A higher derivative/Lee-Wick Standard Model

http://cp3-origins.dk/

Roman Zwicky (Southampton)
January 26 2010, Mainz

Overview

- general introduction -- hierarchy problem - Higgs composite or fund.?

- Lee-Wick QFT -- toy model $L \sim \varphi^3$

  1. Higher derivative formulation
  2. Auxiliary field formulation

- tree-level unitarity -- an instructive example

- The Lee-Wick Standard Model (construction principle)

  absence of quadratic divergences
  fermion sector
  electroweak precision observables (oblique formulation)
  LHC phenomenology I & II

- Ideas behind unitarity -- acausality (brief)

- Epilogue formal & pheno aspects

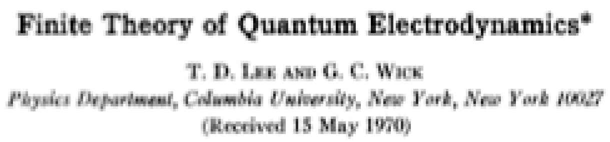
Introduction...

- **ca ’69 renormalization theory not popular with all field theorists**

  prior to: Wilson view renormalization (effective theories)

  ‘t Hooft & Veltman renormalization spont.-broken gauge theories

  strong interaction as a gauge theory known as QCD


Finite Theory of Quantum Electrodynamics*

T. D. Lee and G. C. Wick

Physics Department, Columbia University, New York, New York 10027

(Received 15 May 1970)

declared Pauli-Villars field as physical (at least virtual)

- **ca ’07 omnipresent hierarchy problem ...**

  inspires most new physics models (not flavour ...)

  Grinstein O’Connel Wise ... apply Lee-Wick field theories to SM

  ⇒ *only logarithmically divergent (cure to hierarchy problem)*

Lee-Wick field theories

Lee & Wick ~ 1970

a theory with (unobservable) ghosts
renders QED finite

☆ Field theory laboratory

.. only candidate: non-trivial **acausal** QFT
1970

unitary?

☆ Curious (LW)SM phenomenology
Grinstein, O'Connel, Wise 2007

$|V_{tb}| > 1$!!

interference effects

"solution" to hierarchy problem

Hierarchy Problem

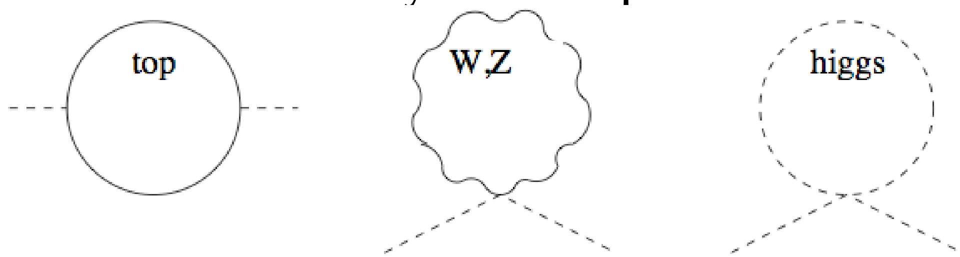
- **If** SM viewed as an effective theory (Wilson)

$$\mathcal{L}^{\mathrm{eff}}_{\Lambda_{\mathrm{EW}}} = m_H^2 h_0^2$$

scalar mass term is relevant operator (grows at low energy)
$\Rightarrow \Lambda_{\mathrm{EW}} \sim m_H$ appears accidental / unnatural

- Viewpoint from perturbation theory -- 1-loop corrections

$\Lambda_{\mathrm{cut-off}} = 10\,\mathrm{TeV}$



$\Delta m_H =$ $\quad -2\,\mathrm{TeV} \quad 0.7\,\mathrm{TeV} \quad 0.5\,\mathrm{TeV}$

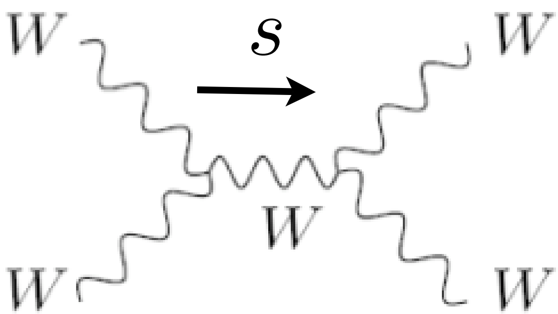
- "Cure": something ought to happen $E_{\mathrm{new}} \approx O(1)\Lambda_{\mathrm{EW}}$
EWPO often $\Rightarrow E_{\mathrm{new}} \approx O(10)\Lambda_{\mathrm{EW}}$ "little hierarchy problem"

In fact: why there really ought to be something .....

