

Weak Isospin and Hypercharge

- QED couples to electric charge; QCD couples to colour charge...
- Electroweak force couples to two “charges”.
 - **Weak Isospin:** total and third component T , T_3 . Depends on **chirality**
 - **Weak Hypercharge, Y** In terms of electric charge Q : $Y = 2(Q - T_3)$
 - All right-handed fermions have $T=0$, $T_3=0$
 - All left-handed fermions have $T=1/2$, $T_3=\pm 1/2$
 - All left-handed antifermions have $T=0$, $T_3=0$
 - All right-handed antifermions have $T=1/2$, $T_3(\bar{f})=-T_3(f)$

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	$1/2$	$+1/2$	-1	u_L, c_L, t_L	$1/2$	$+1/2$	$1/3$
e_L, μ_L, τ_L	$1/2$	$-1/2$	-1	d_L, s_L, b_L	$1/2$	$-1/2$	$1/3$
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	$-2/3$

Weak Isospin Doublets

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	$1/2$	$+1/2$	-1	u_L, c_L, t_L	$1/2$	$+1/2$	$1/3$
e_L, μ_L, τ_L	$1/2$	$-1/2$	-1	d_L, s_L, b_L	$1/2$	$-1/2$	$1/3$
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	$-2/3$

- Neutrinos and left-handed charged leptons form a “weak isospin doublet”:

$$\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad T = 1/2; \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix}$$

- Doublet consists of “charged current flavour change pair”.

➔ They have the same total weak isospin $T=1/2$.

➔ They are differentiated by the third component $T_3=\pm 1/2$.

- Left-handed up-type quarks and left-handed down-type quarks also form isospin doublets

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad T = 1/2; \quad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix}$$

Weak Isospin Currents

- Weak Isospin and Weak Hypercharge couple to a different set of bosons.
- Weak isospin doublets χ_L couple to a set of **three** W -bosons: W^1, W^2, W^3 , with SU(2) symmetry described by the 3 Pauli matrices:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- The W -bosons current is:

$$(j^{W^i})^\mu = [g_W T] \bar{\chi}_L \gamma^\mu \tau_i \chi_L$$

$\tau_{1,2,3}$: Pauli Matrix
 χ_L : weak isospin doublet column vector spinors
 $\bar{\chi}_L$: weak isospin doublet row vectors spinors
 T : weak isospin charge of the doublet
 g_W : weak coupling constant

- e.g for the W^1 boson and the electron doublet:

$$(j^{W^1})^\mu = [g_W T] (\nu_e \ e^-)_L \gamma^\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

- Strength of the fermion interaction with W -bosons is: $g_W T$

Weak Hypercharge Current

- Particles with weak hypercharge couple to **one** B -boson: B^0 with U(1) symmetry.
- Use electron as an example:

$$j_{\mu}^Y = \left(\frac{1}{2}g'_W Y_e\right) \bar{e}\gamma^{\mu}e = \frac{1}{2}g'_W (Y_{eL} \bar{e}_L \gamma^{\mu} e_L + Y_{eR} \bar{e}_R \gamma^{\mu} e_R)$$

Y_e : weak hypercharge of electron
 Y_{eL} : weak hypercharge of left-handed electron
 Y_{eR} : weak hypercharge of right-handed electron
 e : Electron spinor
 e_L : Left-handed electron spinor (u)
 e_R : Right-handed electron spinor (u)
 \bar{e}_L : Left-handed electron spinor (\bar{u})
 \bar{e}_R : Right-handed electron spinor (\bar{u})
 g'_W : coupling constant

- Strength of the fermion interaction with bosons is: $g'_W Y/2$

Physical Bosons

- The physical W^+ , W^- , Z^0 , γ bosons are linear superpositions of the W^1 , W^2 , W^3 and B^0 bosons.

- Use $\cos\theta_W$ and $\sin\theta_W$ to ensure the states are properly normalised

$$W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2) \quad W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2)$$

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W \quad \gamma = W^3 \sin\theta_W + B^0 \cos\theta_W$$

- The coupling of the W^+ , W^- bosons are

$$\frac{1}{\sqrt{2}}(g_W T) = \frac{1}{2\sqrt{2}}g_W$$

- No $(1-\gamma^5)$ term: it integrated into the definition of the χ_L doublet.

