# QCD --- Quantum Chromodynamics

## Outline

### QCD Principles

- Quantum field theory, analogy with QED
- Vertex, coupling constant
- Colour "red", "green" or "blue"
- Gluons

### Quark and Gluon Interactions

- Confinement
- Asymptotic Freedom
- QCD potential

### QCD Experiments

- Experimental evidence for quarks, colour and gluons
- e+e- annihilations
- Charmonium
- Scattering, DIS

### Questions

- Why is strong interaction short range?
- Why are "free quarks" not observed?
- How do quarks and gluons fragment into hadronic jets?

# QCD vs QED

## QED

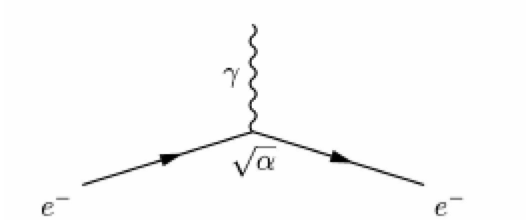
Quantum theory of electromagnetic interactions
mediated by exchange of photons
Photon couples to electric charge e
Coupling strength $\propto e \propto \sqrt{\alpha}$

## QCD

Quantum theory of strong interactions
mediated by exchange of gluons between quarks
Gluon couples to colour charge of quark
Coupling strength $\propto \sqrt{\alpha_S}$

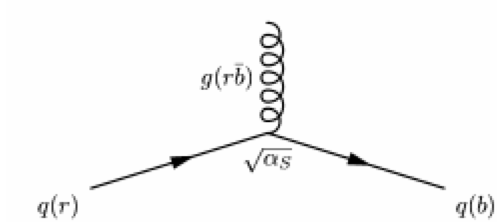
## Fundamental vertices

**QED**




$\alpha = e^2/4\pi \approx 1/137$

**QCD**




$\alpha_S = g_S^2/4\pi \sim 1$

## Coupling constant

Strong interaction probability $\propto \alpha_S > \alpha$
Coupling strength of QCD much larger than QED

# Colour

## What is Colour ?

Charge of QCD

Conserved quantum number

"Red", "green" or "blue"

## Quarks

Come in three colours $r\ g\ b$

Anti-quarks have anti-colours $\bar{r}\ \bar{g}\ \bar{b}$

## Leptons, other Gauge Bosons - $\gamma$, $W^{\pm}$, $Z^0$

Don't carry colour, "zero colour charge"

➔ Don't participate in strong interaction

## Caveat

"... colour is not to be taken literally."

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ "red"}, \qquad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ "green"}, \qquad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ "blue"}.$$

| Interaction | QED | QCD |
|---|---|---|
| Conserved charge | electric charge e | colour charges r, g ,b |
| Coupling constant | $\alpha = e^2/4\pi$ | $\alpha_S = g_S{}^2/4\pi$ |
| Gauge boson | Photon | 8 gluons |
| Charge carriers | fermions ($q \neq 0$) | quarks gluons |

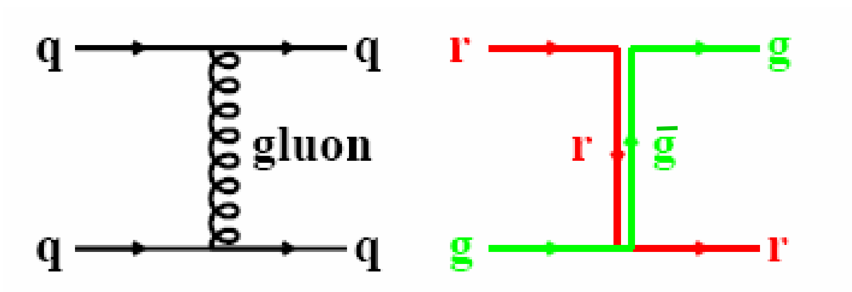
# Gluons

## Gluon Properties

Gluons are massless spin-1 bosons

➔ QCD propagator $1/q^2$

Emission or absorption of gluons by quarks changes colour of quarks - Colour is conserved




➔ Gluons carry colour charge themselves

e.g. $r\bar{g}$ gluon changes red quark into green

QCD very different from QED, q(photon) = 0

## Number of gluons

Naively expect 9 gluons

$r\bar{b}$, $r\bar{g}$, $g\bar{b}$, $g\bar{r}$, $b\bar{r}$, $b\bar{g}$, $r\bar{r}$, $g\bar{g}$, $b\bar{b}$

Symmetry -> 8 octet and 1 singlet states

Octet $\quad r\bar{b} \quad r\bar{g} \quad b\bar{g} \quad b\bar{r} \quad g\bar{r} \quad g\bar{b}$

$$\frac{1}{\sqrt{2}}\left(r\bar{r} - g\bar{g}\right) \quad \frac{1}{\sqrt{6}}\left(r\bar{r} + g\bar{g} - 2b\bar{b}\right)$$

Singlet $\quad \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$

➔ **8 gluons** realised by Nature (colour octet)

# Quark & Gluon Interactions

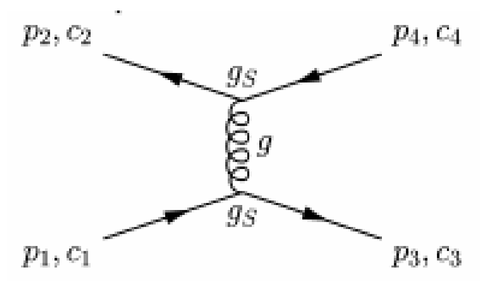
## Quark-Antiquark Scattering

describes a meson

e.g. $\pi^+$ = (u-dbar)

