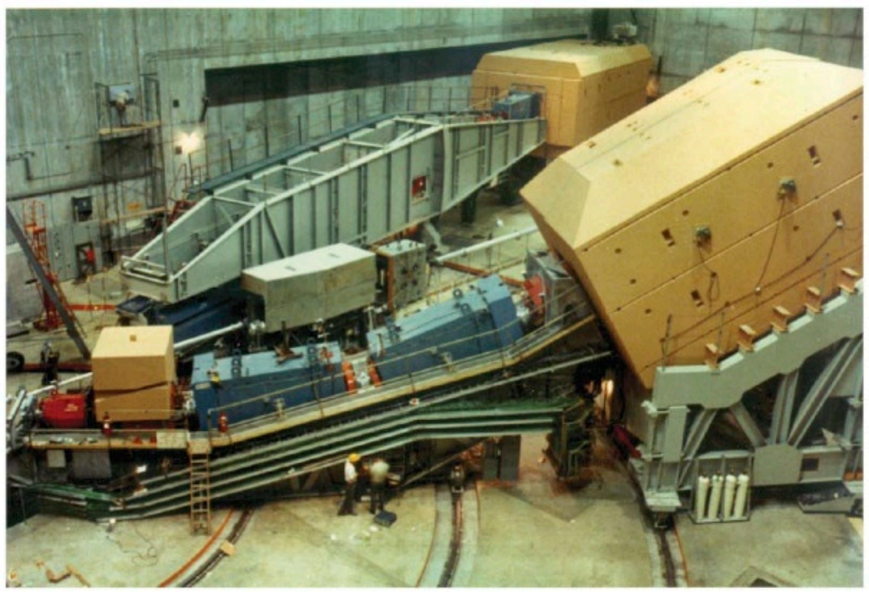
Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 11: *Probing the proton*, or
Deep Inelastic scattering



$$\mathbf{R} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \frac{e_q^2}{e^2}$$

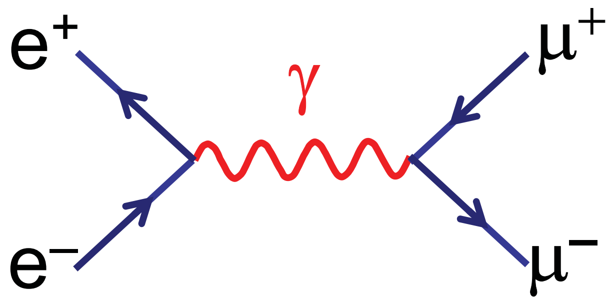
- ★Hadron Colliders
- ★Electron-proton scattering
- ★Deep Inelastic scattering
- ★(Dolly) Partons in the proton

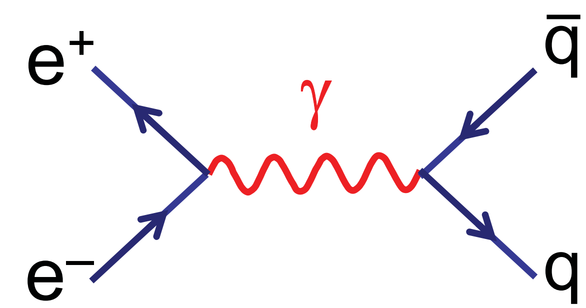
Announcements

- I’m going to a conference next week.
  - Steve Playfer will give lectures in my place.
  - Topics are hadrons, hadron decays and the CKM matrix.

- Tutors will be at the tutorial 3-5pm on Monday, but no new tutorial sheet.

Review from Tuesday: Rate for $e^+e^-\to$hadrons





$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = \frac{e^2}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\mu^+)\gamma^\mu \bar{u}(\mu^-)]$$

$$\mathcal{M}(e^+e^- \to \mathrm{q}\bar{\mathrm{q}}) = \frac{e\,e_q}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\bar{\mathrm{q}})\gamma^\mu \bar{u}(\mathrm{q})]$$

- Ignoring differences in the phase space, ratio, **R** between hadron production and muon production:

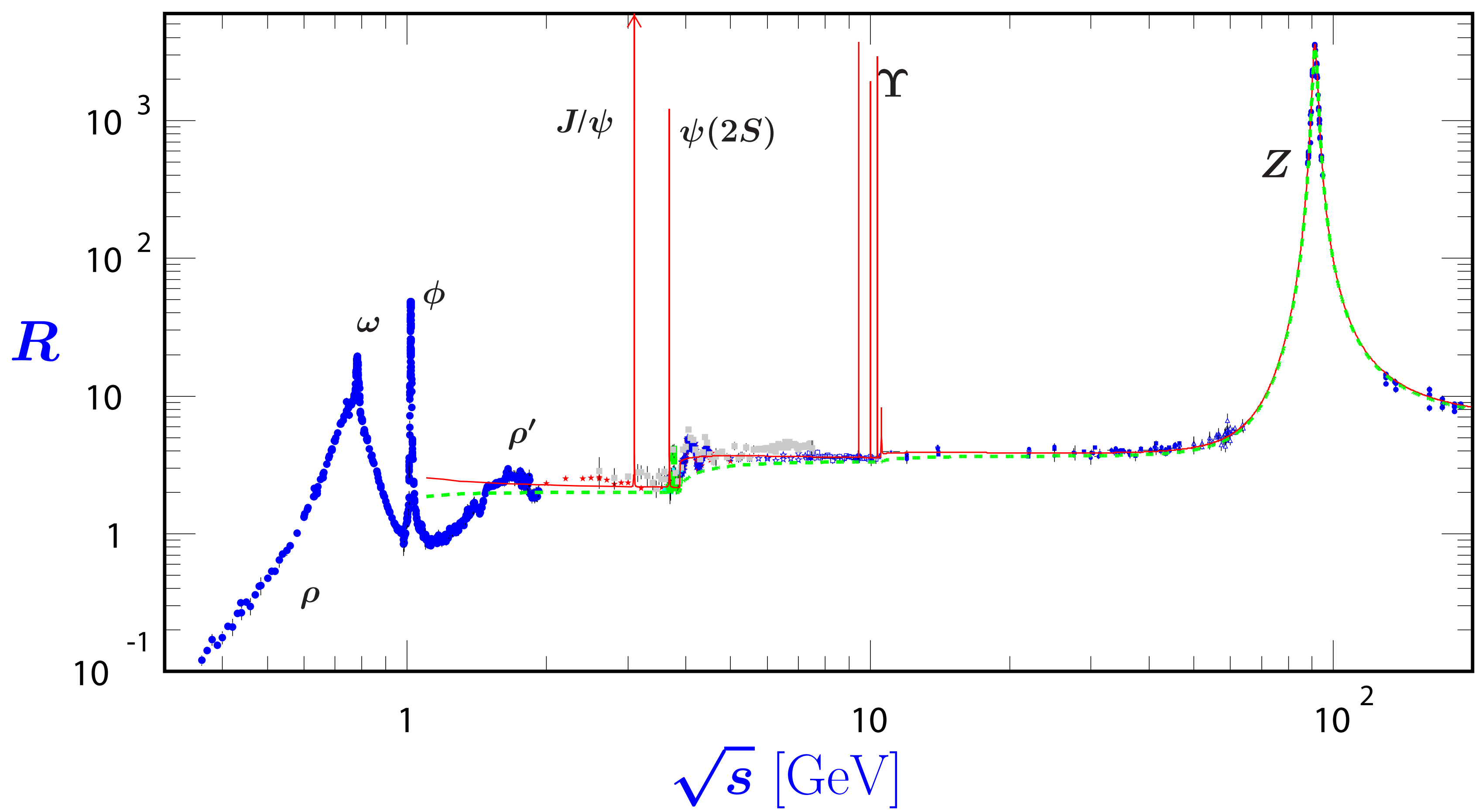
$$\mathbf{R} = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c\frac{e_q^2}{e^2}$$

- N_c=3 is the number of quark colours
- e_q= +⅔, −⅓ is the charge of the quark
- The number of available quark flavours depends on the available $s=q^2$
- $\sqrt{s} > 2\,m_q$ for a quark flavour **q** to be produced.

CM energy (GeV)	Available quark pairs	R
$1 < \sqrt{s} < 3$	u, d, s	2
$4 < \sqrt{s} < 9$	u, d, s, c	10/3
$\sqrt{s} > 10$	u, d, s, c, b	11/3

