Identifying Conformal Gauge Theories (CGT)

Roman Zwicky (Southampton)

30 Nov/1 2010, Marseille/Lyon

Overview

- BSM, Technicolor, Walking-TC $\Rightarrow$ study strongly coupled gauge theories (3)
- General remarks gauge theories - conformal window SUSY & non-SUSY (4)
- conformal gauge theories (CGT) -- observables? (1)
- observables in mass-deformed CGT (8)
  - hyperscaling laws from RG
  - mass scaling from Feynmann-Hellmann thm
  - another look at β-function from trace anomaly
  - trajectory mass & decay constants
  - remarks on S-parameter

Del Debbio & RZ
PRD'10 & arXiv:1009.2894

- Lattice results (3)

$$\Delta_{\bar{q}q} = 3 - \gamma_*$$
where γ_* mass anomalous dimension

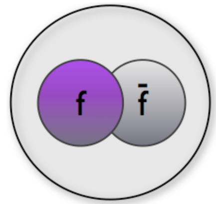
- Epilogue

Beyond the SM

centered around the Higgs mechanism of SSB
⇒ W,Z masses; technical hierarchy problem?*

Is the Higgs (object that unitarizes $W_L W_L$-scattering)
fundamental or composite?

fundamental particle small width	composite particle large width

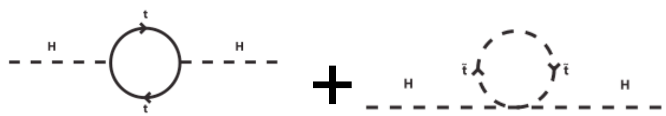


strong dynamics

Supersymmetry
opposite statistics partner

prototype

Technicolour
Higgs sector ⇒ Gauge theory

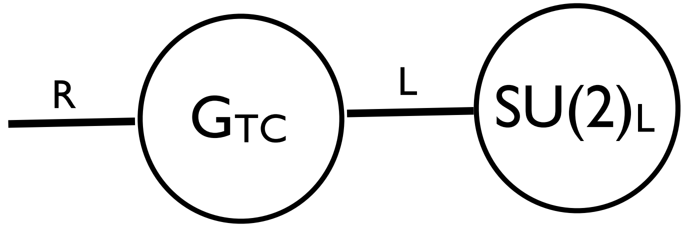


$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_{TC}$

* Flavour sector, where real hierarchies are present, harder for model building

Technicolor

Susskind'79 Weinberg'79



moose notation

- Higgs sector →strongly coupled gauge theory

- χ-symmetry breaking: $\boxed{\langle \bar{Q}_L Q_R \rangle \sim N_{TC} \Lambda^3_{TC}}$

$\Lambda_{TC} \sim 4\pi F_T$

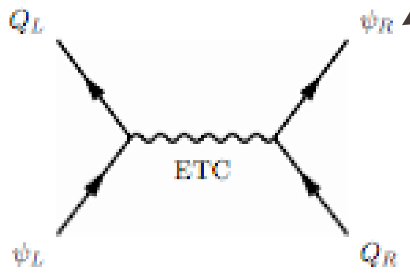
$F_T = v$

masses W,Z bosons through SSB (as in SM!)

- Fermion masses -> Extended TC $G_{\rm SM} \times G_{TC} \subset G_{ETC}$

Dimopoulus Susskind'79 Eichten Lane'80

SM



$$breaking:\ G_{ETC} \to G_{SM} \times G_{TC}$$

$$\mathcal{L}^{\rm eff} = \alpha_{ab} \frac{\bar{Q} T^a Q \bar{\psi} T^b \psi}{\Lambda^2_{ETC}} + \beta_{ab} \frac{\bar{Q} T^a Q \bar{Q} T^b Q}{\Lambda^2_{ETC}} + \gamma_{ab} \frac{\bar{\psi} T^a \psi \bar{\psi} T^b \psi}{\Lambda^2_{ETC}} + \dots$$

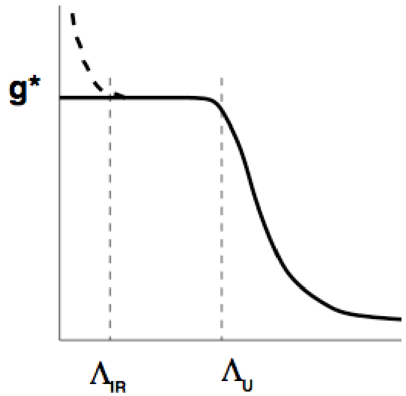
SM fermion masses *FCNC*

* QCD breaks SU(2)L spontaneously and gives mass to W boson (orders of mag. too small)