Single gluon exchange

at short distance ≤ 0.1 fm




## QCD Potential

$$V_{QCD}(r) = -\frac{4}{3}\frac{\alpha_S}{r}$$

at short distance ~ 0.1 fm

attractive - negative sign

QED-like apart from colour factor 4/3

More than one gluon -> colour factor

## Gluon Self-Interactions

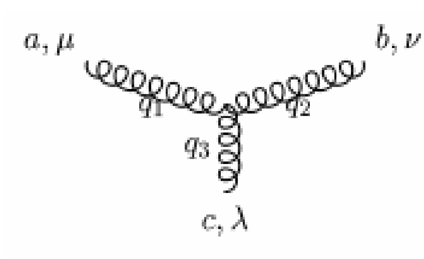
QED versus QCD - So far pretty similar

Photons and gluons – massless spin-1 bosons

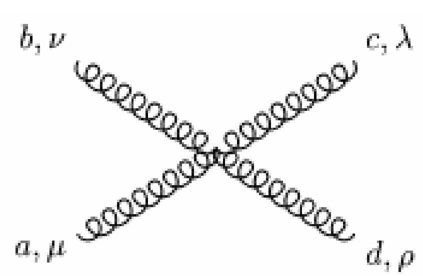
Big difference - gluons carry colour charge

➔ **Gluons interact with each other**

## 3-gluon vertex




## 4-gluon vertex




➔ Origin of huge differences between QCD and QED

# Confinement

## Experimental Evidence

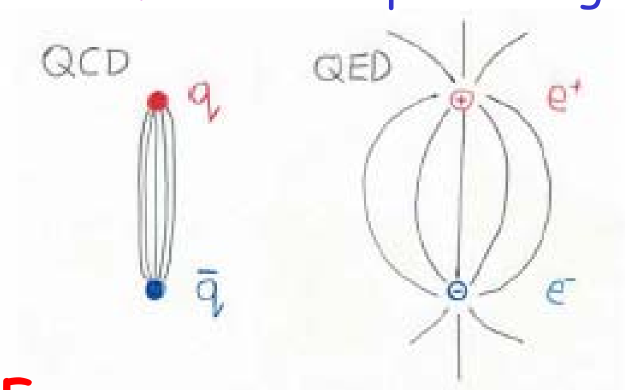
Do not observe free quarks

Quarks confined within hadrons

## Strong Interaction Dynamics

Gluons attract each other - self-interactions

➔ Colour force lines pulled together in QCD




## Colour Force

between 2 quarks at "long" distances O(1 fm)

String with tension k -> Potential $V(r) = kr$

Stored energy/unit length is constant

Separation of quarks

requires infinite amount of energy

## Confinement

Direct consequence of gluon self-interactions

Particles with colour - quarks and gluons -

**confined** inside QCD potential, must combine

into hadrons with zero net colour charge

# QCD Potential

## Mesons

quark-antiquark pair  colour wave fct.

with zero net colour charge  $|\psi\rangle = \frac{1}{\sqrt{3}}|r\bar{r} + g\bar{g} + b\bar{b}\rangle$

Single gluon exchange -> colour of individual q or anti-q can change  $r\bar{r} \to r\bar{r}$  $r\bar{r} \to b\bar{b}$

