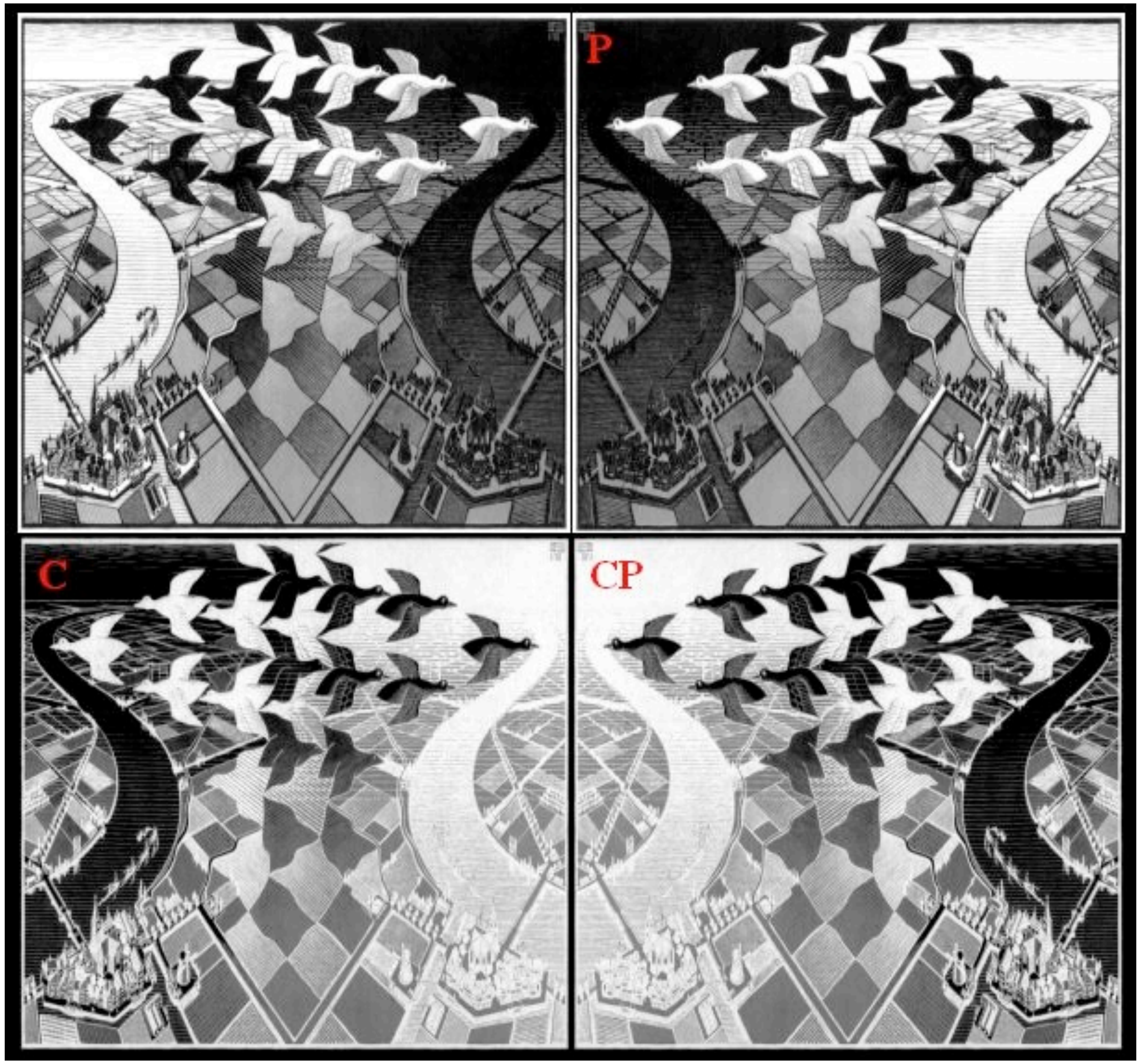
Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 14: Symmetries



- ★Symmetries of QED and QCD
- ★Parity, Charge Conjugation and Time Reversal
- ★Parity Violation in Weak Decay
- ★CP and CPT

Moriond Conference

- I was webcast: http://webcast.in2p3.fr/videos-searches_for_the_beh_boson_into_fermions_at_atlas

Guest Seminars

- From Edinburgh University researchers on their work.

- Next Monday (18th March) in tutorial:
  - Guest seminar from Dr Greig Cowan on B-physics at LHCb

- Following Monday (25th March) in tutorial
  - Guest seminar from Dr Wahid Bhimji on Higgs physicst at ATLAS

Cabibbo-Kobayashi-Maskawa Matrix

- **Mass eigenstates** and **weak eigenstates** of quarks are not identical.

- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix a mixture of the down-type quarks:

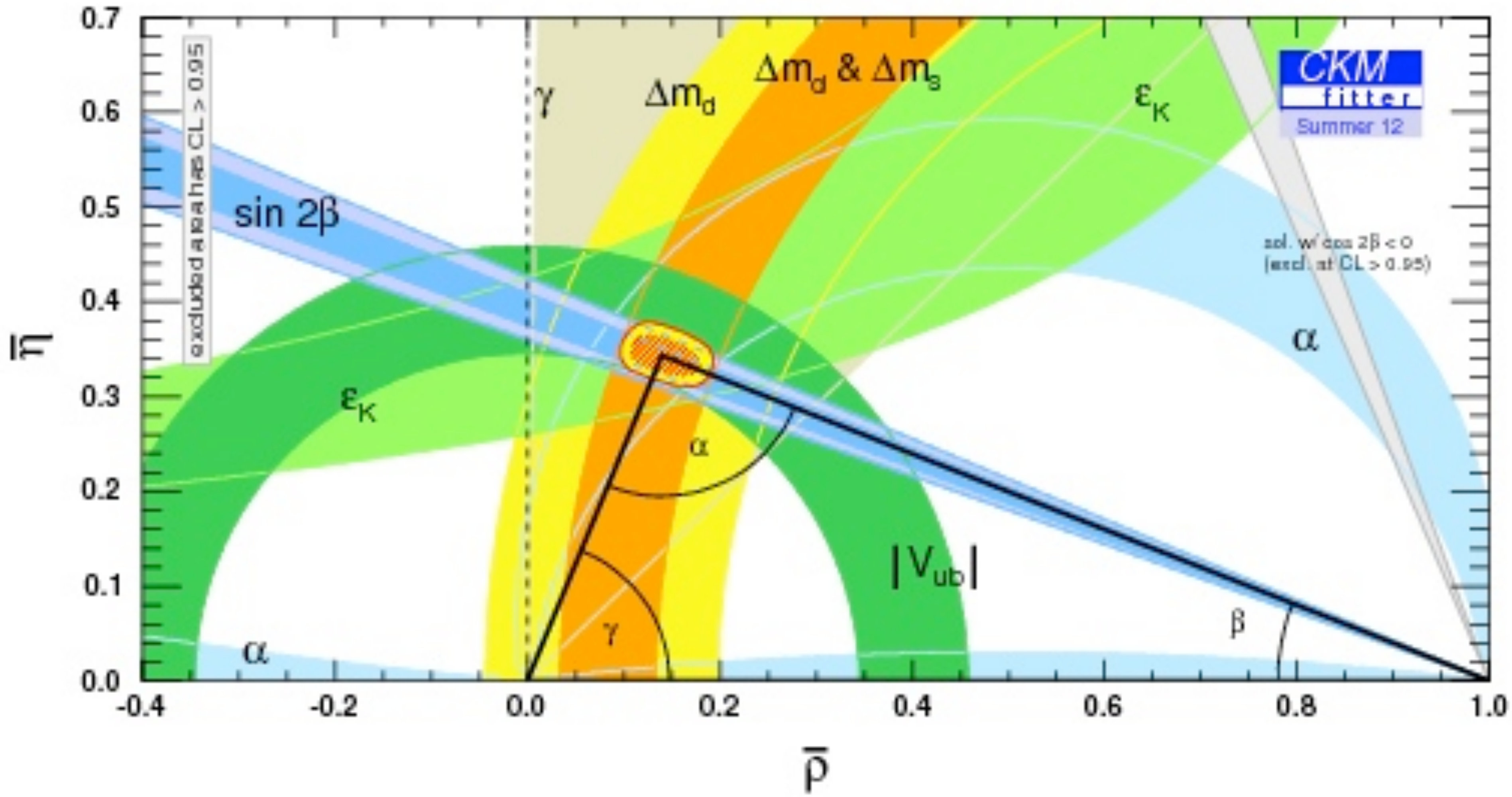
$$\text{weak eigenstates} \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad \text{mass eigenstates}$$

- The CKM matrix is unitary, $V_{CKM}^{\dagger} V_{CKM} = \mathbf{1}$

- **Wolfenstein parameterisation**

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

CKM Fit



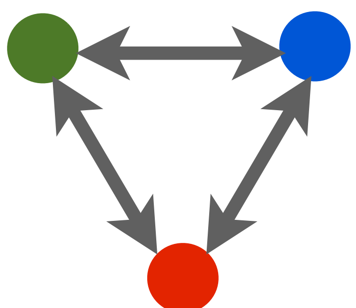
- Results from BaBar, Belle and LHCb experiments used to find best values for η and ρ parameters in Wolfenstein parameterisation.

Introduction: Symmetries in Particle Physics

- Symmetries play a central role in particle physics.
- Symmetries describe operations which leave a physical system unchanged.