W_L-W_L scattering

"perturbative unitarity"

S-wave



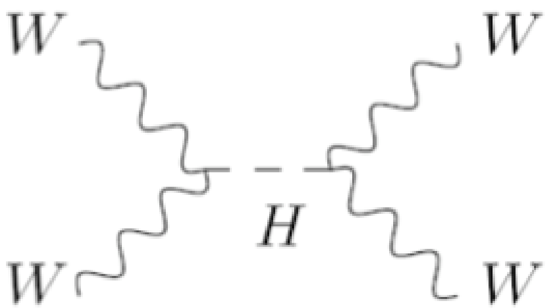
+ ....

$$A_0 = \frac{g_2^2}{16\pi m_W^2} s$$

Unitarity bound

$$\mathrm{Re}[A_0] \leq \frac{1}{2}$$

$$s_{\mathrm{crit}} \sim (1.2\,\mathrm{TeV})^2$$



+ ...

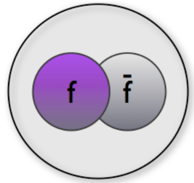
$$\delta_H A_0 = - \frac{g_2^2}{16\pi m_W^2} s$$

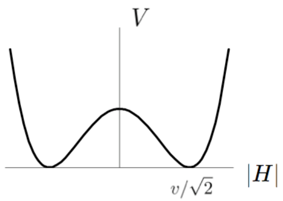
Standard (Model) solution

Beyond the SM

centered around the Higgs mechanism of SSB
⇒ W,Z masses; hierarchy problem?

fundamental particle small width		composite particle large width





SSB $SU(2)_L$
Higgs-mechanism

strong dynamics

$\langle q_R Q_L \rangle \sim (\Lambda_{EW})^3$

need cancelation mechanism

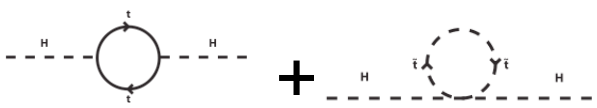
hierarchy problem

quantum corrections
cut off by analogue of Λ_{QCD}

Supersymmetry
opposite statistics partner

Examples

Technicolour
Higgs sector ⇒ Gauge theory



$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_{TC}$

… time to enter Lee-Wick field theories

Toy Model: φ^3

A. Higher derivative formulation

B. Auxiliary field formulation

A. Higher derivative formulation

Add higher derivative term to L

$$\mathcal{L}_{\rm hd} = -\frac{1}{2}\hat{\phi}\,\partial^2\hat{\phi} - \frac{1}{2}m^2\hat{\phi}^2 - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \lambda\hat{\phi}^3$$

Propagator:

$$\hat{D}(p) = \frac{1}{p^2 - p^4/M^2 - m^2} \sim \frac{1}{(p^2 - m_{\rm ph}^2)} - \frac{1}{(p^2 - {M_{\rm ph}}^2)}$$

- ☆ two poles -- one "wrong" sign residue (PV ghost)
- ☆ improved convergence diagrams

B. Auxiliary field formulation

Introduce auxiliary field Φ
(equivalent Lagrangian)

$$\mathcal{L}_{\text{eff}} = \begin{pmatrix} \phi \\ \phi_{LW} \end{pmatrix} \left[\partial^2 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} m^2 & -m^2 \\ -m^2 & m^2 + M^2 \end{pmatrix} \right] \begin{pmatrix} \phi \\ \phi_{LW} \end{pmatrix} + \lambda(\underbrace{\phi - \phi_{LW}}_{\hat{\phi}_{\text{HD}}})^3$$

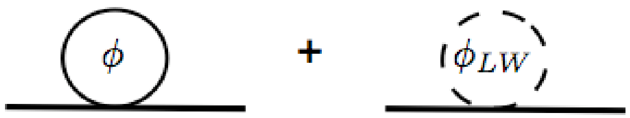
diagonalization $\Rightarrow$ hyperbolic rotations

$$\begin{pmatrix} \phi \\ \phi_{\text{LW}} \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} \phi \\ \phi_{\text{LW}} \end{pmatrix}_{\text{ph}} \qquad \tanh 2\theta = \frac{-2m^2/M^2}{1 - 2m^2/M^2}$$

shift to physical fields couplings & masses

Improved convergence: e.g. self energy

$$\Sigma(p^2) \sim \int_k \frac{1}{k^2 - m^2} - \int_k \frac{1}{k^2 - M^2} = \int_k \frac{(m^2 - M^2)}{(k^2 - m^2)(k^2 - M^2)}$$



“only” log-divergent

Hasty impression (AF-formalism):

P_1: double number of fields

P_2: power counting improves: #(inter lines) (-2)

not as simple for gauge theories!!

