Predicting leptonic widths of Υ mesons using lattice QCD

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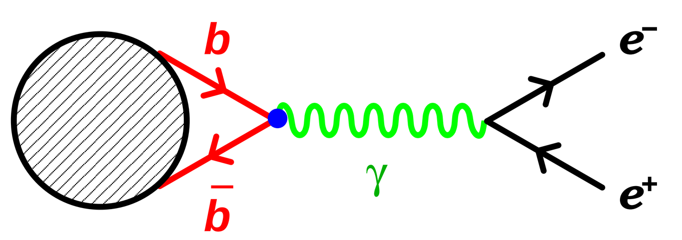
Introduction

Upsilon mesons, Υ, are short-lived particles made of two bottom quarks bound together with no relative orbital angular momentum. The quarks are held together by the Strong Force that is believed to be described by the theory known as quantum chromodynamics (QCD). The lightest meson is known as the $\Upsilon(1s)$, the next the $\Upsilon(2s)$ etc.

The Υ states can decay to charged leptonic particles (e.g. an electron-positron pair): the **$b\bar{b}$ pair** first **annihilate**, forming a **virtual photon** that decays to a **lepton pair**.



$$\Gamma_{ee} = \frac{16\pi}{6M_\Upsilon^2}\, e_b^2\, |M_{\rm ME}|^2\, \alpha_{\rm em}^2$$

The leptonic decays are interesting because:

- They are important for measuring the *strong coupling constant, α_s, at low energies*. Leptonic decays provide a good estimate of background contamination in measurements of

$$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to \gamma gg)} \sim \frac{\alpha_s}{\alpha_{\rm em}} .$$

- They provide a laboratory for *high precision testing* of QCD as a fundamental theory of Nature: measurements of leptonic decays have a high signal-to-noise ratio at particle accelerators such as CLEO, allowing very accurate measurements to be made that can be used to compared to the answers predicted by QCD.
- They allow us to quantify the *systematic errors* in the numerical methods used to obtain experimental predictions from QCD.

The current situation

The *decay width*, Γ_{ee}, gives the lifetime of a particle before it decays to an electron–positron pair. Different states have different widths, depending on the masses of the states and the wavefunctions of the b-quarks.

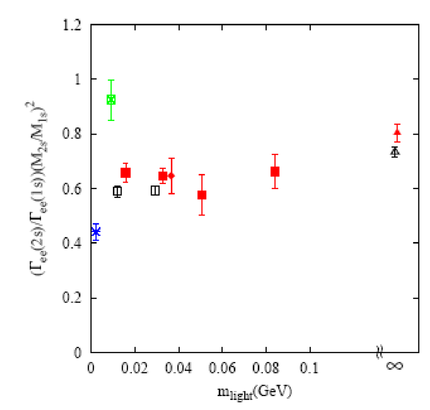
The most accurate experimental measurements and theoretical predictions are for the *ratio* (in which some sources of error cancel in each case):

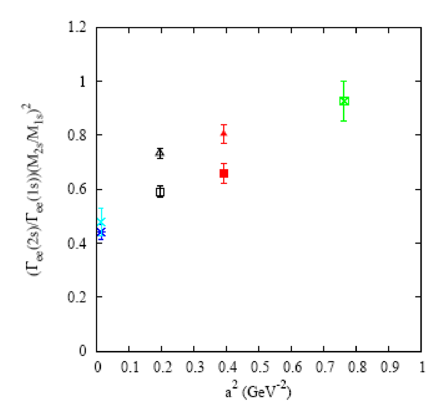
$$\frac{(M_\Upsilon^2\Gamma_{ee})(2s)}{(M_\Upsilon^2\Gamma_{ee})(1s)} = \begin{cases} \text{Experiment}: & 0.44\ (3)\ \text{(PDG 04)}, \\ \text{Theory}: & 0.48\ (5)\ \text{(Gray 05)}. \end{cases}$$

The numbers agree, so QCD works! But...

the CLEO collaboration are analysing 4 fb^{-1} of new data, reducing the experimental error by a factor of 10.

The goal of this work is to make a similar level of improvement to the theoretical prediction by refining the calculation of the QCD matrix element, $M_{\rm ME}$, that represents the probability of the b and $\bar{b}$ quarks coming together and annihilating to form the virtual photon.





