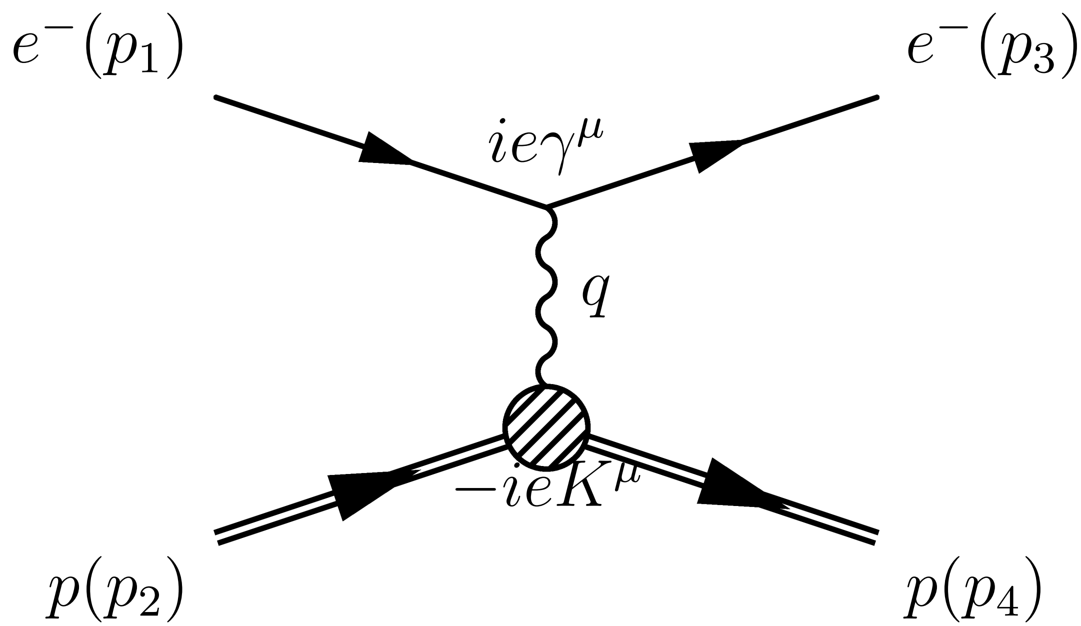
Lecture 6
Electron-Proton Scattering

- Mott Scattering Formula
- Elastic Scattering
- Electric & Magnetic Form Factors
- Deep Inelastic Scattering
- Structure Functions

Electrons (and muons) are used to probe the sub-structure of protons (and neutrons)



Scattering is off quarks by **electromagnetic interactions**

Mott Scattering

Scattering of a relativistic electron by a **pointlike** spin 1/2 proton

Similar to electron muon scattering from last Lecture

Usually described in the Lab frame, where the proton is at rest:

θ is the lab scattering angle of the electron

p_e is the incident electron beam momentum

q^2 is the four-momentum transfer of the virtual photon

Mott scattering formula:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} = \frac{\alpha^2}{4p_e^2 \sin^4 \frac{\theta}{2}} \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2m_p^2} \sin^2 \frac{\theta}{2} \right)$$

This is a modified version of Rutherford scattering

The second term in brackets accounts for the proton recoil

Elastic Scattering Kinematics

Energy lost by electron due to proton recoil ("inelasticity"):

$$\nu = E_1 - E_3 \qquad \nu > 0$$

In **elastic scattering** there is a simple relationship between ν and q^2 from four-momentum conservation:

$$2m_p\nu = Q^2 = -q^2 \qquad q^2 < 0 \quad Q^2 > 0$$

The four-momentum squared of the virtual photon is negative!

Inelastic electron-proton scattering is described by the two variables ν and q^2 (or Q^2)

Be careful not to get confused between q^2 and Q^2

Most textbooks and these lectures use both!

