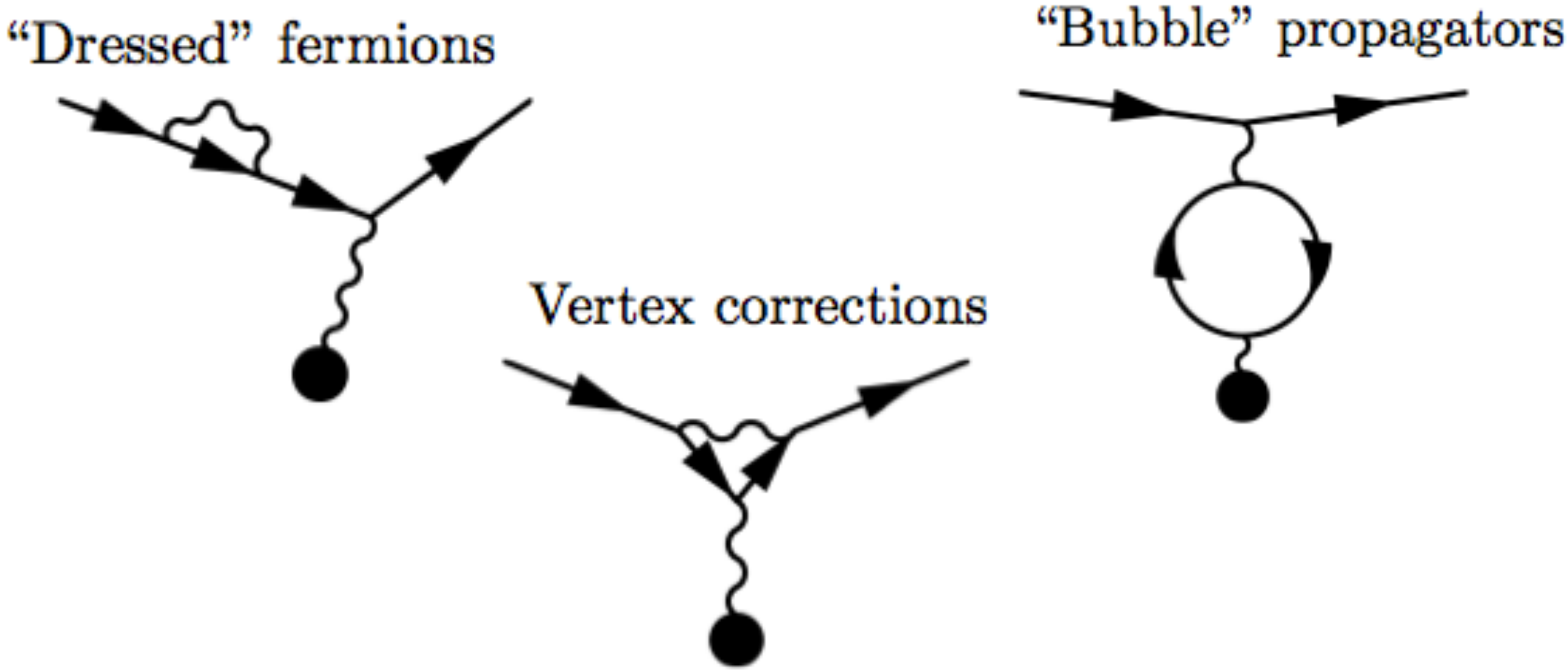
Lecture 4 – Quantum Electrodynamics (QED)



An introduction to the quantum field theory of the electromagnetic interaction

The Feynman Rules for QED

- Incoming (outgoing) fermions have spinors *u ($\bar{u}$) (where $\bar{u} = u^*\gamma^0$)*

- Incoming (outgoing) antifermions have spinors $\bar{v}$ *(v)*

- Incoming (outgoing) photons have polarization vectors ε^μ ($\varepsilon^{\mu *}$)

- Vertices have dimensionless coupling constants $\sqrt{\alpha}$
  At low four-momentum transfers (q^2), $\alpha = e^2/\hbar c = 1/137$

