School of Physics & Astronomy

Thermal Physics

PHYS09061 (SCQF Level 9)

Thursday 15th May, 2014 09:30 – 12:30
(May Diet)

Please read full instructions before commencing writing.

Examination Paper Information

Answer **ALL** questions from Section A and **THREE** questions from Section B & C, answering at least one question from each section.

Special Instructions

- Only the supplied Electronic Calculators may be used during this examination.
- A sheet of physical constants is supplied for use in this examination.
- Attach supplied anonymous bar codes to *each* script book.

Special Items

- School supplied calculators
- School supplied Constant Sheets
- School supplied barcodes

Chairman of Examiners: Prof A Trew
External Examiner: Prof G Lafferty

ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF THIS EXAMINATION.

Section A: Answer ALL of the questions in this Section

A.1 A cup containing 0.5 kg of water at 50°C is left to cool to the temperature of its surroundings: 15°C. Is this a reversible process? **[1]**

What is the entropy change of the water and of the surroundings? **[4]**

(c_P for water = 4.2kJ kg^{-1} K^{-1}.)

A.2 500g of ice at -5^oC is added to 500g of water at 20^oC. What is the final ratio of ice to water in the system? **[3]**

What is the difference in specific internal energy, u, between ice and water? **[2]**

(assume atmospheric pressure throughout, specific heat capacities water c_P =$4.2Jg^{-1}K^{-1}$; ice c_P =$2.1Jg^{-1}K^{-1}$, latent heat of melting $334Jg^{-1}$.)

A.3 Define the coefficient of thermal expansion, β.

The Third Law states that the change in entropy in any process tends to zero as $T \to 0$. Show that this implies that thermal expansion becomes zero as $T \to 0$. ,4 **[1]**

A.4 Two balls are taken at random from a bag containing 10 red and 5 blue balls. What is the probability that

a) both balls are red? **[2]**

b) both balls are of the same colour? **[2]**

c) the two balls are not of the same colour? **[1]**

A.5 a) Give the definitions of a microstate and a macrostate. **[2]**

b) Give the expression for the Fermi-Dirac distribution. Explain the meaning of all the symbols you write down. **[3]**

A.6 Consider N non-interacting atoms in contact with a heat bath at temperature T. Each atom can be either in the ground or excited state with energies 0 or k_BT, respectively.

a) What is the probability for a single atom to be in the excited state? **[3]**

b) On average, how many atoms are going to be in the excited state? **[2]**

Section B: Answer AT LEAST ONE of the questions in this Section

B.1 A fridge is powered by pressurized boiling water. The fridge exchanges heat with the atmosphere (300K), the boiler (400K) and the cold compartment (250K). No external work is provided or produced.

(a) Define a sensible measure of efficiency for the fridge, explaining your choice. **[3]**

The proposed fridge can be regarded as two Carnot engines: A heat engine E running between T=300K and T=400K produces work which is used to drive a refrigerator R running between T=250K and T=300K.

(b) Draw a diagram of the fridge, defining heat and work flows, and write down the relationships between them corresponding to the First Law. **[3]**

(c) What is the maximum efficiency of E and of R ? ,2 **[2]**

(d) Assuming ideal efficiencies, if 200J of heat is extracted from the cool compartment, how much heat is dumped to atmosphere? **[6]**

(e) What is the overall efficiency of the device according to your definition in (a), and how would it change if the water was not pressurised? **[4]**

B.2 (a) Sketch the $P-T$ projection of the PVT surface for a simple substance with solid, liquid and vapour phases, and identify its main features. **[6]**

(b) The chemical potential, μ, is *defined* by adding a term to the Central Equation

$$dU = TdS - PdV + \mu dN$$

In principle, this means that μ can be determined by measuring the change in internal energy when a particle is added to a system: explain why this process is difficult to realise experimentally. **[4]**

(c) Show that for a single component system, μ is the specific Gibbs free energy, i.e. $G = N\mu$. **[4]**

(d) By using the fact that the chemical potentials of coexisting phases are the same, derive the Clausius-Clapeyron equation

$$\frac{dp}{dT} = \frac{l}{T(\nu_2 - \nu_1)}$$

where ν_1 and ν_2 are the specific volumes of the two phases and l is the specific latent heat of the transition. **[4]**

(e) What feature(s) of the PT diagram in section (a) can be calculated using the Clausius-Clapeyron equation **[2]**

B.3 This question pertains to an imaginary material (Tedium Boride, TB) with properties defined for mathematical tractability.