Form Factors

Deviations from the Mott scattering formula describe the charge distribution inside the proton in terms of a **form factor** $F(q^2)$

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} |F(q^2)|^2$$

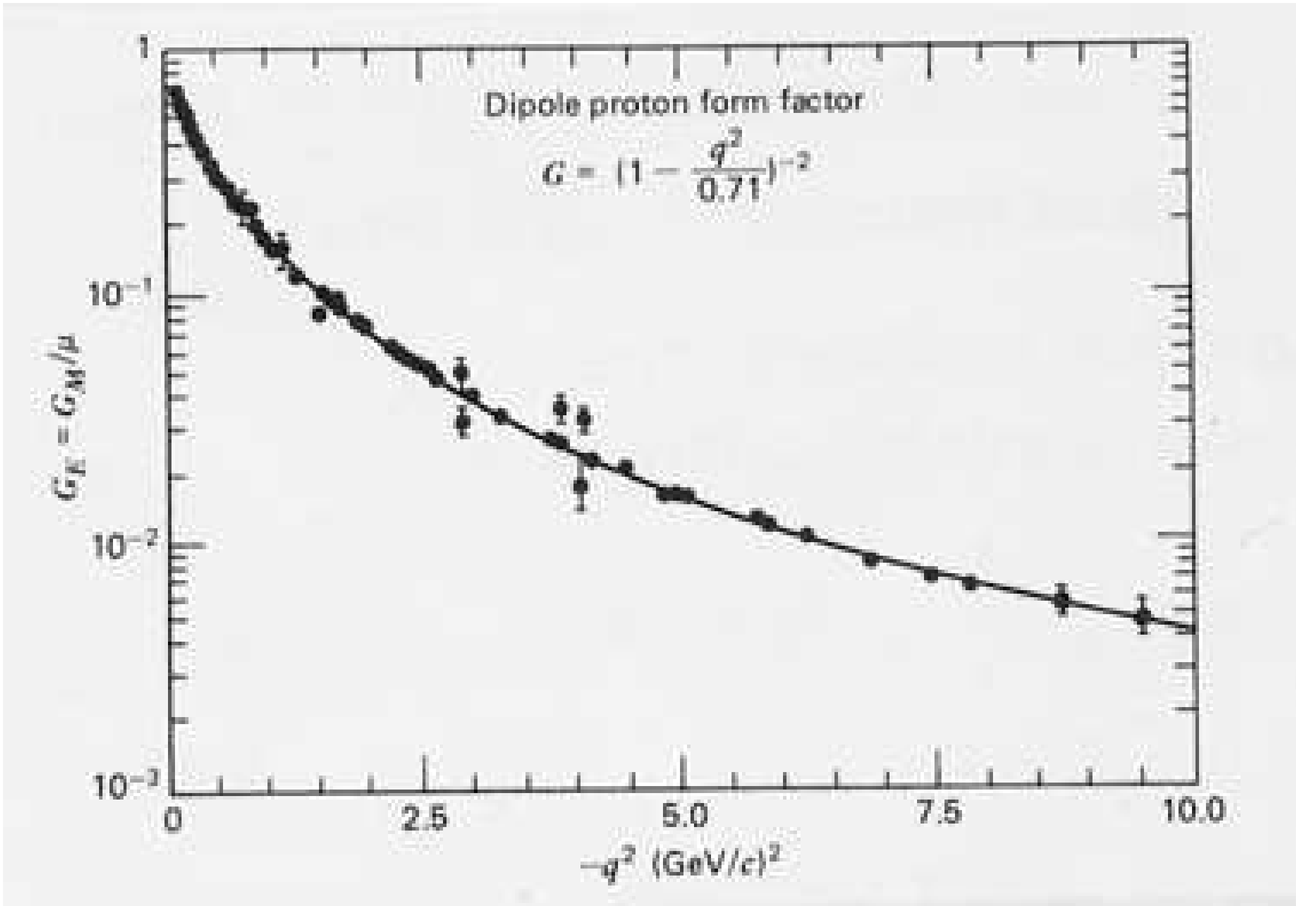
Low q^2 probes distances larger than size of proton ($r \approx 1\text{fm}$)

There is no sensitivity to charge distribution $F(0) = 1$

Large q^2 probes inside the proton and the form factor $F(q^2) < 1$

Form factor is Fourier transform of charge distribution:

$$F(q^2) = \int \rho(\vec{x}) e^{i\vec{q}\cdot\vec{x}} d^3x$$



Mean square charge radius of proton:

$$\rho(r) = \rho_0 e^{-r/r_0} \qquad\qquad < r_0^2 >= 0.8\text{fm}$$

Proton Electromagnetic Current

Matrix element for elastic scattering

$$\mathcal{M}(e^- p \to e^- p) = \frac{e^2}{(p_1 - p_3)^2} \left(\bar{u}_3 \gamma^\mu u_1\right) \left(\bar{u}_4 K_\mu u_2\right) = \frac{1}{q^2} j_e{}^\mu j_{p\,\mu}$$

