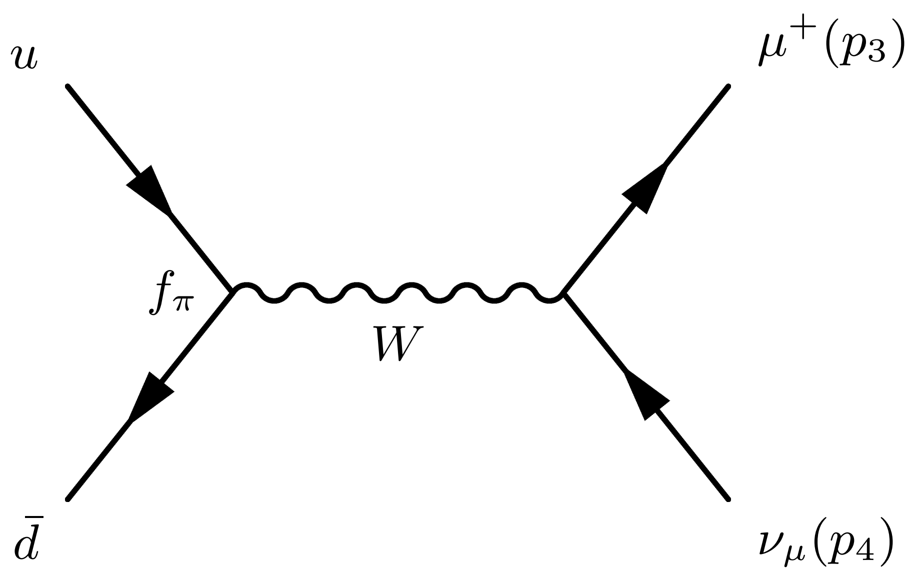
Lecture 12
Weak Decays of Hadrons

- π^+ and K^+ decays
- Semileptonic decays
- Hyperon decays
- Heavy quark decays
- Rare decays
- The Cabibbo-Kobayashi-Maskawa Matrix

Charged Pion Decay

π^+ decay by annihilation of the $u\bar{d}$ into a W^+ boson

π^- decay by annihilation of the $d\bar{u}$ into a W^- boson:



The dominant decay mode is to a muon and a neutrino

$$\pi^+ \to \mu^+ \nu_\mu \qquad \pi^- \to \mu^- \bar{\nu}_\mu$$

Why not to an electron and a neutrino?

Charged Pion Lifetime

The matrix element for the weak decay is:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} f_\pi q^\mu \left(\bar{u}_\mu \gamma^\mu \frac{1}{2}(1-\gamma^5) u_{\nu_\mu} \right)$$

where f_π is the charged pion **decay constant**
(probability that quark-antiquark annihilate inside pion)

The matrix element squared in the rest frame of the pion is:

$$|\mathcal{M}|^2 = 4 G_F^2 f_\pi^2 m_\mu^2 [p_3.p_4]$$

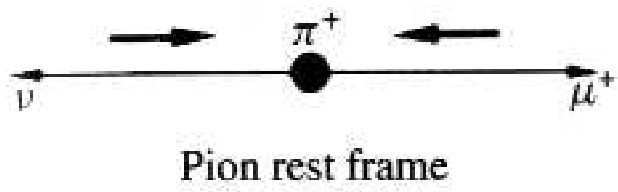
$$\Gamma_\pi = \frac{1}{\tau_\pi} = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)^2$$

Charged pion mass, lifetime and decay constant:

$m_{\pi^+} = 139.6\text{MeV}$ $\tau_{\pi^+} = 26ns$ $f_\pi = 131\text{MeV}$

Helicity Suppression in π^+ Decays

The pion has J=0, so the μ^+ and ν have the same helicities:



The decay to an electron and a neutrino is **helicity suppressed**

$$R = \frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{m_e^2}{m_\mu^2} \frac{1}{(1 - m_\mu^2/m_\pi^2)^2} = 1.275 \times 10^{-4}$$

Experimental result proves $V - A$ theory of weak interactions:

$$R = (1.267 \pm 0.023) \times 10^{-4}$$

Charged Kaon Decay

The main decay modes of the charged Kaon are:

- A leptonic decay (similar to $\pi^+ \to \mu^+\nu_\mu$)

$$\mathcal{B}(K^+ \to \mu^+\nu_\mu) = 63.4\%$$

- A semileptonic decay (with equal amounts of $\ell = e, \mu$)

$$\mathcal{B}(K^+ \to \pi^0\ell^+\nu_\ell) = 8.1\%$$

- Weak hadronic decays to pions

$$\mathcal{B}(K^+ \to \pi^+\pi^0) = 21.1\% \qquad \mathcal{B}(K^+ \to \pi^+\pi^+\pi^-) = 5.6\%$$

Charged Kaon mass, lifetime and decay constant:

$$m_{K^+} = 494\text{MeV} \qquad \tau_{K^+} = 12ns \qquad f_K = 160\text{MeV}$$

$f_K \neq f_\pi$ *is example of breaking of SU(3) flavour symmetry*

Neutral Kaon Decays

For neutral Kaons the decay eigenstates K_S and K_L are not equal to the flavour eigenstates K^0 and $\bar{K}^0$ (more in Lecture 13)

The main decay modes are:

$$\mathcal{B}(K_L \to \pi^- \ell^+ \nu_\ell) = 67.5\% \qquad (\ell = e, \mu)$$

$$\mathcal{B}(K_L \to \pi^+ \pi^- \pi^0) = 12.6\% \qquad \mathcal{B}(K_L \to \pi^0 \pi^0 \pi^0) = 19.6\%$$

$$\mathcal{B}(K_S \to \pi^+ \pi^-) = 69.2\% \qquad \mathcal{B}(K_S \to \pi^0 \pi^0) = 30.7\%$$

Neutral Kaon mass and lifetimes:

$$m_{K^0} = 498\text{MeV} \qquad \tau_S = 0.09ns \qquad \tau_L = 51ns$$

There are two very different lifetimes (short and long)!

The Cabibbo Angle

The couplings of the W boson to $u\bar{d}$ and $u\bar{s}$ quarks are described by the **Cabibbo angle** θ_C

Weak coupling becomes $G_F \Rightarrow G_F V_{ud}$ or $G_F V_{us}$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Interpret this as a rotation matrix between the flavour eigenstates d, s and the weak eigenstates d', s'

The Cabibbo angle is measured to be:

$$\theta_C = 12.7^\circ \qquad \sin\theta_C = 0.220 \qquad \cos\theta_C = 0.976$$

Semileptonic Decays & Selection Rules

The matrix element for semileptonic Kaon decays is:

$$\mathcal{M} \propto f_+ \left[(p_K + p_\pi)^\mu \bar{\ell} \gamma^\mu (1+\gamma^5)\nu \right] + f_- \left[m_\ell \bar{\ell} (1+\gamma^5)\nu \right]$$

where f_+ and f_- are semileptonic decay **form factors** which describe the hadronic transitions $K \to \pi$

The f_- term multiplying m_ℓ is negligible for electrons

Semileptonic kaon decays obey the selection rules:

$$\Delta \mathrm{I} = \Delta \mathrm{I}_3 = \frac{1}{2} \qquad \Delta \mathrm{Q} = \Delta S = 1$$

Hadronic kaon decays obey the selection rules:

$$\Delta \mathrm{I} = 1/2, 3/2 \ \ (1/2 \text{ preferred}) \quad \Delta \mathrm{I}_3 = \frac{1}{2} \quad \Delta S = 1$$

Hyperon Decays

Baryons containing strange quarks are known as **hyperons**

With one exception they all have weak decays:

Hyperon	Quark Content	Decay Modes	Lifetime
Λ	uds	$p\pi^-$, $n\pi^0$	$0.26ns$
Σ^+	uus	$p\pi^0$, $n\pi^+$	$0.80ns$
Σ^0	uds	$\Lambda\gamma$	$7 \times 10^{-20}s$
Σ^-	dds	$n\pi^-$	$0.15ns$
Ξ^0	uss	$\Lambda\pi^0$	$0.29ns$
Ξ^-	dds	$\Lambda\pi^-$	$0.16ns$
Ω^-	sss	ΛK^-, $\Xi^0\pi^-$	$0.08ns$

Lifetimes $\approx 10^{-10}s$ Decay Lengths $\approx$ 1cm are observable

Heavy Quark Decays

Charm quark decays are mainly $c \to s$ (a few percent $c \to d$)

Examples $D \to K\ell\nu$, $D \to K\pi$, $D \to K\pi\pi$

Lifetimes $\tau_{D^+} = 1.04ps$, $\tau_{D^0} = 0.41ps$

Bottom quark decays are mainly $b \to c$ (a few percent $b \to u$)

Examples $B \to D\ell\nu$, $B \to D\pi$, $B \to J/\psi K_S^0$

Lifetimes $\tau_{B^+} = 1.64ps$, $\tau_{B^0} = 1.53ps$

The proper decay lengths of b and c hadrons are $100 - 500\mu m$

The top quark decays almost completely $t \to b$

Its lifetime is too short to form hadrons!

