Particle Physics

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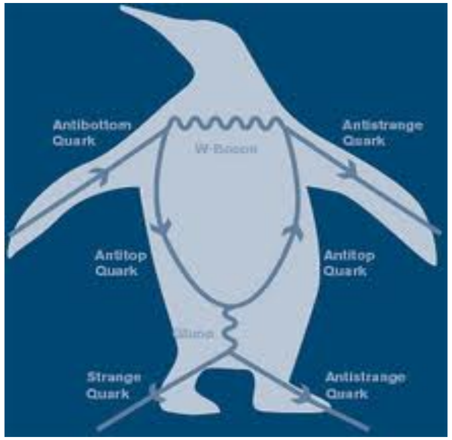
Spring Semester 2013

Lecture 13: Hadron Decays



- ★Decays of Hadrons
- ★Selection Rules
- ★Weak decays of light hadrons
- ★CKM matrix
- ★Neutral Meson Mixing

Decays of Hadrons

Force	Typical τ (s)
QCD	10^{-20} - 10^{-23}
QED	10^{-20} - 10^{-16}
Weak	10^{-13} - 10^{3}

- The proton is the only completely stable hadron
- The free neutron has a weak decay (τ~15 mins)
- **Decay length** of a particle is the distance it travels before decaying $L=\beta\gamma c\tau$

- $\pi^{\pm}$, $K^{\pm}$, K_L^0 mesons are long-lived (τ~10 ns) and have weak decays

  ➡ Live long enough to travel outside radii of collider detectors (L~10 m)
- K_S^0 mesons and Λ^0 hyperons are less long-lived (τ~100 ps) and have weak decays with decay lengths of L~ cm which are inside collider detectors

- $\pi^0\rightarrow\gamma\gamma$, $\eta\rightarrow\gamma\gamma$ are electromagnetic decays, reconstructed from pairs of photons

- ρ, ω, ϕ, K^*, Δ, Σ^*, Ξ^* are resonances with strong decays.

  ➡Reconstructed as broad structures with widths Γ~100 MeV.

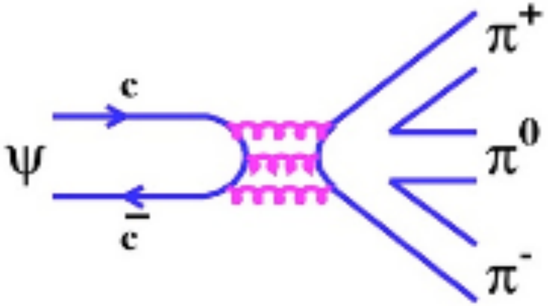
Decay Conservation Laws

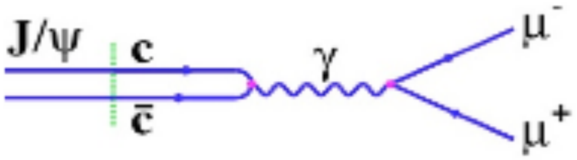
- Relevant quantum numbers are:
  - strong isospin (I, I_3)
  - parity (P)
  - quark flavour: described using strangeness ($S=N(s)-N(\overline{s})$), charm ($C=N(c)-N(\overline{c})$), beauty ($B=N(b)-N(\overline{b})$)
  - Baryon number and lepton numbers are always conserved!

	baryon number	Strong Isospin, I	Strong Isospin, I_3	Flavour, S, C, B	Parity, P
Strong	Y	Y	Y	Y	Y
EM	Y	N	Y	Y	Y
Weak	Y	N	N	N	N

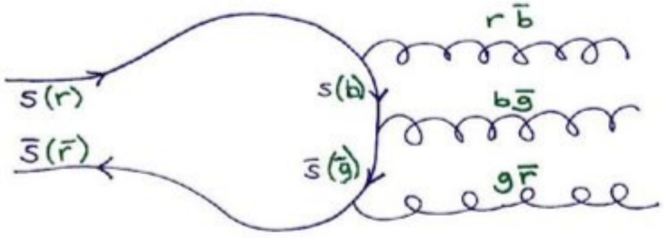
Decays of Charmonium

- The **J/ψ** meson is a $c\bar{c}$ state. It must decay to particles without charm quarks as **M(J/ψ) < 2 M(D).**
- Two options: decay via three gluons or one photon.





