Lecture 13
Discrete Symmetries P, C, and T

- Parity (P)
- Charge Conjugation (C)
- Time Reversal (T)
- CP Violation
- P and C violation in weak decays
- Tests of CPT invariance

Parity Operator P

P performs a **spatial inversion**

$$P\psi(\vec{r}) = \psi(-\vec{r})$$

Applying P twice restores initial state: $P^2 = 1$

Parity of ψ can be either odd $(P = -1)$ or even $(P = +1)$
$\psi(x) = \sin kx$ is odd, and $\psi(x) = \cos kx$ is even.

Angular part of atomic wavefunctions (Legendre polynomials):

$$Y_l^m = \mathrm{P}_l^m(\cos\theta)e^{im\phi}$$

$$PY_l^m = (-1)^{l+m} \times (-1)^m \times Y_l^m = (-1)^l \times Y_l^m$$

Parity associated with orbital angular momentum is $(-1)^L$

Intrinsic Parity

Intrinsic parity of fermions $P_f = +1$ (even)
Intrinsic parity of antifermions $P_{\bar{f}} = -1$ (odd)

Deduced by applying a spatial inversion to the Dirac equation

$$\left(i\gamma^0 \frac{\delta}{\delta t} - i\vec{\gamma} \cdot \vec{\nabla} - m\right) \psi(-\vec{r}, t) = 0$$

There is a change in the sign of the spatial derivative!

Parity operator is identified as $P = \gamma^0$

$$\psi(\vec{r}, t) = \gamma^0 \psi(-\vec{r}, t) = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix} \psi(-\vec{r}, t)$$

$$Pu_1 = Pu_2 = +1 \qquad Pv_1 = Pv_2 = -1$$

Charge Conjugation Operator C

C reverses the sign of the charge and magnetic moment.

C is equivalent to particle $\rightarrow$ antiparticle

Applying C twice restores initial state: $C^2 = 1$

C-parity can be either odd ($C = -1$) or even ($C = +1$)

Fermions are not eigenstates of C because a fermion is transformed into an antifermion.

Cannot define intrinsic C-parity of fundamental fermions

Can define C-parity of composite states:

$$C = (-1)^{L+S} \qquad C(\pi^0) = +1 \qquad C(\rho^0) = -1$$

Decays of Positronium

Positronium is an e^+e^- atom with parity states:

$$P(e^+e^-) = P_{e^-} P_{e^+} (-1)^L = (-1)^{L+1}$$

Note this is the opposite parity to the Hydrogen atom states because of the intrinsic parities of the e^+ and e^-

Charge conjugation symmetry depends on the spin orientations:

$$C(\uparrow\downarrow) = (-1)^L \qquad C(\uparrow\uparrow) = (-1)^{L+1}$$

S=0 is ortho-positronium $\uparrow\downarrow$, S=1 is para-positronium $\uparrow\uparrow$

Photon has $C = -1$ from symmetry of electromagnetic fields

Ortho-positronium states with even(odd) L decay to 2(3) photons
Para-positronium states with odd(even) L decay to 2(3) photons

C-parity is conserved in electromagnetic interactions

Time Reversal Operator T

T reverses the direction of the time axis:

$$T\psi(\vec{r}, t) = \psi(\vec{r}, -t)$$

T changes the sign of time-dependent properties of a state, e.g. spin $\vec{\sigma}$ and momentum $\vec{p}$

Applying T twice restores initial state: $T^2 = 1$

T-parity can be either odd ($T = -1$) or even ($T = +1$)

Test of time-reversal in strong interactions by detailed balance (Martin & Shaw p.104)

$$d\sigma(pp \to \pi^+ d) \propto \left(\frac{p_\pi}{p_p}\right)^2 d\sigma(\pi^+ d \to pp)$$

The Universe does have an **arrow** of time! Entropy increases!

Summary of Discrete Symmetry Operations

Quantity	Notation	P	C	T
Position	$\vec{r}$	-1	+1	+1
Momentum (Vector)	$\vec{p}$	-1	+1	-1
Spin (Axial Vector)	$\vec{\sigma} = \vec{r} \times \vec{p}$	+1	+1	-1
Helicity	$\vec{\sigma}.\vec{p}$	-1	+1	+1
Electric Field	$\vec{E}$	-1	-1	+1
Magnetic Field	$\vec{B}$	+1	-1	-1
Magnetic Dipole Moment	$\vec{\sigma}.\vec{B}$	+1	-1	+1
Electric Dipole Moment	$\vec{\sigma}.\vec{E}$	-1	-1	-1
Transverse Polarization	$\vec{\sigma}.(\vec{p_1} \times \vec{p_2})$	+1	+1	-1

Parity Violation in Weak Decays

First observed in β decay of polarized ^{60}Co

Angular distribution of decay electrons relative to nuclear spin:

$$\frac{dN_e}{d\Omega} = 1 - \frac{\vec{\sigma}.\vec{p}}{E}$$

Electrons prefer to be emitted opposite to spin direction

Parity operation reverses $\vec{p}$ but not $\vec{\sigma}$

Second term in angular distribution violates parity

Parity is *maximally* violated in weak interactions

C-parity is also *maximally* violated

Helicity States of Neutrinos

Neutrinos are (almost) massless fermions

Solutions of Dirac equation give only two helicity states:

$$\mathcal{H}(\psi_L) = -1 \qquad \mathcal{H}(\psi_R) = +1 \qquad \mathcal{H} = \frac{\vec{\sigma}.\vec{p}}{|\vec{\sigma}||\vec{p}|}$$

where $\vec{\sigma}$ are the Pauli spin matrices.