- **Noether's theorem**: every symmetry corresponds to a conservation law.
  - Conservation laws can be experimentally verified

- We use the group theory to describe the symmetries of the three forces:
  - SU(3) symmetry of QCD ⇒ conservation of colour charge
  - SU(2) symmetry of weak force ⇒ conservation of weak isospin
  - U(1) symmetry of QED ⇒ conservation of electric charge
  - Translations in time and space; rotations in space ⇒ conservation of angular and four-momentum



- Some symmetries are observed to only hold under certain conditions, e.g. conservation of quark flavour for strong and electromagnetic interactions

- Breaking of symmetries can be either "dynamical" or "spontaneous" (e.g. Higgs mechanism).

Parity (P)

- Parity, P, is a spatial inversion through the origin: $P\psi(\vec{r}) = \psi(-\vec{r})$
  (not a mirror reflection in a plane!)

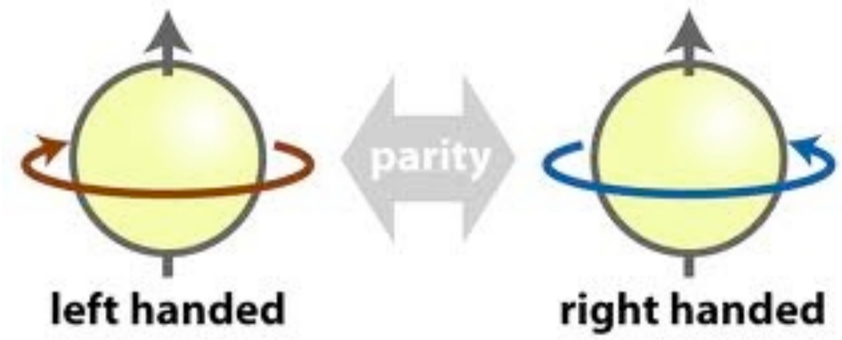
- If you act P twice on a state you get the original state back $\Rightarrow P^2 = 1$.
  - Eigenvalues of P are either $+1$ (even) or -1 (odd)
    - e.g. $\psi(x) = \sin kx$ is odd $\quad \psi(x) = \cos kx$ is even

- e.g. Hydrogen atom states described by Legendre polynomials:
  $Y_L{}^m = P_L{}^m(\cos\theta)\, e^{im\phi}$

  - Parity of state described by $Y_L{}^m$ is $(-1)^L$ (depends on orbital angular momentum L)
  - S (L=0) and D (L=2) states are even, P (L=1) states are odd

Intrinsic Parity

- Intrinsic parity is property of the elementary particles, we **define**:
  - Fermions to be eigenstates with $P = +1$ (even)
  - Antifermions to be eignestates w. $P = -1$ (odd)



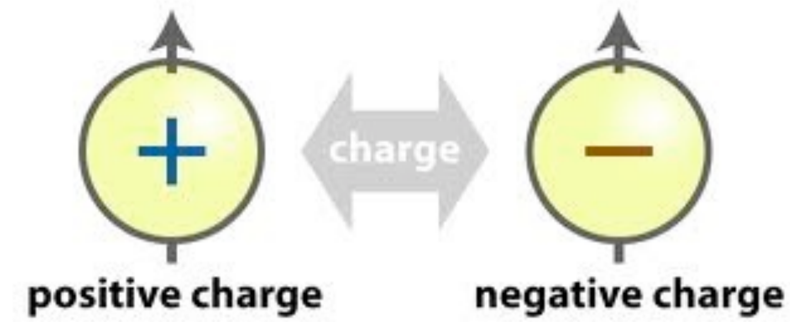
- Parity operator (P) can be represented by $\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix}$
  - Check this by looking at Dirac spinors (on problem sheet)

- Photons have odd parity, $P = -1$
  - Parity operation changes the direction of Electric fields.
  - Check this from transformation of electromagnetic fields (A^μ)