- Strong rate is suppressed by $\alpha_s^6(q_{\mathrm{gluon}})$. This is comparable to α^2 for EM decay
  - ➡ Both strong and electromagnetic final states have large branching ratios.
- The **J/ψ** meson lives for a relatively long time, giving rise to narrow resonance in e.g. $e^+e^- \rightarrow$ **hadrons.**



- Similar phenomena occur in decays of $s\bar{s}$ and $b\bar{b}$ mesons.

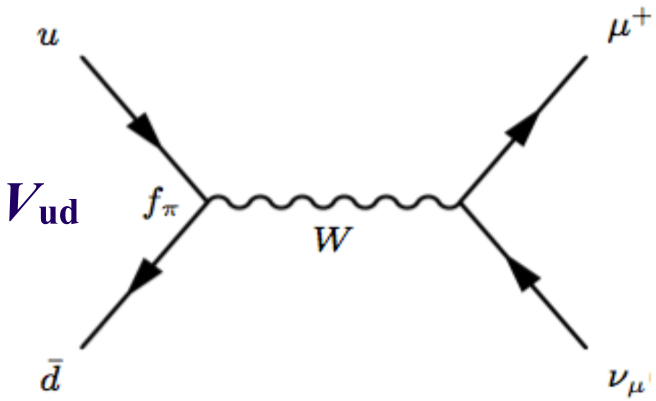
Charged Pion Decay

- See problem sheet 1
- π^+ consists of $\mathbf{u\bar{d}}$, lightest charged meson
- Decays via weak force to change quark flavour u->d

$$\tau(\pi^+) = 26\ \text{ns}$$

➡ CKM matrix element factor $V_{\rm ud}$.

➡ Hadronic decay constant $f_\pi \sim m_\pi$ to account for finite size of pion



$$\begin{aligned}\mathcal{M} &= \left[\bar{v}(\bar{\rm d}) g_W V_{\rm ud} f_\pi \gamma^\mu (1-\gamma^5) u({\rm u})\right] \frac{1}{q^2 - m_W^2} \left[\bar{u}(\nu_\mu) g_W \gamma^\mu (1-\gamma^5) v(\mu^+)\right] \\ &\approx V_{\rm ud} f_\pi \frac{g_W^2}{m_W^2} \left[\bar{v}(\bar{\rm d}) \gamma^\mu (1-\gamma^5) u({\rm u})\right] \left[\bar{u}(\nu_\mu) \gamma^\mu (1-\gamma^5) v(\mu^+)\right] \\ |\mathcal{M}|^2 &= 4\, G_F^2 |V_{\rm ud}|^2 f_\pi^2\, m_\mu^2\, [p_\mu \cdot p_\nu]\end{aligned}$$

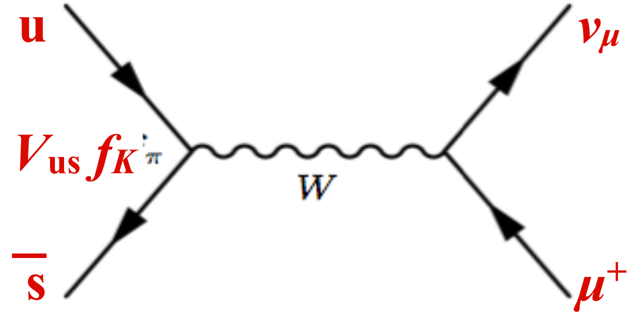
μ preferred to e

$$\Gamma = \frac{|\vec{p}^{\,*}|}{8\pi m_1^2} |\mathcal{M}|^2 = \frac{G_F^2}{8\pi} |V_{\rm ud}|^2 f_\pi^2\, \overset{\downarrow}{m_\mu^2} (m_\pi^2 - m_\mu^2)$$