Higher derivative (HD) formulation:

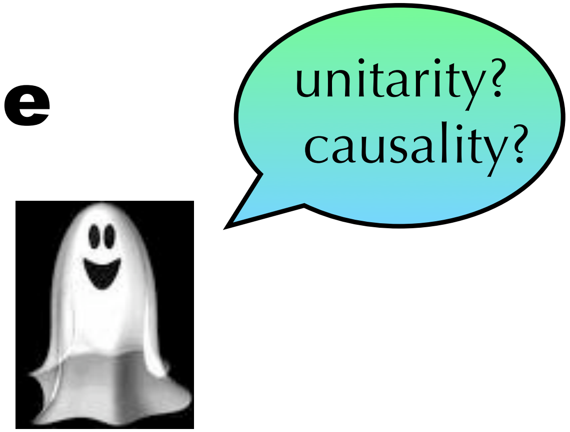
- construction principle of LWSM
- power counting (divergences)

Auxiliary field (AF) formulation:

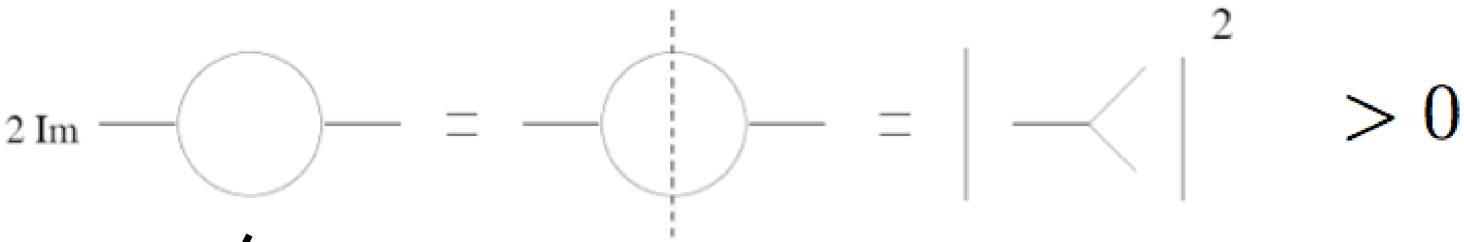
- explicit calculations
- contact with experiment

use them both ⇒ check your results

Interpretation: First example



s-channel unitarity *(1-loop)*



sum all bubbles (use narrow width approximation)

$$\tilde{D}(p^2) = \frac{-1}{p^2 - M^2} + \frac{-1}{p^2 - M^2}\,\Sigma(p^2)\,\frac{-1}{p^2 - M^2} + \dots = \frac{-1}{p^2 - M^2 + \Sigma(p^2)}$$

$$\mathrm{Im}[\tilde{D}(p^2)] \sim (-)_{\mathrm{prop}}\mathrm{Im}[-\Sigma(p^2)] = (-)^2 M\Gamma > 0$$

$(-1)^2 = 1$
negative residue and
wirdth cancel!

The Lee-Wick Standard Model (LWSM)

Grinstein, O'Connel, Wise
(0704.1845)

Construction principle in HD formalism

$$\delta\mathcal{L}_{scalar} = -\frac{1}{M_\phi^2}\left|\hat{D}^2\hat{\phi}\right|^2$$

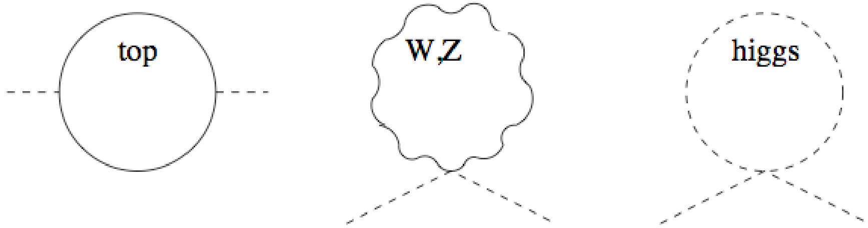
$$\delta\mathcal{L}_{\chi-fermion} = +\frac{1}{M_\psi^2}\,\overline{\hat{\psi}}\, i\hat{\not{D}}\hat{\not{D}}\hat{\not{D}}\,\hat{\psi}$$

$$\delta\mathcal{L}_{gauge} = +\frac{1}{M_A^2}\,\mathrm{Tr}\big((\hat{D}\hat{F})_\mu^2\big)$$

power counting!!

- gauge invariance (minimal coupling) enforces derivatives in L^{int}
- L^{gauge} could be used as PV regulator for non-abelian gauge theories

Absence of quadratic divergences -- HD formalism



the Λ^2_{UV}-graphs

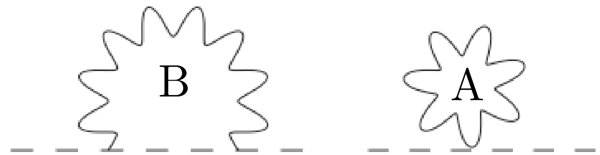
- top and Higgs loop: no new "∂" simply log convergent
- gauge boson loop: new "∂" ⇒ closer look

P_1

$$d_{div} = 6 - 2L - E_S - E_V - 2E_g = 2$$

(naive) power counting!!

$$\mathcal{L}^{int} = h^\dagger D^4 h \sim h^\dagger (\partial^2 + \partial\cdot A + A\cdot\partial \ + A^2)^2 h$$



- in Lorentz gauge (∂·A=0) & partial-∫ ⇒ log-convergent

Grinstein, O'Connel, Wise
(0704.1845)