Pheno improvement via Walking TC

- Issues: 1. electroweak precision parameter S ~'95 Lep
  2. dynamical generation of fermion masses and FCNC Extended-TC

- 'Improvement' Walking-TC: almost reaches IR fixed-point



$$\boxed{1.\ |S_{WTC}| << |S_{TC}| \quad 2.\ \gamma_{mass}^{*} \text{ large}}$$

- Walking results enhancement of
  No parametric definition (challenge)

$$\frac{\langle \bar{Q}Q \rangle_{\mathrm{TC}}}{f^3_{\pi(\mathrm{TC})}}$$

⇒ *need to know more about strongly coupled near-conformal gauge theories ..*

Gauge theories

- what theorists can adjust:

N_c

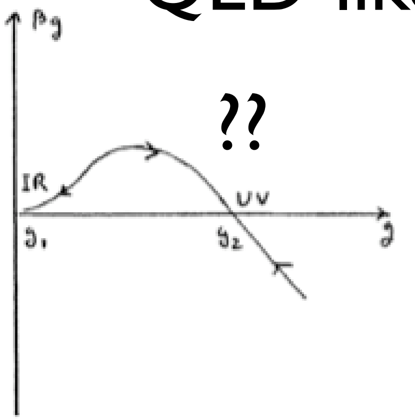
N_F

representation

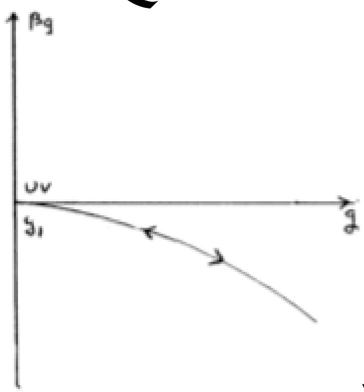
- one coupling theory:
  $g^* = 0$ either IR (QED-like) or UV (QCD-like asymptotic freedom) fixed-point