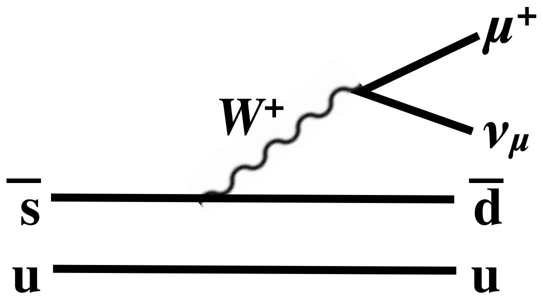
Charged Kaon Decays

- Charged kaon is $\bar{s}\,u$ with $m_K = 498$ MeV
- lightest mesons containing strange quarks ⇒ must decay by weak force

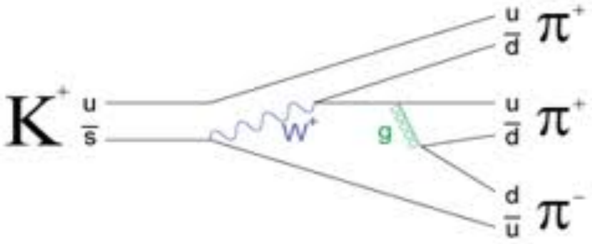
$\tau(K^\pm) = 12$ ns



- Leptonic decays
  - $BR(K^+ \to \mu^+ \nu_\mu) = 63\%$
  - Kaon decay constant, $f_K = 160$ MeV
  - $V_{us} = 0.22$ (Cabibbo angle)



- Semileptonic decays
  - $BR(K^+ \to \pi^0 \mu^+ \nu_\mu) = 3.8\%$
  - $BR(K^+ \to \pi^0 e^+ \nu_e) = 5.1\%$



- Hadronic Decays
  - $BR(K^+ \to \pi^0 \pi^+) = 21\%$
  - $BR(K^+ \to \pi^+ \pi^+ \pi^-) = 5.6\%$

Cabibbo-Kobayashi-Maskawa Matrix

- **Mass eigenstates** and **weak eigenstates** of quarks are not identical.
  - ➡ Decay properties measure mass eigenstates with a definite lifetime and decay width
  - ➡ The weak force acts on the weak eigenstates.
- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix to mix the down-type quarks:

weak eigenstates

$$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

mass eigenstates

- *e.g.* weak eigenstate of the strange quark is a mixture between down, strange and bottom mass eigenstates

$$\mathrm{s}' = V_{\mathrm{cd}}\mathrm{d} + V_{\mathrm{cs}}\mathrm{s} + V_{\mathrm{cb}}\mathrm{b}$$

- The CKM matrix is unitary, $V_{\mathrm{CKM}}{}^{\dagger} V_{\mathrm{CKM}} = \mathbf{1}$; standard parameterisation in terms of three mixing angles ($\theta_1, \theta_2, \theta_3$) and one complex phase (δ) is:

$$\begin{pmatrix} \cos\theta_1 & \sin\theta_1\cos\theta_3 & \sin\theta_1\sin\theta_3 \\ -\sin\theta_1\cos\theta_3 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2\cos\theta_3 - \cos\theta_2\sin\theta_3 e^{i\delta} & -\cos\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3 e^{i\delta} \end{pmatrix}$$

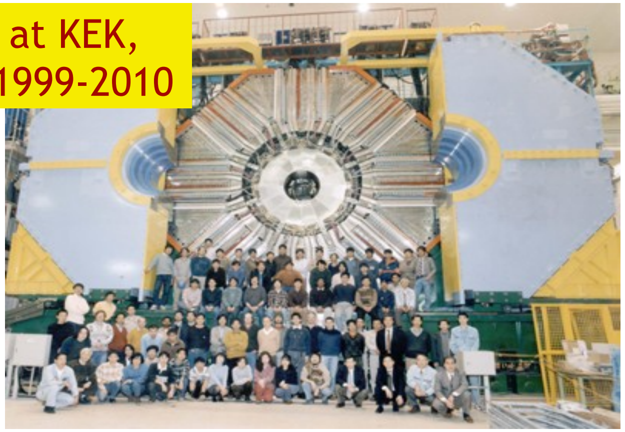
Nobel Prize in Physics 2008

- Awarded to **Makoto Kobayashi**, High Energy Accelerator Research Organization (KEK), Tsukuba, Japan and **Toshihide Maskawa**, Yukawa Institute for Theoretical Physics (YITP), Kyoto University, and Kyoto Sangyo University, Japan
- *"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"*