Measurements of R

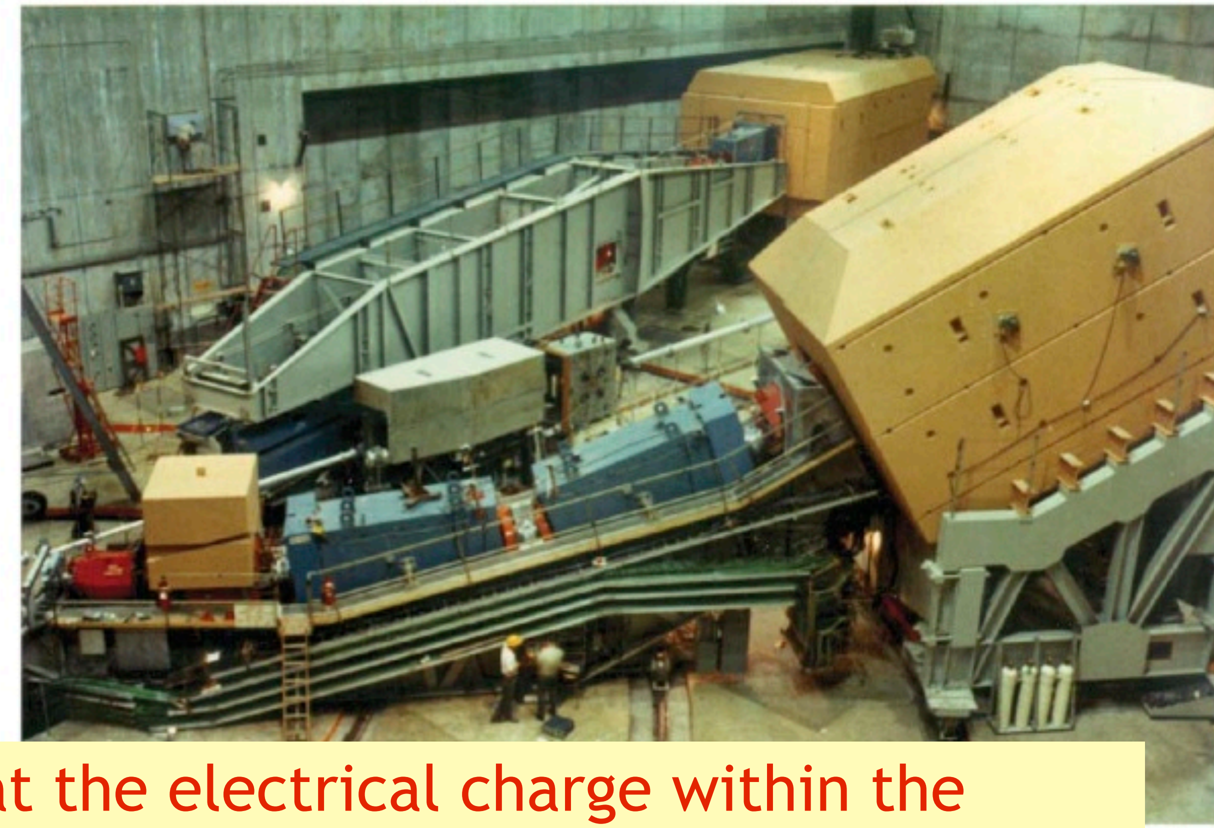
- Compendium of measurements from many lepton colliders.



- Consistent with N_C=3, this is one of the key pieces of evidence for three quark colours.
- At quark thresholds, $\sqrt{s} \sim 2m_q$ "resonances" occur as bound states of $q\bar{q}$ more easily produced (see next lecture).
- Steps at ~4 and ~10 GeV due to charm and bottom quark threshold
- At $\sqrt{s} \sim 100$ GeV, Z-boson exchange takes over.

Electron Proton Scattering Experiments

- **SLAC-MIT experiment ('67)**
- **Electron beam on liquid hydrogen target**



Won the 1990 Noble prize for: Jerome I. Friedman, Henry Kendall, Richard E. Taylor "for their pioneering investigations concerning **deep inelastic scattering** of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