- focus AF-theories ($-\beta_0 < 0$) we know how to handle

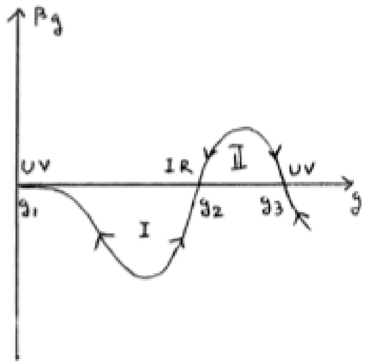
QED-like

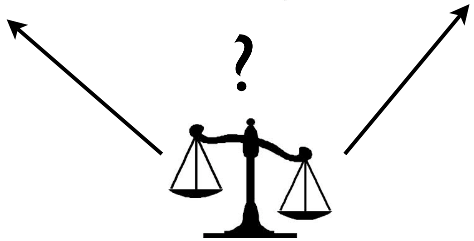


QCD-like



IR-conformal





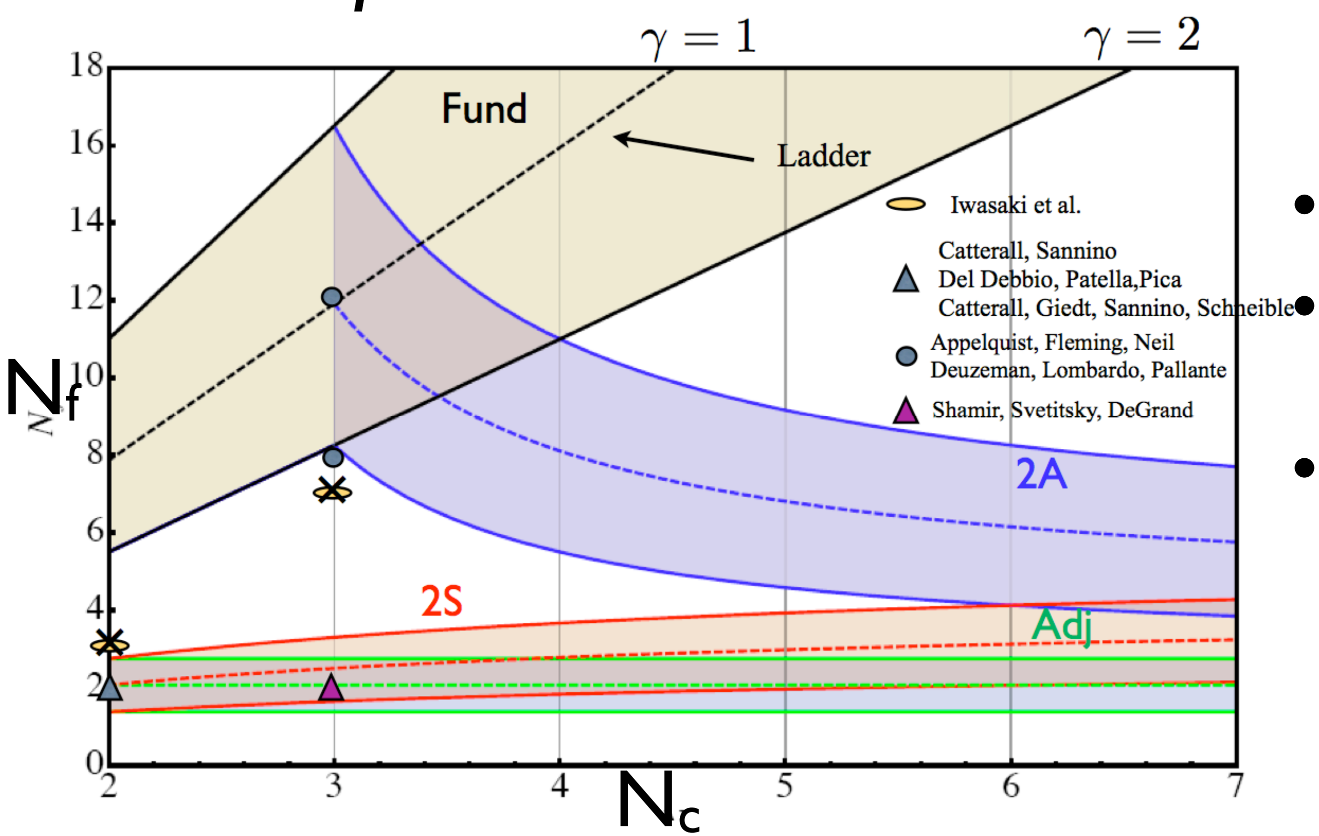
Facts non-SUSY Conformal Window

proof of principle

- ⭐ QCD chiral symmetry is broken (empirical) ⇒ not in CW!
- ⭐ Banks-Zaks'82 (Belavin-Migdal'76) perturbative IR fixed point (conformal)

If β_0 tuned small $\frac{\alpha_s^*}{2\pi} = \frac{\beta_0}{-\beta_1} \ll 1$ $\qquad \beta(\alpha_s) = -\beta_0 \frac{\alpha_S^2}{2\pi} - \beta_1 \frac{\alpha_S^3}{(2\pi)^2} + \ldots$

$\alpha_s^* \sim 0.02$



- upper line AF (ok)
- dashed Dyson-Schwinger $\Delta_{\bar{q}q} = 3 - \gamma \simeq 2$
- lower unitary bound $\Delta_{\bar{q}q} \geq 1$ via conjectured β-fct

SUSY Conformal Window

☆ Exact NSVZ'83 β-fct:

$$\beta(g) = -\frac{1}{16\pi^2}\frac{3t_2(A) - \sum_i t_2(i)(1-\gamma_i)}{1 - t_2(A)g^2/8\pi^2}.$$

