Explicit and spontaneous breaking of SU(3) into its finite subgroups

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based on arXiV.1100.4891v1 in collaboration Alexander Merle (Stockholm)
*along Mathematica package **SUtree** lots of features*
http://theophys.kth.se/~amerle/SUtree/SUtree.html

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0. Prologue -- finite groups

(might want to hear anyway) **not** required for talk

⭐ Finite group F = group finite number of elements

- Example: *three permutations* **S_3**: Order $|S_3|=3!=6$ ( ),(12),(23),(13),(123),(132)

⭐ Dimensionality thm: $\boxed{|F| = \sum |\mathrm{IRREPS}(F)|^2}$ ⇒ finite many of them

- $|S_3| = |\mathbf{1}|^2 + |\mathbf{1'}|^2 + |\mathbf{2}|^2$

⭐ Characterized by charactertable (Brauer hypothesis ....)

- conjugacy classes $c' \sim gcg^{-1}$ $g \in F$
  character $\chi = \mathrm{tr}[c]$

S_3	$\{e\}$	$\{a,b,c\}$	$\{d,f\}$
Γ_1	1	1	1
Γ_2	1	α	β
Γ_3	2	γ	δ

- Almost all information e.g. Kronecker products $(\mathbf{2} \times \mathbf{2})_{S3} = (\mathbf{1}+\mathbf{1}+\mathbf{2})_{S3}$

⭐ Many others than permutation groups -- any finite group can be embedded in a permutation group

Outline

- I. Introduction
- II. Main ideas -- using SO(3) → S_4 as an example
  - II.a Classification of invariants
  - II.b Exemplified S_4
  - II.c Problem of sufficient condition
- III. Finite subgroups of SU(3) -- denoted by F_3
- IV. Database
- V. Example criteria for breaknig SU(3) → F_3
- VI. Complex spherical harmonics
- VI. Tensor-generating function (generalization of Molien function)
- Epilogue (talk about open ends)

1. Introduction

Groups in "disguises"

consider S_4: the group of permutation of four objects

- geometric definition through irreps: $4! = |S_4| = \mathbf{1}^2 + \mathbf{1}'^2 + \mathbf{3}^2 + \mathbf{3}'^2$ (dim thm)
  (to each of four objects assign orthogonal vectors -- implement permutation linearly)
- abstract algebraic definition $S_4 = \langle\langle \text{ words } a,b, \mid a^3=1, b^2=1, (ab)^4=1 \rangle\rangle$

today's particle physicist more familiar with Lie groups e.g. O(3)

- $O(3) = \langle\langle M_3 \mid M_3^T M_3 = 1 \rangle\rangle$

equivalently all linear operations in three variables that leave $x^2 + y^2 + z^2$ invariant.

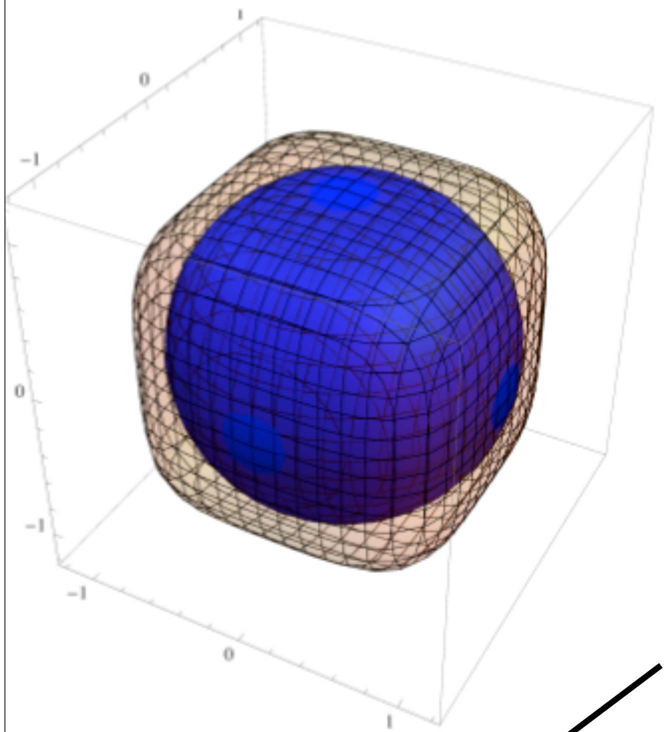
$\boxed{\text{is there an analogous way to think about finite groups?}}$

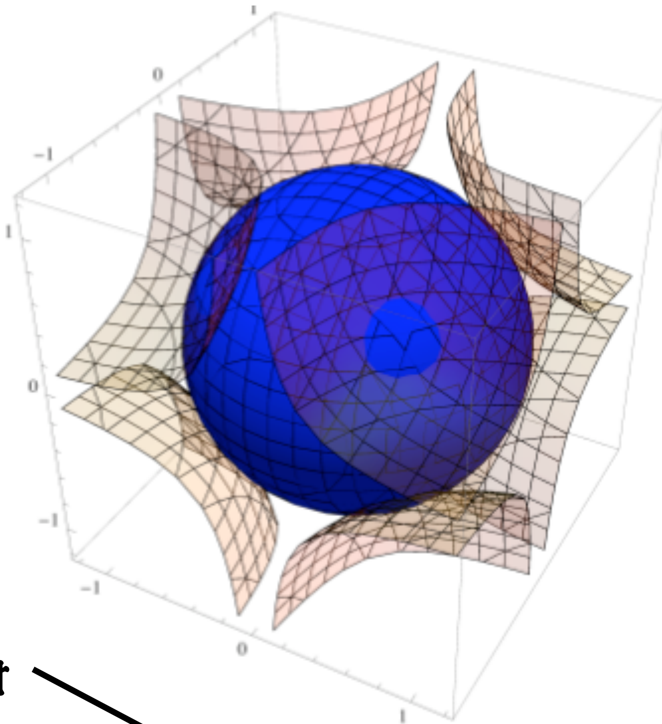
Yes: visual "proof" for SO(3) → S_4

S_4 isomorphic cubic-symmetry
(dual octahedron)

