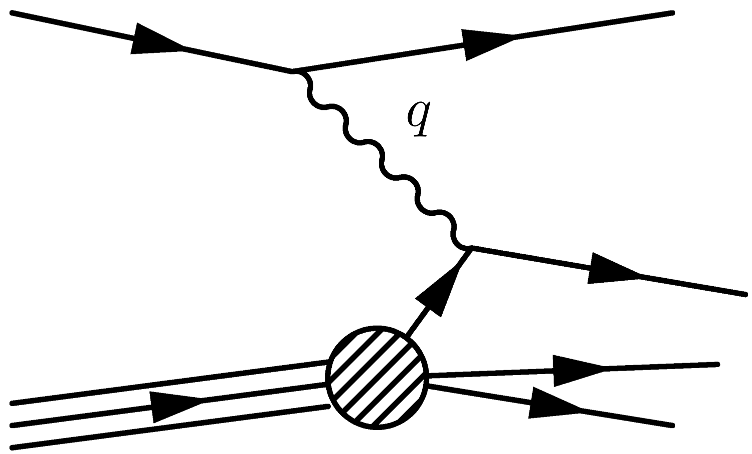
Lecture 7 - The Parton Model

- Parton Distribution Functions
- Bjorken Scaling
- Valence and Sea Quarks
- Neutrino Deep Inelastic Scattering
- Gluons & Scaling Violation

Parton-level Scattering



At high Q^2 the underlying process is "elastic" scattering off a "pointlike" parton of mass m and charge Q_i:

$$2m\nu + q^2 = 0 \qquad\qquad x = \frac{Q^2}{2m_p\nu} = \frac{m}{m_p}$$

$$\frac{d\sigma}{d\Omega} = \frac{\mathrm{Q}_i^2\alpha^2}{4E_1^2\sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)$$

Parton Distribution Functions

The probability of a parton of type i having a fraction x of the proton energy is the **parton distribution function (pdf)** $f_i(x)$

Pointlike partons have structure functions that are δ functions:

$$W_1^i = \frac{Q^2}{4m^2}\delta(\nu - \frac{Q^2}{2m}) \qquad W_2^i = \delta(\nu - \frac{Q^2}{2m})$$

The proton structure functions are sums over the parton pdfs:

$$W_1^p = \frac{F_1(x)}{m_p} \qquad F_1^p(x) = \frac{1}{2}\sum_i \mathrm{Q}_i^2 f_i(x)$$

$$W_2^p = \frac{F_2(x)}{\nu} \qquad F_2^p(x) = \sum_i x\mathrm{Q}_i^2 f_i(x)$$

For $S = 1/2$ quarks must have: $2xF_1^p(x) = F_2^p(x)$

Valence Quark Fractions

Proton structure function from valence quarks uud:

$$F_2^p(x) = \sum_i x Q_i^2 f_i(x) = \frac{4}{9} x u(x) + \frac{1}{9} x d(x)$$

Neutron has valence quarks $udd \Rightarrow$ interchange $u(x)$ and $d(x)$

$$F_2^n(x) = \sum_i x Q_i^2 f_i(x) = \frac{4}{9} x d(x) + \frac{1}{9} x u(x)$$

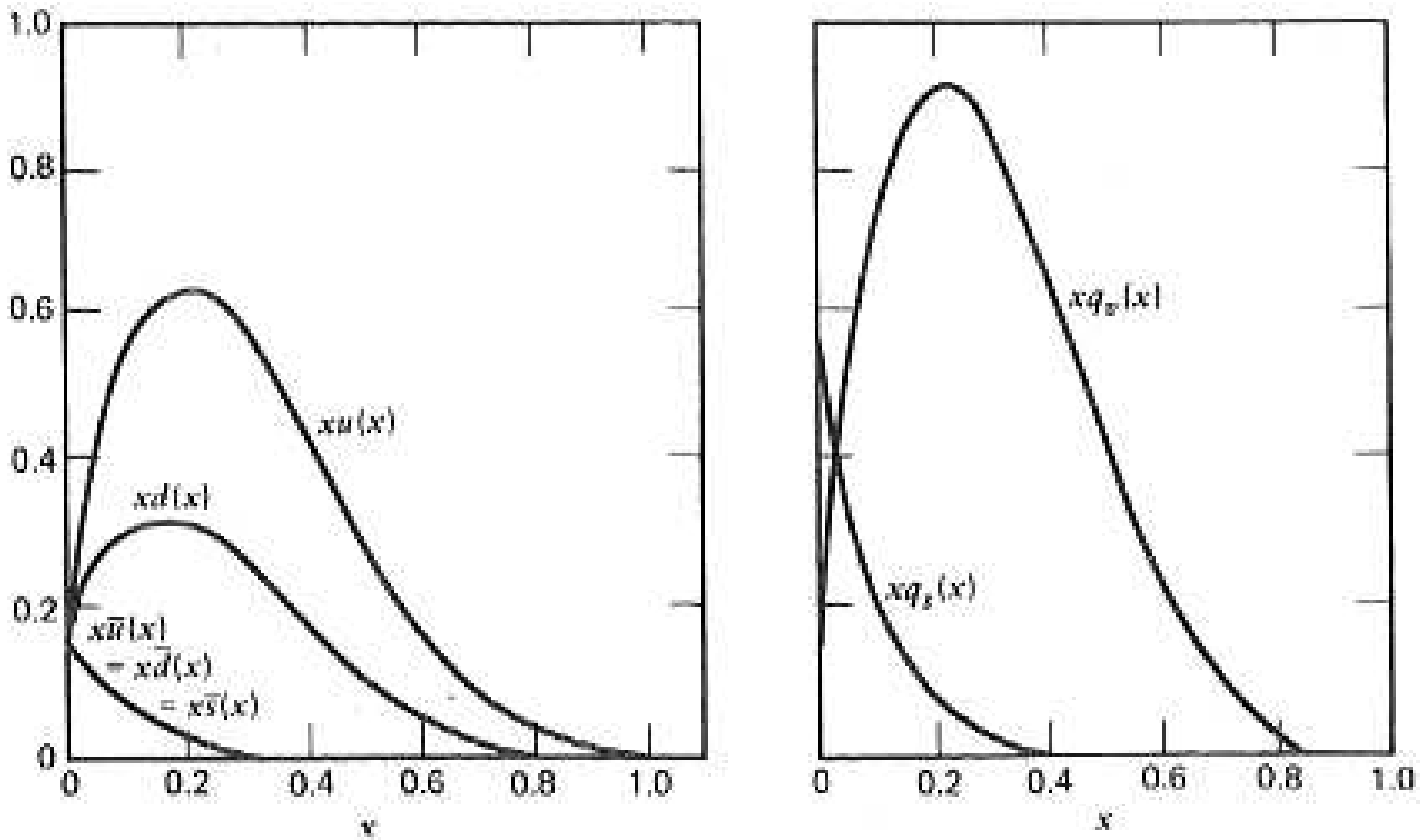
Integrate measured form factors F_2 in electron DIS:

$$\int_0^1 F_2^p(x) dx = \frac{4}{9} f_u + \frac{1}{9} f_d = 0.18 \qquad \int_0^1 F_2^n(x) dx = \frac{4}{9} f_d + \frac{1}{9} f_u = 0.12$$

where $f_u = \int_0^1 x u(x) dx = 0.36$ and $f_d = \int_0^1 x d(x) dx = 0.18$

Valence quarks are only 54% of the proton!

Valence and Sea Quarks in Protons



Valence quarks are u and d at high x

Sea contains quark-antiquark pairs $u\bar{u}, d\bar{d}, s\bar{s}$ at low x

Neutrino (Anti)quark Scattering

Cross-sections for charged current (CC) scattering of ν_μ at the parton level (via virtual W^+ or W^- exchange):

$$\frac{d\sigma}{dy}(\nu_\mu d \to \mu^- u) = \frac{d\sigma}{dy}(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}) = \frac{G_F^2 xs}{\pi}$$

$$\frac{d\sigma}{dy}(\bar{\nu}_\mu u \to \mu^+ d) = \frac{d\sigma}{dy}(\nu_\mu \bar{u} \to \mu^- \bar{d}) = \frac{G_F^2 xs}{\pi}(1-y)^2$$

Virtual $W^\pm$ boson distinguishes between up and down quarks

Neutrinos and antineutrinos distinguish quarks and antiquarks

Neutrino cross-sections are very small $\approx 10^{-42}\mathrm{m}^2$ at $s = 1\mathrm{GeV}^2$