from $\beta = 0$ get γ^*

1. Unitarity bound on squark-bound state $\Delta_{QQ} = 2 - \gamma^* \geq 1 \Rightarrow \gamma^* \leq 1$

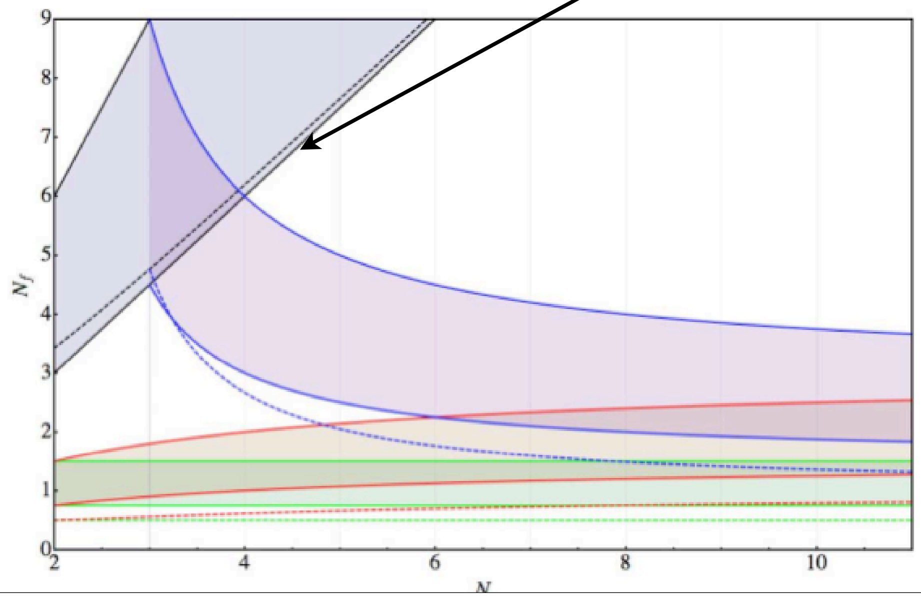
lower bdry
unitarity bound

2. Electric magnetic duality $N^{dual} = N_F - N$

perturbative electric BZ-fixed point upper boundary (like QCD)

perturbative magnetic BZ-fixed point lower boundary !!

⇒ weak-strong coupling duality ⇒ exist strongly coupled CGT (also from γ^*)



Two objectives (almost repetition)

- AF gauge theories ≈ 1/2 non-CFT + 1/2 CFT (SUSY)

  (N.B. only known CFT in 4D are GT, coheres with Coleman-Gross Thm)

size of conformal window?

- strong coupling -- value of γ_*

strongly coupled? size γ_*

- SUSY $\mathcal{N} = 1$ tells $\Delta_{\bar{q}q} = 3 - \gamma_* \geq 2$
- Dyson Schwinger eqn: chiral symmetry breaks $\Delta_{\bar{q}q} \simeq 2$
- unitarity bound (Mack'77) $\Delta_{\bar{q}q} \geq 1$

Is the unitarity bound ever reached?

1. SUSY its because of the squark $\Delta_{QQ} = 2 - \gamma$
2. DS-eqs. truncation -- ladder approximation ... NJL
3. N.B. Δ=1 free field (very strong force ....)

$\Rightarrow$ *we want answers* $\Rightarrow$ *lattice simulations*

Observables in a CFT?

Or how to identify a CFT

1. Observables: vanishing β-function & $\langle O(x)O(0)\rangle \sim (x^2)^{-\Delta}$;$\Delta = d + \gamma_*$
2. Lattice computation finite m_{quark} (& volume anyway)

$\Rightarrow$ look mass-deformed conformal gauge theories (mCGT)*

$$\mathcal{L} = \mathcal{L}_{\mathrm{CGT}} - m\bar{q}q$$

* hardly related to 2D CFT mass deformation a part of algebra and 'therefore' integrability is maintained

Obersvables in mCGT

- Goal: analytic guidance for lattice (parametric laws)

- If strongly coupled ⇒ hadronic spectrum ⇒beloved hadronic observables

signature of such a theory: each hadronic observable

$$\mathcal{O} \sim m^{\eta_{\mathcal{O}}} \,, \qquad \eta_{\mathcal{O}} > 0 \,, \eta = f(\gamma_*)$$

- Let's settle some notation:

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} \,, \quad \text{scaling} = \text{physical} + \text{anomalous dimension}$$

$$\gamma_m = -\gamma_{\bar{q}q} \,, \quad \text{denoted by } \gamma_* \text{ at fixed-point}$$

$$\Rightarrow \Delta_{\bar{q}q} = 3 - \gamma_*$$

Hyperscaling laws

physical states
no anomalous dim.

Consider matrix element: $\mathcal{O}_{12}(g,\hat{m},\mu) \equiv \langle \varphi_2|\mathcal{O}|\varphi_1\rangle$

RG -transformation*
$\mu = b\mu'$

1. $$\mathcal{O}_{12}(g,\hat{m},\mu) = b^{-\gamma_{\mathcal{O}}}\mathcal{O}_{12}(g',\hat{m}',\mu')\ ,$$

$$g' = b^{y_g} g \quad \hat{m}' = b^{y_m}\hat{m}\ , \quad y_m = 1+\gamma_*\ , \quad y_g < 0 \text{ (irrelevant)}$$

change
physical units

2. $$\mathcal{O}_{12}(\hat{m}',\mu') = b^{-(d_{\mathcal{O}}+d_{\varphi_1}+d_{\varphi_2})}\mathcal{O}_{12}(\hat{m}',\mu)$$