$x^4 + y^4 + z^4$

$(xyz)^2$





intersections of sphere SO(3)
and add. invariant = constant

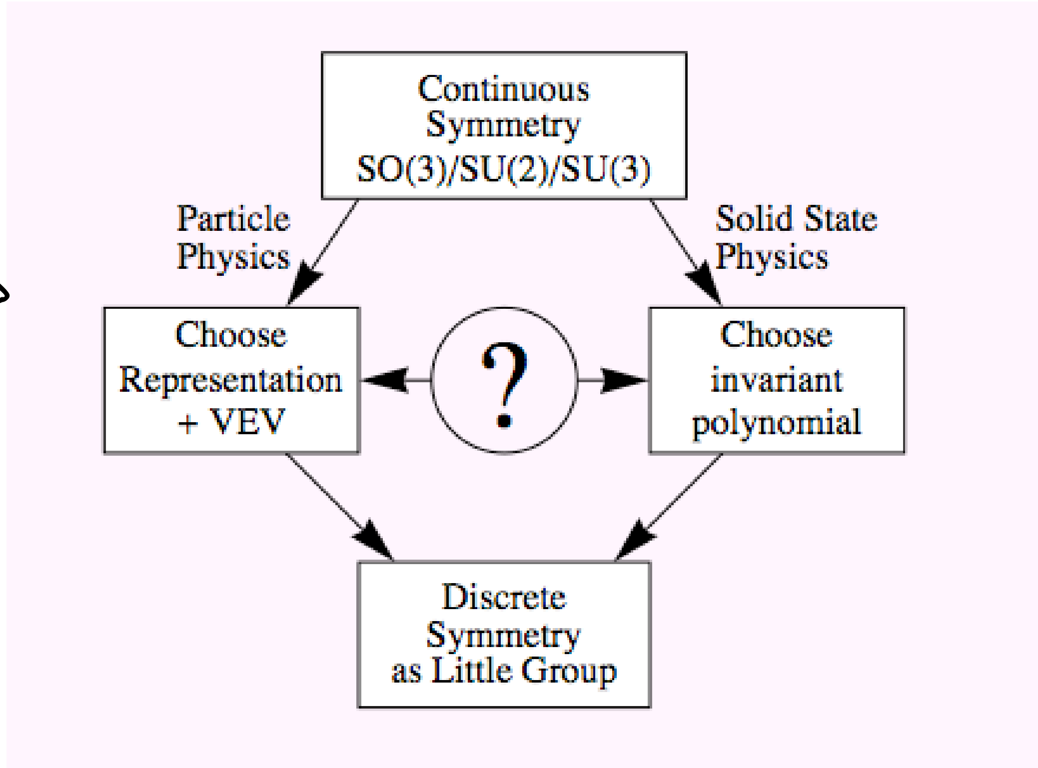
vertices of octahedron

vertices of cube

Turning to the physicist's vocabulary

Definition of a group ⇔ Conditions breaking into this group

Spontaneous symmetry breaking



Explicit symmetry breaking

➥ This talk: - link between SSB & explicit breaking
- classification invariants,
- necessary conditions to define group

for SU(3)

II. Main ideas ...

... discussed within $SO(3) \to S_4$

Spontaneous symmetry breaking (through vacuum)

☆ Consider fundamental irrep SO(3) i.e. $\mathbf{3}_{(l=1)}$

vacuum vector $\uparrow$ $\Rightarrow$ SO(2) symmetry remains ; not enough

☆ Consider higher dimensional irrep l = 2,3,4,5 ; abandon geometric picture

Proceeding abstract manner:
Choose vacuum v, then $H_v = \{g \in SO(3) \mid R(g)v = v\} \subset R(SO(3))$ defines a subgroup

➥ $SO(3) \to H$ through VEV v

Explicit symmetry breaking (through invariants)

☆ Consider $\varphi_i \in \mathbf{3}_{(l=1)}$ $L_{tot} = L_{SO(3)}(|\varphi|^2) + L_{break(H)}(\varphi_1, \varphi_2, \varphi_3)$

Again any $L_{break(H)}$ will break $SO(3) \to H$ some subgroup

From our geometric "proof" $L_{S4} = \varphi_1{}^4 + \varphi_2{}^4 + \varphi_3{}^4$

One-to-one link between spontaneous & explicit symmetry breaking

- How can polynomials be linked to vector spaces?
  ⇒ Groups have polynomial representation functions

spherical harmonics $Y_{l,m}$
for SO(3)
= complete set of irreps

$$\mathcal{I}[S_4] = x^4 + y^4 + z^4 = c\left(Y_{4,-4} + \sqrt{\frac{14}{5}}Y_{4,0} + Y_{4,4}\right)$$

- This means that choosing a VEV:

$$v \sim (1,0,0,0,\sqrt{\tfrac{14}{5}},0,0,0,1) \quad \text{then} \quad \mathbf{9}_{l=4}|_{S_4} \to \mathbf{1}_{S_4} + \ldots$$

branching rule

  which is easily verified explicitly

- Extension to SU(3) involves finding SU(3) representation function
  ⇒ complex spherical harmonics (studied in 60's) (discuss latter)

II.a Classification of invariants

- all polynomial invariants
- algebraic dependencies etc

general

Molien's theorem (1897)

$$M_{\mathcal{R}(H)}(P) \equiv \frac{1}{|\mathcal{R}(H)|} \sum_{h \in \mathcal{R}(H)} \frac{1}{\det(\mathbf{1} - P\,h)} = \sum_{m \geq 0} h_m P^m \ ,$$

easy to compute

R(H) is an irrep of a finite group H.
Thm: Positive coefficients h_m count the number of polynomial invariants of degree m.

Algebraic dependence

- ☆ For n variables there are n algebraically independent invariants (Noether 1916)
  Those we call primary and all the other secondary invariants.