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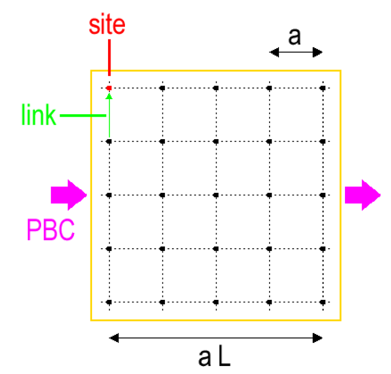
Ways we might calculate the matrix element

The theory of QCD is easy to write down but harder to work with, and there is no exact mathematical result for $M_{\rm ME}$. There are various approaches we can try:

Perturbation theory: gives $M_{\rm ME}$ as a power series in α_s. The series converges very poorly, however, as we ignore the *non-perturbative* physics that confines the quarks within the meson.

Lattice QCD computer simulation: includes the non-perturbative effects. Space and time are divided up so the quarks live on a finite grid with neighbouring points distance a apart. We can then do numerical simulations of QCD on supercomputers, but they get very time-consuming when a gets small. Even on world-leading machines like QCDOC, we cannot get much below $a = 0.05$ fm.



Unfortunately, the b-quarks are very heavy: $m_b \simeq 5$ GeV/c^2. The Compton wavelength of the Υ meson is so small that that they slip through such gaps in the lattice. This is another way of saying that the results we obtain from the simulation are irreversibly contaminated by the effects of discretising space and time.

Lattice NRQCD: the heavy b-quarks move slowly, with $v^2 \simeq \alpha_s \simeq 0.1$. Expanding QCD as a power series in the velocity yields the theory known as NRQCD. The big advantage is that we can now treat the large rest mass separately from the small kinetic energy, avoiding the problem of the small Compton wavelength. Now we use lattice simulations to capture the non-perturbative characteristics. Truncating the NRQCD power series after $\mathcal{O}(v^2)$ makes the computation easier, at the cost of systematic errors at the level of a few %.

The large error in the leptonic width prediction comes from having neglected the $\mathcal{O}(v^2)$ terms in this case. This poster describes how to include them.

The need for matching

- We know how to represent the $b\bar{b}$ quark-antiquark pair annihilating to form a photon: we calculate the matrix element of the QCD current, $\boldsymbol{J}^{\rm QCD} = \bar{v}(-\boldsymbol{p})\boldsymbol{\gamma} u(\boldsymbol{p})$. We just can't do it.
- NRQCD, however, does not predict the corresponding current. We can, however, identify some likely suspects whose matrix elements we *can* calculate using lattice simulations:

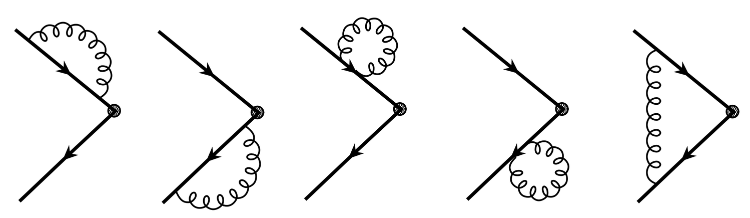
$$\left\langle 0 \left| \boldsymbol{J}_n^{\rm NRQCD} \right| b\bar{b} \right\rangle \qquad \text{with} \qquad \boldsymbol{J}_n^{\rm NRQCD} = \chi^\dagger \boldsymbol{\sigma} \left(\overleftrightarrow{D}^2/M^2 \right)^n \psi .$$

- The desired result, $M_{\rm ME}$, is a particular linear combination of these:

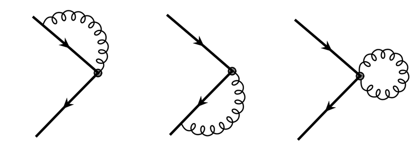
$$M_{\rm ME} = \left\langle 0 \left| \boldsymbol{J}^{\rm QCD} \right| b\bar{b} \right\rangle = \sum_n a_n \left\langle 0 \left| \boldsymbol{J}_n^{\rm NRQCD} \right| b\bar{b} \right\rangle .$$

  which ensures that QCD and NRQCD give the same answer for very slow-moving quarks.
- So what we really need are the *matching coefficients* a_n for $n = 0, 1$. All of the non-perturbative physics has been captured in the already–calculated lattice NRQCD matrix elements, so it is sufficient to use *lattice perturbation theory* for the matching coefficients, and a one-loop calculation will give an answer as accurate as the experimental result from CLEO.