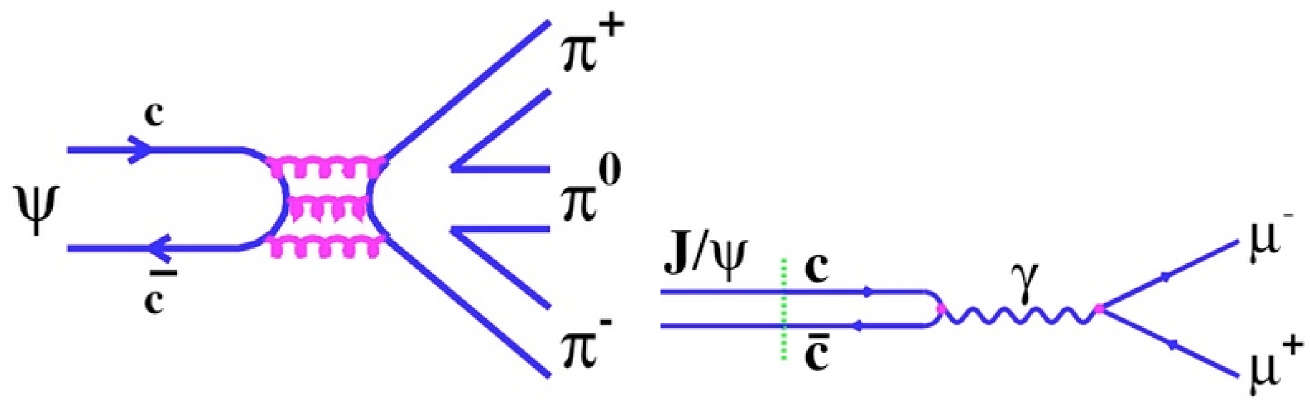
- In general:
  - Scalar ($\bar{\psi}\,\psi$) and axial-vector ($\bar{\psi}\,\gamma^5\,\gamma^\mu\,\psi$) quantities have $P = +1$ (even)
  - Pseudoscalar ($\bar{\psi}\,\gamma^5\psi$) and vector ($\bar{\psi}\,\gamma^\mu\,\psi$) quantities have $P = -1$ (odd)

Charge Conjugation (C)

- A change from particle to antiparticle: $C\bar{f} = f$; $Cf = \bar{f}$
- $C^2 = 1 \Rightarrow$ eigenvalues of C are either $+1$ (even) or -1 (odd)
  - (C can be represented by $i\gamma^2$)
- Fermions and antifermions are not eigenstates of C !



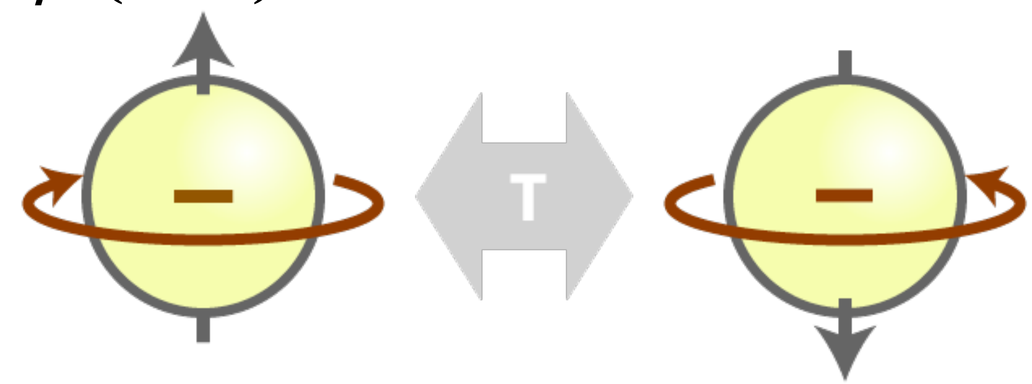
- Photons have $C = -1$. C changes sign of the electric charges, and therefore of the electromagnetic field. Similarly gluons have $C = -1$.
- C and P observed to be conserved in electromagnetic and strong interactions.
- Mesons ($q\bar{q}$) have $C = (-1)^{L+S}$ and $P = (-1)^{L+1}$
- Lightest mesons pseudoscalars (↑↓) with $J^{PC} = 0^{-+}$ Second-lightest states are vectors (↑↑ or ↓↓) with $J^{PC}=1^{--}$



- e.g. J/ψ ($c\bar{c}$) meson has $J^{PC}=1^{--}$. EM and strong decays via odd number of photons or gluons to conserve C and P

Time Reversal (T)

- A reversal of the arrow of time: $T\psi(t) = \psi(-t)$



- $T^2 = 1$ so eigenvalues of T are either $+1$ (even) or -1 (odd)

- T corresponds to interchanging the initial and final states in a Feynman diagram.

- T reverses the sign of time-dependent variables including momentum and angular momentum (spin)