- ☆ Dependence of secondary invariants as follows:

$$\overline{\mathcal{I}}_{n_i}^{\,2} = f_0(\mathcal{I}_{m_1}, \mathcal{I}_{m_2}, \mathcal{I}_{m_3}) + \sum_j f_1^{(j)}(\mathcal{I}_{m_1}, \mathcal{I}_{m_2}, \mathcal{I}_{m_3}) \cdot \overline{\mathcal{I}}_{n_j} \ ,$$

syzygy

☆ Fact: If degrees primary & secondary invariants known then the Molien fct assumes ..

$$\{\mathcal{I}_{m_1}, \mathcal{I}_{m_2}, \mathcal{I}_{m_3}, \overline{\mathcal{I}}_{n_i}, ..\} \quad \Rightarrow \quad M_{H(\mathbf{3})}(P) = \frac{1 + \sum_i a_{n_i} P^{n_i}}{(1 - P^{m_1})(1 - P^{m_2})(1 - P^{m_3})} \; .$$

The form of the Molien fct is <u>not unambiguous</u> thus no ⇐ implication

☆ ⇒ establishing primary & secondary invariants is non-trivial

In practice: 1) guess form of Molien fct as above*
2) generate invariants
3) verify syzygies (great sport)

Thm: $$\text{number of secondary invariants} \equiv 1 + \sum_i a_{n_i} = \frac{m_1 \cdot m_2 \cdot m_3}{|H|} \; ,$$

☆ Generating invariants: symmetrize over group (Reynold operator)

$$\mathcal{I}(x, y, z) = \frac{1}{|\mathcal{R}(H)|} \sum_{h \in \mathcal{R}(H)} f(h \circ x, h \circ y, h \circ z) \; ,$$

for any ansatz f(x,y,z), I is an invariant

II.b Exemplified with S_4

- Molien function takes form (level of ambiguity low)

$$M_{S_4}(P) = \frac{1+P^9}{(1-P^2)(1-P^4)(1-P^6)} \ ,$$

- The following (candidate) primary and secondary invariants are found

$$\mathcal{I}_2[S_4] = x^2+y^2+z^2 \ , \quad \mathcal{I}_6[S_4] = (xyz)^2 \ , \quad \mathcal{I}_4[S_4] = x^4+y^4+z^4 \ ,$$

$$\overline{\mathcal{I}}_9[S_4] = xyz(x^2-y^2)(y^2-z^2)(z^2-x^2) \ ,$$

- The one and only syzygy is:

$$\overline{\mathcal{I}}_9^2 = \mathcal{I}_2^4\mathcal{I}_4\mathcal{I}_6 - \frac{1}{4}\mathcal{I}_2^6\mathcal{I}_6 - \frac{5}{4}\mathcal{I}_2^2\mathcal{I}_4^2\mathcal{I}_6 + \frac{1}{2}\mathcal{I}_4^3\mathcal{I}_6 + 5\mathcal{I}_2^4\mathcal{I}_6^2 - 9\mathcal{I}_2\mathcal{I}_4\mathcal{I}_6^2 - 27\mathcal{I}_6^3 \ ,$$

II.c Problem of sufficient criteria for breaking $G \to H$

- Is the hard problem (in the sense that there's no general stratetgy)
  SO(3) famous Michel criterion '79 counterexamples found

- Illustration of the problem:
  Fact 1: A_4, S_4 both leave $I_4 = x^4 + y^4 + z^4$ invariant
  Fact 2: A_4 is a subgroup of S_4
  $\Rightarrow$ I_4 breaks SO(3) into S_4 (if at all) but <u>not</u> into A_4

- $\Rightarrow$ imposing I_X, SO(3) breaks into maximal subgroup for which I_X is an invariant.

- ought to know entire subgroup tree from G to H (and their invariants)
  not known in general .....
  finding subgroups of say SU(n) seems case by case study -- more thought later

no general strategy → look example

III. Finite subgroups of SU(3)

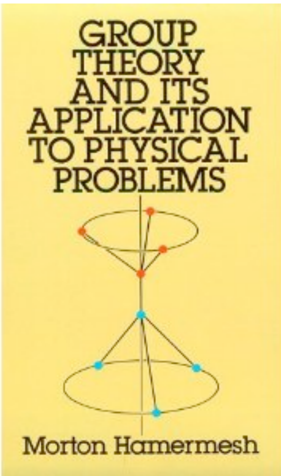
- Of interest flavour model building
- Alternatives to $SU(3)_F$ (eighfold way)
- Discretization of $SU(3)_c$ for lattice e.g. Michael et al

Finite subgroups of SU(3) denoted by F_3

- Classified in a classic book Miller, Dickson, Blichfeld '1916 , analyzed further 8-fold way Fairbairn, Fulton, Klink '64 Further analyzed (lattice ...) Bovier, Luling, Wyler '80 Rescrutinized tri-bi-hype Luhn, Nasri Ramand; Fischbacher RZ, Ludl, Grimus 03' onwards

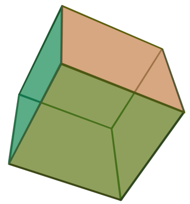
- First SO(3) subgroups (3d irreps) -- then algebraic abstraction SU(3)

Dihedral $D_n = Z_n \rtimes Z_2$



- symmetries of a molecule
- irreps smaller equal to 3

Crystallographic



cube=S_4

tetrahedron=A_4

icosahedron=A_5

"better" approximation to SO(3)

Algebraic abstraction to SU(3) (not simple factor groups Z_3 x ....)