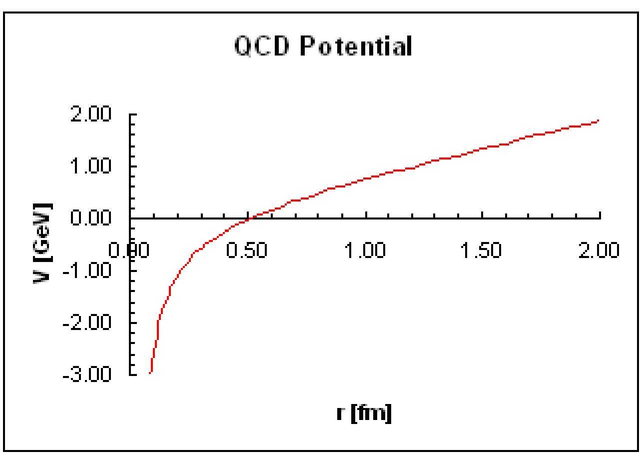
## QCD potential

$$V_{QCD}(r) = -\frac{4}{3}\frac{\alpha_S}{r} + kr$$

QED-like at short distance  $r \leq 0.1$ fm

Quarks are tightly bound  $\alpha_S \approx 0.2 .. 0.3$

String tension -> Potential increases linearly at large distance  $r \geq 1$ fm




$\alpha_S = 0.2$

$k = 1$ GeV/fm

Potential similar for quarks in baryons

## Force

Between two quarks at large distance

$F = |dV/dr| = k = 1.6\ 10^{-10}$ J/ $10^{-15}$ m = 16000 N

Equivalent to weight of large car

# Coupling Constant $\alpha_s$

## Properties

$\alpha_s$ --- coupling strength of strong interaction

Recall QED - coupling constant varies with distance - running $\alpha$

In QED – bare electron charge is screened by cloud of virtual e-e+ pairs

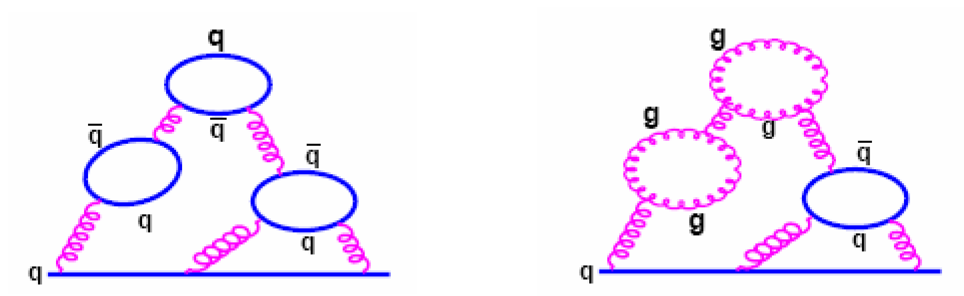
In QCD – similar effects

## QCD Quantum Fluctuations

Cloud of virtual q-anti-q pairs around a quark

→ Screening of colour charge

Colour charge decreases with distance




Cloud of virtual gluons --- no equivalent in QED due to gluon self-interactions

Colour charge of gluons contributes to effective colour charge of quark

→ Anti-screening of colour charge

Colour charge increases with distance

# Running of $\alpha_S$

## Screening and Anti-screening

Anti-screening dominates

Effective colour charge increases with distance

At large distances / low energies $\alpha_S$ ~ 1 - large

Higher order diagrams -> $\alpha_S$ increasingly larger

Summation of diagrams diverges

Perturbation theory fails

## Asymptotic Freedom

Coupling constant

$\alpha_S = 0.12$ at $q^2 = (100\ \mathrm{GeV})^2$

small at high energies

$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{\left(1 + \beta\,\alpha_S(\mu^2)\ln\frac{q^2}{\mu^2}\right)}$$