The Photon

$$\gamma = W^3 \sin \theta_W + B^0 \cos \theta_W$$

- The electron current associated with the γ is:

$$\begin{aligned}
 & (j^{W^3})^\mu \sin \theta_W + (j^Y)^\mu \cos \theta_W \\
 &= [g_W T \sin \theta_W] \bar{\chi}_L \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \chi_L + \left[\frac{1}{2} g'_W Y_e \cos \theta_W \right] \bar{e} \gamma^\mu e \\
 &= - \left[\frac{1}{2} g_W \sin \theta_W \right] \bar{e}_L \gamma^\mu e_L + \left[\frac{1}{2} g'_W \cos \theta_W \right] (-\bar{e}_L \gamma^\mu e_L - 2\bar{e}_R \gamma^\mu e_R) \\
 &= - \underbrace{\left[\frac{1}{2} g_W \sin \theta_W + \frac{1}{2} g'_W \cos \theta_W \right]}_{=e} \bar{e}_L \gamma^\mu e_L - \underbrace{\left[g'_W \cos \theta_W \right]}_{=e} \bar{e}_R \gamma^\mu e_R
 \end{aligned}$$

- Consistent with the photon coupling if $e = g'_W \cos \theta_W = g_W \sin \theta_W$

$\sin^2\theta_W$ and Z -boson couplings

- The mixing angle between g_W and g'_W is not a prediction of the model, it must be measured experimentally.

$$\sin^2\theta_W = \frac{g_W'^2}{g_W^2 + g_W'^2} \approx 0.23$$

- The Z -boson the orthogonal mixture to the γ :

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W$$

- predicts the couplings of the Z^0 boson in terms of T_3 and $Y = 2(Q - T_3)$
- e.g. for electron:

$$\begin{aligned}(j^Z)^\mu &= \frac{g_W}{\cos\theta_W} [(T_3 - Q \sin^2\theta_W)(\bar{e}_L \gamma^\mu e_L) - (Q \sin^2\theta_W)(\bar{e}_R \gamma^\mu e_R)] \\ &= \frac{g_Z}{2} \bar{e} \gamma^\mu (c_V^e - c_A^e \gamma^5) e\end{aligned}$$

- if:

$$g_Z = \frac{g_W}{\cos\theta_W} \quad c_V = T_3 - 2Q \sin^2\theta_W \quad c_A = T_3$$

Summary of Electroweak Unification

- We have recovered the behaviour of the W^\pm , Z and γ
 - ➔ We introduced an SU(2) symmetry (3 bosons) coupling to weak isospin with a coupling constant g_W
 - ➔ We introduced a U(1) symmetry (1 boson) coupling to weak hypercharge with a coupling constant g'_W
 - ➔ Together predicts four bosons we identify with W^+ , W^- , Z and γ
 - ➔ Electroweak Theory is often called **SU(2) \otimes U(1)** model
- All of the properties of electroweak interactions described by:
 - the intrinsic charges of the fermions
 - the SU(2) \otimes U(1) symmetry
 - g_W and g'_W : free parameters that need to be measured
- Along with QCD, Electroweak Theory is the Standard Model.