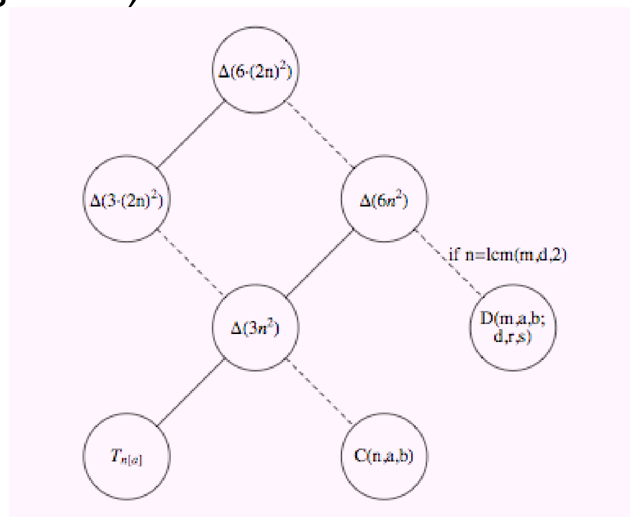
C,D-groups
= dihedral-like
= trihedral

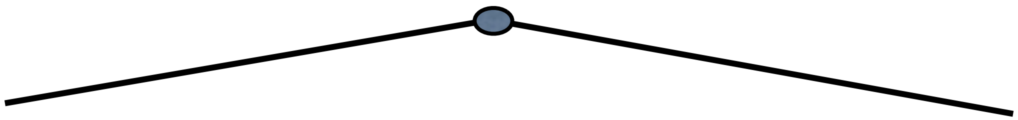
crystallographic type

Group	Generators C-,D-type	$\Sigma(X)$-type
$C(n,a,b)$	$E,\ F(n,a,b)$	
$D(n,a,b;d,r,s)$	$E,\ F(n,a,b),\ G(d,r,s)$	
$\Delta(3n^2) = C(n,0,1),\ n \geq 2$	$E,\ F(n,0,1)$	
$\Delta(6n^2) = D(n,0,1;2,1,1),\ n \geq 2$	$E,\ F(n,0,1),\ G(2,1,1)$	
$T_{n[a]} = C(n,1,a),\ (1+a+a^2) = n\mathbb{Z}$	$E,\ F(n,1,a)$	
$\Sigma(60) = A_5 = I = Y$	$E,\ F(2,0,1)$	H
$\Sigma(168) = PSL(2,7)$	$E,\ M \equiv F(7,1,2)$	N
$\Sigma(36\phi)$	$E,\ J \equiv F(3,0,1)$	K
$\Sigma(72\phi)$	$E,\ J \equiv F(3,0,1)$	$K,\ L$
$\Sigma(216\phi)$	$E,\ J = F(3,0,1),\ P \equiv F(9,2,2)$	K
$\Sigma(360\phi)$	$E,\ F(2,0,1),\ Q \equiv G(6,3,5)$	H

known generators

- general C,D-groups not everything is known (continuous progress ...)
  known: topological structure irreps smaller 3/6 respectively
  unknown: order group subgroup (partial results)





General

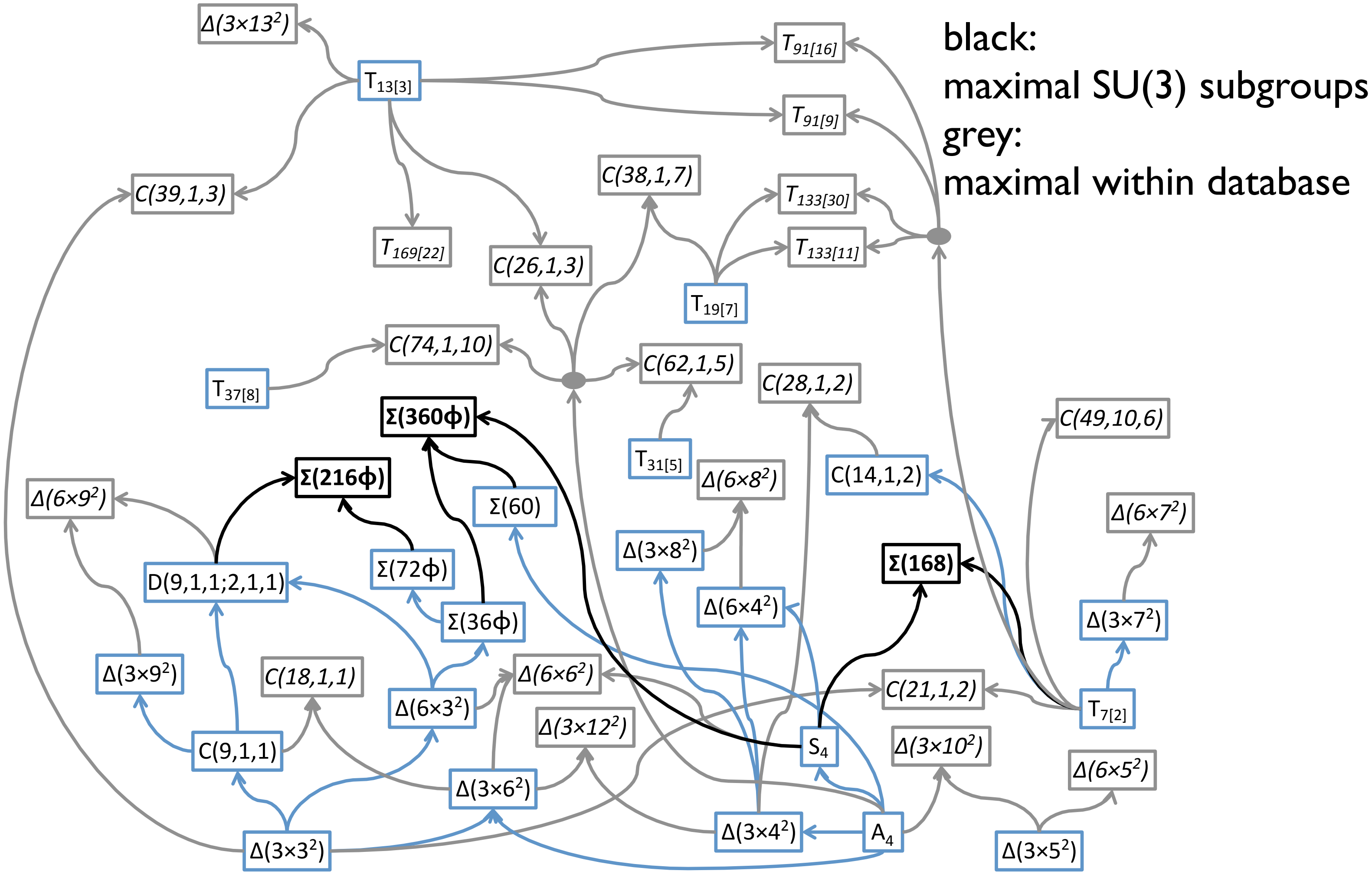
Database....