- Virtual photons have propagators $1/q^2$

- Virtual fermions have propagators $(\gamma^\mu q_\mu + m)/(q^2 - m^2)$

Summing Diagrams

- The Matrix Element for a transition is the sum of ***all possible*** Feynman diagrams connecting the initial and final states

- A minus sign is needed to asymmetrise between diagrams that differ by the interchange of two identical fermions

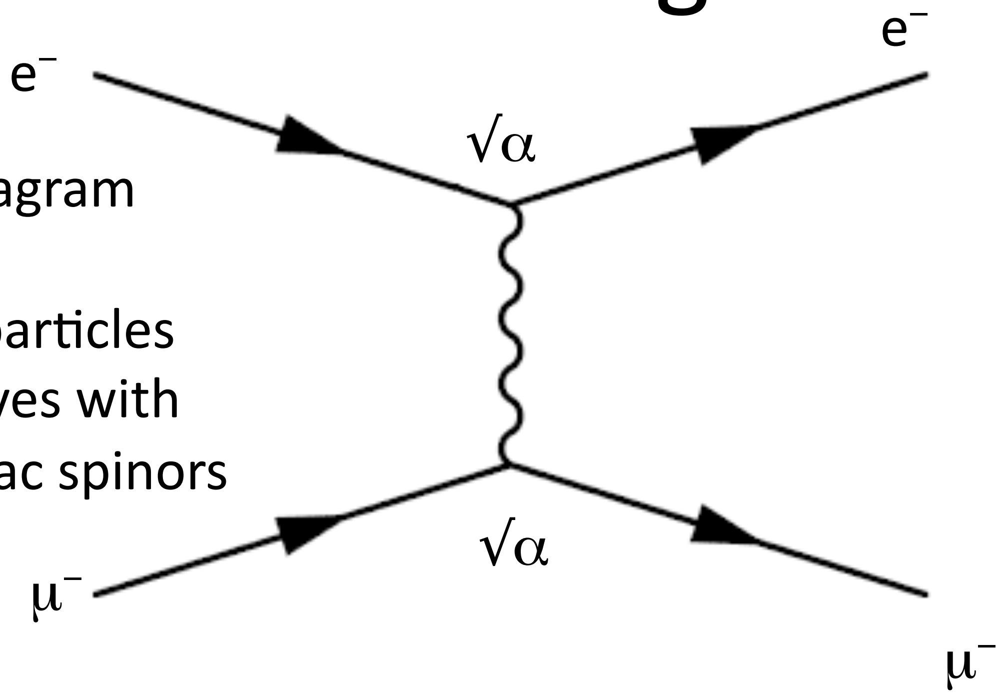
- For QED the sum of higher order diagrams ***converges.*** There are more diagrams with higher numbers of vertices … but for every two vertices you have a factor of α=1/137

- The most precise QED calculations go up to O(α^5) diagrams

Electron Muon Scattering

There is only one lowest order diagram

Start from scattering of spinless particles (Lecture 2) and replace plane waves with electromagnetic currents and Dirac spinors



Vertex Couplings

$$M = \frac{\alpha}{q^2} \left(\bar{u}_3 \gamma^\mu u_1\right)\left(\bar{u}_4 \gamma^\mu u_2\right)$$

Photon Propagator

Electron current

Muon current

For unpolarized cross-section need to average over initial state spins and sum over final states spins

High Energy e-μ scattering

In the limit E>>m it can be shown that fermion helicity is conserved during the scattering process:

$$\bar{u}\,\gamma^\mu\,u \;=\; \bar{u}_L\,\gamma^\mu\,u_L \;+\; \bar{u}_R\,\gamma^\mu\,u_R$$

There are only four possible spin configurations:

$$\mathcal{M}(\uparrow\downarrow\uparrow\downarrow) \quad \mathcal{M}(\downarrow\uparrow\downarrow\uparrow) \quad \mathcal{M}(\uparrow\uparrow\uparrow\uparrow) \quad \mathcal{M}(\downarrow\downarrow\downarrow\downarrow)$$