Flavour sector ... CKM -- AF formalism

- massive fermion = 2×chiral femion
  ⇒ three LW fermion generations !! $\boxed{P_2}$

$$\begin{array}{ccc} Q_L & \leftrightarrow & (\tilde{Q}_L, \tilde{Q}'_R) \\ q_R & \leftrightarrow & (\tilde{q}'_L, \tilde{q}_R) \end{array}$$

$$\begin{pmatrix} m_t & -m_t & 0 \\ -m_t & m_t & -M_t \\ 0 & -M_T & 0 \end{pmatrix}$$

top mass matrix

$$\begin{pmatrix} m_t & -m_t & 0 \\ -m_t & m_t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

top Yukawa matrix

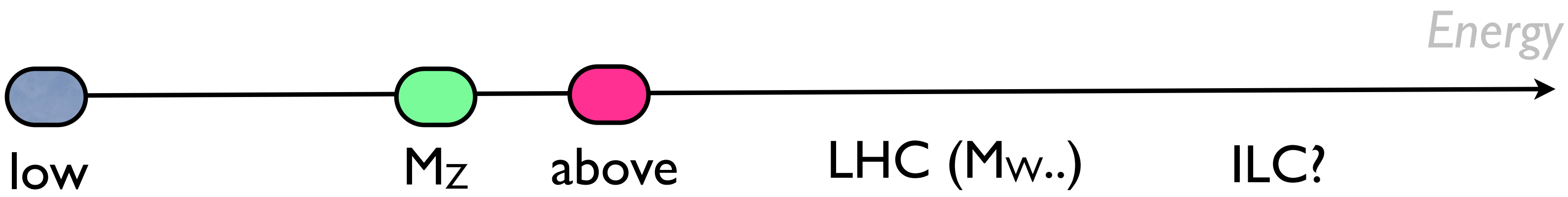
- are <u>not</u> simultaneously diagonal

non-unitary correction to CKM matrix (generalize unitarity)

- e.g. $|V_{\rm tx}| \to |V_{\rm tx}|(1 + \frac{1}{2}\left(\frac{m_t}{m_{\tilde{t}}}\right)^2) > 1$ possible (x=t)

Krauss, Underwood, RZ '07

Electroweak (precision) constraints



- low: ν-e, ν-N scattering, atomic parity
- Z-pole: LEP1/SLC asymmetries FB, polarization initial/final state scattering
- above: LEP2 asymmetries/cross section (contact interactions)

Generally: trade EW parameters (g_1,g_2,v) for three best measured parameters

m_Z = 91.1976(21) GeV , $\alpha^{-1}(m_Z)$=127.918(18), G_F=1.16637(1)10^{-5}GeV^{-2}

$$\mathbf{Quantity X}: \quad X = X(G_F^{\rm SM}, ...|M_{\rm LW}, ...) = X(G_{\rm SM}, M_{\rm LW}, ...)$$

1. Lagrangian ("theory")

2. $G_F = G_F^{SM}(1+\delta_{LW})$ ("experiment")

Work out effective Lagrangians (step 1)

∫-out heavy d.o.f. & diagonalize (hyperbolic)

$$\mathcal{L}_Z^{\rm eff} = -\left(\rho_{\rm f}\sqrt{2}G_F\right)^{1/2} 2\,\mathbf{m_Z}\, J_Z\cdot Z\,, \qquad J_Z^\mu \equiv (J_3 - \mathbf{s}_*^2(\mathbf{m_Z})\, J_Q)^\mu$$

$$\mathcal{L}_{\rm Low}^{\rm eff} = -\frac{4\mathbf{G_F}}{\sqrt{2}}\left(J_+\cdot J_- + \rho_*(\mathbf{0}) J_{\rm nc}^2\right) + C_Q J_Q^2 \qquad J_{\rm nc}^\mu \equiv (J_3 - \mathbf{s}_*^2(\mathbf{0}) J_Q)^\mu$$