work out sufficient criteria
SU(3) → F_3 (later)

IV. Database: groups of order smaller 512 (61 of them)

- Find all syzygies and thus primary & secondary invariants, Molien function, tensor-generating functions.. Finding syzygies is an interesting problem complexity (use polynomial basis ...) Especially crystallographic ones of interest for mathematicians

- and here's the tree:



V. Example criteria for breaking $SU(3) \rightarrow F_3$

☆ SU(3) → F_3 ..... problem to know $F_3 \subset H \subset SO(3)$

- H might be continuous SO(3) and SU(2) and subgroups thereof
  a) SO(3) know all the subgroups ok
  b) notice all subgroups have cyclic generator E (x,y,z)→(y,z,x) SU(2) out of the game
- H mixed ... out for the same reason
- H finite one of our list ⇒ work with explicit generators

justify
as generators
specific embedding

$$E = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad F(n,a,b) = \begin{pmatrix} \eta^a & 0 & 0 \\ 0 & \eta^b & 0 \\ 0 & 0 & \eta^{-a-b} \end{pmatrix}, \quad G(d,r,s) = \begin{pmatrix} \delta^r & 0 & 0 \\ 0 & 0 & \delta^s \\ 0 & -\delta^{-r-s} & 0 \end{pmatrix},$$

$$H = \frac{1}{2}\begin{pmatrix} -1 & \mu_- & \mu_+ \\ \mu_- & \mu_+ & -1 \\ \mu_+ & -1 & \mu_- \end{pmatrix}, \quad J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad K = \frac{1}{\sqrt{3}\, i}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

$$L = \frac{1}{\sqrt{3}\, i}\begin{pmatrix} 1 & 1 & \omega^2 \\ 1 & \omega & \omega \\ \omega & 1 & \omega \end{pmatrix}, \quad M = \begin{pmatrix} \beta & 0 & 0 \\ 0 & \beta^2 & 0 \\ 0 & 0 & \beta^4 \end{pmatrix}, \quad N = \frac{i}{\sqrt{7}}\begin{pmatrix} \beta^4-\beta^3 & \beta^2-\beta^5 & \beta-\beta^6 \\ \beta^2-\beta^5 & \beta-\beta^6 & \beta^4-\beta^3 \\ \beta-\beta^6 & \beta^4-\beta^3 & \beta^2-\beta^5 \end{pmatrix},$$

$$P = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon\omega \end{pmatrix}, \quad Q = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -\omega \\ 0 & -\omega^2 & 0 \end{pmatrix}. \qquad \text{(B.1)}$$

$$\eta \equiv e^{2\pi i/n}, \quad \delta \equiv e^{2\pi i/d}, \quad \mu_\pm \equiv \frac{1}{2}\left(-1 \pm \sqrt{5}\right), \quad \omega \equiv e^{2\pi i/3}, \quad \beta \equiv e^{2\pi i/7}, \quad \epsilon \equiv e^{4\pi i/9}.$$

V.a Crystallographic groups

- Module embedding rather straighforward (just apply all generators to them.._

Group	Molien function	Invariant of lowest degree that breaks $SU(3) \to \Sigma(X)$
$\Sigma(60)$	$\frac{1+P^{15}}{(1-P^2)(1-P^6)(1-P^{10})}$	$(\phi_0^2 x^2 - y^2)(\phi_0^2 z^2 - x^2)(\phi_0^2 y^2 - z^2)$
$\Sigma(36\phi)$	$\frac{1+P^9+P^{12}+P^{21}}{(1-P^6)^2(1-P^{12})}$	$(x^6 + 2x^3y^3 - 6x^4yz + \text{cy.}) - 18x^2y^2z^2$
$\Sigma(168)$	$\frac{1+P^{21}}{(1-P^4)(1-P^6)(1-P^{14})}$	$x^3z + z^3y + y^3x$
$\Sigma(72\phi)$	$\frac{1+P^{12}+P^{24}}{(1-P^6)(1-P^9)(1-P^{12})}$	$x^6 + y^6 + z^6 - 10x^3y^3 - 10y^3z^3 - 10z^3x^3$
$\Sigma(216\phi)$	$\frac{1+P^{18}+P^{36}}{(1-P^9)(1-P^{12})(1-P^{18})}$	$x^6(y^3 - z^3) + y^6(z^3 - x^3) + z^6(x^3 - y^3)$
$\Sigma(360\phi)$	$\frac{1+P^{45}}{(1-P^6)(1-P^{12})(1-P^{30})}$	$x^6 + y^6 + z^6 + ax^2y^2z^2 + b_+(x^4y^2 + \text{cy.}) + b_-(x^4z^2 + \text{cy.})$

famous **Klein-quartic**

$$\phi_0 \equiv \frac{1+\sqrt{5}}{2}\ ,\ a = 3\left(5 - i\sqrt{15}\right)\ ,\ b_\pm = \frac{3}{8}\left[5 \mp 3\sqrt{5} + i\left(\sqrt{15} \pm 5\sqrt{3}\right)\right]$$

V.b $\Delta(6n^2)$, $\Delta(3n^2)$, $T_{n[a]}$-series

- are the known series amongst C/D subgroups -- dihedral-like
- insight comes from invariants & syzygies -- doable for general n (exceptional)!