can be factorized into lepton and proton electromagnetic currents

Proton current can be written in terms of a "Dirac charge" form factor F_1 and an "anomalous magnetic" form factor F_2:

$$j_p^\mu = e \bar{u}_4 \left(\gamma^\mu F_1(q^2) + \frac{i\kappa_p}{2m_p} F_2(q^2) \sigma^{\mu\nu} q_\nu \right) u_2$$

where κ_p is the *anomalous magnetic moment* of the proton

$$\mu_p = \frac{(1+\kappa_p)e}{2m_p} \qquad \kappa_p = 1.79$$

Differential Cross-section

Differential cross-section for elastic scattering is written in terms of the form factors F_1 and F_2:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \left\{ \left(F_1^2 - \frac{\kappa_p^2 q^2}{4m_p^2} F_2^2 \right) \cos^2 \frac{\theta}{2} - \frac{q^2}{2m_p^2} \left(F_1 + \kappa_p F_2 \right)^2 \sin^2 \frac{\theta}{2} \right\}$$

Low q^2 limit has $F_1^p(0) = 1, F_2^p(0) = 1$

A pointlike proton would have $F_1(q^2) = 1$ for all q^2 and $\kappa_p = 0$
(This is Mott scattering)

Even though the neutron has no charge it does have an anomalous magnetic moment $\kappa_n = -1.91$, $\mu_n = \kappa_n e/2m_n$!
In the low q^2 limit $F_1^n(0) = 0, F_2^n(0) = 1$

Electric and Magnetic Form Factors

Define linear combinations of F_1 and F_2:

$$G_E = F_1 + \frac{\kappa q^2}{4m_p^2} F_2 \qquad\qquad G_M = F_1 + \kappa F_2$$

Differential cross-section becomes:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

where $\tau = Q^2/4m_p^2$ and G_M is associated with the proton recoil

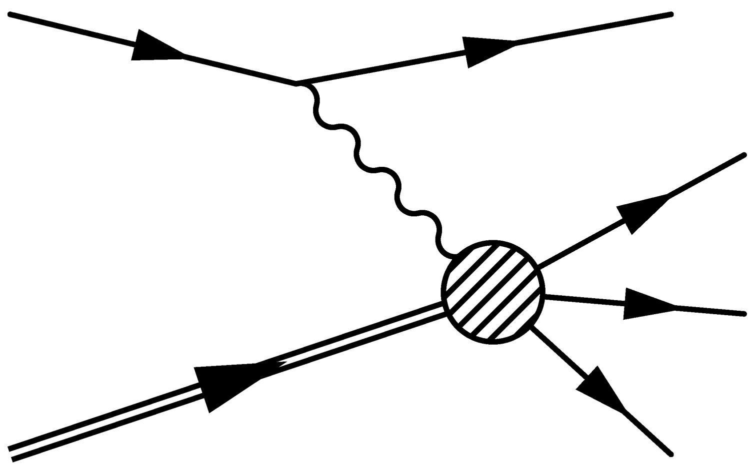
G_E and G_M are known as **electric** and **magnetic** form factors

The ratios of the proton form factors are constrained to be:

$$G_E = \frac{G_M}{\mu_p} \qquad\qquad F_1 = \kappa_p \frac{(1+\mu_p\tau)}{(\mu_p - 1)} F_2$$

Deep Inelastic Scattering

During deep inelastic scattering (DIS) at high q^2 the proton breaks up into its constituent quarks:



The quarks form a **hadronic jet** with invariant mass W

Kinematics of DIS

The invariant mass squared of the hadronic jet is:

$$W^2 = m_p^2 + 2m_p\nu + q^2$$

Since $W \neq m_p$, the four momentum transfer q^2 and inelasticity ν are independent variables.