$$\beta = \frac{11n - 2f}{12\pi}$$

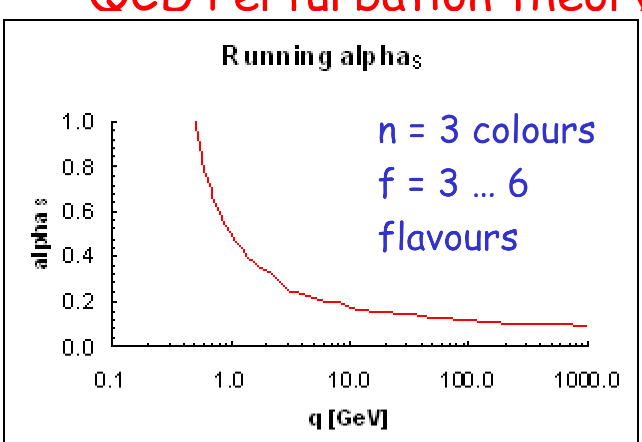
## Running of $\alpha_S$

depends on $q^2$ and # of colours and flavours

Energetic quarks are (almost) free particles

Summation of all diagrams converges

QCD Perturbation theory works



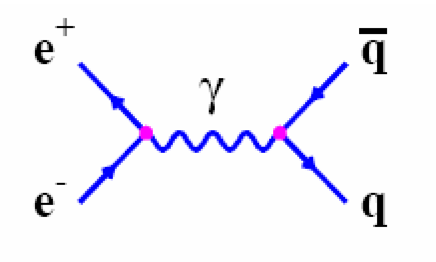

Nobel prize 2004

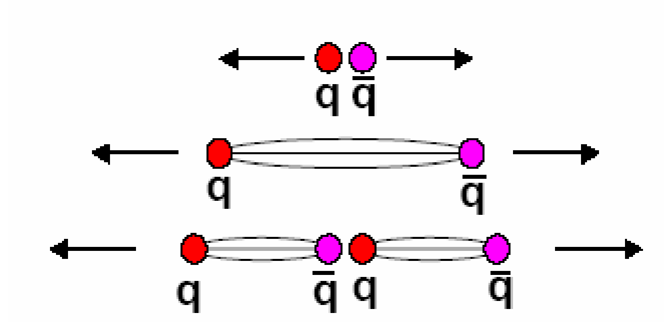
Gross, Politzer, Wilczek

# Hadronisation & Jets

## What happens when quarks separate?

Example: $e^+e^- \to q\bar{q}$ annihilation



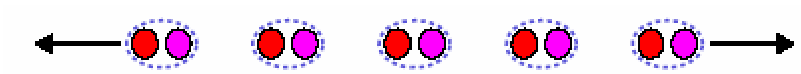




Quarks separate

$E_{string}$ increases - when $E_{string} > 2\ m_q$

**String breaks up into $q\bar{q}$ pairs - fragmentation**

## Hadronisation



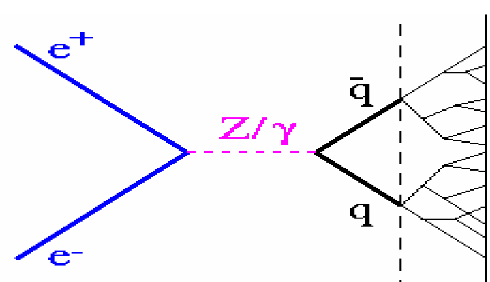
As energy decreases

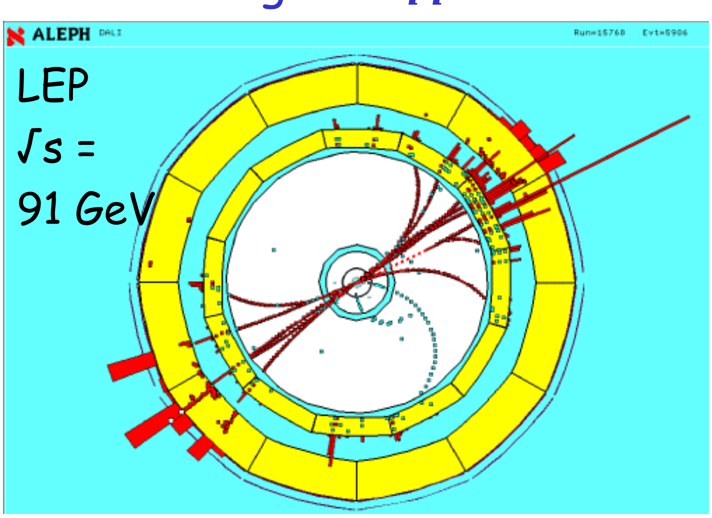
Formation of hadrons

(mesons and baryons)

Hadrons follow direction

of original $q\bar{q}$




## Jets

$e^+e^- \to q\bar{q}$ **hadronisation**

$e^+e^- \to$ **hadrons**

Observe collimated jets

back-to-back

in CoM frame

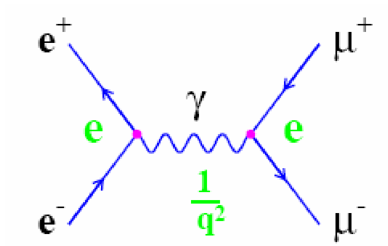
# e+e- Annihilation

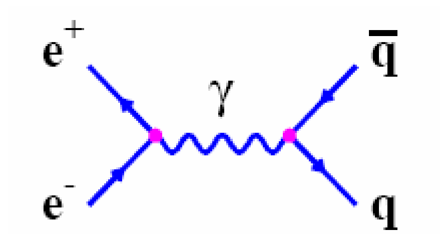
## Feynman Diagrams

$e^+e^- \to q\bar{q}$ and $e^+e^- \to \mu^-\mu^-$

Quark and muon masses are neglected

Only difference in coupling of virtual photon to final state fermion pair is charge $Q_f$




muons $Q_\mu = \pm 1$ $\mu^+\mu^-$

quarks $Q_q = \pm 2/3$ or $\pm 1/3$ $q\bar{q}$

## Cross section

For a single quark flavour --- without colour expect cross section ratio

$$R_q = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{Q_q^2}{Q_\mu^2} = Q_q^2$$

With colour – each quarks has $N_C = 3$ final states

Rule is to sum over all available final states

$$R_q = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_C Q_q^2 = 3Q_q^2$$

## Hadronisation

Measure $e^+e^- \to$ **hadrons** **not** $e^+e^- \to q\bar{q}$

$q\bar{q}$ -pairs fragment and form hadronic jets

Jets from different $q\bar{q}$ -pairs are similar at high energies compared to quark masses

# e+e- Annihilation

## Ratio R

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

Sum is over all quark flavours (u, d, s, c, b, t) kinematically accessible at CoM energy, √s, of collider, and 3 colours (r,g,b) for each flavour

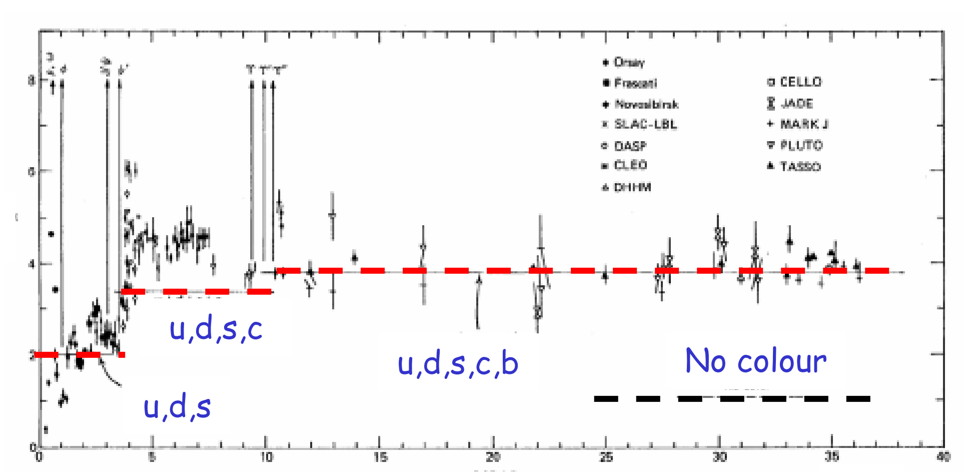
$$R(\sqrt{s} > 2m_s \sim 1\,\text{GeV}) = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right) = 2 \qquad u,d,s$$

$$R(\sqrt{s} > 2m_c \sim 3\,\text{GeV}) = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right) = \frac{10}{3} \qquad u,d,s,c$$

$$R(\sqrt{s} > 2m_b \sim 10\,\text{GeV}) = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right) = \frac{11}{3} \qquad u,d,s,c,b$$