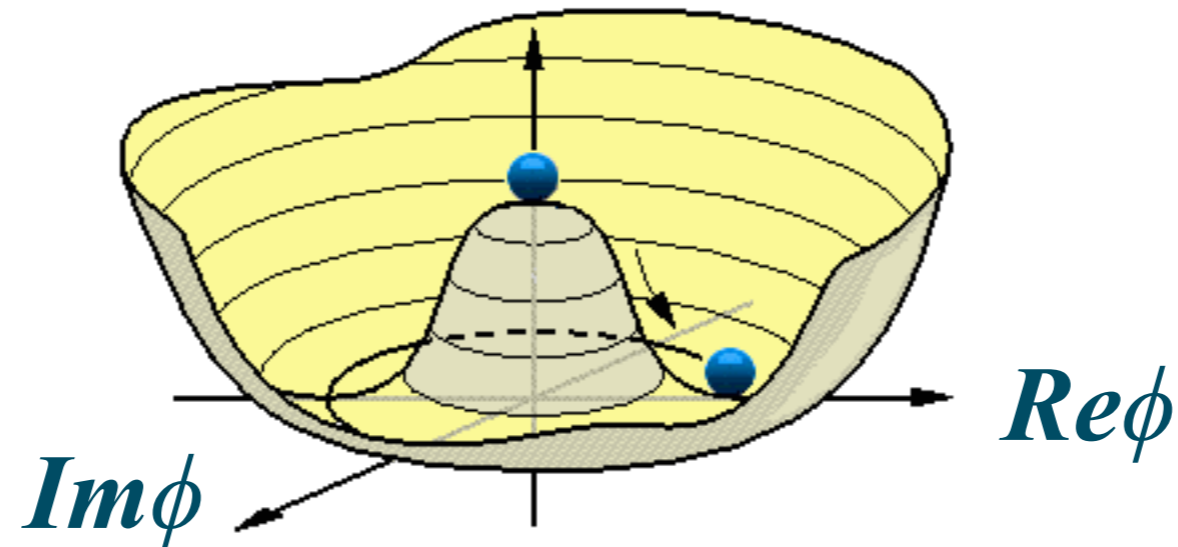
The Higgs Mechanism: Introduction

- The Higgs Mechanism was proposed in 1964 separately by Higgs and Brout & Englert.
- It introduces an extra field, ϕ , which interacts with the electroweak currents. The potential of the field is:

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \quad \text{with } \mu^2 > 0, \lambda > 0$$

- The Higgs mechanism allows the W and Z bosons to have a mass. (Otherwise forbidden by the external symmetries.)
- Provides an explanation for fermion masses ($e, \mu, \tau, u, d, s, c, t, b$).
- P.W. Higgs pointed out that a further consequence would be the existence of a spin-0 boson: the Higgs boson, H .

