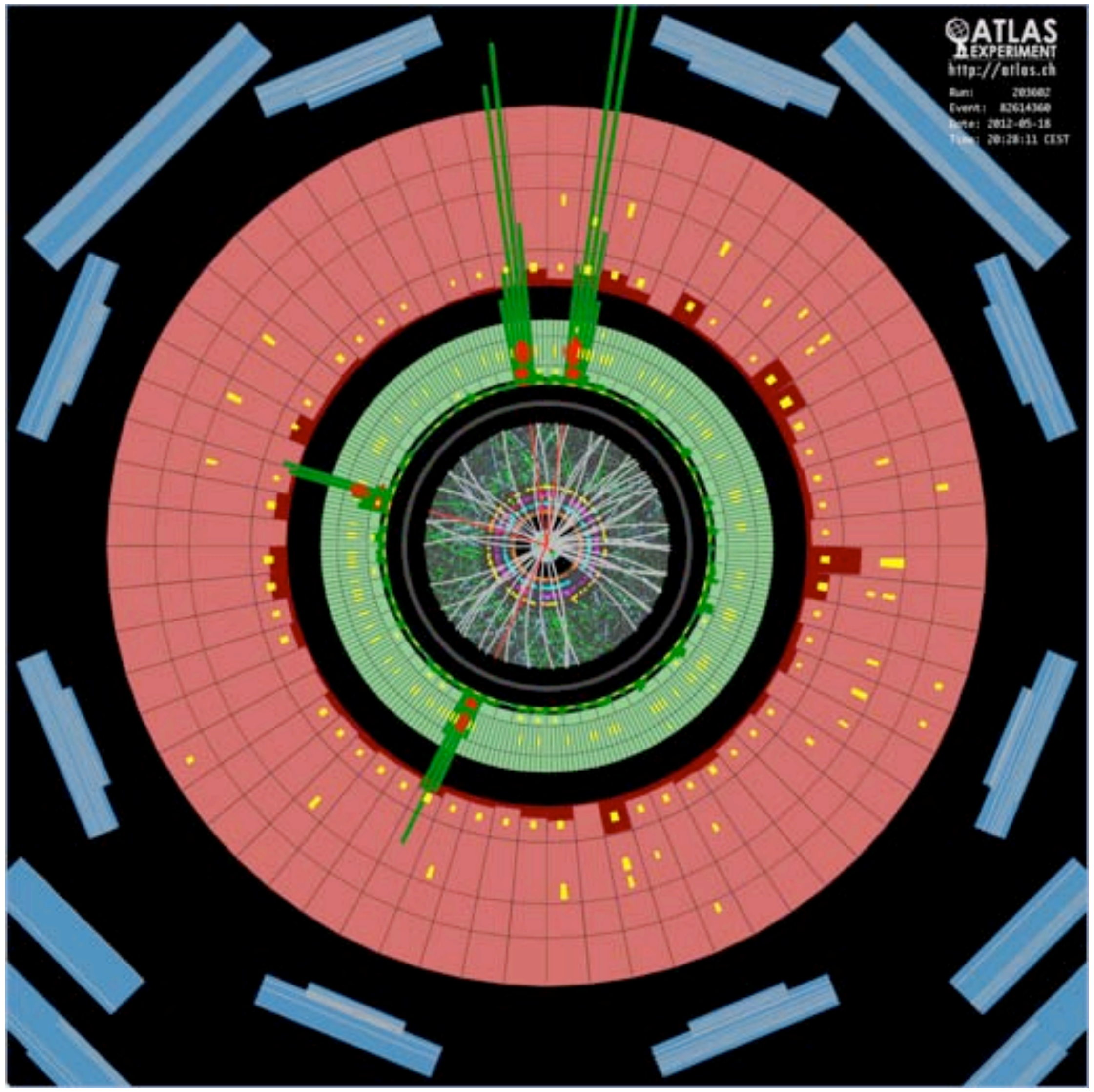
Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 1: The Mysteries of Particle Physics,
or “Why should I take this course?”