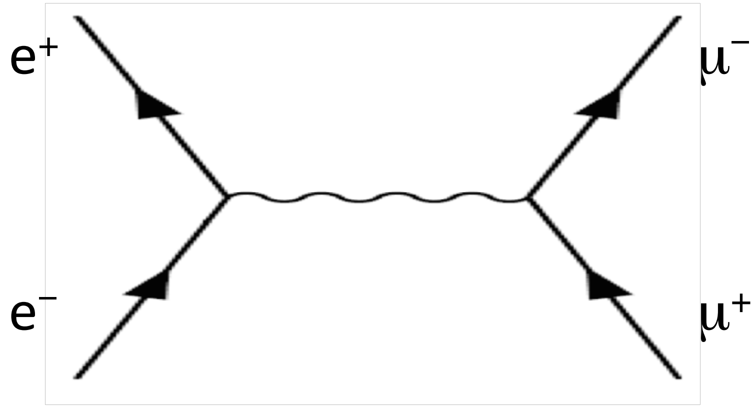
The unpolarized matrix element squared is:

$$|M|^2 = \frac{1}{(2S_1+1)(2S_2+1)}\,\frac{\alpha^2}{q^4}\,\Sigma_{S_3,S_4}\,(\bar{u}_3\,\gamma^\mu\,u_1\,)^*\,(\bar{u}_3\,\gamma^\mu\,u_1\,)\,(\bar{u}_4\,\gamma^\mu\,u_2\,)^*(\bar{u}_4\,\gamma^\mu\,u_2\,)$$

$$|M|^2 \;=\; 2e^4\,\frac{(s^2+u^2)}{t^2} \;=\; 2e^4\,\frac{(1+4\cos^4\theta/2)}{\sin^4\theta/2}$$

Cross-section for $e^+e^- \rightarrow \mu^+\mu^-$

Related to e-μ scattering by *crossing symmetry* t $\leftrightarrow$ s (90^o rotation of Feynman diagram)



$$|M|^2 = 2e^4 \frac{(t^2 + u^2)}{s^2} = e^4 (1+\cos^2\theta)$$

Separated into allowed spin configurations:

$$\mathcal{M}(\uparrow\downarrow\uparrow\downarrow) = \mathcal{M}(\downarrow\uparrow\downarrow\uparrow) = e^2(1+\cos\theta)$$

$$\mathcal{M}(\uparrow\downarrow\downarrow\uparrow) = \mathcal{M}(\downarrow\uparrow\uparrow\downarrow) = e^2(1-\cos\theta)$$

Differential cross-section: $\dfrac{d\sigma}{d\Omega} = \dfrac{e^4}{4s}(1 + \cos^2\theta)$

Total cross-section: $\sigma = \dfrac{4\pi\alpha^2}{3s}$ (decreases with increasing CM energy)