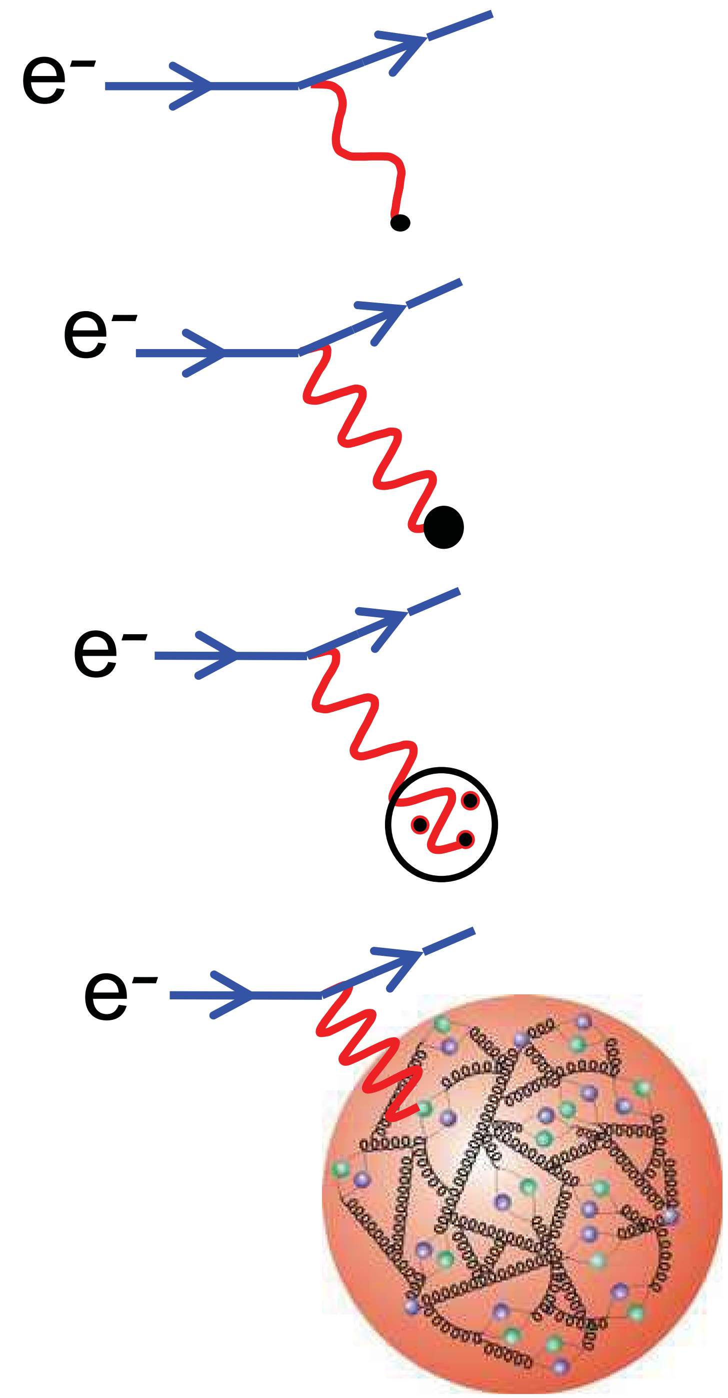
The investigations gave the surprising result that the electrical charge within the proton is concentrated to smaller components of negligible size.

- **DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany**

- HERA was the world's only electron - proton collider, ran 1992 - 2007
- **E(e^-) = 30 GeV, E(p) = 820 GeV**
- 6.3 km in circumference
- Three experiments:
  - Two general purpose experiments: ZEUS, H1
    - ➡ Probe proton at very high Q^2 and very low x

Probing the Structure of the Proton

- In $e^-p \to e^-p$ scattering the nature of the interaction of the virtual photon with the proton depends strongly on wavelength, $\lambda = ch/E$



- At very low electron energies $\lambda \gg r_p$: the scattering is equivalent to that from a "point-like" spin-less object
- At low electron energies $\lambda \sim r_p$ the scattering is equivalent to that from a extended charged object
- At high electron energies $\lambda < r_p$: the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks
- At very high electron energies $\lambda \ll r_p$: the proton appears to be a sea of quarks and gluons.

Form Factors

- Extended object - like the proton - have a matter density ***ρ(r)***, normalised to unit volume:

$$\int d^3\vec{r}\,\rho(\vec{r}) = 1$$

- Fourier Transform of ***ρ(r)*** is the **form factor, *F(q)***:

$$F(\vec{q}) = \int d^3\vec{r}\,\exp\{i\vec{q}\cdot\vec{r}\}\,\rho(\vec{r}) \Rightarrow F(0) = 1$$

