Understanding Υ mesons using lattice QCD

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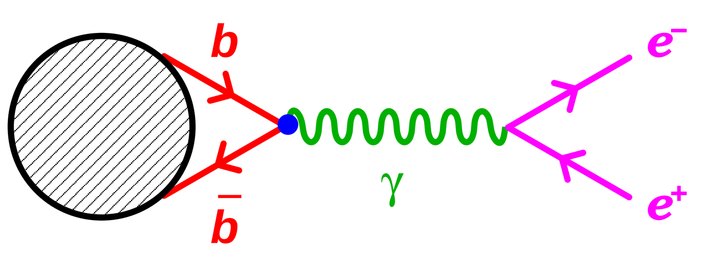
Introduction

Upsilon mesons, Υ, are short-lived particles made of two bottom quarks bound together with no relative orbital angular momentum. The quarks are held together by the Strong Force that is believed to be described by the theory known as quantum chromodynamics (QCD). The lightest meson is known as the $\Upsilon(1s)$, the next the $\Upsilon(2s)$ etc.

The Υ states can decay to an electron-positron ("lepton") pair. The mechanism for the process is that the **$b\bar{b}$ pair** first **annihilate**, forming a **virtual photon** that decays to the **lepton pair**.



$$\Gamma_{ee} = \frac{16\pi}{6M_\Upsilon^2}\, e_b^2\, |M_{\rm ME}|^2\, \alpha_{\rm em}^2$$

The *leptonic decay width*, Γ_{ee}, gives the lifetime before decaying to an electron–positron pair. The leptonic widths are interesting because:

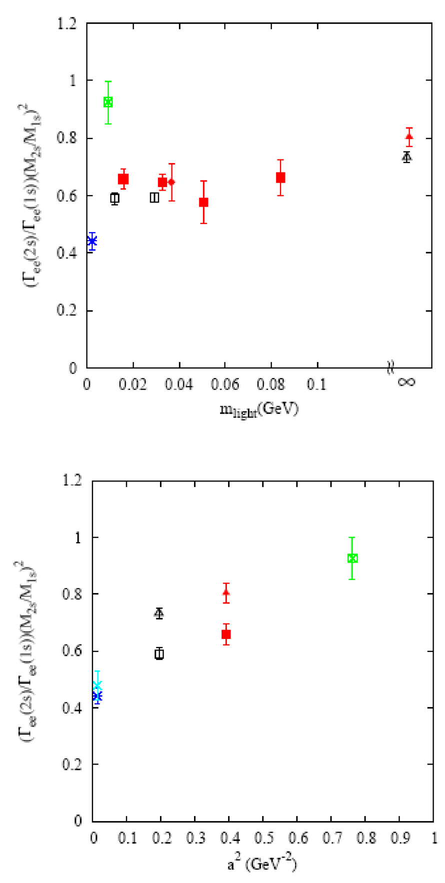
- They can be experimentally measured to high accuracy at particle accelerators such as CLEO.
- They can be theoretically predicted from to similar accuracy using numerical lattice NRQCD.

Comparing the two therefore allows a *high precision test of QCD* as a fundamental theory of Nature. It also gives a way of quantifying the systematic uncertainties in the numerical methods used.

Currently, the most accurate results are for the *ratio* of decay widths, where some systematic errors cancel:

$$\frac{(M_\Upsilon^2\Gamma_{ee})(2s)}{(M_\Upsilon^2\Gamma_{ee})(1s)} = \begin{cases} \text{Experiment}: & 0.517\ (10)\quad \text{(CLEO 05)},\\ \text{Theory}: & 0.48\ (5)\quad \text{(Gray 05)}.\end{cases}$$

So, high precision testing of QCD is held back by the large uncertainty in the theoretical prediction.



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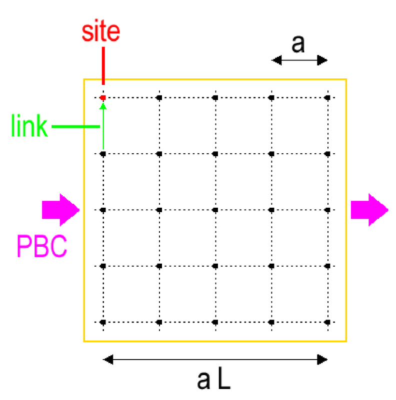
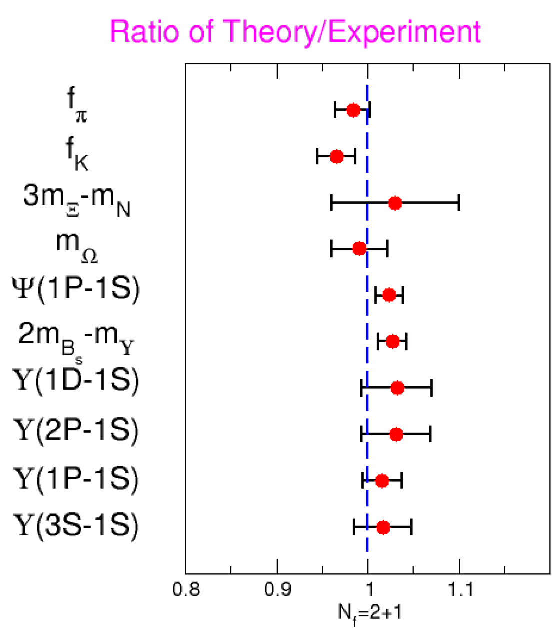
Our goal is to reduce the theoretical error by a factor of five, which we have done by refining the calculation of the *QCD matrix element*, $M_{\rm ME}$, that represents the probability of the b and $\bar{b}$ quarks coming together and annihilating.