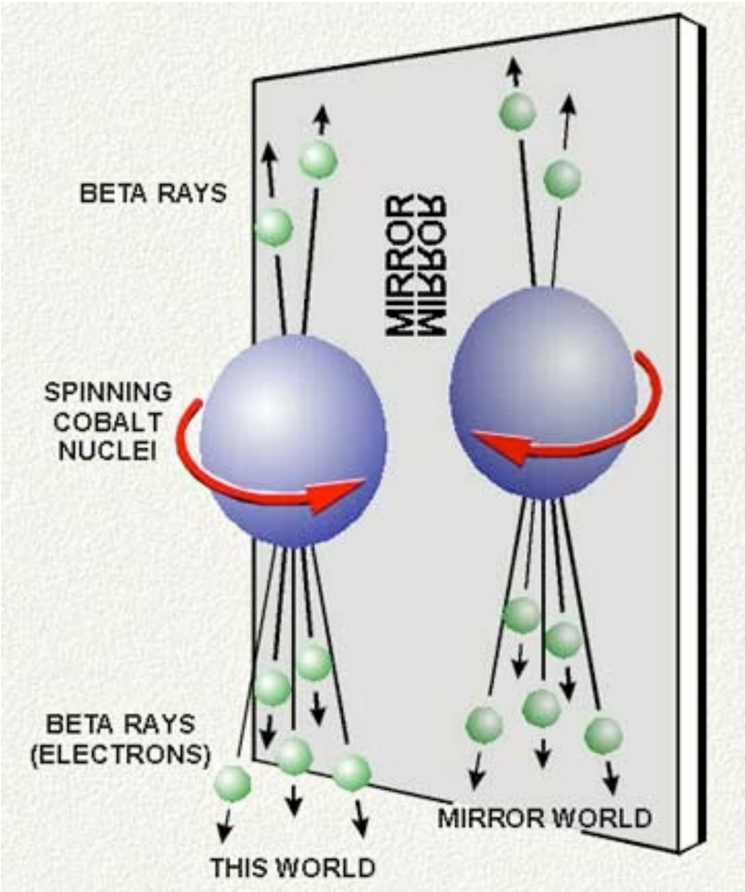
- It is evident that the Universe is not T symmetric!
  - Big Bang, Hubble expansion, increase in Entropy …

Discrete Symmetries of Physical Quantities

Quantity	Notation	P	C	T
Position	$\vec{r}$	-1	$+1$	$+1$
Momentum (Vector)	$\vec{p}$	-1	$+1$	-1
Spin (Axial Vector)	$\vec{\sigma} = \vec{r} \times \vec{p}$	$+1$	$+1$	-1
Helicity	$\vec{\sigma} \cdot \vec{p}$	-1	$+1$	$+1$
Electric Field	$\vec{E}$	-1	-1	$+1$
Magnetic Field	$\vec{B}$	$+1$	-1	-1
Magnetic Dipole Moment	$\vec{\sigma} \cdot \vec{B}$	$+1$	-1	$+1$
Electric Dipole Moment	$\vec{\sigma} \cdot \vec{E}$	-1	-1	-1
Transverse Polarization	$\vec{\sigma} \cdot (\vec{p_1} \times \vec{p_2})$	$+1$	$+1$	-1

Parity Violation in Weak Decays

- First observed by Chien-Shiung Wu in 1957 through β-decay of polarised ^{60}Co nuclei: $^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^{-} + \bar{\nu}_e$
  - Recall: under parity changes momentum sign but not spin $\vec{p} \rightarrow -\vec{p}$ $\vec{S} \rightarrow \vec{S}$
- Electrons were observed to be emitted to opposite to nuclear spin direction
- Particular direction is space is preferred!



- P is found to be violated maximally in weak decays
- P is a good symmetry in strong and QED.

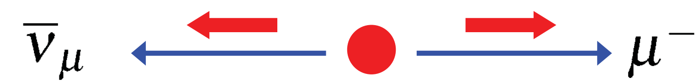
- Recall the vertex term for the weak force is $g_w \gamma^\mu(1-\gamma^5)/\sqrt{8}$
- Fermion currents are proportional to $\bar{\psi}(e)\,(\gamma^\mu - \gamma^\mu\gamma^5)\,\psi(\nu_e)$ **"vector – axial vector"**
- Under parity, $P\,(\bar{\psi}\,(\gamma^\mu - \gamma^\mu\gamma^5)\,\psi) = \bar{\psi}\,(\gamma^\mu + \gamma^\mu\gamma^5)\,\psi$ (mixture of eigenstates)

- Compare to QED and QCD vertices: $P\,(\,\bar{\psi}\,\gamma^\mu\,\psi\,) = \bar{\psi}\,\gamma^\mu\,\psi$

(pure eigenstates - no P violation)

Parity Violation in Pion Decays

- Consider charged pion decay at rest, $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$