## Measurements

R




√s

R increases in steps with √s

R ≈ 3.85 ≈ 11/3 at √s ≥ 10 GeV

➔ **Overwhelming evidence for colour**

√s < 10 GeV -- resonances (c-cbar and b-bar)

# Charmonium

## Discovery of Charm Quark

1974 Brookhaven and SLAC

Narrow resonance at 3.1 GeV

decays into e+e-, μ+μ-, hadrons

did not fit in existing schemes

## J/ψ Meson

Mass $m_{J/\psi}$ = 3.1 GeV/c$^2$

Narrow width, smaller than experimental resolution

Total width $\Gamma$ = 0.087 MeV

Lifetime $\tau = \hbar/\Gamma = 7.6 \cdot 10^{-21}$ s

= 1000 x expected for strong interaction process

## Branching Fraction

J/ψ decays

many final states with partial decay width $\Gamma_i$

Total decay width

$$\Gamma = \sum_i \Gamma_i$$

Branching fraction

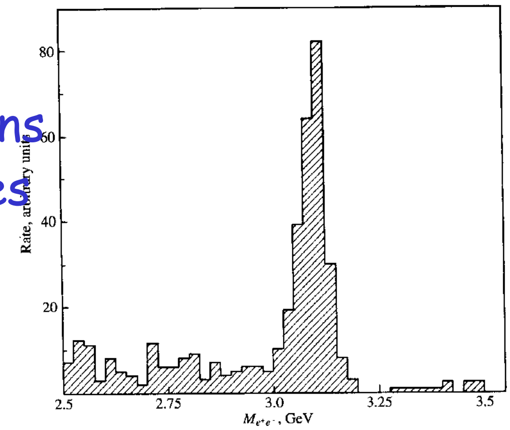
$$B_i = \frac{\Gamma_i}{\Gamma}$$

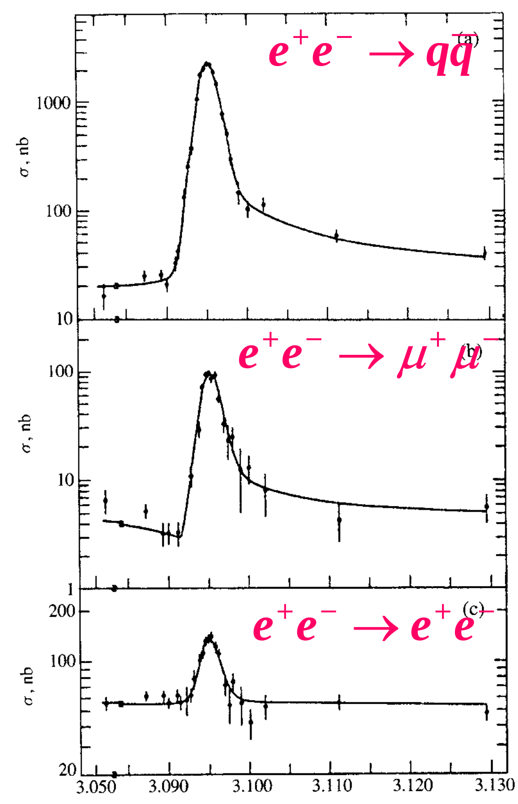
$$B(J/\psi \to q\bar{q}) = (87.7 \pm 0.5)\%$$

$$B(J/\psi \to \mu^+\mu^-) = (5.88 \pm 0.10)\%$$

$$B(J/\psi \to e^+e^-) = (5.93 \pm 0.10)\%$$

$p\,Be \to e^+e^- X$

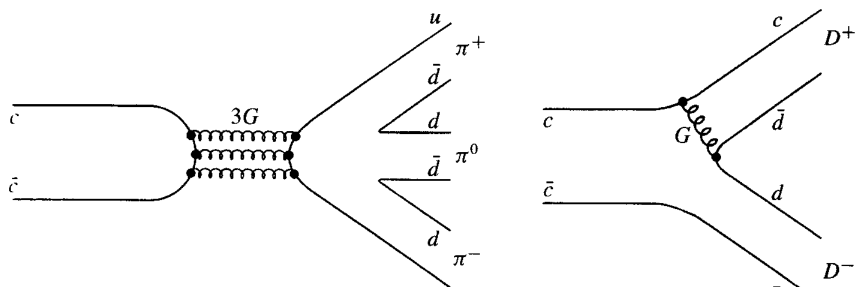


# Charmonium

## Quark Model Explanation

J/ψ is new quark (c-cbar) bound state



(a) (b)

Strong decay for J/ψ (diagram b) is forbidden by energy conservation at $\sqrt{s} = m_{J/\psi} < 2m_D$