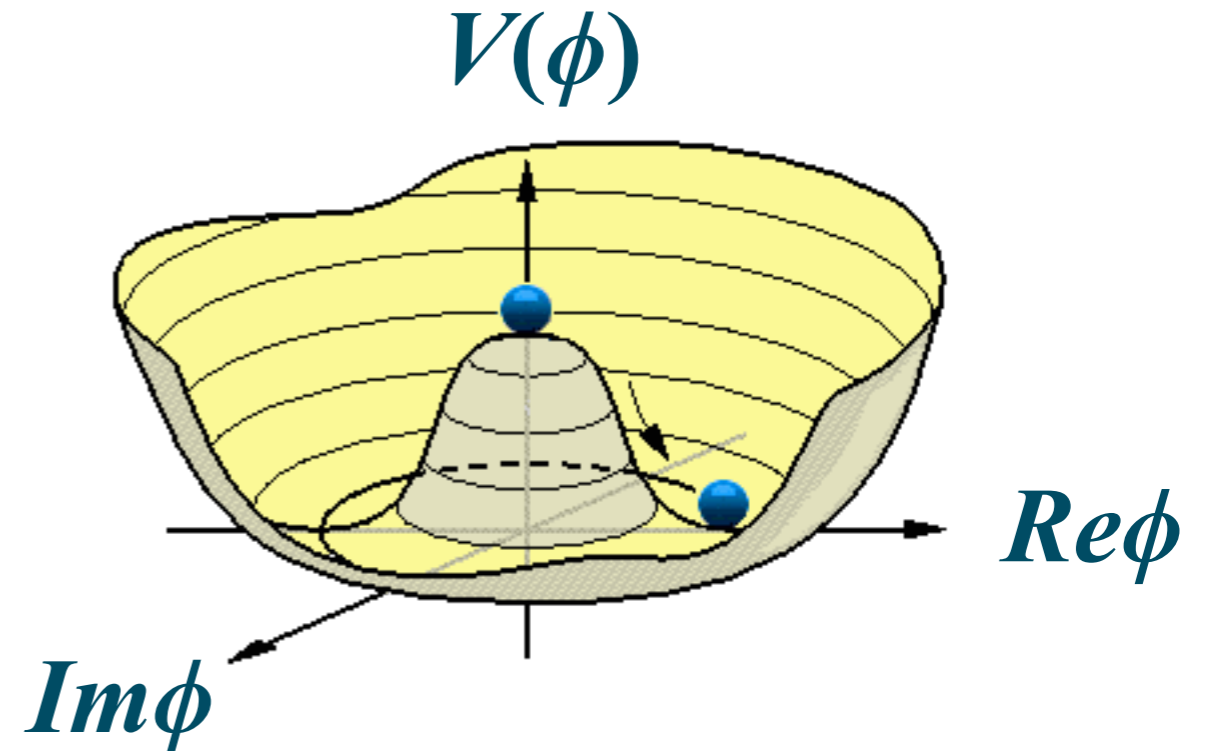
Spontaneous Symmetry Breaking



- Start with a system that has an intrinsic symmetry
 - ➔ Choosing a particular ground state configuration the symmetry is broken
 - ➔ If the choice is arbitrary, i.e. no external agent is responsible for the choice, then the symmetry is “spontaneously” broken
- **Everyday example:** A circle of people are sitting at a dining table with napkins between them. The first person who picks up a napkin, either with their left or right hand spontaneously breaks the L/R symmetry. All the others must do the same if everyone is to end up with a napkin.
- **Physics example:** In a domain inside a ferromagnet all the spins align in a particular direction. If the choice of direction is random, the underlying theory has a rotational symmetry which is spontaneously broken. The presence of an external magnetic explicitly breaks the symmetry and defines a preferred direction.

Higgs Potential

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$



- ϕ is complex function.
- $V(\phi)$ is symmetric: the maximum symmetry occurs at $\phi=0$.
- A circle of values minimise the potential at $\phi=\phi_0 \equiv -v/\sqrt{2}$ with $|\phi_0| = \frac{\mu}{\sqrt{2\lambda}}$
 - Any coordinate around the circle minimise the potential: $\arg(\phi_0) = [0, 2\pi)$
- The choice of which complex value of ϕ_0 is chosen spontaneously breaks the symmetry.
- The value of v , related to the value of $|\phi|$ at the minimum of the potential known as the vacuum expectation value. Measured to be $v = 246 \text{ GeV}$

Standard Model Higgs Field

- In the Standard Model, the Higgs field is a complex isospin doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad T = 1; \quad \begin{matrix} T_3 = +1 \\ T_3 = 0 \end{matrix}$$

- Higgs field has four degrees of freedom.

ϕ^+ : +ve charged field
 ϕ^0 : neutral field
 ϕ_0 : minimum of field

- In the Higgs mechanism (when the symmetry is spontaneously broken) three of these degrees of freedom are used to give mass to W^+ , W^- , Z^0 .
- This fixes three of the degrees of freedom: two charged and one neutral.
- The minimum of the potential ϕ_0 for ground state can then be written in terms of the remaining free parameter:

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Where v is related to the value of ϕ which minimises V : $v = \frac{\mu}{\sqrt{2\lambda}}$

Introducing the Higgs Boson

- Consider a fluctuation of the Higgs field about its minimum:

$$\phi(x) = \phi_0 + h(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Substitute $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$ into $V(\phi)$ and expand to second order in $h(x)$:

$$V(\phi) = -\mu^2 \left(\frac{v + h(x)}{\sqrt{2}} \right)^2 + \lambda \left(\frac{v + h(x)}{\sqrt{2}} \right)^4 = \dots = V(\phi_0) + \underbrace{\lambda v^2 h^2}_{= \frac{1}{2} m_H^2} + \mathcal{O}(h(x)^3)$$

- In quantum field theory a term quadratic in the field describes a particle's mass.
- This fluctuation around the minimum of the potential describes a spin-0 particle with a mass $m = \sqrt{2\lambda}v$

- **The Higgs boson!**

Higgs Couplings



- The Higgs mechanism predicts that the Higgs boson interacts with the W and Z bosons and massive fermions, in proportion to their mass.
- The dotted line is the prediction.
- The points are the measured values from the CMS collaboration at the LHC.