$$\mathcal{L}_{\rm QED}^{\rm eff} = -\frac{1}{16\pi\alpha}F^2 + ..$$

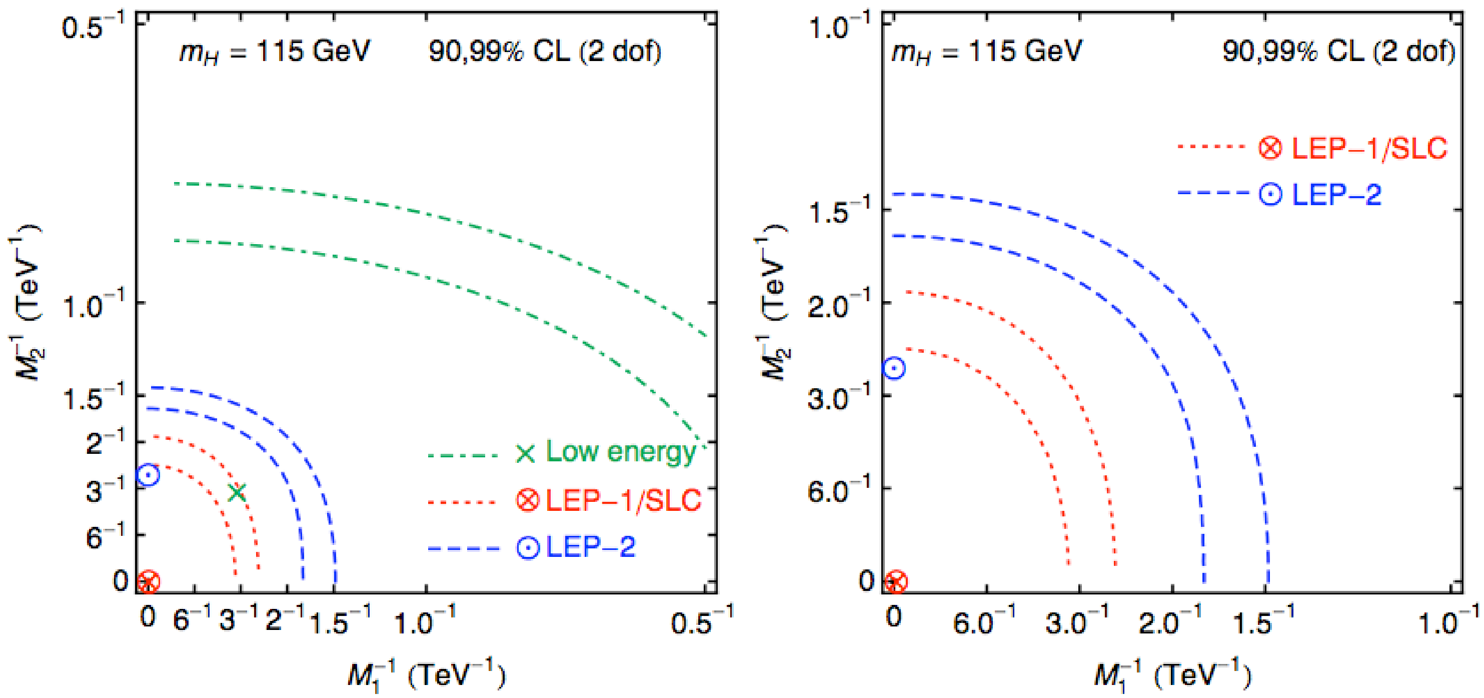
- Out of (G_F, m_Z, α) only m_Z affected
- Low energy not affected (formally) -- " $\partial^4 / M_{LW}^2 \Rightarrow 0$ "

Match with experiment (step 2)

$$s_W^2 c_W^2 = \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2}\,\frac{(1+\cancel{\delta_G})(1+\delta_Z)}{1+\cancel{\delta_\alpha}}$$

low energy is (finally) affected

perform the χ^2-fit .....



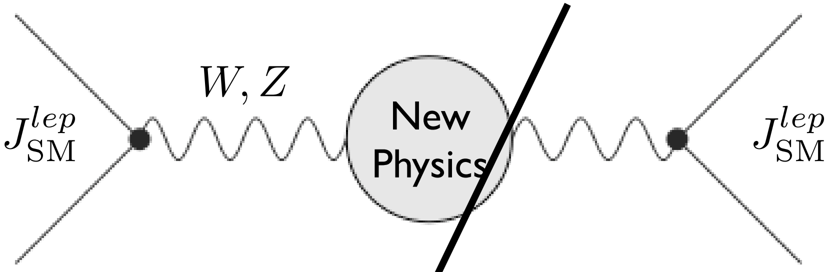
⭐ traces of the “little hierarchy” problem ?

like many other models

Oblique formalism (S,T,U)

Kennedy & Lynn, Altarelli & Barbieri, Peskin & Takeuchi ~90'

- If "new physics" in (W,Z) self energy (e.g. 4[th] family, technicolor)



"universal" corrections

- If $\varepsilon=(M_W/M_{New})^2 \ll 1 \Rightarrow$ oblique approximation

$$\Pi_{ab}(\epsilon) = \Pi_{ab}(0) + \Pi'_{ab}(0)\epsilon + \mathcal{O}(\epsilon^2)$$

$$ab \in \{BB, B3, 33, 11\}$$

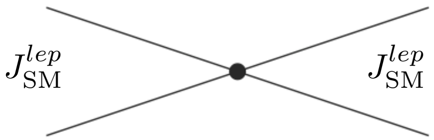
- $\Rightarrow 8 - 3(G_F, m_Z, \alpha) - 2_{U(1)} = 3$ parameters S, (T, U)

e.g.

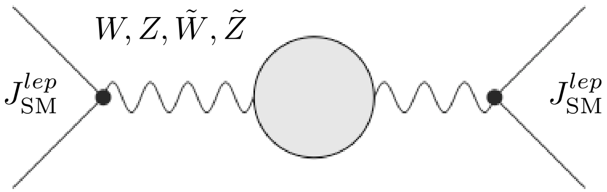
$$\rho_*(0) = 1 + \alpha T\,; \qquad T = \frac{1}{M_W^2 \alpha}(\Pi_{33}(0) - \Pi_{11}(0))$$

Non-universal models ......