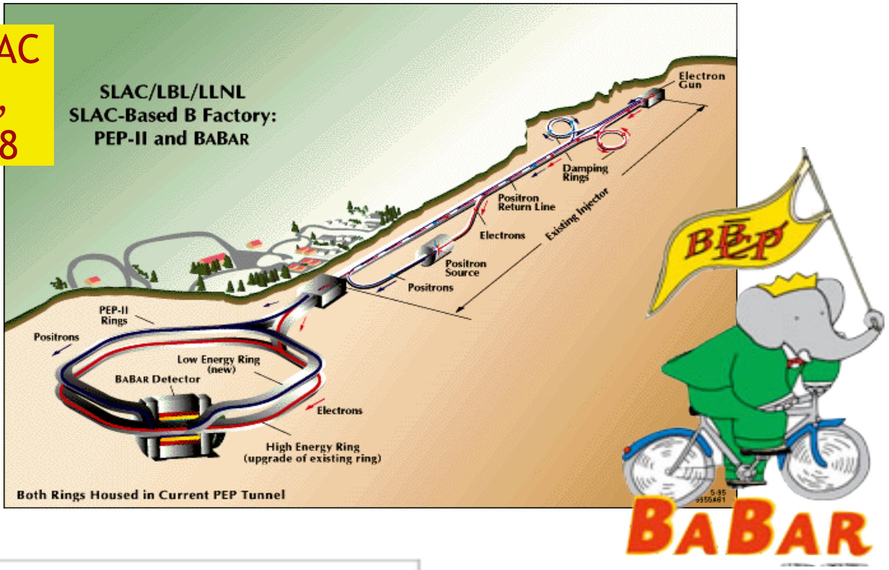
Experimental Measurements of CKM Matrix

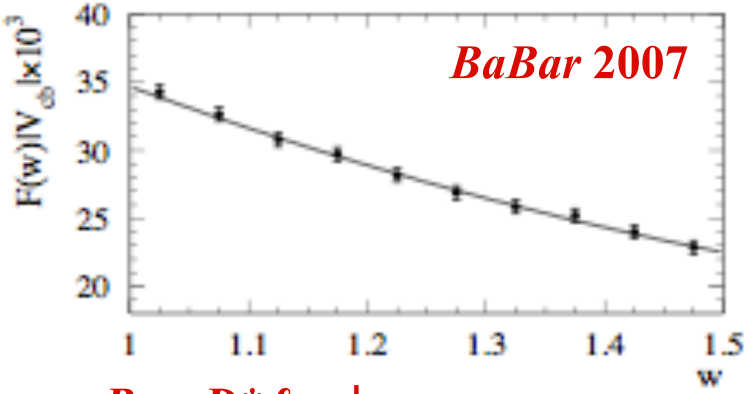
- Many measurements made by the BaBar and Belle experiments.
- Both study $e^+e^- \rightarrow \Upsilon^{(4s)} \rightarrow \mathbf{B^0}\,\mathbf{\overline{B}^0}$ to measure the decays of $\mathbf{b}$ and $\mathbf{c}$ quarks, e.g. V_{cb} and V_{ub}

