

College of Science and Engineering
School of Physics



Thermodynamics

SCQF Level 9, U01359, PHY-3-Thermo

Wednesday 27th April, 2005
9.30 – 11.30 a.m.

Chairman of Examiners
Professor M Cates

External Examiner
Professor P Main

Answer **ALL** of the questions in **Section I**
and **TWO** questions from **Section II**

**The bracketed numbers give an indication of the value assigned
to each portion of a question.**

Only the supplied Electronic Calculators may be used during this examination.

ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF
THIS EXAMINATION.

Section I

Answer **ALL** the questions from this Section

1. The pressure and volume of an ideal gas undergoing a reversible adiabatic process are related by $pV^\gamma = \text{constant}$. Define γ . Assuming that this quantity is a constant, show that the temperature and pressure in such a process are related by

$$\frac{T}{P^{(\gamma-1)/\gamma}} = \text{constant} . \quad [5]$$

2. State the most general, differential form of the first law of thermodynamics, carefully defining your symbols and sign conventions. Explain the conditions under which and the substances for which the expression you have written down can be combined with the second law to give

$$dU = TdS - PdV . \quad [5]$$

3. Derive the following relationship:

$$\left(\frac{\partial T}{\partial P} \right)_S = \frac{V\beta_P T}{C_P} ,$$

where β_P is the constant-pressure thermal expansivity and C_P is the constant-pressure heat capacity. [5]

4. Explain what is meant by the *equation of state* (EOS) of a substance. The EOS of n moles of a simple fluid can be written in the form of a ‘virial expansion’:

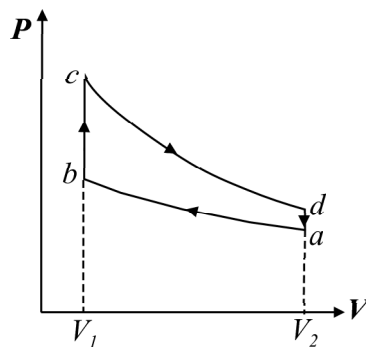
$$\frac{P}{RT} = \frac{n}{V} \left[1 + B_2 \left(\frac{n}{V} \right) + B_3 \left(\frac{n}{V} \right)^2 + \dots \right] ,$$

where the coefficients B_i are temperature-dependent. Explain under what circumstances the second and higher terms inside the square brackets on the right hand side of this equation can be neglected, and interpret the form of the equation of state under these circumstances. [5]

Section II

Answer **TWO** of the questions from this **Section**

5. (a) Explain the difference between exact and inexact differentials, and give a thermodynamic example of each. [4]
- (b) Explain carefully the difference between a reversible and an irreversible process. [4]
- (c) A gas is confined to one half of a thermally isolated container by a dividing wall. The other half is evacuated. A hole is punctured in this wall, allowing the gas to expand freely to fill the whole container.
- (i) Why is there no change in the internal energy of the gas after such a ‘free expansion’? [3]
- (ii) Why is there a net entropy change even though no heat enters or leaves the system? [4]
- (iii) Consider the case of an ideal gas. What is the temperature change? By devising a suitable reversible path linking the initial and final states of the free expansion process, calculate the net entropy change. [5]
6. (a) Clausius’s statement of the second law of thermodynamics is that no process is possible whose sole result is the transfer of heat from a cold to a hot source. Give the statement of the second law due to Kelvin. [4]
- (b) By considering a ‘composite’ heat engine, comprising two separate engines, show that if Clausius’s statement is false so is that due to Kelvin. [6]
- (c) An ideal gas is taken through the reversible cycle shown in the following diagram (the ‘Otto cycle’), where ab and cd are adiabatics. The temperatures at points a, b, c, d are, respectively, T_a, T_b, T_c and T_d .

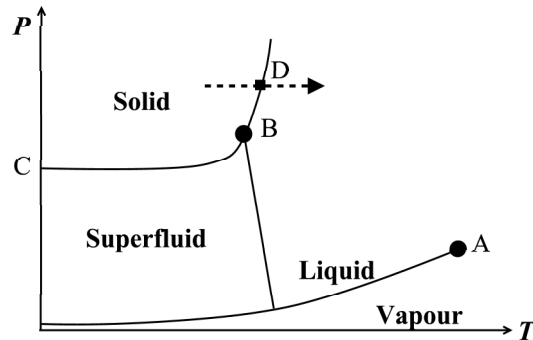


- (i) Briefly describe the working cycle. In particular, identify in your description the processes during the cycle in which heat flows in/out of the system, and *give expressions for these heat flows* in terms of the appropriate heat capacity. Also, *give an expression* for the net work output per cycle, and identify this quantity with the appropriate feature on the P - V diagram. [7]

- (ii) Show that the theoretical efficiency of the Otto cycle is given by

$$1 - \frac{T_d - T_a}{T_c - T_b}. \quad [3]$$

7. The figure shows a schematic representation of the pressure-temperature (P - T) phase diagram of helium.



- (a) Explain the significance of the points marked by A and B . [4]

- (b) Briefly describe the significance of the fact that the phase boundary between the superfluid and solid phases shows $dP/dT = 0$ at the point C . [4]

- (c) Upon heating at constant pressure along the dotted arrow, a first order phase transition occurs at the point D , which is a point on the fluid-solid phase boundary.

- (i) Explain briefly what is meant by a *first order* phase transition. [3]

- (ii) Sketch (on a single diagram) the Gibbs' function of the fluid and solid phases of helium as a function of temperature along the dotted line, indicating where point D occurs. [4]

- (iii) The Clausius-Clapeyron equation gives the slope on this phase boundary

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V}.$$

Explain the significance of the two quantities on the right hand side of this equation. In particular, relate ΔS to the plots you have sketched in part (ii) above. [5]