3. Choose b s.t. $\hat{m}' = 1$

Hyperscaling
relations

$$\Rightarrow \quad \boxed{\mathcal{O}_{12}(\hat{m},\mu) \sim (\hat{m})^{(\Delta_{\mathcal{O}}+d_{\varphi_1}+d_{\varphi_2})/y_m}}$$

* From Weinberg-like RNG eqs on correlation functions (widely used in critical phenomena)

Applications: $\eta_{\mathcal{O}_{12}} = (\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m$

- vacuum condensates: $\langle \bar{q}q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$, $\langle G^2 \rangle \sim m^{\frac{4}{1+\gamma_*}}$ more later on...

- decay constants:
  $|\varphi\rangle = |H(\text{adronic})\rangle$

  N.B. ($\Delta_H = d_H = -1$ choice)

$\mathcal{O}$	def	$\langle 0\vert\mathcal{O}\vert J^{P(C)}(p)\rangle$	$J^{P(C)}$	$\Delta_{\mathcal{O}}$	$\eta_{G[F]}$
S	$\bar{q}q$	G_S	0^{++}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
S^a	$\bar{q}\lambda^a q$	G_{S^a}	0^+	$3-\gamma_*$	$(2-\gamma_*)/y_m$
P^a	$\bar{q}i\gamma_5 q$	G_{P^a}	0^-	$3-\gamma_*$	$(2-\gamma_*)/y_m$
V	$\bar{q}\gamma_\mu q$	$\epsilon_\mu(p)M_V F_V$	1^{--}	3	$1/y_m$
V^a	$\bar{q}\gamma_\mu\lambda^a q$	$\epsilon_\mu(p)M_V F_{V^a}$	1^-	3	$1/y_m$
A^a	$\bar{q}\gamma_\mu\gamma_5\lambda^a q$	$\epsilon_\mu(p)M_A F_{A^a}$	1^+	3	$1/y_m$
		$ip_\mu F_{P^a}$	0^-	3	$1/y_m$

- masses from **trace anomaly**: $\theta_\alpha{}^\alpha|^{q\neq 0}_{\text{on-shell}} = \frac{1}{2}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$

  Adler et al, Collins et al
  N.Nielsen '77 Fujikawa '81

  $\beta = 0 \quad \& \quad \langle H(p)|H(k)\rangle = 2E_p\delta^{(3)}(p-k) \Rightarrow$

$$\boxed{2M_h^2 = N_f(1+\gamma_*)m\langle H|\bar{q}q|H\rangle} \quad \sim m^{\frac{2}{(1+\gamma_*)}}$$

relation reminiscent GMOR-relation

...

- Summarizing:
  scaling laws for <u>entire</u> spectrum, decay constants & condensates
  No SSB of χ-symmetry breaking (no goldstone boson)
  since condensate triggered by explicit χ-breaking

There is no chiral perturbation theory

- Credits (presentation focused last paper):
  lowest mass state Miransky '98
  quark condensate (just stated) DeGrand'09
  all lowest state results DelDebbio RZ'10 May (large time euclidian correlators)
  all state results DelDebbio RZ'10 Sep

- A point that can be clarified:
  $M_H \sim m^{1/(1+\gamma^*)}$ looks a bit like heavy quark physics
  The definite signature is $f_{P(\text{B-meson})} \sim m^{-1/2}$ whereas $f_{P(\text{mCGT})} \sim m^{(2-\gamma^*)/(1+\gamma^*)}$

Mass scaling without RG

Del Debbio, RZ Sep'10

Hellmann-Feynman-Thm

$$\boxed{\frac{\partial E_\lambda}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial \hat{H}(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle}$$

idea: $\frac{\partial \langle \psi(\lambda) | \psi(\lambda) \rangle}{\partial \lambda} = 0$

- applied to our case: $$m \frac{\partial M_h^2}{\partial m} = N_f m \langle H | \bar{q} q | H \rangle$$

- combined with GMOR-like .. $$m \frac{\partial M_H}{\partial m} = \frac{1}{1+\gamma_*} M_H$$

  $2M_H^2 = N_f(1+\gamma_*) m \langle H | \bar{q} q | H \rangle$

$$\boxed{M_H \sim m^{\frac{1}{1+\gamma_*}}}$$

scaling law
without using RG!