Allowed transition (diagram a) has three gluons

Decay rate suppressed $\propto \alpha_S^6$

$J/\psi = \psi(1S)$ resonance established quarks as real particles

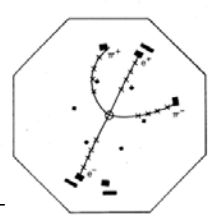
## Excited Charmonium states

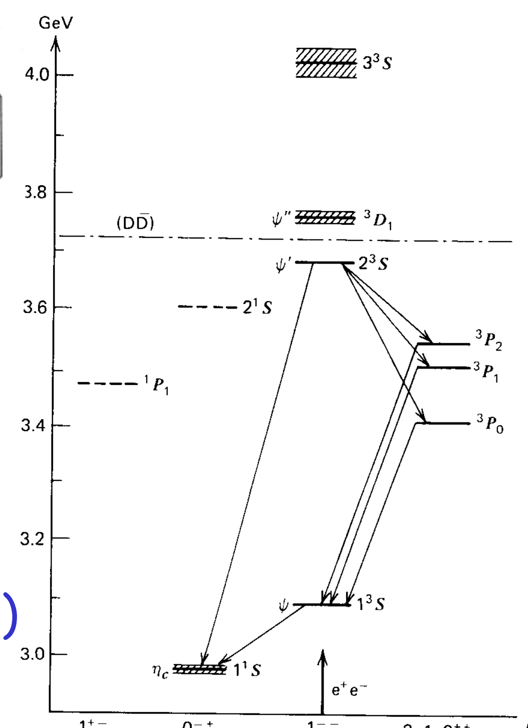
Found more states

ψ(2S), ψ(3S)

e.g. $\psi(2S) \to J/\psi \pi^+ \pi^-$

$J/\psi \to e^+ e^-$






ψ states - spin J = 1 (like γ)

Observe also $\eta_c$ (J = 0)

and P states $\chi_c$ (L = 1)

In agreement with

QCD potential calculations

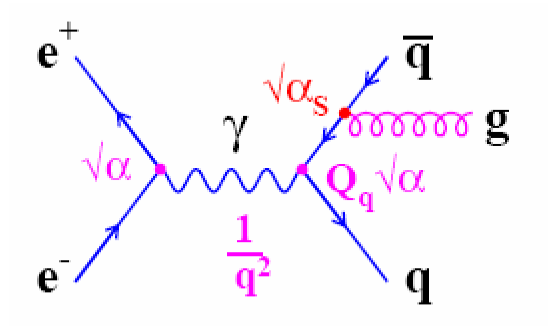
Similar to positronium (e+e-)

# Evidence for Gluons

## Quarks radiate Gluons

2nd order diagram

$e^+e^- \rightarrow q\bar{q}g$

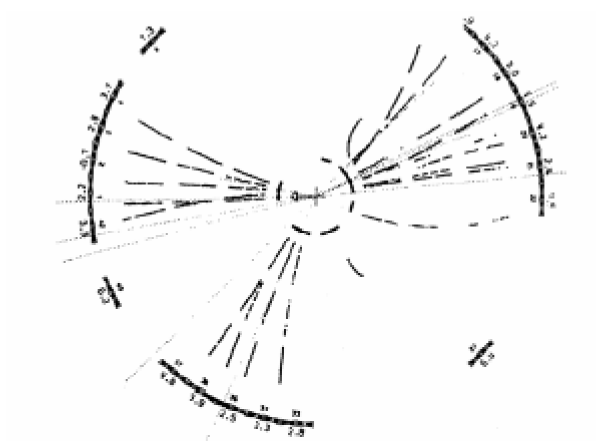



## Experimental Signature

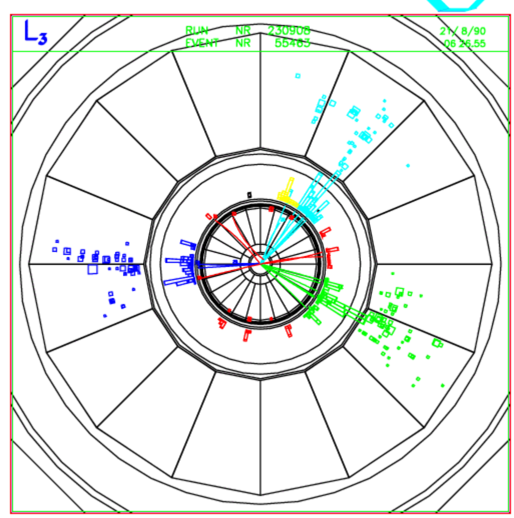
Gluons confined, fragments hadronises into jet

➔ 3-jet events

JADE $\sqrt{s}$ = 35 GeV



LEP $\sqrt{s}$ = 91 GeV




## Measurement of $\alpha_S$

When including gluon radiation additional factor $\sqrt{\alpha_S}$ in matrix element adds term with factor $\alpha_S$ in cross section

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3\sum_q Q_q^2\left(1 + \frac{\alpha_S}{\pi}\right)$$

e.g $R(q^2 = (25\ \text{GeV})^2) \approx 3.85 > 11/3 \rightarrow \alpha_S = 0.15$

# Running of $\alpha_S$

## Measurements

at many energies $\sqrt{s}$ = 1.5 GeV to 200 GeV

e+e- Annihilations

- Ratio $R \propto (1+\alpha_S/\pi)$
- Ratio of 3 jet versus 2 jet events $\propto \alpha_S$
- Event shapes - angular distributions