- additional weak gauge bosons $\tilde{W}, \tilde{Z}$ -- ∫-out



⇒ *4-Fermi contact terms (non-univsersal)*



$$W^{\rm out} = \alpha W + \beta \tilde{W}$$
$$W^{\rm out} \perp W^{\rm in}\,; \qquad \mathcal{L}^{\rm eff} \sim W^{in} \cdot J_{\rm SM}^{lep}$$

⇒ theory “universal” form -- “universalized”

- prize to pay: one higher order ⇒ 3 + 4 = 7 parameters (S,X) (T,U,V) (W,Y)

captures contact terms of LEP-2

$$\Pi_{ab}(\epsilon) = \Pi_{ab}(0) + \Pi'_{ab}(0)\epsilon + \frac{1}{2}\Pi''_{ab}(0)\epsilon^2 + \mathcal{O}(\epsilon^3)$$

Strumia’99 .... Barbieri,Pomarol,Rattazi,Strumia‘04

Underwood,RZ arxiv:0805.3296[hep-ph]

LWSM (AF) formalism

- belongs to this class of models
- "universalize" AF formulation ⇒ HD formulation !!

think backwards

e.g.

$$\Pi(q^2)_{33} = q^2 - m_W^2 - \frac{q^4}{M_2^2}\,; \qquad W = \frac{M_W^2}{2}\Pi''_{33}(0) = -\frac{m_W^2}{M_2^2}$$

All leading order terms

$S=0$, $T=0$, $W = -(m_W/M_2)^2$, $Y = -(m_W/M_1)^2$

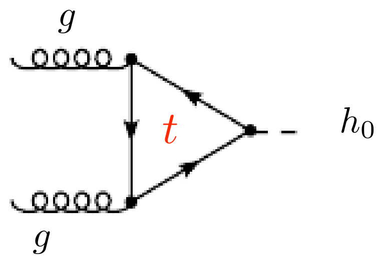
- expand (previous) tree-level exact results in ε -- they do agree!
- N.B. carefully avoided "custodial symmetry" ; ambiguous $g_1 \neq 0$!
- Similar models x-dim. LW ~ KK; but W,Y ~ $(m_W/M_{KK})^2 > 0$ no ghost :)

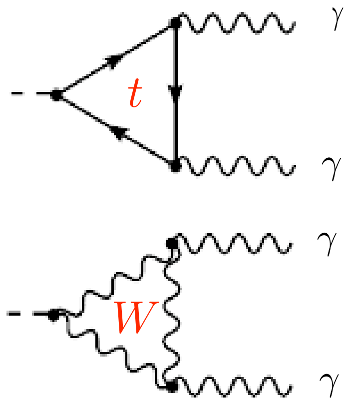
LHC Pheno I -- $gg \to h_0 \to \gamma\gamma$

Krauss, Underwood, RZ '07

- Low Higgs mass discovery channel sensitive to heavy quarks i.e. top

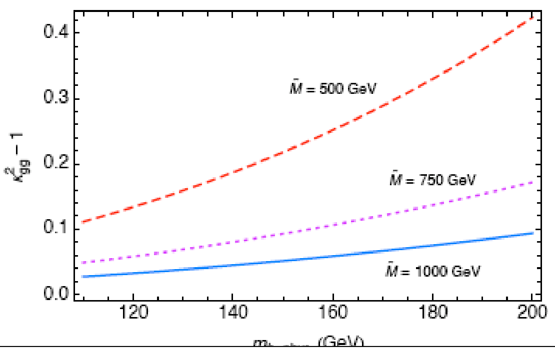
$$\mathcal{A} \sim (1 \pm \mathcal{O}(1/m_{\mathrm{LW}}^p))$$

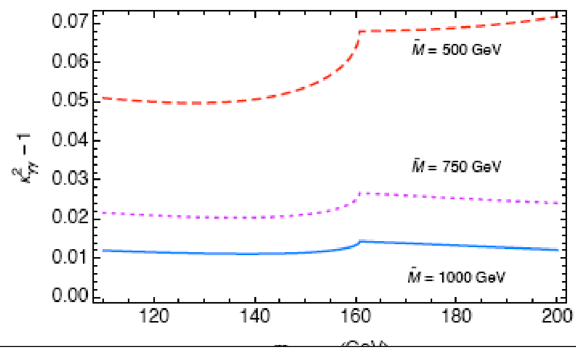




1. ghost reduction -  hyperbolic rotation (V_{tx}) + winner
2. p=2 ; fast quadratic decoupling $(m_W/m_{LW})^2$ -- could have been linear ....