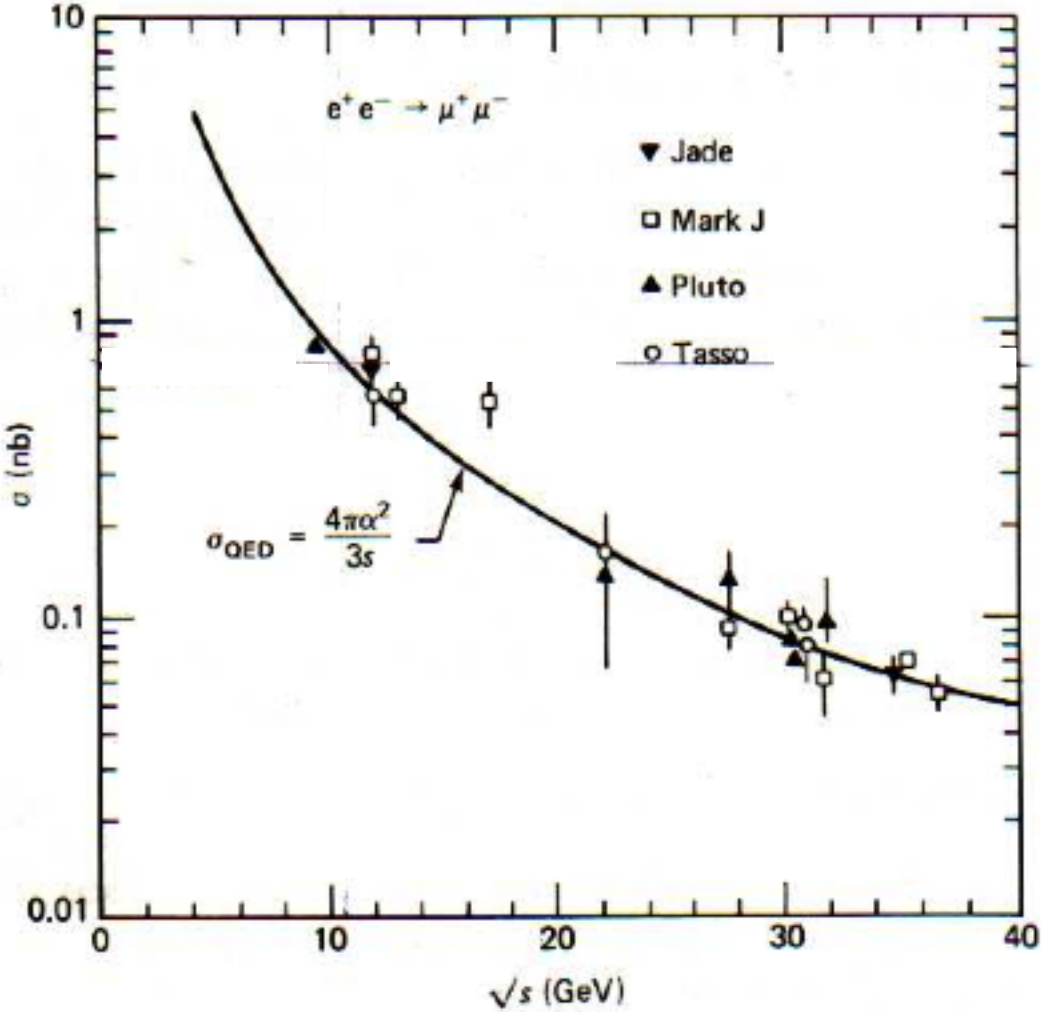
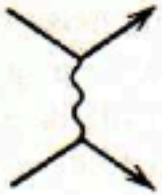
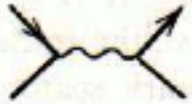


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

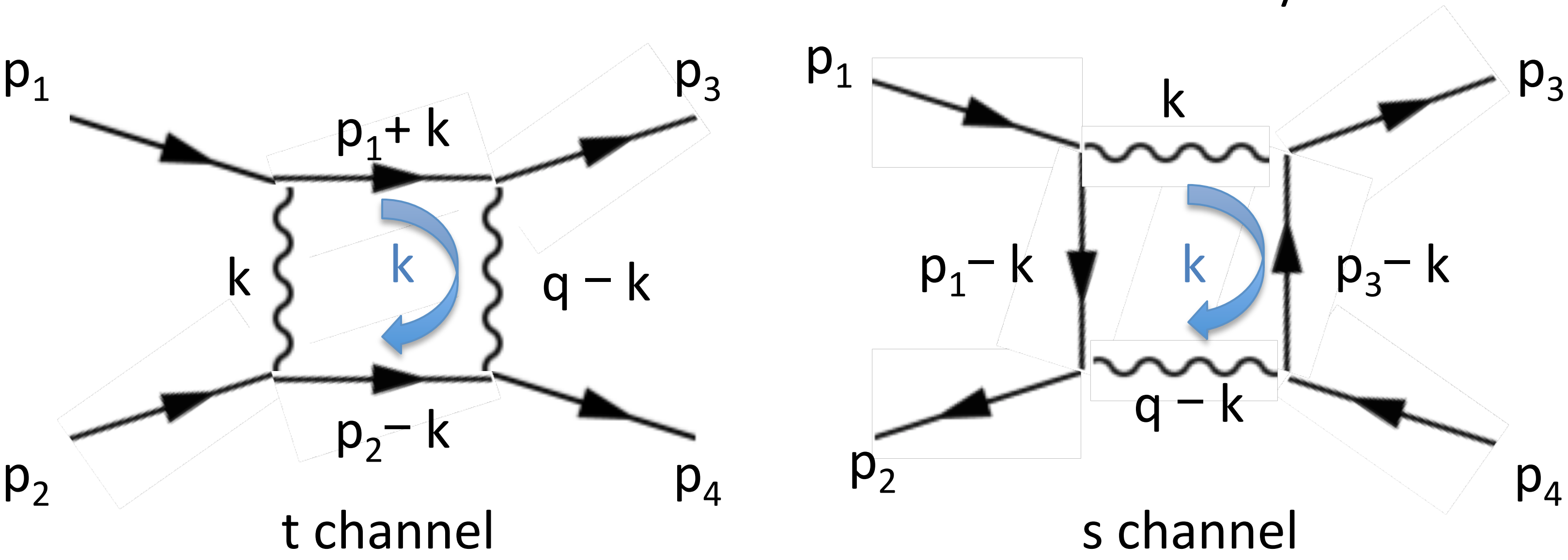
TABLE 6.1
Leading Order Contributions to Representative QED Processes