Liquid TB has Helmholtz Free Energy $F(V,T) = F_l - aT^{3/2} - bT + B(V - V_l)^2/2$

Solid TB has Helmholtz Free Energy $F(V,T) = F_s - aT^{3/2} + B(V - V_s)^2/2$

(a) Consider the constants $F_{ls} = F_l - F_s$ and $V_{ls} = V_l - V_s$. What sign would you expect these quantities to have, and why? **[2]**

(b) Write down expressions for the pressure and entropy of liquid TB as a function of T and V, **[2]**

(c) evaluate the heat capacities C_p and C_v, thermal expansivity and bulk moduli K_s and K_T for both phases **[5]**

(d) What are the Gibbs free energies $G(V,T)$ of the two phases? Taking $P = 0$, sketch the Gibbs free energy G vs T for each phase on the same graph. ,2 **[2]**

(e) Are expressions for S and P consistent with the third law of thermodynamics? Justify your answer. **[3]**

(f) Find an expression for the melting temperature T_m and latent heat of melting at $P = 0$ in terms of V_{ls}, F_{ls}, a, b and B. ,1 **[3]**

Section C: Answer AT LEAST ONE of the questions in this Section

C.1 Consider four spins arranged on a ring with each spin pointing either up or down. Only neighbouring spins are assumed to interact and their interaction energy is given by

$$-Js_is_j,$$

where the spin variable s_i is $+1$ if the spin i is pointing upwards and -1 otherwise.

a) How many microstates does this system have? **[2]**

b) What are the macrostates of this system and how many microstates correspond to each macrostate? **[4]**

c) Write down the expressions for the free energy corresponding to the macrostates with the total interaction energy 0 and $-4J$. **[6]**

d) Give explicitly the temperature range where the system on average will be in a state with all spins pointing in the same direction. **[4]**

e) Discuss qualitatively how this temperature range changes when the number of spins on the ring is increased to infinity and its bearing on phase transitions in 1-dimensional systems. **[4]**

C.2 The partition function of a semi-classical ideal gas of N particles is given by

$$Z = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N,$$

where

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

is the thermal wavelength.

a) Use equipartition to calculate the total energy of the gas. Why is the usage of equipartition justified? **[4]**

b) Calculate the pressure of the gas. **[3]**

c) Using the total energy from a), calculate the entropy of the gas assuming $N \gg 1$ and $\ln(N!) = N\ln N - N$. **[6]**

d) A container is divided by a partition into two equal regions of volume V, each holding N particles of the same gas at temperature T. Calculate the change in entropy after the partition is removed. Assume that the temperature did not change after the partition is removed. Explain the results. **[7]**

C.3 Consider a gas of photons confined to a two-dimensional cavity of typical size L. The allowed frequencies of the standing waves in the cavity are given by

$$\nu = \frac{c}{2L}\sqrt{n_x^2 + n_y^2},$$

where n_x and n_y are quantum numbers.

a) Calculate the density of states. Note that in 2D photons have only one possible polarisation. **[5]**

b) Derive the spectral density per unit area as a function of the wavelength. **[6]**

c) Find the wavelength λ_{max} corresponding do the maximum of this distribution. You may assume that $hc/\lambda_{max}k_BT \gg 1$. **[5]**

d) Using the result from c), calculate what colour our Sun would have in 2D. Assume the surface temperature of the Sun to be $T = 6000K$. **[4]**