$ in the rest frame of the planet. Show that the energy of the particle *in the rest frame of the observer* is equal to $-\underline{\underline{U}}.\underline{\underline{p}}$.

(d) Hence, or otherwise, find the Doppler frequency shift of the photon detected by C. (You may assume that the line between A and C at the moment of light emission is a tangent of the orbit.)

(e) Find the Doppler frequency shift of the photon detected by B.

(f) Explain why the paths followed by the photons are not perfectly straight.

Does this affect your answers to part (d)?