They increase linearly with the CM energy-squared s

Quark and Antiquark Distributions

Neutrino-nucleon scattering can be written in terms of parton density functions for quarks, $Q(x)$, and antiquarks, $\bar{Q}(x)$

$$\frac{d\sigma^{CC}}{dxdy}(\nu N) = \frac{G_F^2 sx}{2\pi}\left[Q(x) + \bar{Q}(x)(1-y)^2\right]$$

$$\frac{d\sigma^{CC}}{dxdy}(\bar{\nu} N) = \frac{G_F^2 sx}{2\pi}\left[\bar{Q}(x) + Q(x)(1-y)^2\right]$$

Ratios of total cross-sections for fermion (antifermion) $f(\bar{f})$:

$$R = \frac{\sigma(\bar{\nu} f)}{\sigma(\nu f)} = \frac{1}{3} \qquad \bar{R} = \frac{\sigma(\bar{\nu}\bar{f})}{\sigma(\nu\bar{f})} = 3$$

Fraction of sea antiquarks is found to be:

$$\frac{\bar{Q}}{Q} = \frac{3R-1}{3-R} \approx 0.1$$

νN Structure Functions

Need three Structure Functions to describe ν-Nucleon DIS:

$$\frac{d\sigma^{CC}}{dxdy}(\nu N) = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu}(x) + y^2 x F_1^{\nu}(x) \pm y\left(1-\frac{y}{2}\right) x F_3^{\nu}\right]$$

The $\pm$ sign refers to ν and $\bar{\nu}$ scattering respectively

The form factor F_2^{ν} in neutrino DIS is given by:

$$F_2^{\nu p}(x) = 2x\left[d(x) + \bar{u}(x)\right] \qquad F_2^{\nu n}(x) = 2x\left[u(x) + \bar{d}(x)\right]$$

For a heavy nucleus N the ratio of ν to e structure functions:

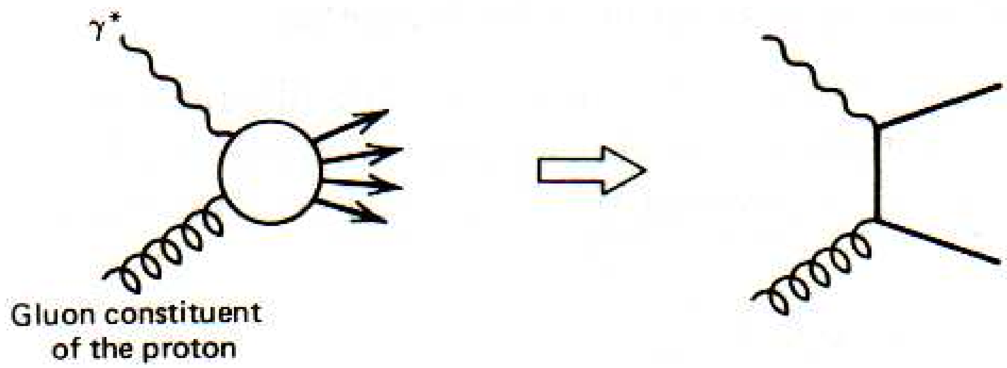
$$F_2^{\nu N} = \frac{18}{5} F_2^{eN} \quad \text{if} \quad Q_u = +2/3e \quad Q_d = -1/3e$$

Total number of valence quarks obtained by integrating F_3:

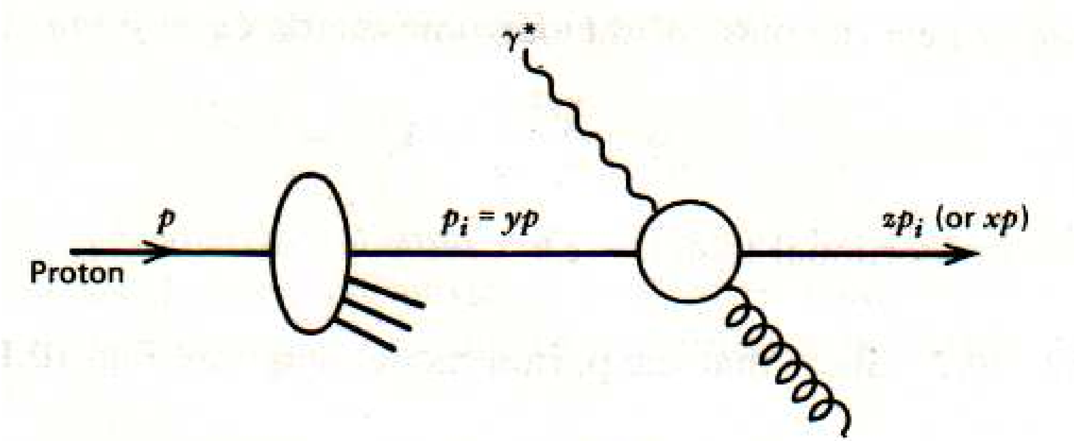
$$\int_0^1 F_3^{\nu N}(x)dx = \int_0^1 \left[u(x) - \bar{u}(x) + d(x) - \bar{d}(x)\right] dx = 3$$