Belle at KEK,
Japan 1999-2010

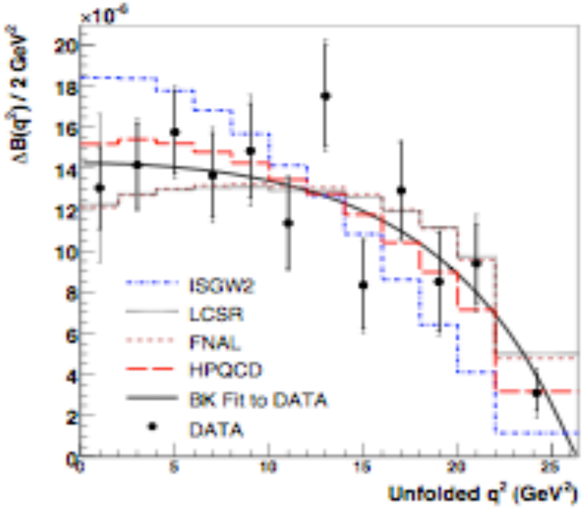


BaBar at SLAC
California,
1999 - 2008





$B \rightarrow D^* \ell\, \nu$ decays
(as function of D^* recoil)
measures $|V_{cb}| = 0.0374 \pm 0.0017$



Exclusive $B \rightarrow \pi\, \ell\, \nu$
(as a function of q^2)
$|V_{ub}| = 0.0034(5)$

The Wolfenstein Parameterisation

- An expansion of the CKM matrix in powers of $\lambda = V_{us} = 0.22$

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Parameterisation reflects almost diagonal nature of CKM matrix:
  - ➡ The diagonal elements V_{ud}, V_{cs}, V_{tb} are close to 1
  - ➡ Elements $V_{us}, V_{cd} \sim \lambda$ are equal and measure λ
  - ➡ Elements $V_{cb}, V_{ts} \sim \lambda^2$ are equal and measure A
  - ➡ Elements $V_{ub}, V_{td} \sim \lambda^3$ are very small

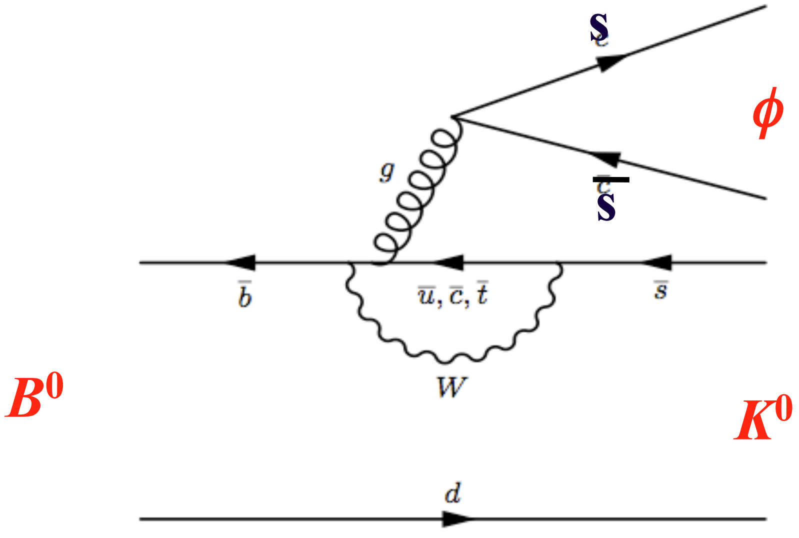
- Note that the parameter ρ and the complex phase η only appear in the very small elements V_{ub} and V_{td}, and are thus hard to measure.

Flavour Changing Neutral Currents

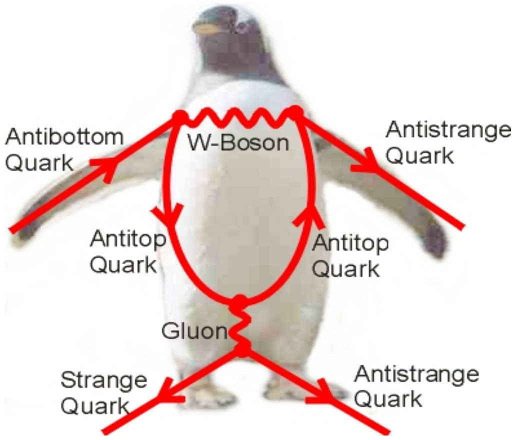
- At 1st order, there are no allowed transitions between quarks of the same charge, e.g. **s↔d, c↔u, b↔s, b↔d**

- Weak neutral current (the Z boson) does not change the flavour of fermions.

