College of Science and Engineering
School of Physics and Astronomy

Thermodynamics

SCQF Level 9, U01359, PHY-3-Thermo

Thursday 21st May, 2009

2.30 – 4.30 p.m.

Chairman of Examiners
Professor R D Kenway

External Examiner
Professor M Green

Answer ALL of the questions in Section A and TWO questions from Section B.

The bracketed numbers give an indication of the value assigned to each portion of a question.

Only the supplied Electronic Calculators may be used during this examination.

ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF THIS EXAMINATION.

Section A: Answer ALL of the questions in this Section

A.1. What is the definition of 'heat' in classical thermodynamics? Give three processes by which heat can be transferred between bodies. **[5]**

A.2. Carnot's theorem states that no closed cycle heat engine working between two fixed temperature reservoirs can be more efficient than a Carnot engine. Show that if such a heat engine could be constructed it would violate the Clausius or Kelvin-Planck statement of the second law of thermodynamics. **[5]**

A.3. State the 3rd law of thermodynamics and explain how it implies that the thermal expansion coefficient

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

must ultimately tend to zero as temperature is decreased towards 0 K. **[5]**

A.4. A working substance is taken once around a <u>closed</u> cycle that involves performing some external work. It absorbs heat, $Q_1 = 1000\,\mathrm{J}$, from a reservoir at $T_1 = 1000\,\mathrm{K}$ and heat $Q_2 = 2000\,\mathrm{J}$ from a reservoir at $T_2 = 2000\,\mathrm{K}$. The only other exchange of heat is with a reservoir at $T_3 = 300\,\mathrm{K}$.

What is the **minimum** heat that must have been transferred from the substance to the reservoir at T_3 (explain your reasoning)? **[5]**

Section B: Answer TWO of the questions in this Section

B.1. This question examines Joule-Kelvin (throttling) expansion.

In a Joule-Kelvin expansion a gas at initial temperature T_1 is pushed through a porous plug from a high pressure chamber maintained at a constant pressure P_1 into a low pressure chamber maintained at constant pressure P_2, both chambers being thermally isolated.

a. Explain why the enthalpy of the gas is unchanged in such a process. **[4]**

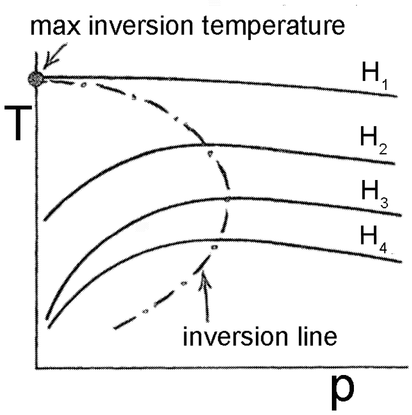

Figure 1: 4 isenthalps labelled H_1 to H_4 (solid lines) for a fluid as a function of temperature and pressure. The inversion line is shown dashed.

b. Figure 1 shows different lines of constant enthalpy (isenthalps) for a fixed mass of gas. Explain the significance of the curve labelled 'inversion line'. **[2]**

c. Show that the gradient of the isenthalps in figure 1 is

$$\left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{C_p}\left(T\left(\frac{\partial V}{\partial T}\right)_p - V\right).$$

C_p is the isobaric heat capacity and V is the volume of gas. **[6]**

d. Show that at any point on the inversion line

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{V}{T}.$$

[2]

e. For helium gas the maximum inversion temperature is 45 K. What consequence does this have for liquifying helium with Joule-Kelvin expansion starting with helium gas at room temperature. **[2]**

f. For the Van der Waals equation of state $(p + a/V^2)(V - b) = RT$ the maximum inversion temperature can be shown to be $RT_{\text{inv max}} = 2a/9b$. From this result and your knowledge of the origin of the constants a and b comment on how the maximum inversion temperature might be expected to change going down the series of noble gases (He, Ne, Ar, Kr, Xe). **[4]**

B.2. This question investigates a material that undergoes a discontinuous change of volume with temperature.

a. Starting from the statement that total entropy ($S_{\text{system}} + S_{\text{surroundings}}$) can only increase show that $G = U - TS + pV$ will attain its minimum value for a system in equilibrium with a fixed pressure and temperature reservoir. **[7]**

b. At atmospheric pressure a particular substance is found to undergo a discontinuous change between two states at temperature T_C when heated. Its volume increases by ΔV and it absorbs latent heat L as its temperature is changed from just below T_C to just above T_C. Explain why at T_C the value of G is the same for the two states with different volumes. **[4]**

c. Derive the following equation relating how T_C changes with pressure.

$$\frac{dT_C}{dP} = \frac{\Delta V T_C}{L} \, .$$

[6]

d. Explain briefly why L must be positive and comment on whether T_C is expected to increase or decrease with pressure. **[3]**

B.3. The speed of sound in a gas c is given in terms of pressure P and mass density ρ by

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_S .$$

a. Show that

$$\left(\frac{\partial P}{\partial \rho}\right)_S = -\frac{V^2}{\bar{M}}\left(\frac{\partial P}{\partial V}\right)_S$$

with V the molar volume and $\bar{M}$ the molar mass. [**3**]

b. Derive the following relation

$$\left(\frac{\partial P}{\partial V}\right)_S = -\frac{c_p}{c_v}\frac{K_T}{V}.$$

c_v and c_p are the constant volume and constant pressure molar heat capacity of the gas and $K_T = -V\left(\frac{\partial P}{\partial V}\right)_T$ is the bulk modulus. [**5**]

c. Show that for an ideal gas

$$K_T = \frac{\rho R T}{\bar{M}}.$$

[**3**]

d. Show that for an ideal gas $c_p - c_v = R$. [**5**]

e. Hence estimate the speed of sound in air at 300 K assuming air to be an ideal diatomic gas (for a diatomic gas $c_v = (5/2)R$) and $\bar{M} = 0.029$ Kg. [**4**]