ψ_L describes a **left-handed neutrino**

ψ_R describes a **right-handed antineutrino**

P,C operations do not give physical (anti)neutrino states

$$P(\nu_L) \rightarrow \nu_R \qquad C(\nu_L) \rightarrow \bar{\nu}_L \qquad CP(\nu_L) \rightarrow \bar{\nu}_R$$

CP changes left-handed neutrino into right-handed antineutrino

Dirac or Majorana Neutrinos?

Dirac neutrinos:

Left-handed ν and right-handed $\bar{\nu}$ are a fermion/antifermion pair.

The other helicity states are suppressed by $m_\nu = 0$

Majorana neutrinos:

Left-handed and right-handed states of the same particle.

The neutrino is its own antiparticle.

Double beta decay can distinguish between these

Majorana: $A^Z \rightarrow B^{(Z+2)} + 2e^- + 0\nu$

Dirac: $A^Z \rightarrow B^{(Z+2)} + 2e^- + 2\bar{\nu}_e$

Weak Charged Current: $J^\mu = \bar{u}\gamma^\mu \frac{1}{2}(1-\gamma^5)u$

$P_L = 1-\gamma^5$ projects out fermion spinor u

$$P_L u^1 = \frac{1}{2}\begin{pmatrix} 1-\beta \\ 0 \\ -(1-\beta) \\ 0 \end{pmatrix} \qquad P_L u^2 = \frac{1}{2}\begin{pmatrix} 0 \\ 1+\beta \\ 0 \\ -(1+\beta) \end{pmatrix}$$

Only the left-handed state u^2 remains in the limit $\beta \to 1$

Effect of Parity operator γ^0:

$$\gamma^0(1-\gamma^5) = (1+\gamma^5)\gamma^0 \qquad \gamma^0\gamma^\mu = -\gamma^\mu\gamma^0$$

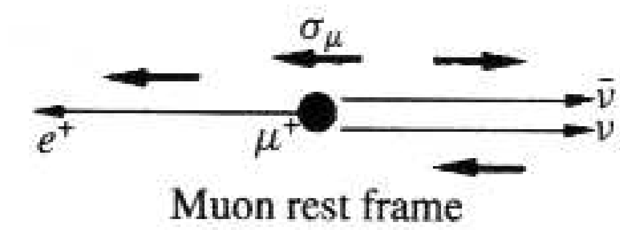
$$P(J^\mu) = \bar{u}\gamma^0\gamma^\mu\frac{1}{2}(1-\gamma^5)u = -\bar{u}\frac{1}{2}(1+\gamma^5)\gamma^\mu\gamma^0 u$$

$P_R = 1+\gamma^5$ projects out right-handed component of $\bar{u}$

Helicities in Muon Decay

Maximum electron (positron) energy is $E_0 = m_\mu/2$

Occurs when ν and $\bar{\nu}$ are in the same direction:



Positron prefers to be right-handed

μ^+ decay with E_0 allowed if muon spin is in positron direction

μ^+ decay with E_0 forbidden if muon spin in neutrino direction

Electron prefers to be left-handed

Reverse direction of muon spin in above

Parity Violation in Muon Decay

Angular distribution of decay electrons relative to μ^- spin:

$$\frac{dN_e}{d\Omega} = 1 - \cos\theta$$

Electrons prefer to be emitted opposite to μ^- spin

Angular distribution of decay positrons relative to μ^+ spin:

$$\frac{dN_e}{d\Omega} = 1 + \cos\theta$$

Positrons prefer to be emitted parallel to μ^+ spin

Parity and Charge Conjugation are violated in weak interactions

... but CP is conserved: changes μ^- decay to μ^+ decay

CP and CPT Operations

The **CPT theorem** is a general result of field theory:

"All interactions described by a local Lorentz invariant gauge theory must be invariant under CPT."

If CPT is violated $\Rightarrow$ non-locality and/or loss of Lorentz invariance

There is no experimental evidence for CPT violation

Assuming CPT conservation these operations are equivalent:

$$CP \leftrightarrow T \qquad CT \leftrightarrow P \qquad PT \leftrightarrow C$$

There is experimental evidence for CP violation
and hence for time reversal violation

CP violation will be discussed in detail in the next Lecture

Tests of CPT Invariance

- Particles and antiparticles have equal masses:

$$[M(K^0) - M(\bar{K}^0)] < 10^{-19}[M(K^0) + M(\bar{K}^0)]$$

- Particles and antiparticles have equal lifetimes

$$[\tau(\mu^+) - \tau(\mu^-)] < 10^{-4}[\tau(\mu^+) + \tau(\mu^-)]$$

- Particles and antiparticles have equal and opposite charges

$$Q(p) + Q(\bar{p}) < 10^{-5}e$$

- Particles and antiparticles have equal magnetic moments

$$[\mu(e^+) - \mu(e^-)] < 10^{-12}[\mu(e^+) + \mu(e^-)]$$

- Hydrogen and Anti-Hydrogen atoms have identical spectra