	Feynman Diagrams		$\overline{\lvert\mathcal{M}\rvert^2}/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^-e^- \to e^-e^-$			$\frac{s^2+u^2}{t^2}$	$+\ \frac{2s^2}{tu}$	$+\ \frac{s^2+t^2}{u^2}$
				($u \leftrightarrow t$ symmetric)	
(Crossing $s \leftrightarrow u$)					
	Forward	"Time-like"	Forward	Interference	Time-like
Bhabha scattering $e^-e^+ \to e^-e^+$			$\frac{s^2+u^2}{t^2}$	$+\ \frac{2u^2}{ts}$	$+\ \frac{u^2+t^2}{s^2}$
$e^-\mu^- \to e^-\mu^-$			$\frac{s^2+u^2}{t^2}$		
(Crossing $s \leftrightarrow t$)					
$e^-e^+ \to \mu^-\mu^+$					$\frac{u^2+t^2}{s^2}$

Higher Order QED Diagrams - I

Two photon exchange diagrams ("box" diagrams)

These add two vertices with a factor of α = 1/137



The four momentum k flowing round the loop can be anything!

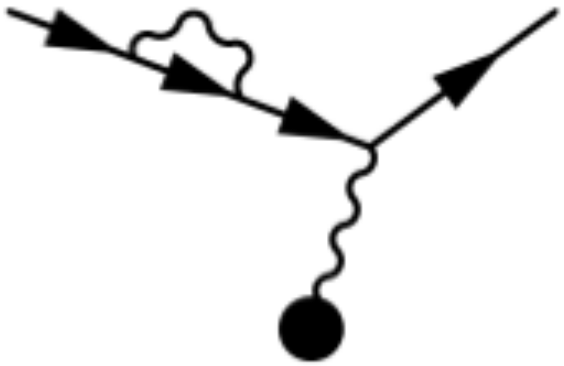
Need to integrate over $\int f(k)\, d^4k$

Unfortunately this integral gives ln(k) which diverges!

This is solved by "renormalisation" in which the infinities are "miraculously swept up into redefinitions of mass and charge" (Aitchison & Hey P.51)

Higher Order QED Diagrams - II

"Dressed" fermions

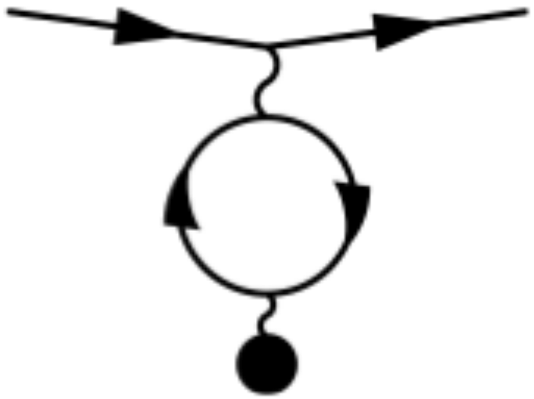


A real (or virtual) fermion
can emit and reabsorb
a virtual photon.
This modifies the
fermion wavefunction.
When the photon is emitted
the fermion becomes virtual
and changes its 4-momentum.