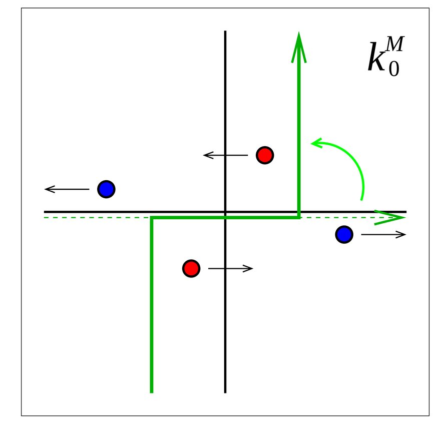
  We calculate the vertex correction diagrams for both continuum QCD and lattice NRQCD:



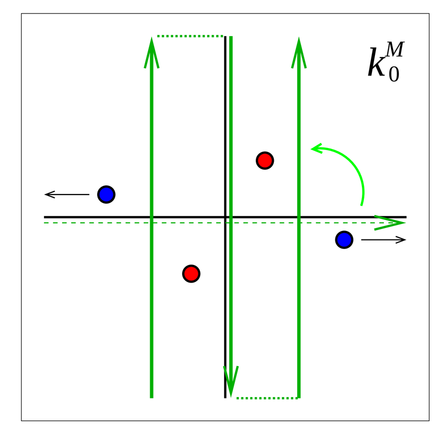
The gruesome details



- We work in the centre–of–mass, Breit frame, and only consider *on-shell matrix elements*. Off-shell contributions are at most $\mathcal{O}(\alpha_s^2)$ and may vanish inside bound states.
- Lattice quantum effects in the diagrams shown lead to *mixing down*: whereas the tree level matrix element $\langle 0|J_n^{\rm NRQCD}|b\bar{b}\rangle^{(0)}$ varies with the velocity as v^{2n}, the $\mathcal{O}(\alpha_s^t)$ quantum correction varies as a lesser power, $v^{2(n-t)}$. We correct for this by defining *tadpole subtracted operators*.
- Careful *Wick rotation*. At small gluon momenta the NRQCD vertex diagrams have a complicated pole structure. The rotation to Euclidean time must be done carefully so the contour does not cross the poles.



- The infrared divergence at low v in the one-loop vertex correction must exactly cancel as we match the theories. We improve the efficiency of our calculations by *analytically removing the Coulomb divergence* from both using a well-chosen mathematical expression



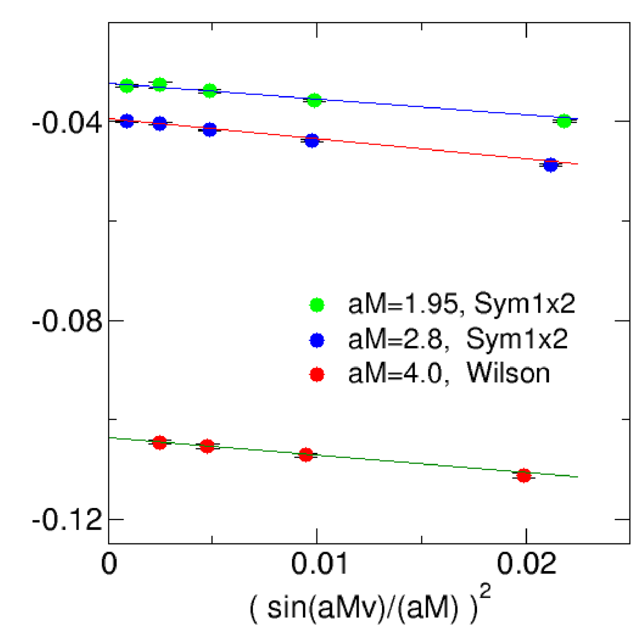
$$I_{\rm odd} = \frac{g(v)}{16v}, \quad g(v) = \frac{(1+2v^2)\left(1+2\sqrt{1+v^2}\right)}{3(1+v^2)} .$$

Results

We estimate the Feynman integrals statistically using VEGAS, using 96 processors of the Sunfire supercomputer at the Cambridge–Cranfield HPCF. 18 hours of running gives a statistical error of 1%: extra runs reduce it further.

We compare the one-loop matrix elements for a range of v. The matching coefficients are then the parameters from a straight line fit as shown:

$$I_{\rm QCD} - I_{\rm NRQCD} = a_0 - a_1 \frac{4\sin^2(aMv)}{(aM)^2}$$



Conclusions

- We now have the NRQCD currents needed to predict the leptonic widths of Υ mesons.
- We have worked perturbatively to one loop, and accuracy

$$\Gamma_{ee} \sim \mathcal{O}(v^2, \alpha_s, \alpha_s v^2) \sim \mathcal{O}(10\ \%) .$$

- In the ratio, some uncertainties cancel to give higher accuracy

$$\frac{(M_\Upsilon^2\Gamma_{ee})(2s)}{(M_\Upsilon^2\Gamma_{ee})(1s)} \sim \mathcal{O}(v^4, \alpha_s^2, \alpha_s v^2) \sim \mathcal{O}(1\ \%).$$

- We have matched theoretically the experimental advances of CLEO
- We await the CLEO results for a high precision showdown between Nature and QCD.