- scaling derived "easily" HD picture
- 1. sign 2. scaling



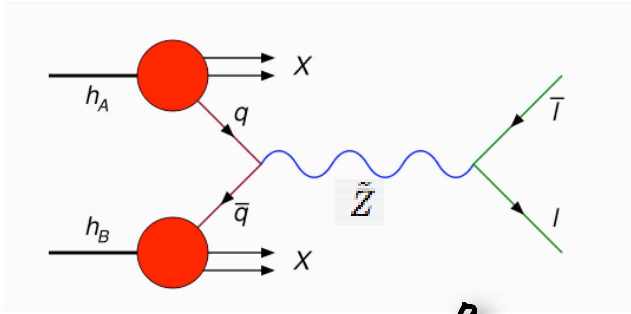


LHC Pheno II -- $qq \to \tilde{Z}(\tilde{W}) \to ll(\nu)X$

Rizzo arxiv:0704.3458[hep-ph]

- Can LW gauge bosons be "uniquely" identified?

wrong sign	1. residue
	2. width



Drell-Yan

- 2. width ~2 GeV small -- at least Γ_{LW}/M_W suppressed + hadronic env.
- 1. measure cross section rather than amplitude!

$$\sigma \sim (-1)_{\text{residue}} \lambda_{qq\tilde{Z}} \lambda_{ll\tilde{Z}} \cdot \lambda_{qqZ} \lambda_{llZ}$$

sign absorbed coupling
Answer: no!

- obvious formal cure: same initial final state vertices!

- gauge bosons to two jets (dijets) -- challenging background
- Bhabba scattering $e^+e^- \to e^+e^-$ for ILC

Rizzo arxiv:0712.1791[hep-ph]

Answer: yes!

Lee-Wick 1970

Idea behind unitarity

negative norm states & unitarity are in straight contradiction

have seen tree-level (improved pert. th) works ... but how about loops? .

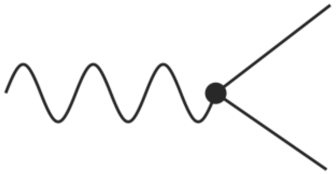
☆ basic-picture: S-matrix block-diagonal on particle/ghost-space

$$\mathcal{S} = \begin{pmatrix} S & 0 \\ 0 & X \end{pmatrix}$$ particle-world
Lee-Wick-resonance

$$S^\dagger S = 1$$

asymptotically 2-worlds

☆ $e \neq 0$, the states $\{|e^+e^-\rangle_{m_{\tilde{A}}}, |\tilde{A}\rangle\}$ mix for $m_{\tilde{A}} > 2m_e$



$$\mathcal{H}^{int} = \begin{pmatrix} 0 & i\Gamma/2 \\ i\Gamma/2 & \end{pmatrix}, \qquad \Gamma = \frac{1}{3}\alpha\, m_{\tilde{A}}$$

$$\Rightarrow |\pm\rangle \sim (|\tilde{A}\rangle \pm |e^+e^-\rangle_{m_{\tilde{A}}}) \text{ with } E_c = m_{\tilde{A}} \pm i\Gamma/2$$

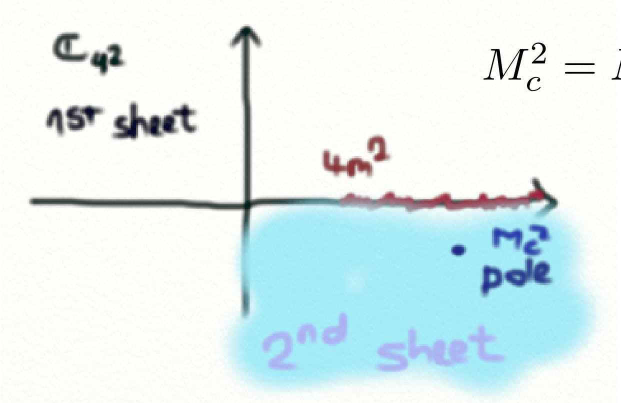
complex di-pole

negative norm states have complex energies

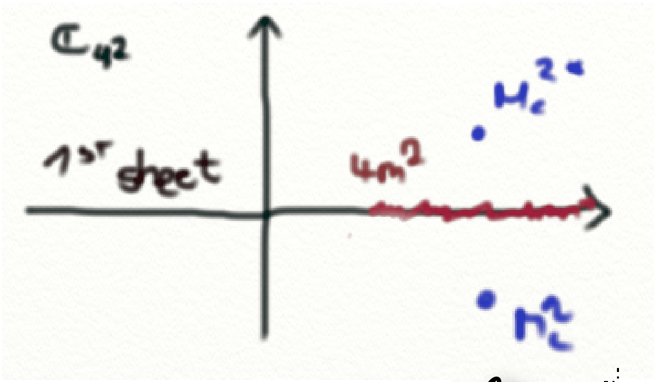
Viewpoint of analyticity

(rather non-analytic points:)

