College of Science and Engineering
School of Physics and Astronomy

Thermodynamics

SCQF Level 9, U01359, PHY-3-Thermo

Monday 3rd May, 2010

2.30 – 4.30 p.m.

Chairman of Examiners
Professor R D Kenway

External Examiner
Professor M Green

Answer ALL of the questions in Section A and TWO questions from Section B.

The bracketed numbers give an indication of the value assigned to each portion of a question.

Only the supplied Electronic Calculators may be used during this examination.

ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF THIS EXAMINATION.

Section A: Answer ALL of the questions in this Section

A.1. What is the defining property of (a) an adiabatic and (b) a diathermal boundary wall? Briefly describe a practical example of each. [**5**]

A.2. What is the definition of heat capacity C_V? Explain how a measurement of C_V can be used to determine the difference in entropy between equal volume equilibrium states at different temperatures. [**5**]

A.3. An infinitesimal change in internal energy of a dielectric particle is given by $dU = TdS + Ed\mathcal{P}$ with E an applied electric field and $\mathcal{P}$ the induced electric dipole moment of the particle. Derive the following relation between partial derivatives explaining carefully your reasoning,

$$\left(\frac{\partial E}{\partial S}\right)_{\mathcal{P}} = \left(\frac{\partial T}{\partial \mathcal{P}}\right)_{S}.$$

[**5**]

A.4. A heat pump delivers 2.9 kW of heat to a building maintained at 17 ^{o}C extracting heat from the sea at 7 ^{o}C. What is the minimum power consumption of the pump?

[**5**]

Section B: Answer TWO of the questions in this Section

B.1. This question examines the Carnot cycle.

The Carnot cycle comprises sequentially: (i) a reversible isothermal expansion of a working substance in contact with a high-temperature reservoir at temperature T_1, (ii) a reversible adiabatic expansion to a final state having a temperature T_2 equal to that of a low-temperature reservoir, (iii) an isothermal reversible compression in thermal contact with the low-temperature reservoir, (iv) a reversible adiabatic compression returning the working substance to its initial state.

a. Sketch the above cycle on a PV indicator diagram, labelling each step (i)-(iv). **[2]**

b. What is the direction of heat transfer for each step of the cycle? **[2]**

c. Starting from the Kelvin-Planck or Clausius statement of the second law of thermodynamics explain why no heat engine working in a closed cycle between two fixed temperature reservoirs can be more efficient than a Carnot engine (Carnot's theorem). **[6]**

d. By considering the working substance in a Carnot cycle to be an ideal gas show that the heat removed from the hot reservoir per cycle Q_1 and heat transferred to the cold reservoir per cycle Q_2 are related by

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}.$$

You may use without proof that $TV^{(\gamma-1)}$ is constant for reversible adiabatic changes of state of an ideal gas. **[6]**

e. A Carnot engine with a non-ideal gas as the working substance works between reservoirs at $T_1 = 400$ K and $T_2 = 300$ K performing 30 J of work per cycle. What is the change of entropy of the working substance during step (i) of the cycle? **[4]**

B.2. This question examines Joule-Kelvin expansion.

In a Joule-Kelvin expansion a gas with an initial volume V_1 is pushed through a porous plug from a thermally isolated high-pressure chamber maintained at a constant pressure p_1 into a thermally isolated low-pressure chamber maintained at constant pressure p_2.

a. Explain why the enthalpy of the gas is unchanged in such a process. **[4]**

b. Show that the isenthalpic compressibility is related to the isothermal compressibility by

$$\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_H = \frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T\left(\frac{1+\Omega_G}{\gamma}\right),$$

with $\Omega_G \equiv \beta_p K_T \bar{v}/c_V$. Here $\gamma = c_p/c_V$ with c_V and c_p the constant volume and constant pressure molar heat capacities of the gas, $\bar{v}$ is the molar volume, β_p is the volume thermal-expansion coefficient and K_T is the bulk modulus. **[8]**

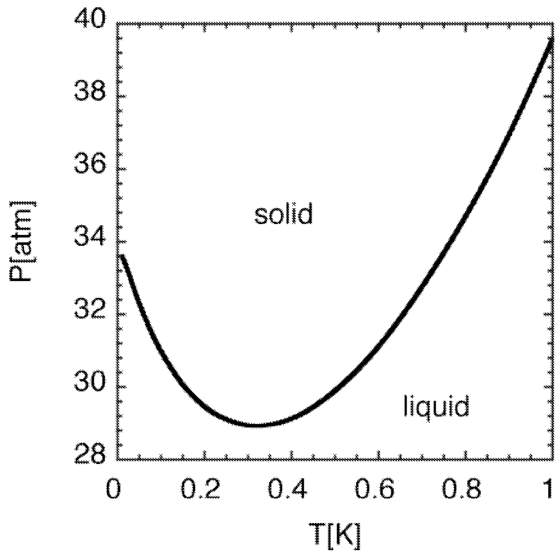
c. Show that for an ideal gas (i) $\beta_p = 1/T$, (ii) $\Omega_G = R/c_V$ and (iii) that the expression given in part (b) reduces to

$$\left(\frac{\partial V}{\partial p}\right)_H = -\frac{V}{p}.$$

You may use without proof the result $c_p - c_V = R$ for an ideal gas. **[5]**

d. Hence, derive an expression for the final volume of an ideal gas V_2 in terms of p_1, p_2 and V_1 following a Joule-Kelvin expansion from p_1,V_1 to p_2,V_2. Hence deduce the change of temperature for an ideal gas undergoing a Joule-Kelvin expansion. **[3]**

B.3. This question looks at the first order liquid-solid transition in ^{3}He.



The figure shows the measured solid-liquid phase transition line for ^{3}He for temperatures from 0.010 K to 1 K. Pressure is measured in atmospheres.

a. The transition from liquid to solid helium is a first order transition. Give two generic characteristics of first order transitions. **[2]**

b. Derive the Clausius-Clapeyron equation that relates the slope of the liquid-solid transition line to the change in entropy and volume across the transition,

$$\left(\frac{dp}{dT}\right)_{transition} = \frac{\Delta S}{\Delta V}.$$

[6]

c. Based on the data shown in the figure does the solid or liquid have the higher entropy (i) at 0.2 K and (ii) at 0.5 K, at the transition? Explain carefully your reasoning. **[4]**

d. From the figure it might be tempting to linearly extrapolate the phase transition line from the data between 0.1 K and 0.01 K down to 0 K. Explain why such a linear extrapolation would not be consistent with the laws of thermodynamics. **[4]**

e. In the Pomeranchuk method of refrigeration liquid ^{3}He is compressed adiabatically and as reversibly as possible converting the liquid into solid (starting from a low temperature below 0.32 K). For temperatures in the range 0.01 K - 0.1K the molar entropy of the solid is approximately constant with value $s_s = Rlog_e(2)$ while the molar entropy of the liquid increases approximately linearly with temperature as $s_l = R(T/T_0)$ with $T_0 \approx 0.2$ K; both entropies are only weakly pressure dependent. Estimate the lowest temperature that can be obtained starting with liquid at 0.1 K and solidifying 50% of the mass of liquid by adiabatic compression. **[4]**