Generalized Banks-Casher relation

- Banks & Casher '80 a la Leutwyler & Smilga 92':

Green's function: $\langle q(x)\bar{q}(y)\rangle = \sum_n \frac{u_n(x)u_n^\dagger(y)}{m-i\lambda_n}$, where $\not{D}u_n = \lambda_n u_n$

$$\langle \bar{q}q\rangle_V = \frac{1}{V}\int dx\,\langle \bar{q}(x)q(x)\rangle = -\frac{2m}{V}\sum_{\lambda_n>0}\frac{1}{m^2+\lambda_n^2} \overset{V\to\infty}{=} -2m\int_0^\infty d\lambda \frac{\rho(\lambda)}{m^2+\lambda^2}$$

- UV-divergences later -- focus IR-physics

$$\langle \bar{q}q\rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}}$$

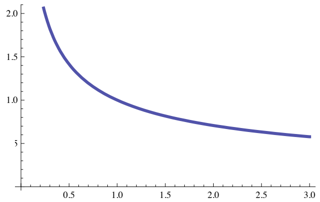
- QCD : $\eta_{\bar{q}q} = 0 \Rightarrow \rho(0) = -\pi\langle \bar{q}q\rangle$
  mCGT: another way to measure anomalous dimension

Banks, Casher'80

DeGrand'09
DelDebbio RZ'10 May

Heuristic look

- Deconstruct the continuous spectrum of a two point function Stephanov'07
  Infinite sum of adjusted particles can mimick continuous spectrum



$$\bar{q}q(x) \sim \sum_n f_n \varphi_n(x)\,; \qquad \langle \varphi_n | \bar{q}q | 0 \rangle \sim f_n\,, \qquad \begin{cases} f_n^2 = \delta^2\,(M_n^2)^{\Delta_{\bar{q}q}-2} \\ M_n^2 = n\delta^2 \end{cases}$$

- Adding mass term looks like tadpole.
  ⇒ find new minimum -- add M_n to potential

$$\mathcal{L} = -m \sum_n f_n \varphi_n - 1/2 \sum_n M_n^2 \varphi_n^2$$

Delgado, Espinoso, Quiros'07

- Solve $\quad m f_n + M_n^2 \varphi_n = 0 \quad \Rightarrow \quad \langle \varphi_n \rangle = -m f_n / M_n^2$ and reinsert:

$$\langle \bar{q}q \rangle \sim \sum_n f_n \langle \varphi_n \rangle = -m \sum_n \frac{f_n^2}{M_n^2} \overset{\delta \to 0}{\to} -m \int_{\Lambda_{\rm IR}^2}^{\Lambda_{\rm UV}^2} s^{\Delta_{\bar{q}q}-3} ds$$

  - Λ_{UV}: $\Delta_{qq} = 3$ find quadratic divergence known from Leutwyler-Smilga rep.
  - Λ_{IR}: 1) $\Lambda_{IR} \sim M_H \sim m^{1/(1+\gamma)}$ or use $(M_{dyn})^{\Delta qq} \sim$ <qq> generalizing Politzer OPE.
    and confirm $\eta_{qq} = \Delta_{qq}/(1+\gamma)$!

A few additional topics

Another look at the β-function

- Consider the again the trace (scale) anomaly:

$$\theta_\alpha{}^\alpha|_{\mathrm{on-shell}} = \frac{1}{2g}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$$

- Evaluate it on any hadronic state |H> and solve for β:

$$\boxed{\beta = \frac{A_H + \gamma_m B_H}{G_H}}$$

$$A_H = 2M_H^2 - mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$B_H = mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$G_H = \langle H|G^2|H\rangle\,.$$

- Ratios of A_H/G_H & B_H/G_H independent

- Form β-function close to NSVZ β (for N=1 SUSY gauge theories)

$$\beta(g) = -\frac{1}{16\pi^2}\frac{3t_2(A) - \sum_i t_2(i)(1-\gamma_i)}{1 - t_2(A)g^2/8\pi^2}.$$

Mass & decay constant trajectory

- At large-N_c neglect width

$$g_{H_n} \equiv \langle 0|\mathcal{O}|H_n\rangle \text{ (decay constant)}$$

$$\Delta(q^2) \sim \int_x e^{ixq} \langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle = \sum_n \frac{|g_{H_n}|^2}{q^2+M_{H_n}^2}$$

- In limit m → 0 (scale invariant correlator)

$$\Delta(q^2) = \int_0^\infty \frac{ds\, s^{1-\gamma_*}}{q^2+s} + \text{s.t} \propto (q^2)^{1-\gamma_*}$$