- At 2nd order so-called “Penguin Diagrams” can cause transitions such as **b↔s**
  - *e.g.* $\mathbf{b \to s\, s\, \bar{s}}$, $B^0 \to \phi\, K^0$



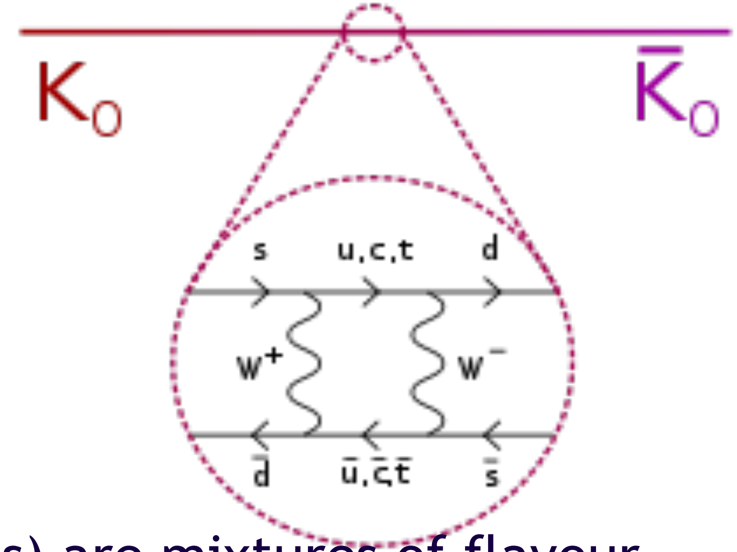
$B^0 \to J/\psi\, K^0$



Neutral Meson Mixing

- Second order weak interactions mix long-lived neutral mesons with their antiparticles:

  $K^0\,(\bar{s}\ d)$, $D^0\,(\bar{c}\ u)$, $B^0\,(\bar{b}\ d)$, $B_s\,(\bar{b}\ s)$



➡ $K^0 \leftrightarrow \bar{K}^0$ $D^0 \leftrightarrow \bar{D}^0$ $B^0 \leftrightarrow \bar{B}^0$ $B_s \leftrightarrow \bar{B}_s$

Observed particles (weak decay eigenstates) are mixtures of flavour eigenstates:

$K_S = 1/\sqrt{2}\,(K^0 + \bar{K}^0)$ with $\tau_S = 0.09$ ns

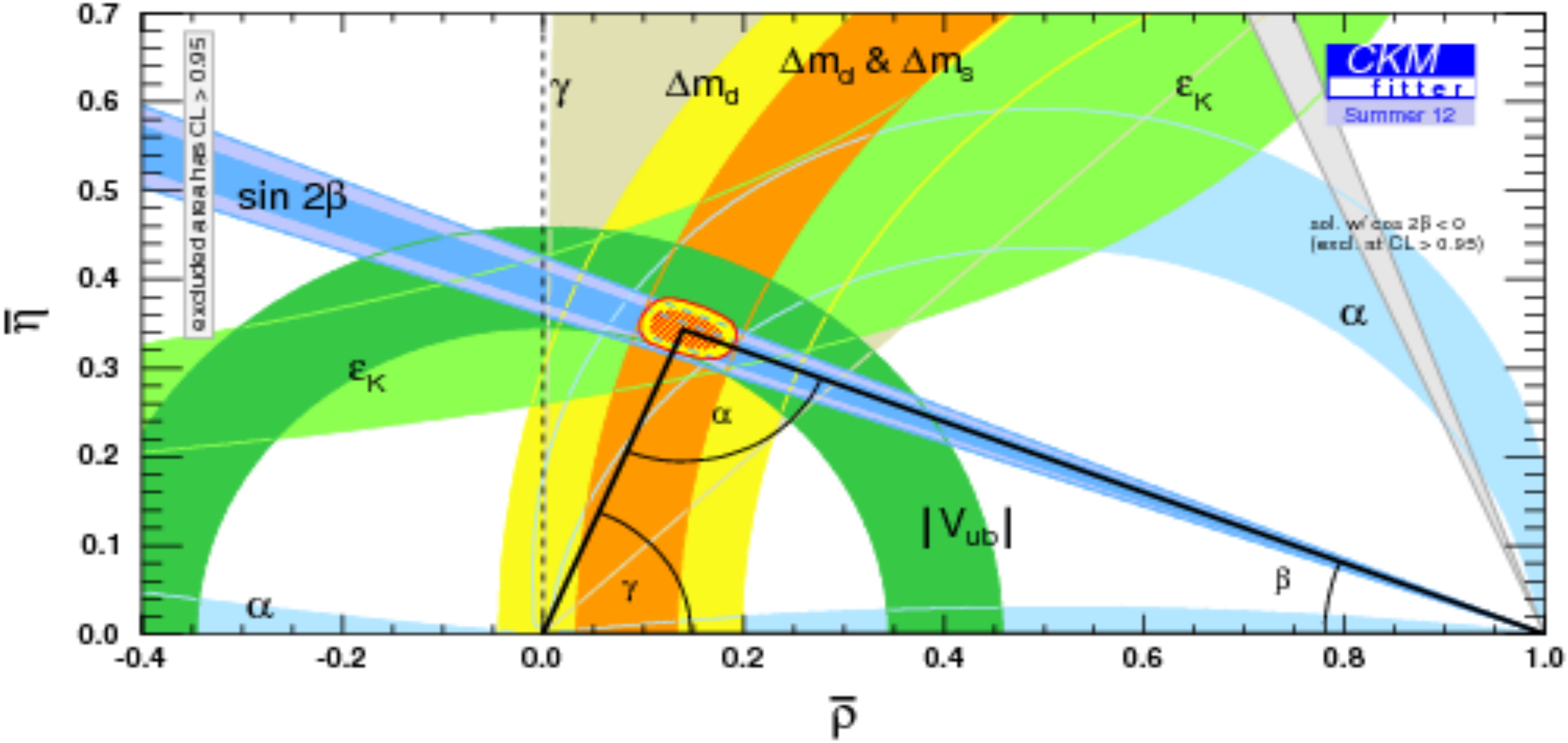
$K_L = 1/\sqrt{2}\,(K^0 - \bar{K}^0)$ with $\tau_L = 51$ ns

Mass difference $\Delta m_K = m_L - m_S = 3.52(1)\ \mathrm{x}10^{-12}$ MeV $= 0.53\ \mathrm{x}\ 10^{-10}\ \mathrm{s}^{-1}$

This is the oscillation frequency of the mixing

More about this next week, when we talk about CP violation

CKM Fit



- Many measurements, including results from BaBar, Belle, Tevatron and LHCb experiments. Semileptonic b->u decays, penguin diagrams, neutral meson mixing and CP violation are used to find best values for **η** and **ρ** parameters in Wolfenstein parameterisation.

Summary: Decays of Hadrons

- **Strong decays** are characterised by very short lifetimes, $\tau \sim 10^{-20}$ - 10^{-23} s appearing as **resonances** with a large width $\Gamma \sim \text{MeV}$.
  - ➡ Final states are hadronic. All quantum numbers are conserved.
- **Electromagnetic decays** are characterised by $\tau \sim 10^{-20}$ - 10^{-16} s.
  - ➡ Decays containing photons are electromagnetic.
  - ➡ All quantum numbers conserved except total isospin, I.
- **Weak decays** characterised by long lifetimes, $\tau \sim 10^{-13}$ - 10^{3} s.
  - ➡ Only decays that allow change of quark flavour (including s, c, b decays).
  - ➡ Responsible for most light meson and baryon decays.
  - ➡ Particles can live long enough to reach the detector.
  - ➡ Final states may be leptonic, semi-leptonic or hadronic.
  - ➡ Strong Isospin, I, I_3, Parity, P, Flavour quantum numbers not conserved.
- **CKM matrix** relates the quark mass eigenstates to the weak eigenstates
  - ➡ Non-diagonal: mixes quark flavours. Off-diagonal elements get smaller.
  - ➡ Allows higher order penguin diagrams, and neutral meson mixing.
  - ➡ Contains four free parameters, including a complex phase (leads to CP violation).