Hadronic collisions

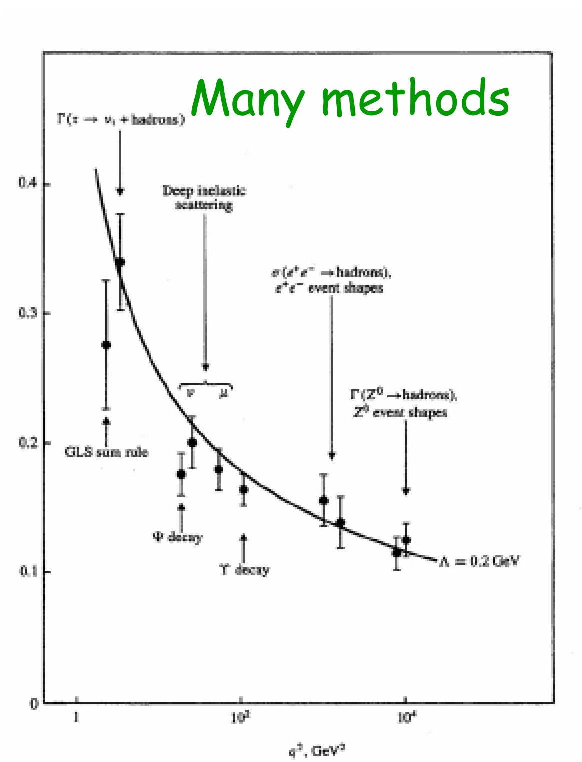
Deep Inelastic scattering

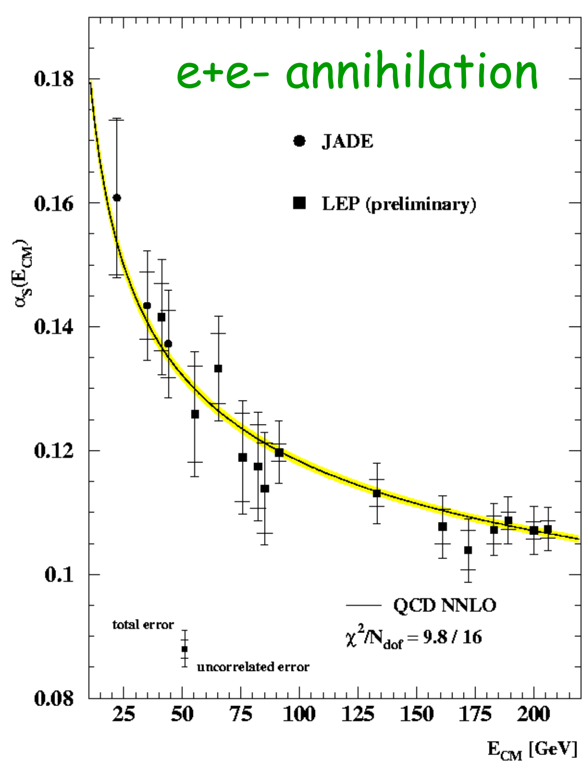
Charmonium and Upsilon

Tau decays

Lattice QCD calculations

➔ $\alpha_S$ **is running** $\alpha_S(M_Z) = 0.1187 \pm 0.002$



# Evidence for Colour

## Ratio R

Discussed in previous slides

## $\Delta^{++}$ Baryon

Strong interaction resonance - spin 3/2

Quark model explains $\Delta^{++}$ as (uuu)

Wave function for $(u\uparrow u\uparrow u\uparrow)$ is symmetric under interchange of identical quarks

Appears to violate Pauli Principle

➔ Led to introduction of colour

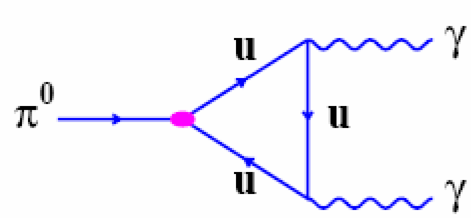
1964 Greenberg

Antisymmetric colour wave function for baryons

$$(rgb - rbg + gbr - grb + brg - bgr)/\sqrt{6}\ .$$

Same arguments for $\Delta^-$ (ddd) and $\Omega^-$ (sss)

## Decay rate $\pi^0 \to \gamma\gamma$



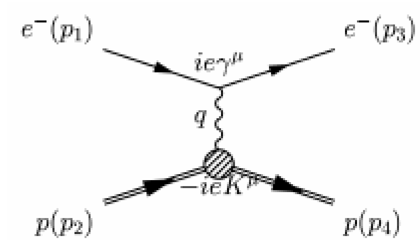

$\Gamma(\pi^0 \to \gamma\gamma) \propto N^2_{colour}$

Measurement: $N_{colour} = 2.99 \pm 0.12$

# Elastic e-p Scattering

## e-p → e-p

Probe structure of proton with electron beam




## Kinematics

Laboratory frame, proton at rest

Energy and momentum transfer $\nu, \vec{q}$

$$\nu = E_1 - E_3 \qquad q^\mu = (\nu, \vec{q})$$

$$(q + p_2)^2 = p_4^2 \qquad p_2 \cdot q = (M, \vec{0}) \cdot (\nu, \vec{q}) = M\nu$$

$$q^2 + M^2 + 2p_2 \cdot q = M^2 \qquad \Rightarrow \qquad \boxed{q^2 = -2M\nu < 0}$$

$q^2$ and $\nu$ not independent, $E_3$ and scattering angle $\theta$ related, only need to measure $E_1$ and $\theta$