Finding the Gluons

A virtual photon can **hard scatter** off a gluon inside a proton:

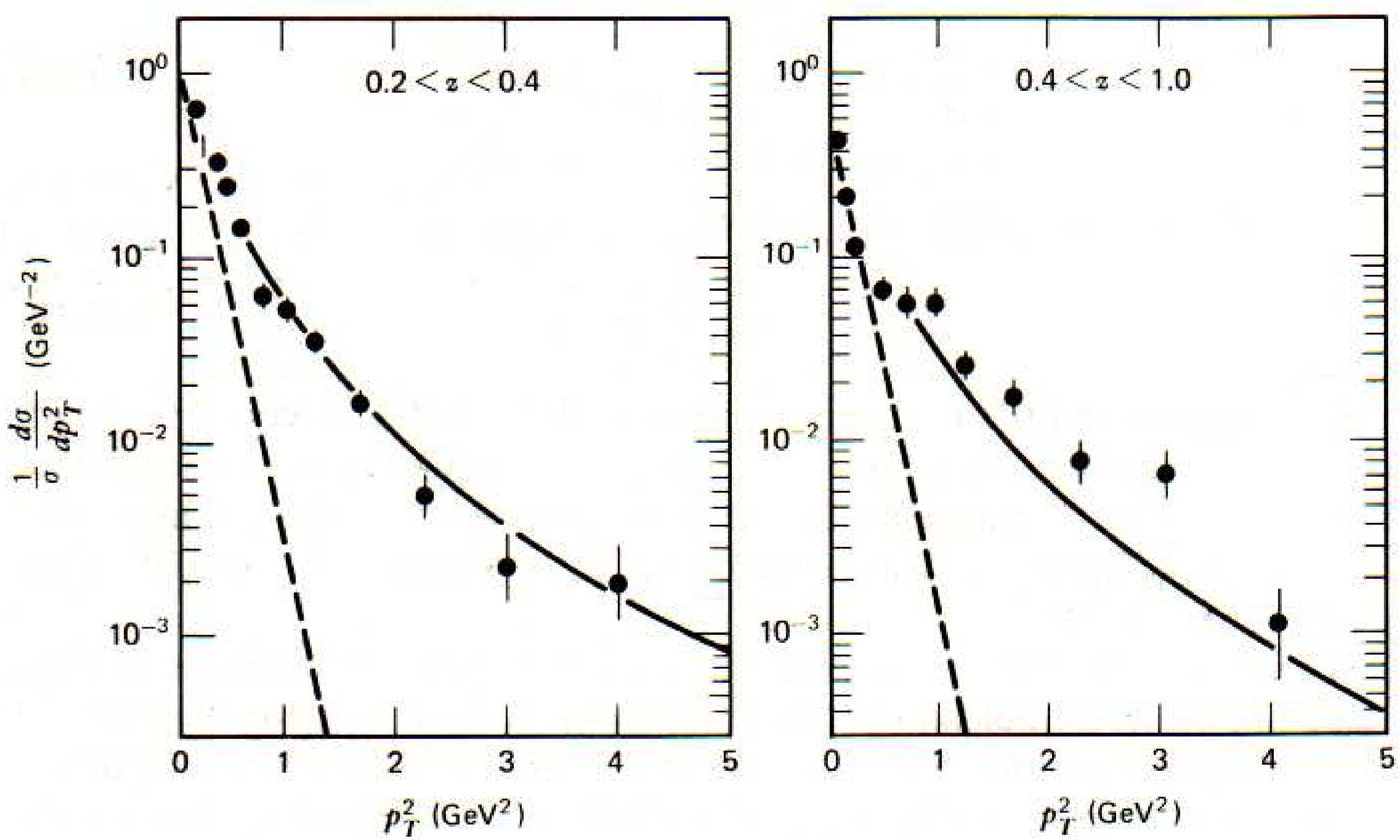


and there is **gluon emission** from a scattered quark:



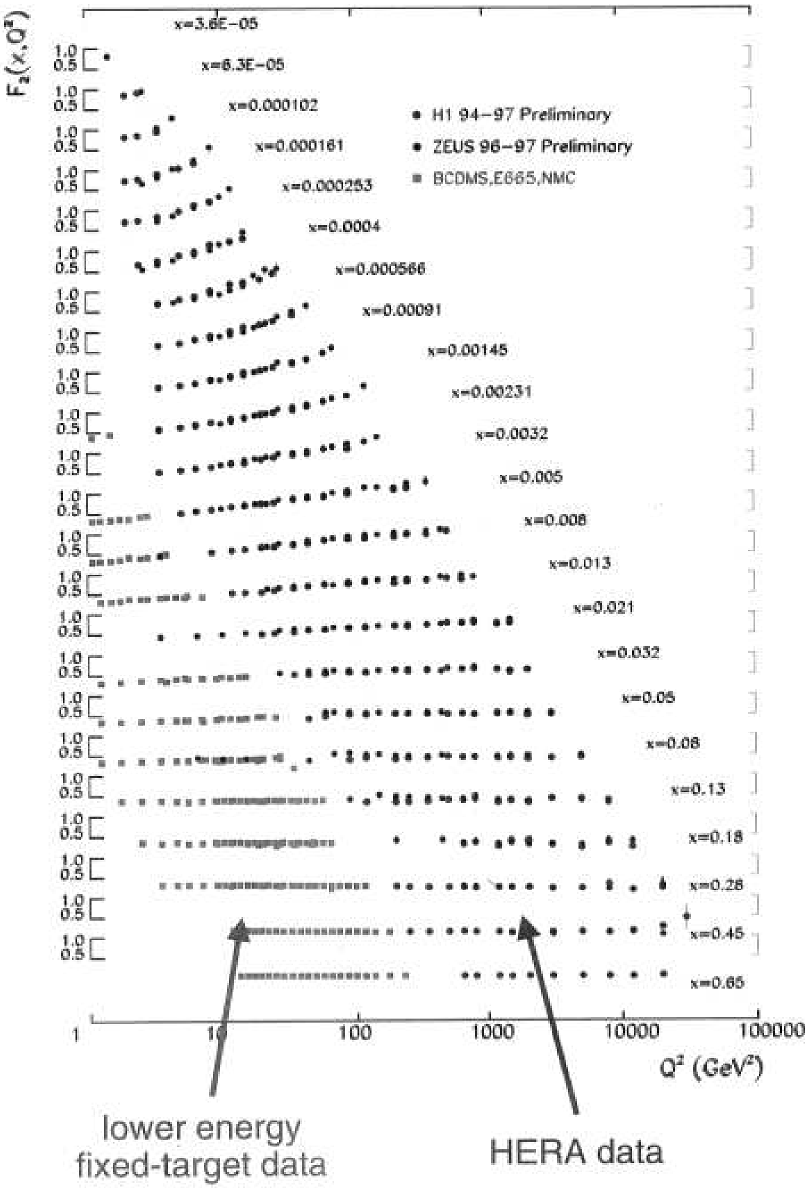
Gluon emission and hard scattering lead to **scaling violation**

Evidence for Gluon Emission



The hadronic jet in DIS has a larger *transverse momentum*, p_T^2, than would be expected from lepton-quark scattering.

$F_2\,(x,q^2)$ from HERA



Bjorken Scaling:

Structure function $F_2(x)$
is independent of Q^2

HERA measured
collisions between
30 GeV electrons and
830 GeV protons

Scaling breaks
down at low x

Structure Functions with Gluons

Gluons change kinematics of lepton-quark scattering

Change in the structure function F_2 due to gluon emission:

$$F_2(x, Q^2) = x\mathrm{Q}_i^2[q(x) + \Delta q(x, Q^2)]$$

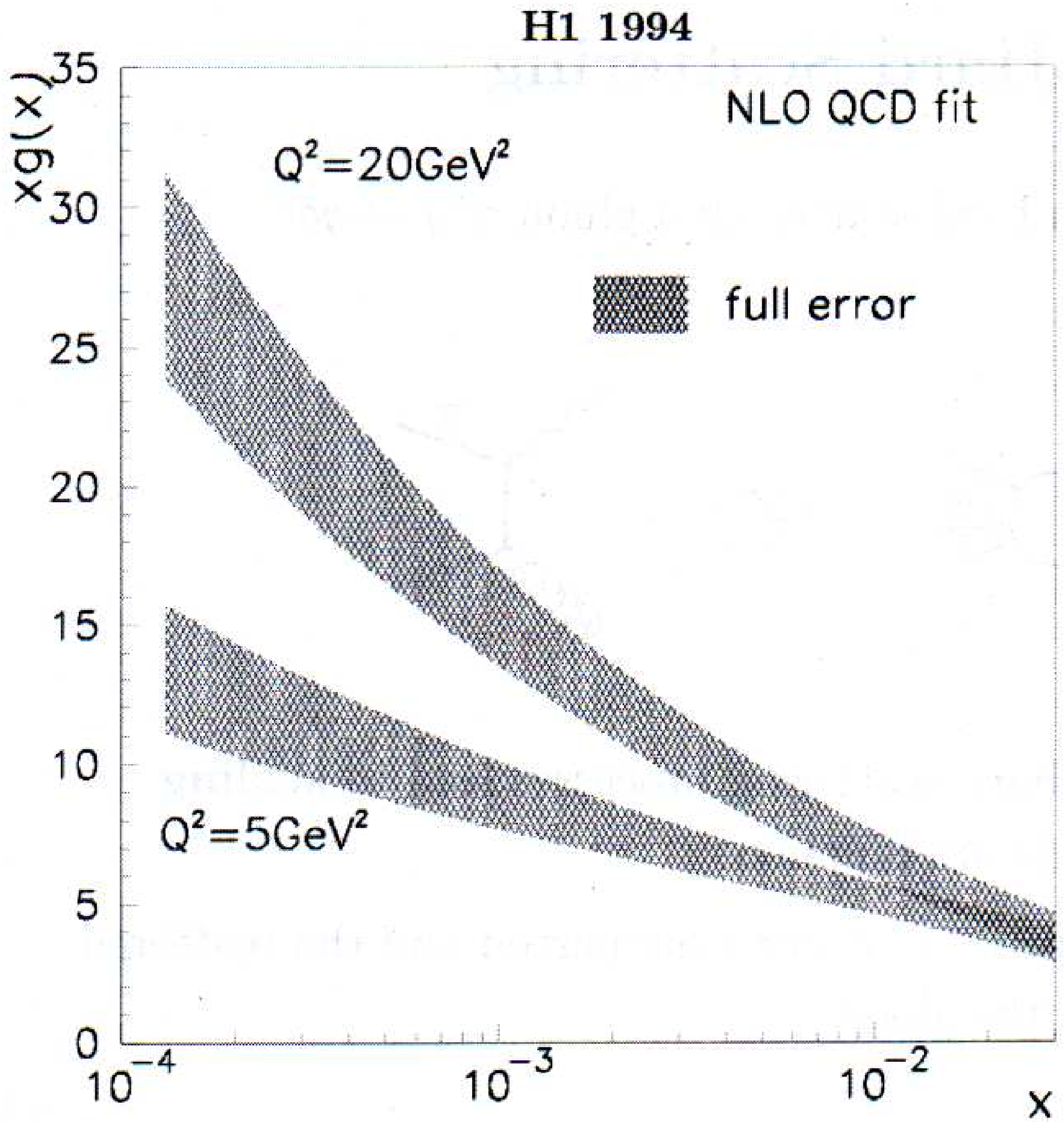
$$\Delta q = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} q(y) P(z) log\left(\frac{Q^2}{\mu^2}\right)$$

P is the probability of a gluon emission that changes the parton energy from y to x, i.e. reduces it by a fraction $z = x/y$

Change in F_2 due to hard scattering:

$$\Delta F_2(\gamma^* g \rightarrow q\bar{q}) = \mathrm{Q}_i^2 \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y) P(z) log\left(\frac{Q^2}{\mu^2}\right)$$

where $g(y)$ is the gluon parton density function


H1 1994
xg(x)
NLO QCD fit
Q²=20GeV²
full error
Q²=5GeV²
35
30
25
20
15
10
5
0
10⁻⁴
10⁻³
10⁻²
x