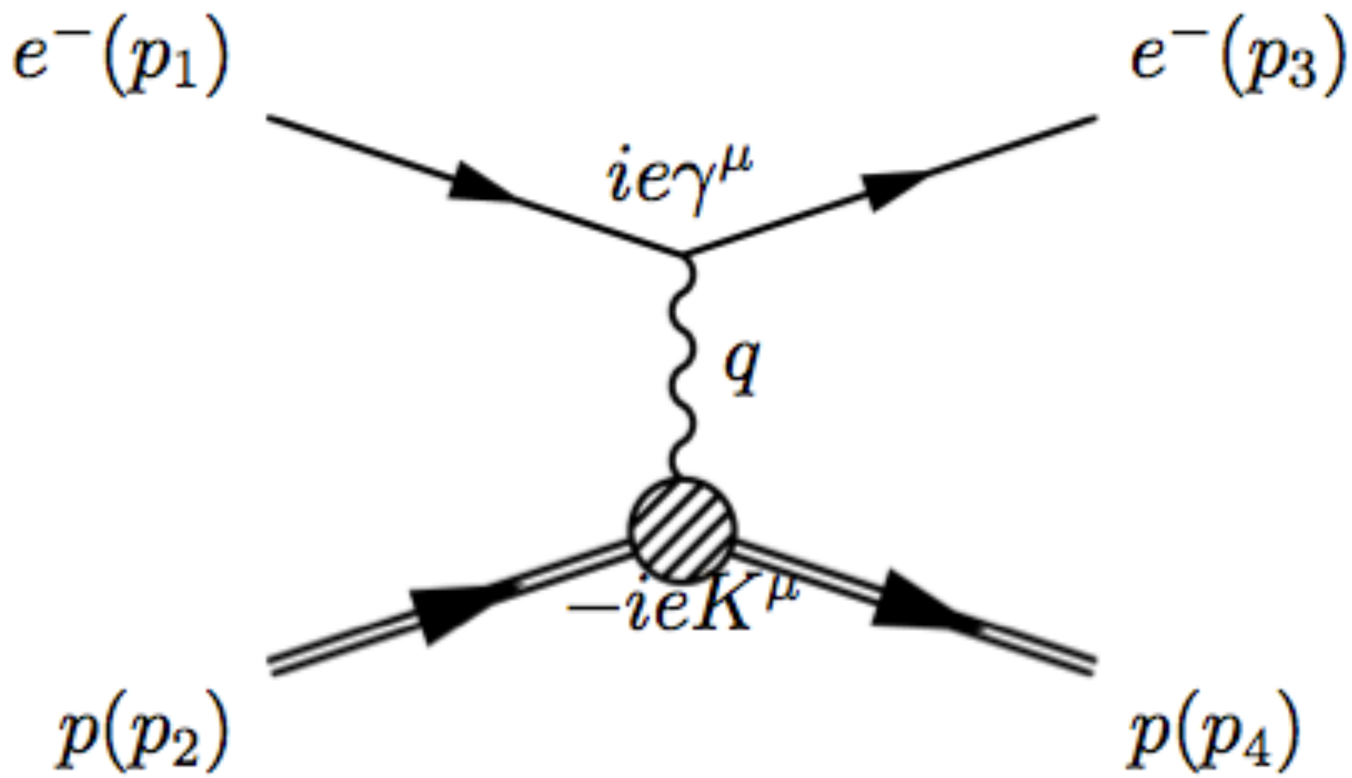
- Cross section are modified by the form factor:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{extended}} \approx \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{point-like}} |F(\vec{q})|^2$$

- For ***ep→ep*** scattering we need two form factors:
  - F_1 to describe the distribution of the electric charge
  - F_2 to describing the recoil of the proton

Elastic Electron Proton Scattering

Scattering of high energy electrons by electromagnetic interactions probes the charge distribution of the proton



In elastic scattering the proton remains a proton, but the proton current is modified by K^μ because the proton is not a pointlike particle

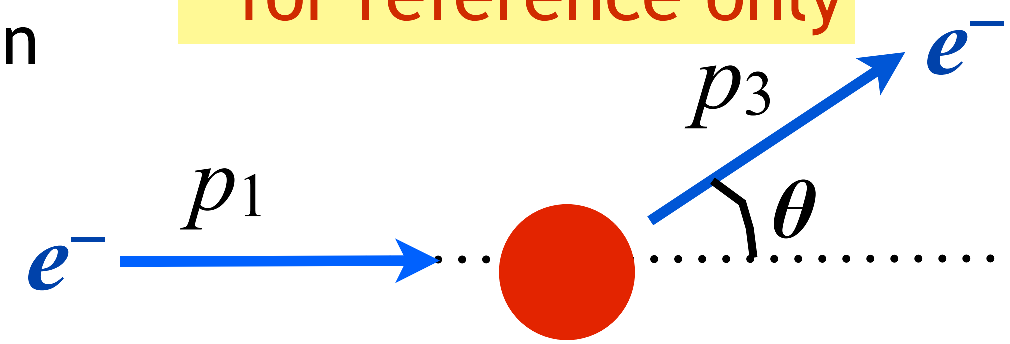
$$\mathcal{M}(e^- p \to e^- p) = \frac{e^2}{(p_1 - p_3)^2} \left(\bar{u}_3 \gamma^\mu u_1\right) \left(\bar{u}_4 K_\mu u_2\right)$$

$$K^\mu = \gamma^\mu F_1(q^2) + \frac{i\kappa_p}{2m_p} F_2(q^2) \sigma^{\mu\nu} q_\nu$$

