

College of Science and Engineering
School of Physics & Astronomy



Thermodynamics
SCQF Level 9, PHYS09021
Thursday 28th April, 2011
9.30 – 11.30 a.m.

Chairman of Examiners

Professor R D Kenway

External Examiner

Professor D McMorrow

**Answer ALL of the questions in Section A
and TWO questions from Section B.**

**The bracketed numbers give an indication of the value assigned
to each portion of a question.**

Only the supplied Electronic Calculators may be used during this examination.

A sheet of standard constants is available for use during this examination.

**ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF
THIS EXAMINATION.**

Section A: Answer ALL of the questions in this Section

A.1. (a) What is a triple point and why is it useful for calibrating thermometers? [2]

(b) A constant-volume ideal-gas thermometer gives a pressure reading of 0.010 atmospheres in contact with a cell at the triple point temperature of water (273.16 K). What is the temperature when the pressure reading is 0.012 atmospheres? [3]

A.2. (a) Give the expression for the maximum possible efficiency of a closed cycle engine working between higher temperature T_H and lower temperature T_L heat reservoirs. [2]

(b) Sketch a diagram showing the heat flows into and out of the working substance from the reservoirs per cycle. Explain why setting $T_L = 0$ would contradict the Kelvin statement of the second law of thermodynamics. [3]

A.3. Kirchoff's law relates the emissivity $\epsilon_\lambda(T)$ of a surface at wavelength λ to the absorption coefficient $\alpha_\lambda(T)$. It can be written

$$\epsilon_\lambda(T) = \frac{\alpha_\lambda c u_\lambda(T)}{4}$$

in which c is the speed of light and $u_\lambda(T)$ is a universal function of temperature.

(a) What is the physical significance of the quantity $cu_\lambda/4$ appearing on the right hand side of the equation? [2]

(b) What is the thermodynamic justification for equating the left and right sides of the equation? [2]

(c) Explain why wrapping a hot sandwich in silver metal foil will keep it warmer for longer than wrapping it in anodized (blackened) foil. [1]

A.4. Two rigid cylinders, A and B with respective volumes V_A and V_B ($V_A \ll V_B$) are connected by a capillary. Initially cylinder B is empty (ie a vacuum) and cylinder A contains an ideal monatomic gas at 300 K and pressure 1 bar (ie atmospheric pressure). A moveable piston in A is used to maintain the pressure in A at 1 bar completely expelling all the gas into B through the capillary. The system is thermally isolated and the walls and capillary have negligible heat capacity. The internal energy of n moles of an ideal monatomic gas is $U = \frac{3}{2}nRT$. What is the final temperature of the gas once it has reached thermal equilibrium in B ? [5]

Section B: Answer TWO of the questions in this Section

- B.1.** The Clausius inequality applies to the heat absorbed from a series of subsidiary bodies having temperatures T_{sub} when a system is taken around a closed path.

$$\oint \frac{\delta q}{T_{\text{sub}}} \leq 0.$$

- (a) By considering a reversible cycle, explain how the above inequality implies the existence of a state function S , the entropy. [4]

Two fixed volume bricks of mass $M_1 = 2$ kg and $M_2 = 1$ kg with initial temperatures $T_1 = 400$ K and $T_2 = 100$ K are enclosed in a system thermally isolated from the surroundings. The bricks are made from a material with temperature independent heat capacity $c_v = 1$ kJ/kg/K.

- (b) In process A the bricks are brought into direct thermal contact. What is their final temperature once they have reached thermal equilibrium with each other? [4]
- (c) Process B starts from the same initial conditions but the maximum amount of work is extracted from the system (for example with the use of a heat engine). What is the final temperature of the bricks in this case? [4]
- (d) What is the change in entropy for process B? [2]
- (e) What is the change in entropy for process A? [4]
- (f) How much work was extracted in process B? [2]

B.2. This questions considers a constant volume elastic string. Changes of internal energy are given by $dU = TdS + fdL$ where f is the tension of the string, L its length, T its temperature, and S its entropy.

(a) Derive the following Maxwell relation

$$\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L$$

explaining carefully each step of your derivation.

[6]

The elastic string is found to obey the following two equations.

$$\left(\frac{\partial f}{\partial L}\right)_T = \alpha_1 T$$

$$\left(\frac{\partial f}{\partial T}\right)_L = \alpha_2 L$$

where α_1 and α_2 are constants.

(b) Explain why thermodynamics requires $\alpha_1 = \alpha_2$.

[2]

(c) Derive an expression for $f(T, L)$ explaining carefully your reasoning. Explain whether your expression gives a complete thermodynamic description of the system.

[4]

(d) The heat capacity at constant length is found to have a constant value C_L independent of temperature and length. The string is stretched adiabatically starting from length L_0 and $T = T_0$ to twice its initial length. Derive an expression for the string's final temperature in terms of α_1 , C_L , L_0 and T_0

[8]

B.3. This question looks at phase transitions.

(a) Compare and contrast the main features of a first-order (discontinuous) phase transition with those of a continuous phase transition stating clearly which thermodynamic variables are continuous crossing each transition. [4]

(b) Give two examples of a continuous phase transition. [2]

(c) A second order transition is an idealisation of a continuous phase transition in which the region dominated by critical fluctuations is negligibly small and second derivatives of the Gibbs free energy have apparent discontinuities crossing the transition.

Derive the following relation applicable to such a transition

$$\left(\frac{dp}{dT}\right)_{\text{transition}} = \frac{\Delta\beta_p}{\Delta k_T}$$

which relates the slope of the transition line on a P-T indicator diagram to changes in the thermal expansion coefficients $\beta_p = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p$ and compressibility $k_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_T$ crossing the transition. [7]

(d) For the above idealised transition explain why the slope of the transition line $\left(\frac{dp}{dT}\right)_{\text{transition}} \rightarrow 0$ as $T \rightarrow 0$. [5]

(e) Sketch the thermal expansion coefficient as a function of temperature (i) crossing an idealised second-order transition and (ii) crossing a continuous phase transition in which the region of critical fluctuations is significant. [2]