A virtual photon can be
emitted and reabsorbed
across a vertex.
This effectively changes
the coupling at the vertex.

Vertex corrections

"Bubble" propagators



In a virtual photon propagator
a fermion-antifermion pair
can be produced and
then annihilate.
This modifies the
photon propagator.
Any charged fermion is allowed
(quarks and leptons)!

Each of these diagram adds two vertices with a factor of $\alpha = 1/137$

Again four momentum flowing round a loop can be anything!

Gyromagnetic Ratio g

Measures the ratio of the magnetic moment to the spin

$$\vec{\mu} = g\,\mu_B\,\vec{S} \quad \text{for an electron}$$

with the Bohr magneton $\mu_B = e\hbar/m_e c = 5.8\text{x}10^{-11}$ MeV/T

From the Dirac equation expect g=2 for a pointlike fermion

Higher order diagrams lead to an anomalous magnetic moment described by g−2

The QED contributions have been calculated to $O(\alpha^5)$

The theoretical calculation is now limited by strong interactions (corrections from the bubble diagrams with quarks)

Accuracy of g-2 Measurements

- Anomalous magnetic moment of electron

$$Experiment: \quad \left[\frac{g-2}{2}\right]_e = 0.0011596521869(41)$$

$$Theory: \quad \left[\frac{g-2}{2}\right]_e = 0.00115965213(3)$$

- Anomalous magnetic moment of muon

$$Experiment: \quad \left[\frac{g-2}{2}\right]_\mu = 0.0011659160(6)$$

$$Theory: \quad \left[\frac{g-2}{2}\right]_\mu = 0.0011659203(20)$$

Some argument about a 2-3σ discrepancy in the muon g-2 at the moment!

Renormalisation

- Impose a “cutoff” mass M on the four momentum inside a loop
  - This can be interpreted as a limit on the shortest range of the interaction
  - It can also be interpreted as possible substructure in pointlike fermions
  - Physical amplitudes should not depend on choice of M
- Assume $M^2 >> q^2$ (but not infinity!)
- $\ln(M^2)$ terms appear in the amplitude
  - These terms are absorbed into fermion masses and vertex couplings

- The masses $m(q^2)$ and couplings $\alpha(q^2)$ are functions of q^2

- Renormalisation of electric charge:

$$e_R = e \left(1 - \frac{e^2}{12\pi^2} \ln (M^2/m^2) \right)^{1/2}$$

N.B. This formula assumes only one type of fermion/antifermion loop

Can be interpreted as a “screening” correction due to the production of electron/positron pairs in a region round the primary vertex

Running Coupling Constant

Converting from electric charge to fine structure constant and allowing for all fermion types:

$$\alpha(q^2) = \alpha(0)\left(1 + \alpha(0)\,\frac{z_f}{3\pi}\,\ln(-q^2/M^2)\right)$$

where $z_f = \Sigma_f\, Q_f^2$ is a sum over the active fermion/antifermion charges (in units of e)

z_f is a function of q^2 : = 1 (1MeV), = 8/3 (1 GeV), = 38/9 (100GeV)

A trick is to replace the M^2 dependence with a reference value μ:

$$\alpha(q^2) = \alpha(\mu^2)\left(1 - \alpha(\mu^2)\,\frac{z_f}{3\pi}\,\ln(-q^2/\mu^2)\right)^{-1}$$

Can choose any value of μ.

For QED usual choice is μ~1 MeV, α=1/137 (from atomic physics)

An alternative is $\mu = M_Z$, $\alpha(M_Z)$=1/129