They are usually replaced by the **parton energy** x:

$$x = \frac{Q^2}{2m_p\nu} = \frac{-q^2}{2m_p\nu}$$

and the **parton rapidity** y:

$$y = \frac{p_2 \cdot q}{p_2 \cdot p_1} = \frac{\nu}{E_1}$$

x, y are dimensionless variables with ranges $0 \leq (x, y) \leq 1$

Matrix element for DIS

The matrix element squared can be factorised into lepton and hadronic current terms:

$$|\mathcal{M}|^2 = \frac{e^4}{q^2} L_e^{\mu\nu} (W_{\text{hadron}})_{\mu\nu}$$

The hadronic part describes the inelastic breakup of the proton in terms of two **structure functions** W_1 and W_2 which depend on the kinematic variables ν and Q^2 (or equivalently on x and y)

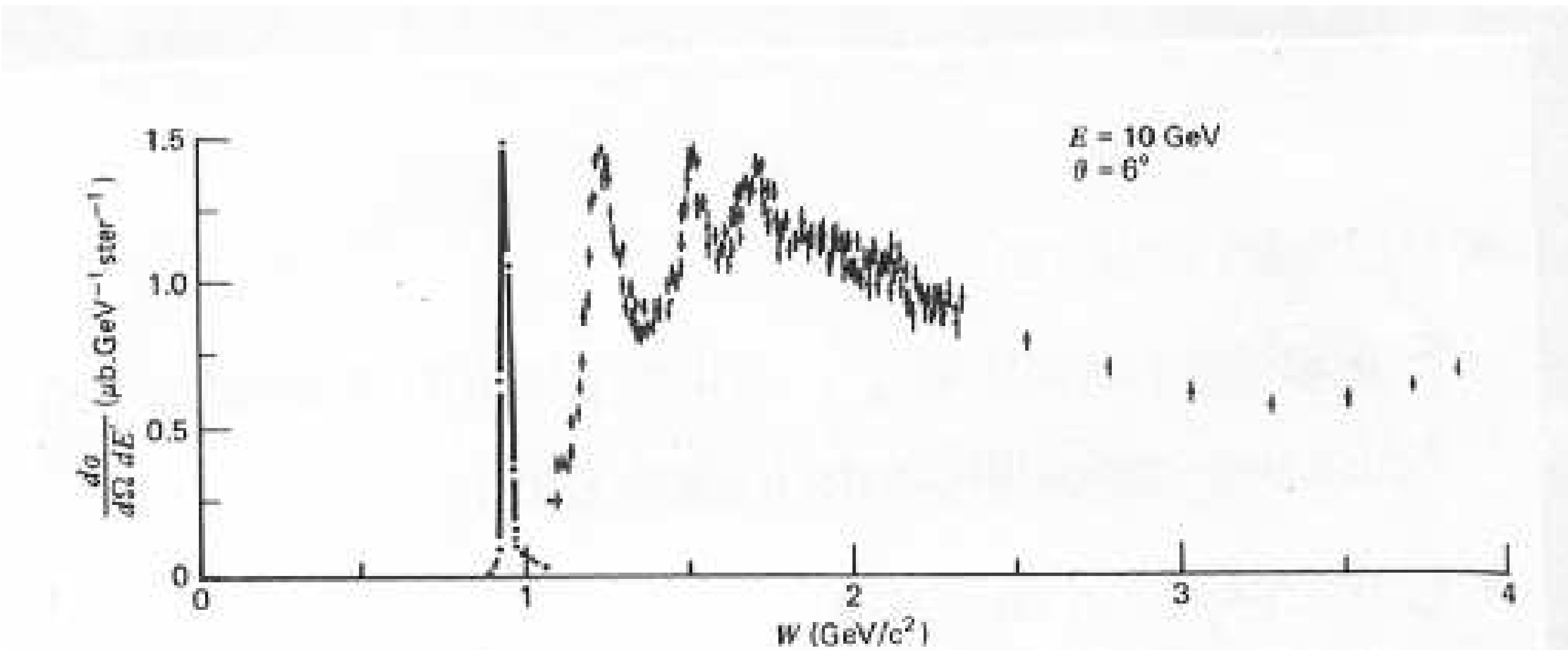
The doubly differential cross-section is:

$$\frac{d\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \left(W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} \right)$$

Measurement of DIS cross-section

First done at SLAC in 1970s

(Friedman, Kendall & Taylor: Nobel Prize 1990)



Peaks are from proton (elastic) and baryonic resonances

Structure Functions F_1 and F_2

In Deep Inelastic Scattering it is usual to replace the structure functions W_1, W_2 with F_1, F_2:

$$m_p W_1(\nu, Q^2) \rightarrow F_1(x) \qquad \nu W_2(\nu, Q^2) \rightarrow F_2(x)$$