- Solution are given by:

$$\boxed{M_{H_n}^2 \sim \alpha_n m^{\frac{2}{1+\gamma_*}} \;, \quad g_{H_n}^2 \sim \alpha_n'(\alpha_n)^{1-\gamma_*} m^{\frac{2(2-\gamma_*)}{1+\gamma_*}}}$$

where α_n arbitrary function (corresponds freedom change of variables in ∫)

- QCD expect $\alpha_n \sim n$ (linear radial Regge-trajectory) (few more words)

- For those who know: resembles deconstruction Stephanov'07
  difference physical interpretation of spacing due to scaling spectrum

remarks S-parameter

Analytical guidance S-parameter: $S = 4\pi\Pi_{V-A}(0) - \text{pion pole}$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\delta_{ab}\Pi_{V-A}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\left(V_a^\mu(x)V_b^\nu(0) - (V \leftrightarrow A)\right)|0\rangle$$

$$\Pi_{V-A}(q^2) \simeq \frac{f_V^2}{m_V^2 - q^2} - \frac{f_A^2}{m_A^2 - q^2} - \frac{f_P^2}{m_P^2 - q^2} + \dots .$$

(conspiracy) cancellations module
improve on ...

$$\Pi_{V-A}^{\mathrm{W-TC}}(0) \sim O(m^{-1})$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(0) \sim O(m^0)$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(q^2) \sim \frac{m^{2/y_m}}{q^2}$$

for $-q^2 \gg (\Lambda_U)^2$

Sannino'10 free theory

$\Rightarrow$ lattice determination coming soon (already some market)

Lattice simulations (generic remarks):

- Ca 7(2) groups UK Swansea/Edbgh), Finnland, Holland, Lin & Onugi
  USA (LSD,deGRand, Knuti,Fodor, Caterall & Sannino .....)

- IR mass is relevant; coupling irrelevant *(principal no tuning necessary)*

- Measure β-fct (stepsize scaling)
  problem: $m \neq 0$ so not fixed-pt β-fct not physical
  measuring zero (cancellations)

- measure enhancement $\langle QQ \rangle / f_\pi^3$ (LSD) parametric control?

- It would seem longterm mass/decay constant parametric scaling should help

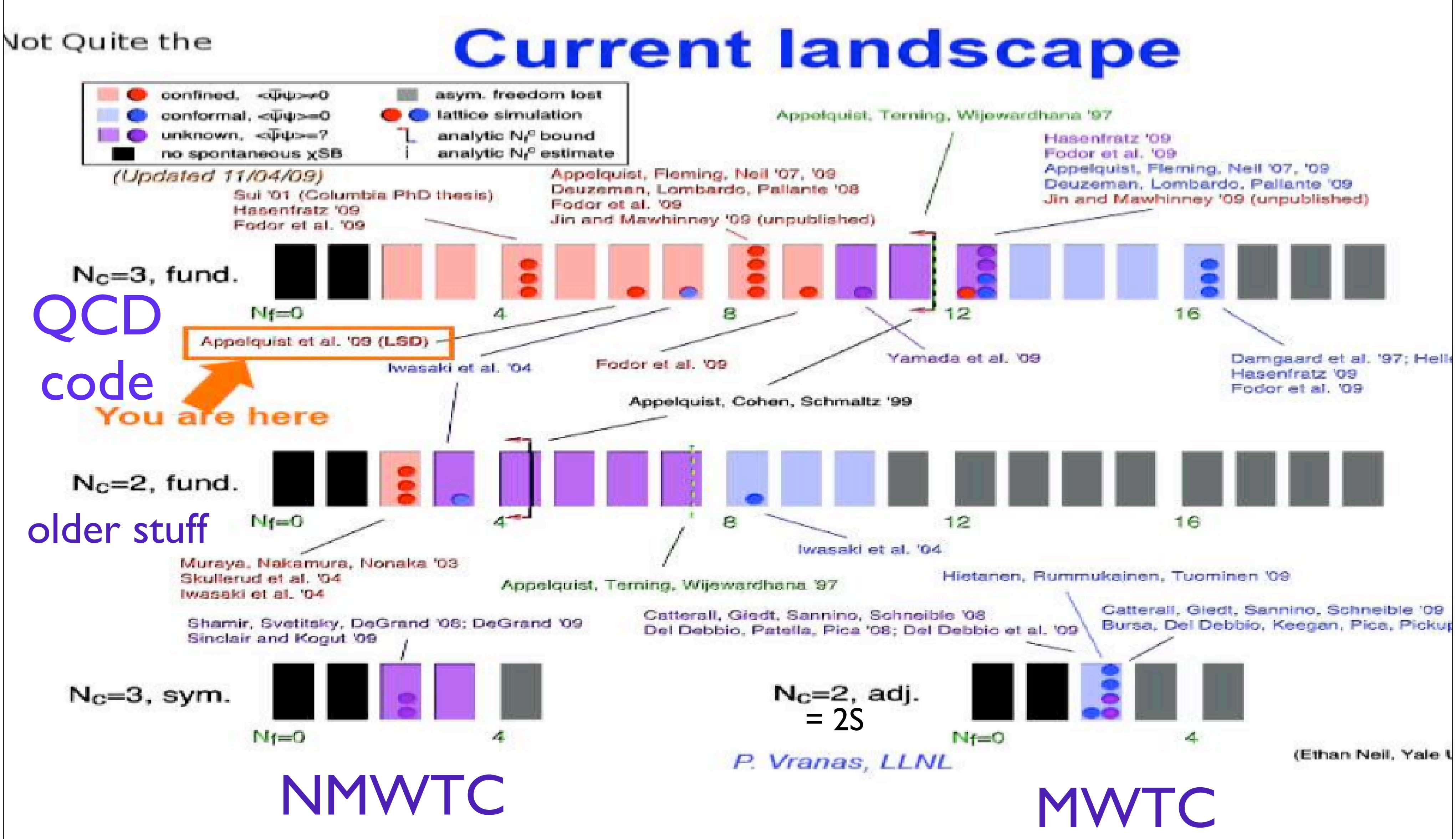
Summary of results:

- See reasonable results scaling in $0^{-+}, 1^{--}$ channels
  0^{++} more noisy (as usual)

- typically $\gamma^* \sim 0.4(3?)$ not too large (upper bound difficult)

- so-called MinimalWTC looks conformal $\Rightarrow$ conformal-TC model building

- Why & What is simulated: next slide ....



Not Quite the
Current landscape
confined, <ψ̄ψ>≠0
conformal, <ψ̄ψ>=0
unknown, <ψ̄ψ>=?
no spontaneous χSB
asym. freedom lost
lattice simulation
analytic N_f^c bound
analytic N_f^c estimate
(Updated 11/04/09)
Sui '01 (Columbia PhD thesis)
Hasenfratz '09
Fodor et al. '09
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '08
Fodor et al. '09
Jin and Mawhinney '09 (unpublished)
Appelquist, Terning, Wijewardhana '97
Hasenfratz '09
Fodor et al. '09
Appelquist, Fleming, Neil '07, '09
Deuzeman, Lombardo, Pallante '09
Jin and Mawhinney '09 (unpublished)
Nc=3, fund.
QCD code
Nf=0
4
8
12
16
Appelquist et al. '09 (LSD)
You are here
Iwasaki et al. '04
Fodor et al. '09
Yamada et al. '09
Damgaard et al. '97; Hell
Hasenfratz '09
Fodor et al. '09
Appelquist, Cohen, Schmaltz '99
Nc=2, fund.
older stuff
Nf=0
4
8
12
16
Iwasaki et al. '04
Muraya, Nakamura, Nonaka '03
Skullerud et al. '04
Iwasaki et al. '04
Appelquist, Terning, Wijewardhana '97
Hietanen, Rummukainen, Tuominen '09
Shamir, Svetitsky, DeGrand '08; DeGrand '09
Sinclair and Kogut '09
Catterall, Giedt, Sannino, Schneible '08
Del Debbio, Patella, Pica '08; Del Debbio et al. '09
Catterall, Giedt, Sannino, Schneible '09
Bursa, Del Debbio, Keegan, Pica, Pickup
Nc=3, sym.
Nf=0
4
Nc=2, adj.
= 2S
Nf=0
4
P. Vranas, LLNL
(Ethan Neil, Yale U
NMWTC
MWTC

Epilogue

- ☆ People accept it will take more time to establish CW than foreseen

- ☆ Major goals:
  1) size of conformal window
  2) how large anomalous dimension $1 \leq \Delta_{qq}$ .....
     will unitary-bound be reached? Is $\Delta_{qq} \cong 2$ border DS?
  3) measurement S-parameter

- ☆ Conformal window studies open up theory space model building CTC...

- ☆ More theoretical questions: trajectories etc

- ☆ Maybe by understanding mCGT better we learn sthg about QCD

Thanks for your attention!

Backup slides ...