Semileptonic & Rare b Decays

Inclusive and exclusive semileptonic decays (BaBar/Belle)

$$\mathcal{B}(b \to c\ell\nu_\ell) = (10.75 \pm 0.15)\%$$

$$\mathcal{B}(B \to D\ell\nu_\ell) = (2.2 \pm 0.1)\% \qquad \mathcal{B}(B \to D^*\ell\nu_\ell) = (5.6 \pm 0.5)\%$$

$$\mathcal{B}(b \to u\ell\nu_\ell) = (1.3 \pm 0.1) \times 10^{-3} \qquad \mathcal{B}(B \to \pi\ell\nu_\ell) = (1.4 \pm 0.1) \times 10^{-4}$$

Determine CKM couplings V_{cb} and V_{ub}

Flavour-changing neutral currents (CLEO/BaBar/Belle)

$$\mathcal{B}(b \to s\gamma) = (3.5 \pm 0.3) \times 10^{-4} \qquad \mathcal{B}(B \to K^*\gamma) = (4.5 \pm 0.2) \times 10^{-5}$$

These are second-order weak decays ("penguin" loops)

Set limits on many New Physics models

Leptonic Decays of Heavy Quarks

Leptonic D decays measured by CLEO-c experiment (2008)

$$\mathcal{B}(D^+ \to \mu^+ \nu_\mu) = (4.4 \pm 0.6) \times 10^{-4}$$

$$\mathcal{B}(D_s^+ \to \mu^+ \nu_\mu) = (6.2 \pm 0.6) \times 10^{-3}$$

$$\mathcal{B}(D_s^+ \to \tau^+ \nu_\tau) = (6.6 \pm 0.6) \times 10^{-2}$$

Leptonic B decay measured by BaBar/Belle experiments (2006)

$$\mathcal{B}(B^+ \to \tau^+ \nu_\tau) = (1.4 \pm 0.4) \times 10^{-4}$$

Determine decay constants f_D, f_{Ds}, f_B

Set limits on possible charged Higgs couplings

Cabibbo-Kobayashi-Maskawa Matrix

Kobayashi & Maskawa awarded Nobel prize in October 2008!

By extension from the Cabibbo angle, the full description of weak decays of quarks needs the 3×3 CKM matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The CKM matrix is unitary, and its elements satisfy:

$$\sum_i V_{ij}^2 = 1 \qquad \sum_j V_{ij}^2 = 1$$

$$\sum_i V_{ij} V_{ik} = 0 \qquad \sum_j V_{ij} V_{kj} = 0$$

The CKM matrix can be written in terms of just **four parameters**

CKM Parametrizations

With three angles $s_i = \sin\theta_i$, $c_i = \cos\theta_i$, and a complex phase δ:

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_3 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}$$

Wolfenstein parametrisation is expansion in powers of $\lambda = \sin\theta_C$:

$$\begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & A\lambda^2 & 1 \end{pmatrix}$$

Measurements of CKM Elements

- $V_{ud} = 0.976$ from pion and nuclear β decays
- $V_{us} = 0.220$ from Kaon and Hyperon decays
- $V_{cs} = 0.97 \pm 0.12$ from $D \to K\ell\nu$ semileptonic decays
- $V_{cd} = 0.224 \pm 0.012$ from neutrino production of charm
- $V_{cb} = 0.0420 \pm 0.0007$ from $b \to c\ell\nu$ semileptonic decays
- $V_{ub} = 0.0044 \pm 0.0004$ from $b \to u\ell\nu$ semileptonic decays
- V_{td} and V_{ts} are measured in B meson mixing (Lecture 13)
- $V_{tb} \approx 1$ is measured in top decays at the Tevatron

Wolfenstein parameters:

$\lambda = 0.2265 \pm 0.0025 \quad A = 0.80 \pm 0.03 \quad \rho = 0.19 \pm 0.08 \quad \eta = 0.36 \pm 0.04$