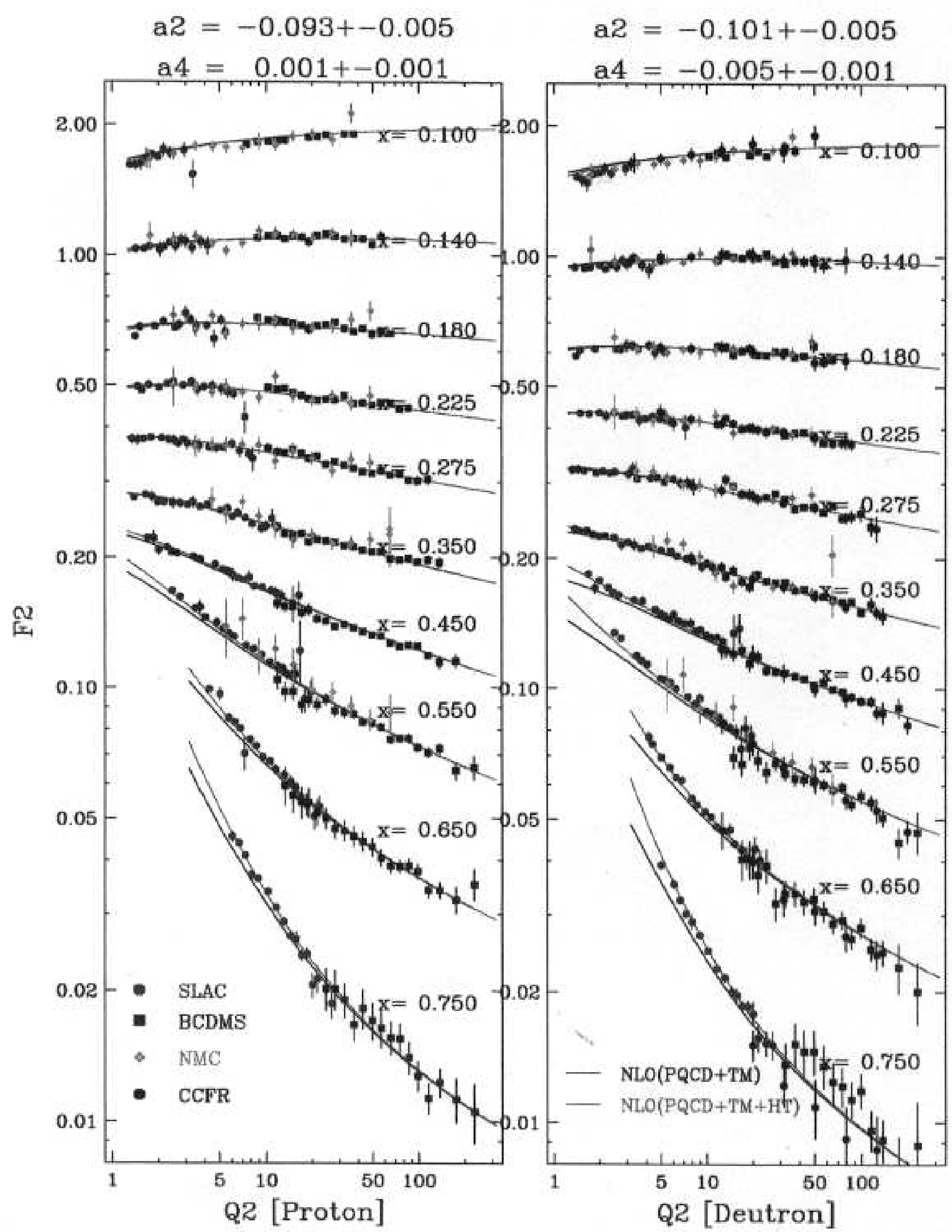
Warning - $F_{1,2}(x)$ in DIS are **not** the same as $F_{1,2}(q^2)$ in elastic scattering!

Note that F_1 and F_2 are only written as functions of x!

x is fraction of the proton energy off which the scattering occurs:

$$x = \frac{Q^2}{2m_p\nu} = \frac{m}{m_p} \qquad 2m\nu + q^2 = 0$$

Implies that the virtual photon interacts with a point-like spin 1/2 **parton** inside the proton (i.e. a quark)



Measurements of Structure functions

Are data for a given x independent of Q^2?

Some dependence on Q^2 (more next lecture)