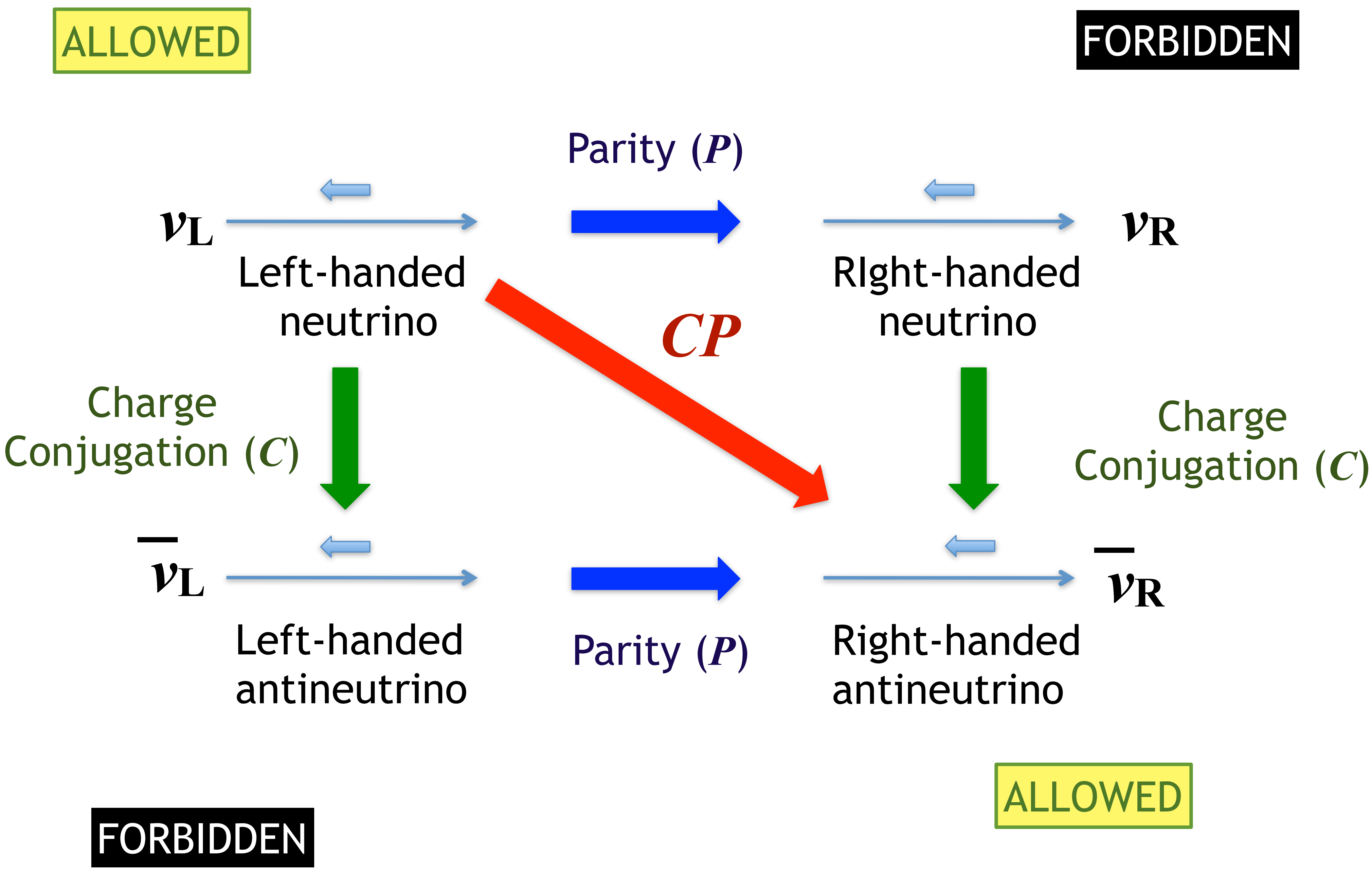


- Charged pion has $S = 0 \Rightarrow$ muon and neutrino produced with equal & opposite spin
  - ➡ The muon and neutrino will have identical helicities.
  - ➡ Experiments observe muon helicity is always right-handed
  - ➡ Conclusion: Only right-handed anti-neutrinos exist! (or only right-handed anti-neutrinos interact)

- Similarly for $\pi^+ \rightarrow \mu^+ + \nu_\mu$ anti-muon helicity is always left-handed
  - ➡ only left-handed neutrinos exist!

- This observation also explains why $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$ is preferred over $\pi^- \rightarrow e^- + \overline{\nu}_e$
  - ➡ The weak interaction only acts on LH **chiral** states, pion must decay to RH **helicity** electron or muon.
  - ➡ The change from LH chiral to RH helicity for lepton (ℓ) mass: $\propto \dfrac{m_\ell}{m_\pi + m_\ell}$

Neutrino States



CPT Theorem

- ***CPT*** is the combination of ***C***, ***P*** and ***T***.
  Turns a forward-going particle with LH helicity into backward-going antiparticle with RH helicity.
- ***CPT*** Theorem states:

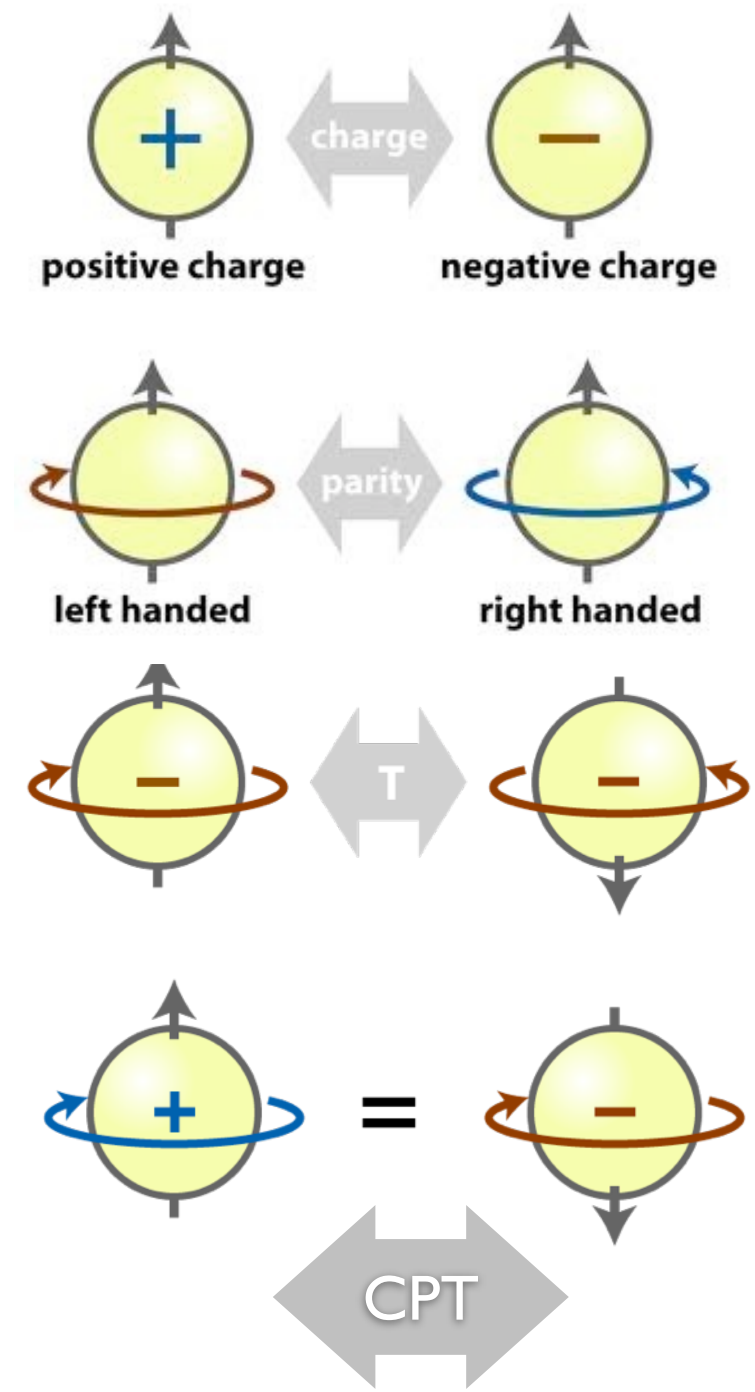
  "All interactions described by a local Lorentz invariant gauge theory must be invariant under the combined operation ***CPT***"

  ➡ ***CPT*** violation would imply non-locality and/or loss of Lorentz invariance

  ➡ Impossible to write down relativistic quantum field theories?

  ➡ Impossible to describe interactions in terms of Feynman diagrams?

  ➡ ***CPT*** conservation implies that ***CP*** violation is equivalent to ***T*** violation



- The Universe needs ***CP*** violation for the matter-antimatter asymmetry and it needs ***T*** violation for the arrow of time

Tests of *CPT* Invariance

- ***CPT*** invariance implies particles and antiparticles must have equal masses:

$$\frac{M(\mathrm{K}^0) - M(\overline{\mathrm{K}}^0)}{\frac{1}{2}[M(\mathrm{K}^0) + M(\overline{\mathrm{K}}^0)]} < 10^{-18}\ \mathrm{GeV}$$

Don't learn all these limits, just for your appreciation

- Particle and antiparticles must have equal lifetimes:

$$\frac{\Gamma(\mathrm{K}^0) - \Gamma(\overline{\mathrm{K}}^0)}{\frac{1}{2}[M(\mathrm{K}^0) + M(\overline{\mathrm{K}}^0)]} < \times 10^{-17}$$