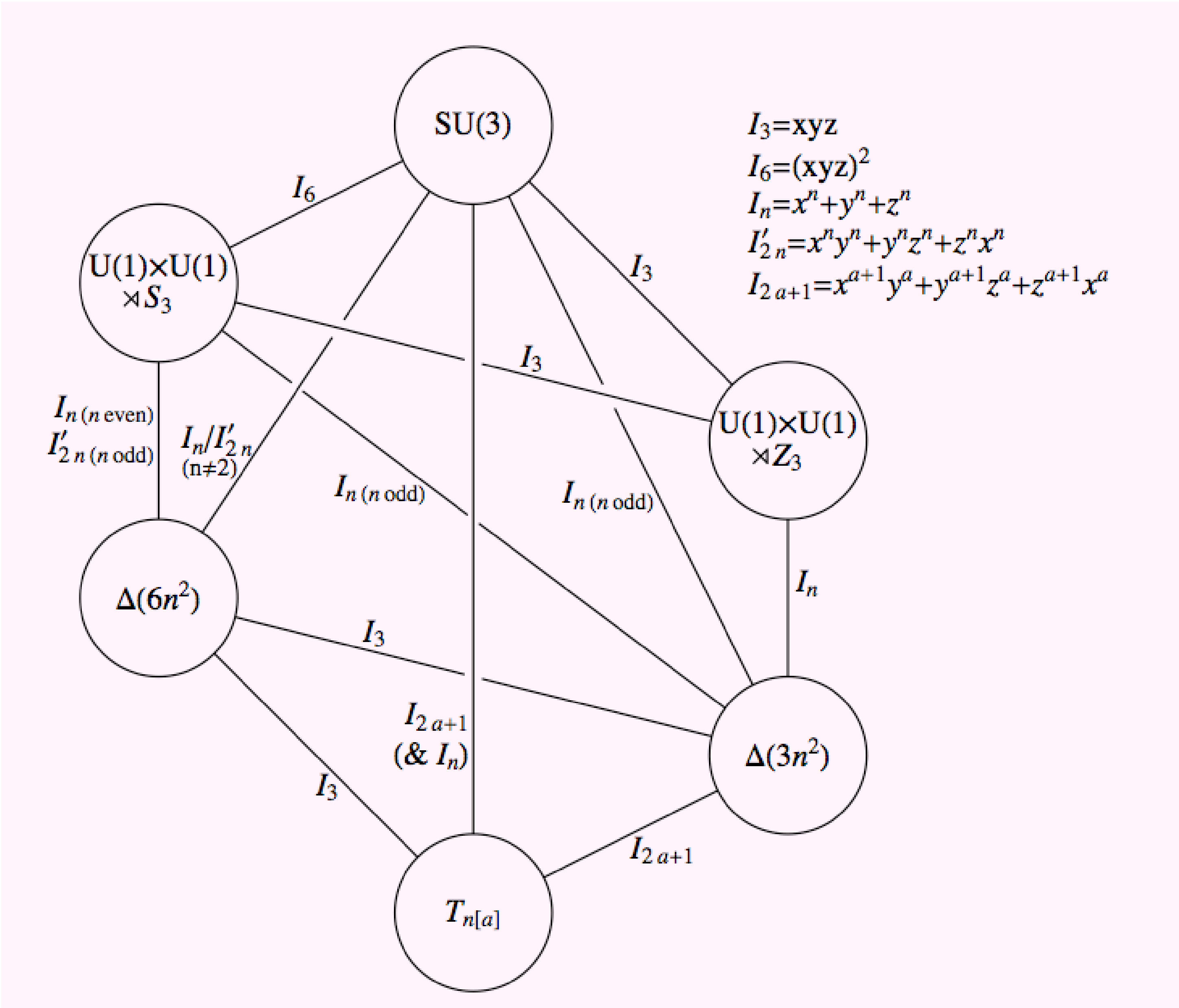
Group	Type	Invariants
$\Delta(3n^2)$	primary	$\mathcal{I}_3 = xyz$, $\mathcal{I}_n = x^n + y^n + z^n$ $\mathcal{I}_{2n} = x^{2n} + y^{2n} + z^{2n}$
	secondary	$\mathcal{I}_{3n} = x^{3n} + y^{3n} + z^{3n}$
	syzygy	$\overline{\mathcal{I}}_{3n}^2 = 9\mathcal{I}_3^{2n} + 9\mathcal{I}_3^n\mathcal{I}_n\mathcal{I}_{2n} + \frac{9}{4}\mathcal{I}_n^2\mathcal{I}_{2n}^2 - 3\mathcal{I}_3^n\mathcal{I}_n^3 - \frac{3}{2}\mathcal{I}_n^4\mathcal{I}_{2n} + \frac{1}{4}\mathcal{I}_n^6$
$\Delta(6n^2)$ even n	primary	$\mathcal{I}_6 = (xyz)^2$, $\mathcal{I}_n = x^n + y^n + z^n$ $\mathcal{I}_{2n} = x^{2n} + y^{2n} + z^{2n}$
	secondary	$\overline{\mathcal{I}}_{3n+3} = xyz(x^n - y^n)(y^n - z^n)(z^n - x^n)$
	syzygy	$\overline{\mathcal{I}}_{3n+3}^2 = \mathcal{I}_6\left[\frac{1}{2}\mathcal{I}_{2n}^3 - 27\mathcal{I}_6^n - 9\mathcal{I}_6^{n/2}\mathcal{I}_n\mathcal{I}_{2n} + 5\mathcal{I}_6^{n/2}\mathcal{I}_n^3 + \mathcal{I}_n^4\mathcal{I}_{2n} - \frac{1}{4}\mathcal{I}_n^6 - \frac{5}{4}\mathcal{I}_n^2\mathcal{I}_{2n}^2\right]$

suggests the that the dihdral-like groups are generalizations tetrahedron/cube by changing the euclidian metric

$$A_4; S_4 : \quad \rightarrow \quad \Delta(3n^2); \Delta(6n^2)_{n\in 2\mathbb{N}} : S_3 : \quad \rightarrow \quad \Delta(6n^2)_{n\in 2\mathbb{N}+1} :$$

$$x^2 + y^2 + z^2 \quad \rightarrow \quad x^n + y^n + z^n \,. \qquad xy + yz + zx \quad \rightarrow \quad x^n y^n + y^n z^n + z^n x^n \,,$$

after tedious (yet fun -- geometric intuition) work we were able to show:



V.c Hint at questions of embedding

equivalent embedding
= similarity transformation

$g' = AgA^{-1}$
using Schur's Lemma
& subgroup tree show
not "lost" anything

inequivalent embedding
= distinct irrep

e.g. A_5 **3**,**3**'
show image same or complex conjugate
(latter case particle/anti-particle)

VI. The complex spherical harmonics

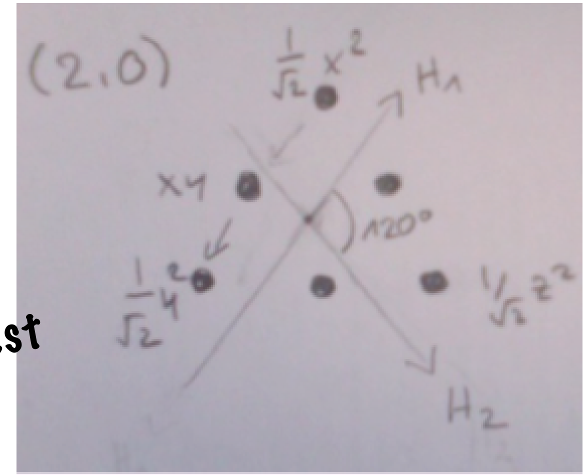
SU(3)-representation functions

- eigenfunctions of Laplacian on SU(3)/SU(2)

- formal construction SU(3) rank 2 -- basis Cartan subalgebra $\{T_3,T_8\}$
  $T_3 = \frac{1}{2}(x\partial_x - y\partial_y)\ , \quad T_8 = ..$ highest weight (p,0) $\leftrightarrow$ x^p

- (p,q) = { polynom degree (p,q) in $(\{x,y,z\},\{x^*, y^*, z^*\})\}/\{xx^*+ yy^*+ zz^*\}$

simplest



- comparison of SO(3) vs SU(3)

group	$SO(3)$	$SU(3)$
rank	$1 \leftrightarrow l$	$2 \leftrightarrow (p,q)$
repres. fct.	$Y_{l,m}$	$h^{rst}_{(p,q)}$
fct. on manifold	$SO(3)/SO(2) \simeq S_2$	$SU(3)/SU(2) \simeq S_5$
embedding	$\hookrightarrow \mathbb{R}^3$ with $x^2+y^2+z^2=r^2$	$\hookrightarrow \mathbb{C}^3$ with $z_1\bar{z}_1+z_2\bar{z}_2+z_3\bar{z}_3=\rho^2$
labelling irrep	$(l) \in \mathbb{N}_0$	$(p,q) \in \mathbb{N}_0^2$
dim(irrep)	$(2l+1)$	$(p+1)(q+1)(p+q+2)/2$
labelling states irrep	$m=-l..l$	$r=0..q\ ,\ s=0..p\ ,\ t=0..(p+r-s)$

Fun example $(1,1)_{SU(3)} \longrightarrow S_3$

- Branching rule $(1,1)_{SU(3)} \longrightarrow (\mathbf{1} + \mathbf{1}' + 3\ \mathbf{2})_{S_3}$ $\Rightarrow$ one S_3-invariant in (1,1) basis

- This smells of eightfold way let's guess the invariant
  S_3 is a discrete flavour symmetry exchanging the x,y,z or u,d,s flavours

$$
\begin{aligned}
\mathcal{I}[S_3]_{1,1} &= xy^* + yx^* + z^*y + y^*z + x^*y + xz^* = v[S_3]_{1,1} \cdot \mathcal{B}_{(1,1)} \ , \\
v[S_3]_{1,1} &= (1, -1, 0, 1, 0, -1, -1, -1) \ , \\
\mathcal{B}_{(1,1)} &= \left\{ xz^*, -yz^*, \frac{xx^* + yy^* - 2zz^*}{\sqrt{6}}, xy^*, \frac{xx^* - yy^*}{\sqrt{2}}, -x^*y, -y^*z, -x^*z \right\}
\end{aligned}
$$

- Construct Gell-Mann basis basis above (well in this case it's just the structure constants
  and check that S_3-generators lead to a representation of S_3
  (this representation is reducible $(1,0)_{SU(3)} \longrightarrow (\mathbf{1}+\mathbf{2})_{S_3}$

VII. The tensor-generating function

- The Molien function counts the number of invariants
  Is there an object that counts the number of covariants (=tensors)?

- The (tensor)-generating function

$$M_H(\mathbf{c},\mathbf{f};P) = \frac{1}{|\mathcal{R}_f(h))|}\sum_{h\in H}\frac{\chi_c[h]^*}{\det(\mathbf{1}-P\mathcal{R}_f(h))} = \sum_{n\geq 0} c_n P^n$$

Thm: positive c_n number of **c**-tensor in the irrep **f** where $\chi_c[h]=tr[R_c(h)]$ is the character
N.B. reduces to Molien fct for **c=1**, **f=3**; since $\chi_1[h] = 1$

- Similar program of syzygies, primary and secondary covariants etc applies (details paper....)