## Cross Section

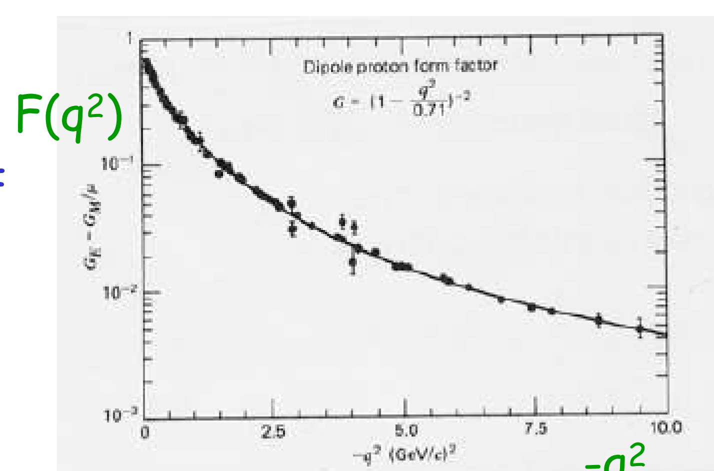
$$\boxed{\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} \left|F(q^2)\right|^2}$$

**Form factor $F(q^2)$**

describes deviation from a point charge

$F(q^2)$ is Fourier transform of charge distribution inside proton,

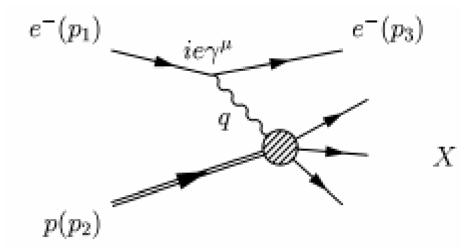
see Nuclear Physics



# Deep Inelastic Scattering

## e-p → e-X Scattering

At high $|q^2|$ proton breaks up into hadrons




$$W^2 = q^2 + M^2 + 2p_2 \cdot q \neq M^2$$

$$x = \frac{-q^2}{2M\nu}$$

$q^2$ and $\nu$ independent, hadronic mass W

define dimensionless variable x with $0 < x < 1$

Form factor $F(q^2)$ → **Structure function** $F_2(\nu, q^2)$

## Experimental Results

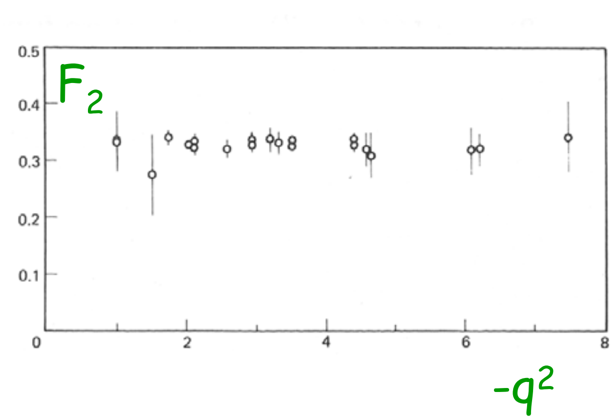
Inelastic cross section

independent of $q^2$

dependent on x → $F_2(x)$

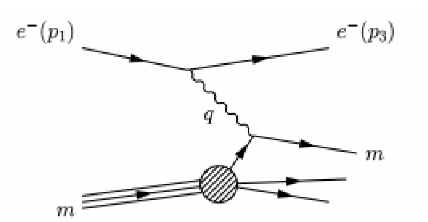
Evidence for point-like

particles inside proton




## Partons

Point-like constituents inside nucleons Feynman




$$\nu + \frac{q^2}{2m} = 0 \quad \Rightarrow \quad x = \frac{-q^2}{2M\nu} = \frac{m}{M}$$

$$m^2 = x^2E_2^2 - x^2\vec{p}_2^{\,2} = x^2M^2$$

electron scatters off "free" parton with mass m

x is fraction of proton 4-momentum

# Partons

## Parton Distribution Functions $f_i(x)$

Probability that parton i carries fraction x of particle momentum

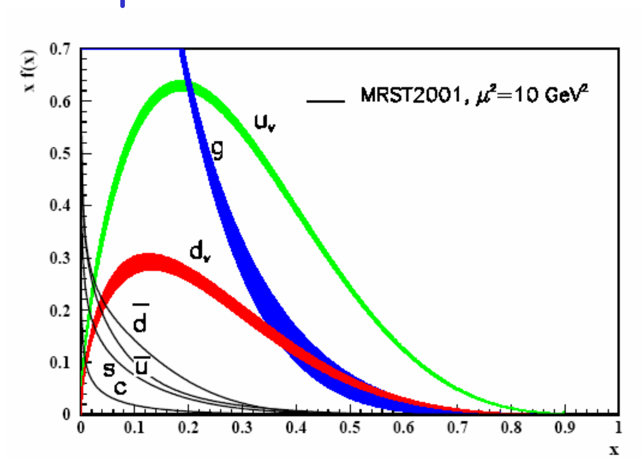
i = u,d,s (valence quarks), sea quarks, gluons

Require $\sum_i \int x\, f_i(x)\,dx = 1$

Quarks carry only 54% of proton momentum

Gluons carry remaining 46%

➔ Partons are quarks and gluons




## Quark- Quark Scattering

2 jet events at p-pbar collider √s = 315 GeV

$|q^2| \approx 2000\ \text{GeV}^2$ see QCD points

$$M \propto \frac{\sqrt{\alpha_S}\sqrt{\alpha_S}}{q^2}$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} \propto \frac{\alpha_S^2}{\sin^4(\theta/2)}$$

QED points are Geiger & Marsden (1911) Rutherford scattering