Lattice QCD

The matrix element is calculated using the theory of QCD, but while the QCD is easy to write down mathematically, to get answers we need *lattice QCD* (lQCD) computer simulation.
Space and time are divided up so the quarks live on a finite grid with neighbouring points distance a apart. We then do numerical simulations of QCD on supercomputers such as the world-leading QCDOC at various values of a, before extrapolating away the discretisation effects.
This graph shows the ratio of experimental measurements to lattice QCD predictions.





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That this is unity for such a wide range of quantities shows:

- QCD is broadly correct as a fundamental theory: the possible *Beyond the Standard Model* effects we are looking for must be small, and will require high accuracy in both theoretical predictions and experimental data
- the lattice method is the only one allowing such high precision tests.

Non–Relativistic QCD

The b-quarks are very heavy, $m_b \simeq 5$ GeV$/c^2$, and therefore move slowly with $v^2 \simeq \alpha_s \simeq 0.1$. To include them in lattice simulations, we exploit this using *lattice NRQCD*. Expanding QCD as a power series in the velocity allows us to treat the large rest mass separately from the small kinetic energy, so avoiding the problem of small Compton wavelengths on a discrete lattice.

Truncating the power series in v^2 makes the computation easier, at the cost of introducing quantifiable systematic errors. These are currently large in the QCD leptonic widths, as the power series has been truncated at $\mathcal{O}(v^2)$. In this work we have extended the series to $\mathcal{O}(v^4)$, reducing the uncertainties to only a few %.

The matching calculation

- To calculate the leptonic width we need to calculate a matrix element of a current that represents the $b\bar{b}$ quark-antiquark pair annihilating to form a photon: $\boldsymbol{J}^{\rm QCD} = \bar{v}(-\boldsymbol{p})\boldsymbol{\gamma}u(\boldsymbol{p})$.
- In NRQCD, $M_{\rm ME}$, is given to order v^2 by a linear combination of currents:

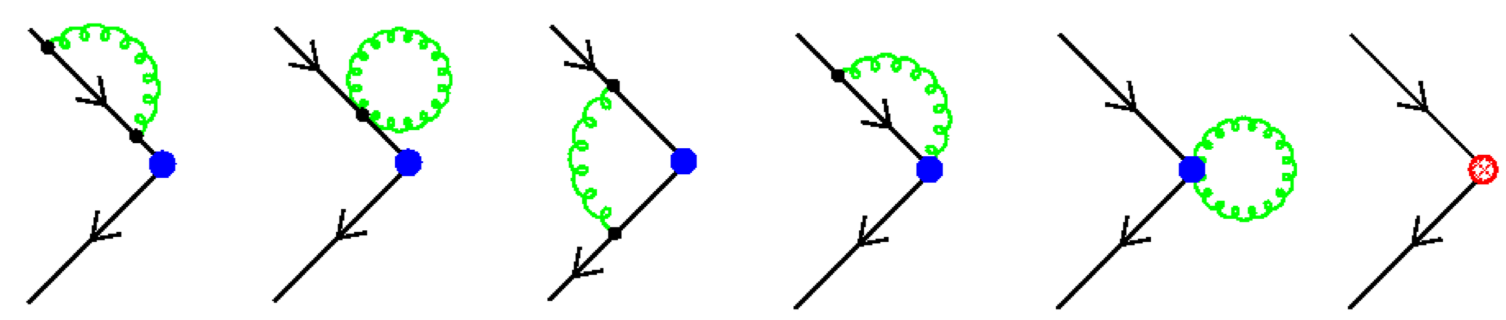
$$M_{\rm ME} = \left\langle 0 \left| \boldsymbol{J}^{\rm QCD} \right| b\bar{b} \right\rangle = a_0 \left\langle 0 \left| \boldsymbol{J}_0^{\rm NRQCD} \right| b\bar{b} \right\rangle + a_1 \left\langle 0 \left| \boldsymbol{J}_1^{\rm NRQCD} \right| b\bar{b} \right\rangle .$$

where

$$\boldsymbol{J}_0^{\rm NRQCD} = \chi^\dagger \boldsymbol{\sigma} \psi\ , \qquad \boldsymbol{J}_1^{\rm NRQCD} = \chi^\dagger \boldsymbol{\sigma} \left(\frac{4D^2}{M^2}\right) \psi\ .$$