Important application: branching rules

- Invariant generating fct = Molien fct ⇒ (p,q) → $(n^{1}_{(p,q)}\mathbf{1} + \quad)_{F3}$ number of invariants in branching rule

- ⇒ **c**-Tensor generating fct ⇒ (p,q) → $(n^{c}_{(p,q)}\mathbf{c} + \quad)_{F3}$ number of **c**-tensors in branching rule

$$\boxed{\text{get the branching rules!!!}}$$

- computed all tensor-generating fcts for database -- example how it works:

```
In[1]:= SetDirectory["...(your directory).../SUtree_v1p0/"];

In[2]:= $RecursionLimit=260;
        <<SUtree.m

In[3]:= BranchingSU3[{3,0}, "A4"];
Out[3]= {{3, 0}, 10, {1, 1}, {3, 1}, {3, 1}, {3, 1}}
```

Epilogue

open ends

☆ U(3) rather than SU(3)
- classification not done (Ludl'10 some progress)
- thought $U(3) = U(1) \times SU(3)$ more complicated than $F_1 x F_3$ (there can be twists)

☆ Generalization to SU(n) ... how much is known
- Hanney & He '99 "A Monograph on the classification of the discrete subgroups of SU(4)"
- Bet on "quadrihedral" groups $Z_n \times Z_n \times Z_n \rtimes Z_4$ with invariants $x^n + y^n + z^n + w^n$ and alike

☆ Language between explicit and spontaneous breaking
- how does potential look like which breaks $SU(3) \to F_3$? *"What's the landscape?"*
- suppose explicit breaking terms are non-renormalizable
  can potential in SSB-picture be renormalizable?
  (examples in the literature give no answer ..)

Bibliography

- Classics

Chapter XII in G. A. Miller, H. F. Blichfeldt, L. E. Dickson, "Theory and Applications of Finite Groups", John Wiley & Sons, New York, 1916, and Dover Edition, 1961.

W. Burnside, "Theory of groups of finite order," Cambridge University Press, second edition, 1897.

- Modern

B. Sturmfels, "Algorithms in Invariant Theory," Texts and Monographs in Symbolic Computation, Springer.

J. Patera, R. T. Sharp, "Generating Functions For Characters Of Group Representations And Their Applications" (Lecture Notes in Physics 94), New York: Springer, pp. 17583.

- Finite SU(3) subgroups (modern)

P. O. Ludl, "Systematic analysis of finite family symmetry groups and their application to the lepton sector" arXiv:0907.5587 [hep-ph].

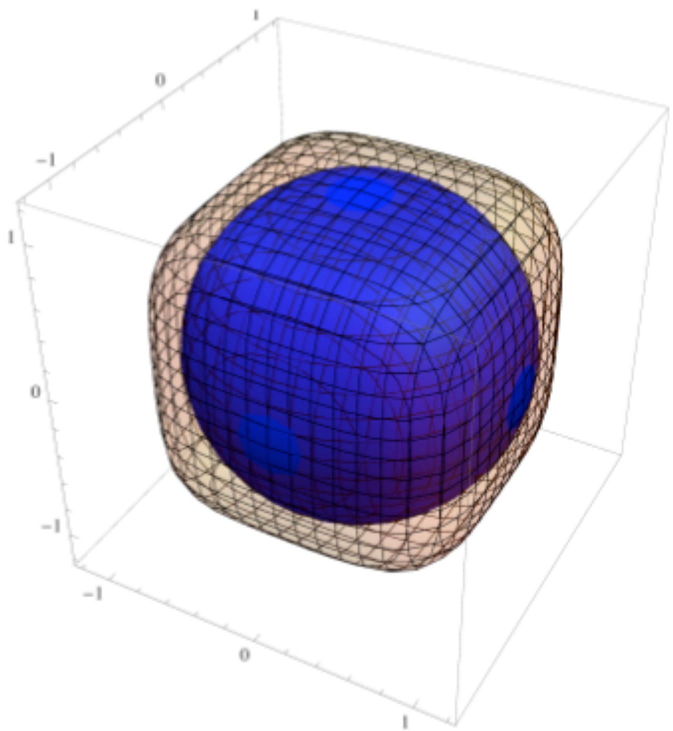
W. Grimus and P. O. Ludl, "Finite flavour groups of fermions," arXiv:1110.6376

- Computer Algebra

The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.4.12*, 2008,
http://www.gap-system.org

H. U. Besche, B. Eick and E. A. O'Brien, *SmallGroups - a GAP package*, 2002,

Backup Slides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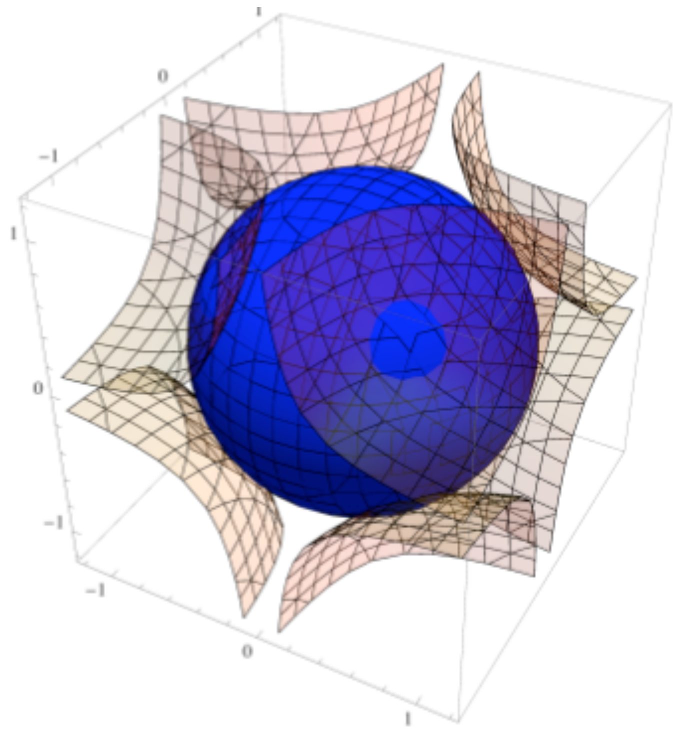
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