Symmetries of Classical Mechanics

[PHYS10016]

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- Lecture: Tuesday 11:00–12:00 LTC
- Lecture: Friday 11:00–12:00 LTC
- Workshop: Friday 16:00–18:00 5327

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Contents

Syllabus

The course provides an introduction to rotational and space–time symmetries in classical physics using the concept of tensors.

- Vectors, bases, matrices, determinants, Einstein summation convention, the delta and epsilon symbols.
- Rotations of bases, composition of two rotations, reflections, projection operators, passive and active transformations, the $SO(3)$ symmetry group.
- Cartesian tensors:
  - definition/transformation properties and rank
  - quotient theorem, pseudotensors, the delta and epsilon symbols as tensors
  - isotropic (pseudo)tensors
- Taylor's theorem: the one- and three-dimensional cases, the multipole expansion.
- Examples of tensors I:
  - conductivity tensor
  - moment of inertia tensor
    * diagonalisation of rank-2 tensors (ie eigenvalues/eigenvectors)
    * principal axes
- Examples of tensors II: Continuum Mechanics
  - the strain tensor, stretching and shear
  - the stress tensor, and some properties
  - elastic deformations of solid bodies, generalised Hooke's Law, isotropic media (and the various parameterisations for constants, ie Lamé constants; Young's modulus and Poisson's ratio; bulk and shear modulus)
  - fluid mechanics: the Navier–Stokes equation
- Examples of tensors III: Special Relativity

- Physical basis of Special Relativity
  - inertial systems, Galilean transformations, Michelson-Morley experiment
  - Einstein's postulates, Lorentz transformations
  - time dilation and Fitzgerald contraction, addition theorems for frames and particle velocities, rapidity
  - Doppler effect and aberration
  - Minkowski diagrams
- Non-orthogonal co-ordinates, covariant and contravariant tensors
- Covariant formulation of classical mechanics, position, velocity, momentum and force 4 vectors, particle collisions
- Relativistic formulation of electromagnetism from the Lorentz force
  - Maxwell tensor
  - covariant formulation of Maxwell's equations, Lorentz transformation of the electric and magnetic fields, invariants

(Mathematical or Theoretical) Physics:
Make a model to explain facts already known and make new predicitions. The tools/language needed are provided by (non-rigorous) mathematical methods. The course follows on from MfP3/MfP4 *a knowledge of which will be assumed.*

Books

Any Mathematical Methods book that you are comfortable with.

- K. F. Riley, M. P. Hobson and S. J. Bence,
  Mathematical Methods for Physics and Engineering, (CUP 1998).
- P. C. Matthews,
  Vector Calculus, (Springer 1998).
- M. L. Boas,
  Mathematical Methods in the Physical Sciences, (Wiley 2006).
- G. B. Arfken and H. J. Weber,
  Mathematical Methods for Physicists, (Academic Press 2001).
- D. E. Bourne and P. C. Kendall,
  Vector Analysis and Cartesian Tensors, (Chapman and Hall 1993).
- B. Lautrup,
  Physics of continuous matter : exotic and everyday phenomena in the macroscopic world, (CRC Press 2011).
- W. Rindler,
  Introduction to Special Relativity, (Oxford 1989).
- W. D. McComb,
  Dynamics and Relativity, (Oxford 1999).
- Often books on electromagnetism or general relativity have a chapter on special relativity, eg

  J. D. Jackson,
  Classical electrodynamics, Wiley (1975) [The classic book]