☆ positive norm: negative norm:



$$M_c^2 = M_{\mathrm{LW}}^2 - iM_{\mathrm{LW}}\Gamma$$



Schwartz-reflection principle

complex di-pole

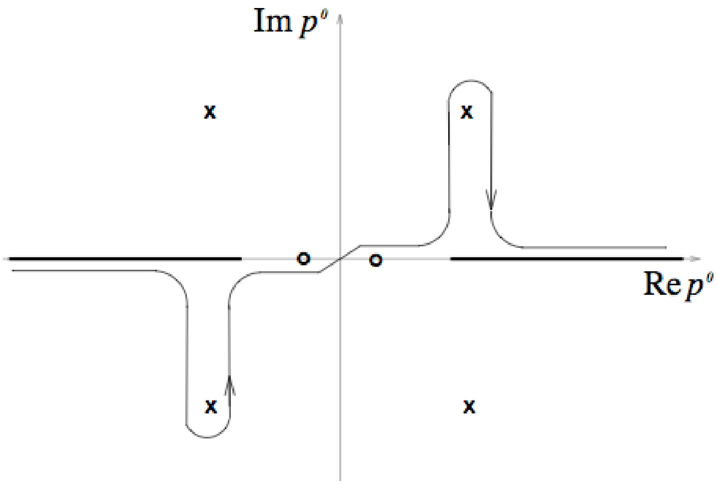
$$i\Delta(q^2) = \frac{1}{q^2 - M_c^2} + \frac{1}{q^2 - M_c^{*2}} + \int_{4m^2} \frac{ds\rho(s)}{s - q^2 - i0} \overset{q^2 \simeq M^2 \gg \Gamma^2}{\longrightarrow} \frac{1}{q^2 - M_c^2}$$

Ready for Loops

(rather non-analytic points:)

Lee-Wick '69
CLOP '69 (refined)

$$i\Delta(q^2) = \frac{1}{q^2 - M_c^2} + \frac{1}{q^2 - M_c^{*2}} + \int_{4m^2} \frac{ds\rho(s)}{s - q^2 - i0} \overset{q^2 \simeq M^2 \gg \Gamma^2}{\longrightarrow} \frac{1}{q^2 - M_c^2}$$



$$\int_p \Delta(p^2)\Delta((p+q)^2)$$

Examples: Lee-Wick poles in C_{p0}

$$p_0 = \pm\sqrt{\vec{p}^2 + M_c^2}$$

$$p_0 = \pm\sqrt{\vec{p}^2 + M_c^{*2}}$$

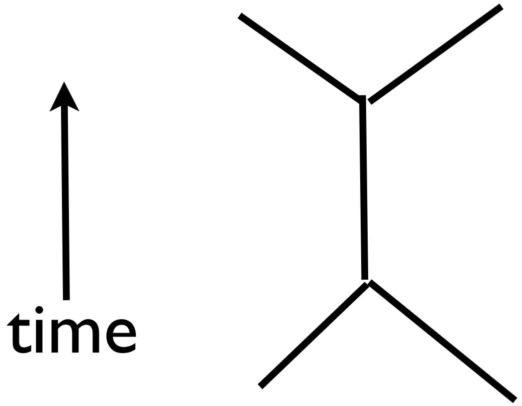
- Status: no examples known in literature where this does not work!

Coleman's Eridice lecture ....
Lee-Wick'69

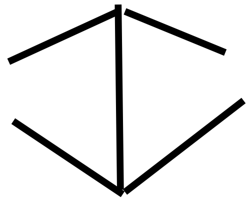
(A)Causality

(In proofs of analyticity causality is ingredient ...may expect trouble)

- Classical level: higher derivatives .. need future BC to eliminate unstable modes
- Quantum level: acausality is microscopic phenomenon



reverse sign width



- Distance of backward propagation prop. to $1/\Gamma_{LW}$ .. microscopic

Alvarez et al '09 proposal to measure with primary & secondary vertex ... lee-wick electron cand. since it is lightest.

Epilogue I (formal)

- Unitarity to all order in perturbation theory?
- Non-perturbative definition *Boulaware, Gross '84*
  path-∫ via canonical formalism does not yield CLOW prescription
- Is LWFT Lorentz invariant?
  No with LW prescription -- Yes? with CLOW prescription
- Is LWFT effective description of sthg beyond field theory?
  (borne out in desperation of the hierarchy problem)
- Proposal path-∫ quantum fields restricted test fcts space
  yields CLOW prescription *Tonder, Dorca '06*
- Not discussed high T physics e.o.s. w=1 ($p = w\,\rho$) ......

Epilogue II (pheno)

- 2007 GOW extended LWFT to SM: chiral fermions -- non-abelian gt

  counting of d.o.f. not trivial when $M_{SM} = 0$ ($M_{LW} \neq 0$ by construction

- not suprisingly LWSM has curious phenomenology

  e.g. possible $|V_{tb}| > 1$

  negative residue "cought" in: $xx \to Z_{LW} \to xx$; xx=(qq,ll)

- EWPO constrains ~ 2-3TeV for weak LW gauge bosons

  again distinguished other models e.g. x-dim

little hierarchy problem

- Issues left out:

  LWFT as lattice regulator (avoids discretization effects cut-off) used 1990

  LWFT running of beta-fct, LWFT & massive vector boson scattering

Merci pour votre attention!