- Particle and antiparticles must have equal and opposite charges and magnetic moments

$$Q(p) + Q(\bar{p}) < 10^{-21} e$$

$$\frac{g(e^+) - g(e^-)}{\frac{1}{2}[g(e^+) + g(e^-)]} < 2 \times 10^{-12}$$

- Hydrogen and anti-hydrogen atoms have identical spectra

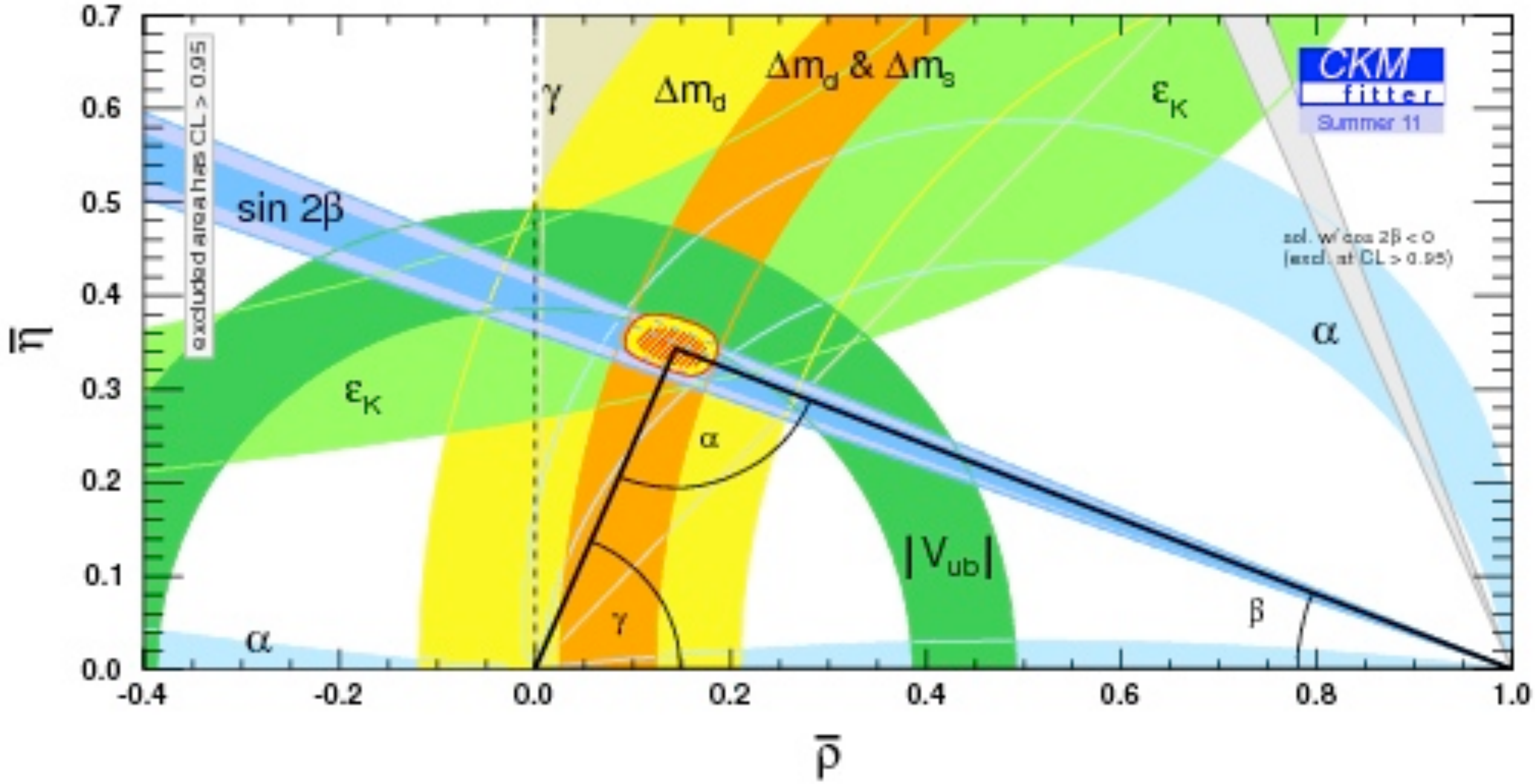
Summary

- Symmetries play a key role in describing interactions in particle physics.
- QED and QCD obey Gauge symmetries in the Lagrangian corresponding to symmetry groups. These lead to conservation of electric and colour charge.
- Three important discreet symmetries: Charge Conjugation (C), Parity (P) and Time reversal (T).
  - C: changes the sign of the charge
  - P: spatial inversion, reserves helicity. Fermions have P=+1, antifermions P=−1
  - T: changes the initial and final states
  - Gluons and photons have C =−1, P=−1

- C and P are conserved in QED and QCD, maximally violated in weak
- Only LH neutrinos and RH anti-neutrinos are found in nature.

- CPT is thought to be absolutely conserved.

CKM Fit



- All measurements, including results from BaBar and Belle experiments, penguin diagrams, neutral kaon mixing used to find
- Best values for **η** and **ρ** parameters in Wolfenstein parameterisation.