Symmetries of Classical Mechanics (SoCM) – Example sheet VIII: 18.11.11

Semester 1 2011-2012

1. S and S' are two frames moving with $\underline{v} = v\underline{e}_x$ = const. with respect to each other.

   (a) A rod at rest in S makes an angle 45° with the x-axis. At what angle does it appear in S'?

   (b) A particle has a velocity of $\underline{u} = (2v, v, 0)$ in S. Which angle does its path make with the x-axes in the S and S' frames?

   (c) A photon leaves the origin of S at time $t = 0$ in a direction that makes an angle of 45° with the x-axis. Which angle does it make in S'?

2. (a) For the following pairs of events

   i. P_1: $x_1 = 1\,\text{m}$, $y_1 = 2\,\text{m}$, $z_1 = 3\,\text{m}$, $t_1 = 3 \times 10^{-8}\,\text{s}$
   P_2: $x_2 = 4\,\text{m}$, $y_2 = 2\,\text{m}$, $z_2 = 7\,\text{m}$, $t_2 = 6 \times 10^{-8}\,\text{s}$ or

   ii. P_1: $x_1 = 7\,\text{m}$, $y_1 = 0\,\text{m}$, $z_1 = -2\,\text{m}$, $t_1 = 1.1 \times 10^{-7}\,\text{s}$
   P_2: $x_2 = 4\,\text{m}$, $y_2 = 5\,\text{m}$, $z_2 = 3\,\text{m}$, $t_2 = 0.9 \times 10^{-7}\,\text{s}$

   is there a causual connection?

   (b) Is it possible to find an inertial system in which these events are simultaneous? With which speed and in which direction would this inertial system move?

3. A spaceship is launched from earth and when it reaches velocity v with respect to the earth, it launches a second spaceship, which accelerates until it reaches velocity v with respect to the first spaceship, and so on, all spaceships moving in the same direction. By using the rapidity variable, show that the velocity of the n^{th} space-ship relative to the earth is given by

$$v_n = c \tanh(n \tanh^{-1} \beta)\,,$$

   where $\beta = v/c$.
   Given that $\beta = 0.01$, estimate the value of n required to reach a velocity of $v_n = 0.99c$.

4. A rocket of length L_0 flies with constant velocity v (in frame S' relative to a frame S in the x-direction. At time $t = 0 = t'$, the capsule on top of the rocket passes the point P_0 in S. At this moment a light signal is sent from the top of the rocket to the bottom.

   (a) In the rest frame of the rocket, how long does it take the light signal to reach the end of the rocket?

   (b) In the rest frame of the observer, S, at which time does the signal reach the end of the rocket?

   (c) At what time the observer see the end of the rocket passing P_0?

5. In a given frame S a particle moves on the x-axis with variable velocity $v = dx/dt$. The 'proper' time τ (as measured by an 'ideal' clock moving with the particle) is defined by the condition that it varies at any instant at the same rate as that of the instanteously coinciding clock fixed in an inertial frame S' relative to which the particle is at rest at that instant (ie $d\tau/dt' = 1$ at the instant when $dx'/dt' = 0$). Show that

$$\frac{d\tau}{dt} = \frac{1}{\gamma(v)}\,.$$

The 'proper' acceleration a' at any instant is defined as the value of d^2x'/dt'^2 in the instantaneous rest frame S'. Show that

$$a' = \frac{d}{dt}\left[\gamma(v)v\right] .$$

If the particle starts from rest at the origin of S and moves with *constant* acceleration a' show that

$$\frac{v}{c} = \frac{a't}{c}\left(1+\frac{a't^2}{c^2}\right)^{-\frac{1}{2}} = \tanh\frac{a'\tau}{c} ,$$

and hence that

$$x = \frac{c^2}{a'}\left[\left(1+\frac{a't^2}{c^2}\right)^{\frac{1}{2}}+1\right] = \frac{c^2}{a'}\left[\cosh\left(\frac{a'\tau}{c}\right)-1\right] .$$

6. In order to recreate terrestial living conditions, a space traveller adjusts the propr acceleration of his spaceship so that its magnitude is equal to the gravitational acceleration g on Earth. He sets off from Earth on a return trip following a straight path. In terms of his own proper time his itinerary consists of 4 stages:

   (a) four years of acceleration directly away from the earth

   (b) four years of acceleration in the opposite direction (after which he comes to rest relative to the earth – at a point of maximum distance)

   (c) four years of acceleration back to the earth

   (d) four years of acceleration in the original direction, so that he finally comes to rest on the earth again

   Show that

   - to sufficient accuracy (for this question) using units where time is measured in years and distances in light-years then $g = 1$
   - Using the results from the constant acceleration problem, show that the maximum distance from the earth is about 50 light years and that the earth will have aged about 1 century, during what was in his own proper time a 16 year absence