Low-Energy Scattering *

* for reference only

- Elastic scattering of electron on stationary proton



$$|\vec{p}^*| = |\vec{p}_1| = |\vec{p}_2|$$

- Described by Mott Scattering:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4|\vec{p}^*|^2\beta^2 \sin^4\theta/2}(1-\beta^2\sin^2\theta/2)$$

- $\sin^4(\theta/2)$ term due to photon propagator, $1/q^2$

- At very low energies we have Rutherford scattering: Coulomb scattering on the electric charge of proton ($E_K=p^2/2m_e$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_K^2 \sin^4\theta/2}$$

- At relativistic energies, $\beta\to 1$, influence of spin-½ nature of proton, need to also account for finite size of proton charge distribution through form factor $F(q^2)$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\theta/2}\cos^2\frac{\theta}{2}|F(\vec{q}^2)|^2$$

Higher Energy Elastic Scattering*

* for reference only

- At higher energies need to account for the recoil of the proton …

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2} \underbrace{\frac{E_3}{E_1}}_{\text{from proton recoil}} \left(\cos^2\theta/2 - \underbrace{\frac{q^2}{2M^2}\sin^2\theta/2}_{\text{Magnetic interaction due to spin-spin interaction}}\right)$$

- … and finite size effects:

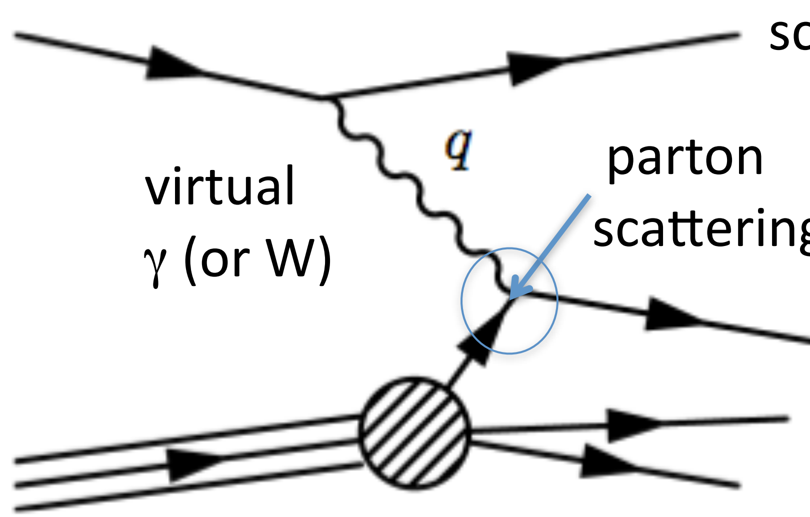
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left\{ \left(F_1^2 - \frac{\kappa^2 q^2}{4m_p^2} F_2^2 \right) \cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2} (F_1 + \kappa F_2)^2 \sin^2\frac{\theta}{2} \right\}$$

- $F_1(q^2)$ and $F_2(q^2)$ are the form factors, which need to be measured.

- Measurement of elastic scattering demonstrate the proton is extended object with rms radius of **~0.8 fm**

Deep Inelastic Scattering (DIS)

- In deep inelastic scattering the proton disintegrates
- The final state hadronic system contains at least one baryon, implying invariant mass of the final state system, $M_X > M_p$



- For deep inelastic scattering introduce new kinematic variables: x, Q^2, ν

$$x \equiv \frac{Q^2}{2p_2 \cdot q} \qquad Q^2 = -q^2 = (p_1 - p_3)^2 > 0 \qquad \nu \equiv \frac{p_2 \cdot q}{M_p}$$

$$M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2 \cdot q + M_p^2$$

$$\Rightarrow \; Q^2 = 2p_2 \cdot q + M_p^2 - M_X^2 \;\; \Rightarrow Q^2 < 2p_2 \cdot q$$

- inelastic: $0 < x < 1$ • elastic: $x = 1$

DIS: Cross Section

- Assume that the photon is elastically scattering off the individual constituents of the proton.
- Proton constituents are called **partons**.
- x is the fraction of the proton's energy carried by the individual partons
- Cross section for DIS is:

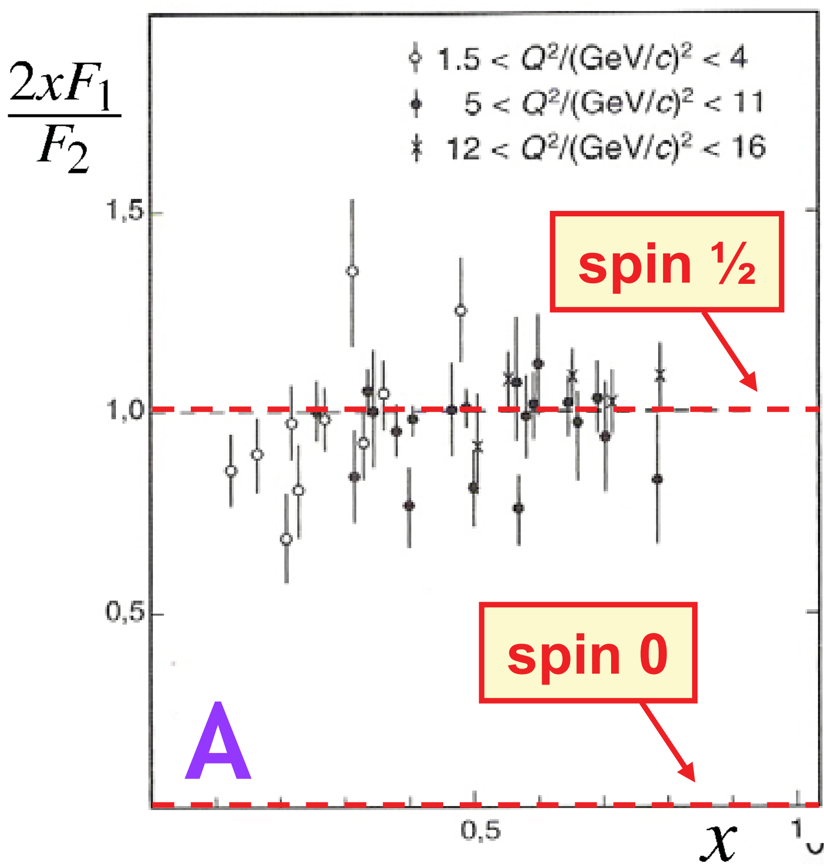
$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \left(\frac{1}{\nu} F_2\underbrace{(x, Q^2)}_{\text{EM structure function}} \cos^2\frac{\theta}{2} + \frac{2}{M} F_1\underbrace{(x, Q^2)}_{\text{Magnetic structure function}} \sin^2\frac{\theta}{2} \right)$$

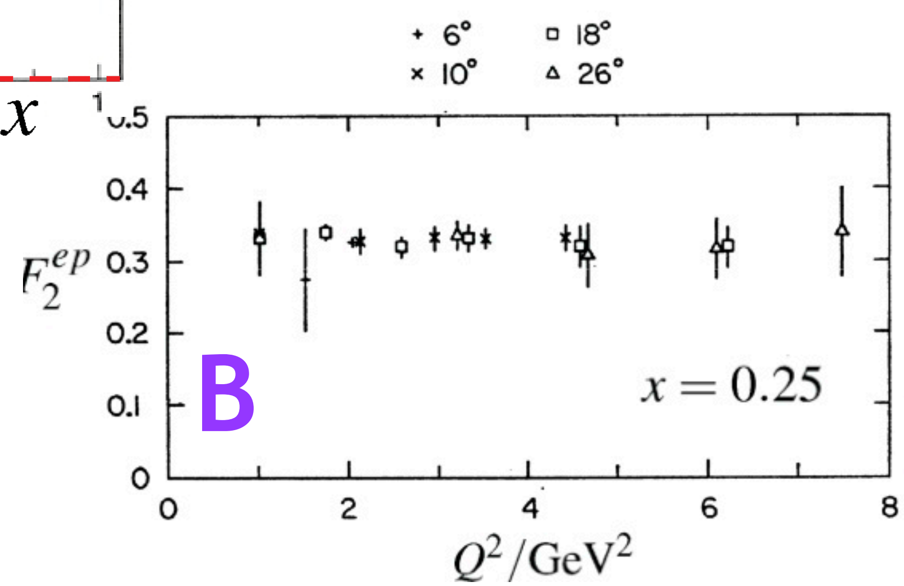
- The structure functions are sums over the charged partons in the proton:

$$2xF_1(x) = F_2(x) = \sum_q x\, e_q^2 \mathrm{q}(\mathrm{x})$$

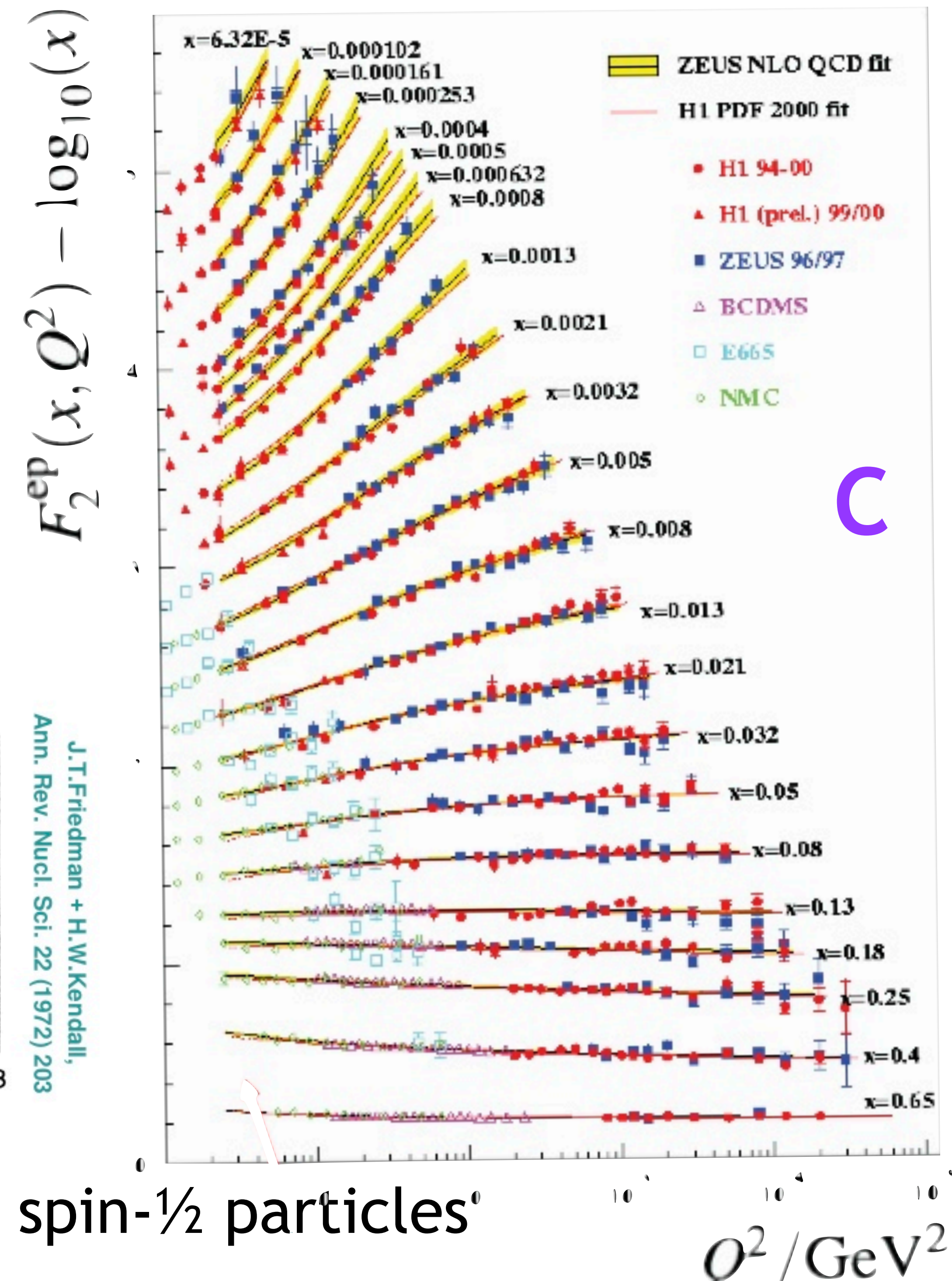
- Partons the proton are:
  - ➡**valance quarks** = **uud**
  - ➡**sea quarks** in quark anti-quark pairs, e.g. $\mathbf{\bar{u}u, \bar{d}d, \bar{s}s, \bar{c}c}$, ...
  - ➡**gluons**, **g**

Experimental Measurements of F_1 and F_2





J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203



A. $2xF_1 = F_2$ is predicted if the constituents are spin-½ particles

B. At low Q^2, F_2 is predicted to be independent of Q^2

C. At very low x, F_2 is not independent of Q^2, as gluons start to take a larger share of the proton momentum.

Summary

- The measurement of

$$\mathbf{R} = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \frac{e_q^2}{e^2}$$

is experimental proof of N_c=**3**, three colours of quarks.

Electron-proton scattering investigates proton substructure

- At lower energies: elastic scattering $e^- p \to e^- p$
  - ➡ proton remains intact, scattering can be described by proton form factor.

- At higher energies: **Deep inelastic scattering** $e^- p \to e^- X$
  - ➡ Scattering from individual quarks within the proton.
  - ➡ Each parton carries the momentum fraction x, described by proton structure functions, $\mathbf{q}(x)$
  - ➡ Only ~50% proton momentum carried by up and down quarks, remainder carried by gluons, sea quarks.