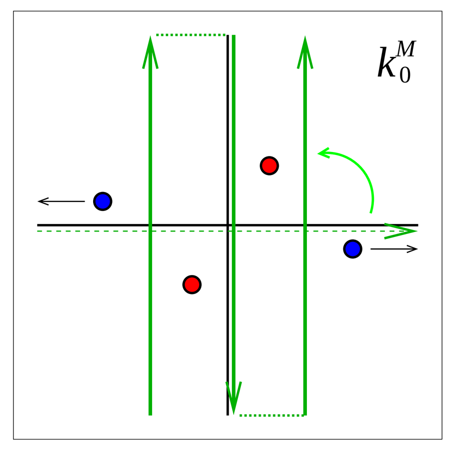
The coefficients are expressed as perturbative series: $a_n = a_n^{(0)} + 4\pi\alpha_s\, a_n^{(1)} + \ldots$

- The *tree level coefficients*, $a_n^{(0)}$, are chosen to ensure that QCD and NRQCD give the same answer for very slow-moving quarks.
- Our goal is to do a matching calculation to determine the *one-loop coefficients*, $a_0^{(1)}$ and $a_0^{(1)}$.
- As $\alpha_s \simeq v^2$, two-loop effects can be ignored.
- The matching is done by calculating the one-loop matrix elements in QCD and NRQCD at various values of v, then adjusting $a_0^{(1)}$ and $a_1^{(1)}$ to ensure agreement. We have to calculate the following Feynman diagrams:



The gruesome details

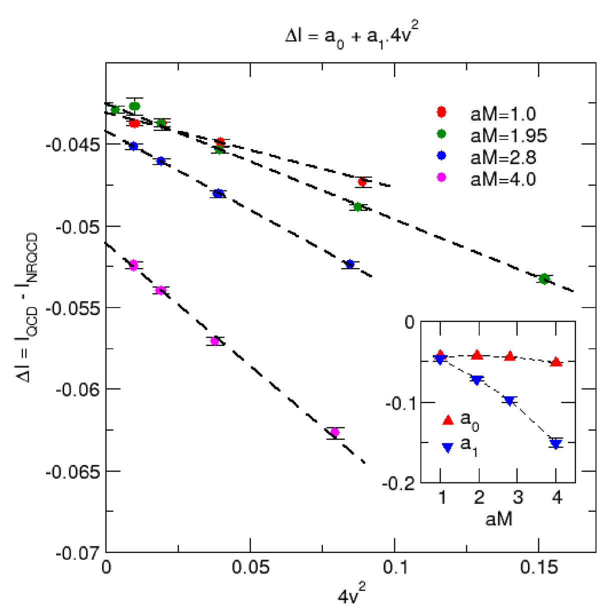
- We work in the centre–of–mass, Breit frame, and only consider *on-shell matrix elements*. Off-shell contributions are at most $\mathcal{O}(\alpha_s^2)$ and may vanish inside bound states.
- Careful *Wick rotation*. At small gluon momenta the NRQCD vertex diagrams have a complicated pole structure. The rotation to Euclidean time must be done carefully so the contour does not cross the poles.
- We improve the efficiency of our calculations by analytically removing the physical *Coulomb divergence* at small v.



Results

We estimate the Feynman integrals statistically with the VEGAS package, using 96 processors of the Sunfire supercomputer at the Cambridge–Cranfield HPCF. 18 hours of running gives a statistical error of 1%: extra runs reduce it further.
We compare the one-loop matrix elements for a range of v. The matching coefficients are then the parameters from a straight line fit as shown:

$$\Delta I = I_{\rm QCD} - I_{\rm NRQCD} = a_0^{(1)} - 4v^2\, a_1^{(1)}$$



Conclusions

- We now have the NRQCD currents needed to predict the leptonic widths of Υ mesons.
- We have worked perturbatively to one loop, and accuracy

$$\Gamma_{ee} \sim \mathcal{O}(v^2, \alpha_s, \alpha_s v^2) \sim \mathcal{O}(10\ \%)\ .$$

- In the ratio, some uncertainties cancel to give higher accuracy

$$\frac{(M_\Upsilon^2\Gamma_{ee})(2s)}{(M_\Upsilon^2\Gamma_{ee})(1s)} \sim \mathcal{O}(v^4, \alpha_s^2, \alpha_s v^2) \sim \mathcal{O}(1\ \%).$$

- These numbers now need to be plugged into the NRQCD simulation code at Glasgow University.
- Then we will have matched theoretically the experimental advances of CLEO, allowing …
- A *high precision showdown* between Nature and QCD.