Contents:

- Review of the Standard Model
  - ➡ What we know
  - ➡ What we don’t know
- Highlights from 2012 particle physics
  - ➡ What we know now that we didn’t know this time last year

Course Organisation

Teaching weeks: 14 January - 15 February; 25 February - 5 April
ILW: 18 - 22 February (no lectures)

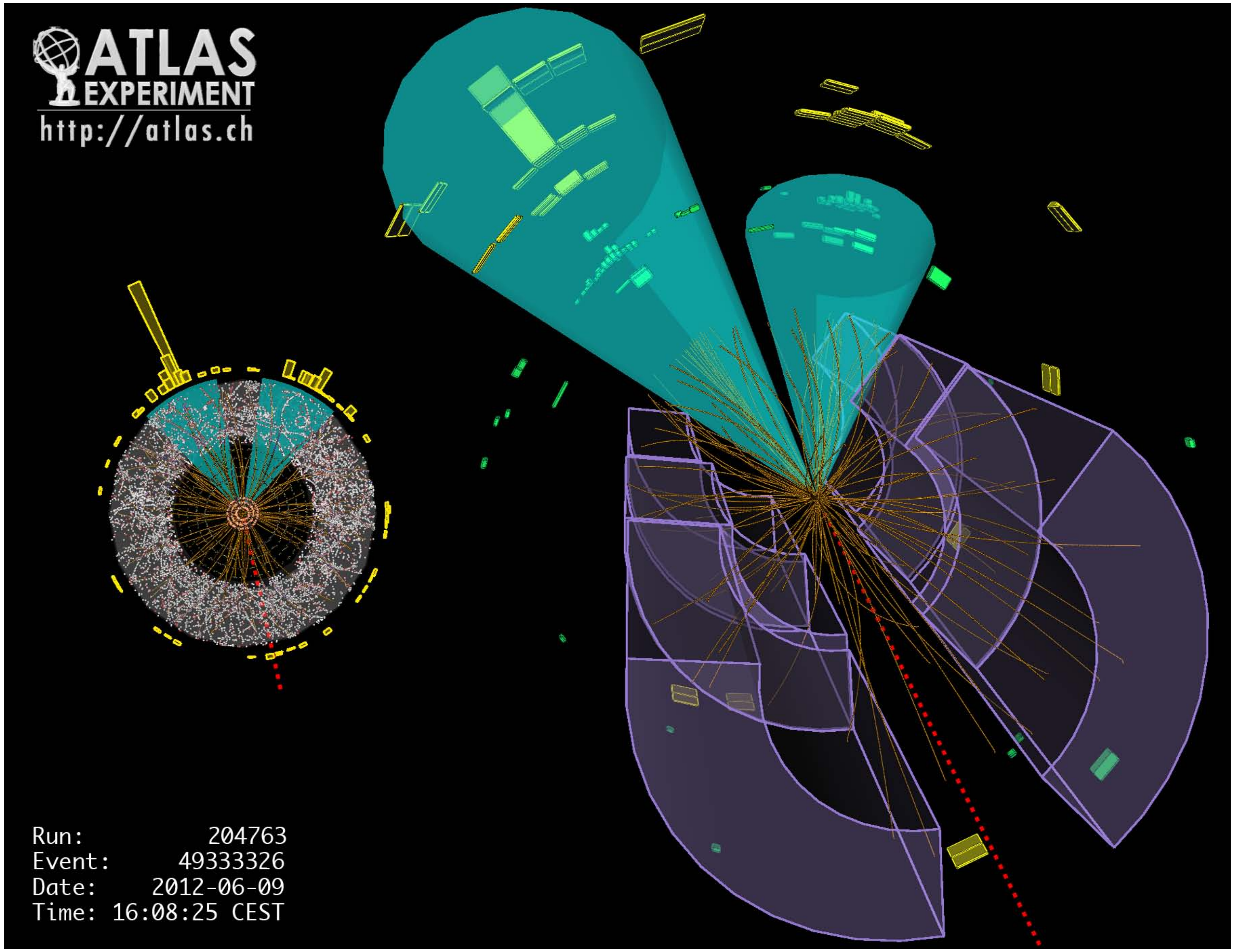
Particle Physics course:

- **18 Lectures**: Tuesday, Friday 12:10-13:00 (JCMB 5215)
  - No PP lectures: 10 Feb, 6 April
  - Two themes: **Particles & interactions of the Standard Model, Current topics in particle physics**
- **Tutorials**: Monday 3-5 in 5326
- I'll try to recommend reading for the course from *Introduction to Elementary Particle Physics* by David Griffiths. (7 copies in Darwin Library)

- Printed notes and problem sheets handed out periodically and available on the web.
- Lecture slides (and eventually solution sheets) will only be available on the web.

Me

- Dr Victoria Martin, JCMB 5419
- I work on the ATLAS experiment at the Large Hadron Collider at CERN.
- I currently lead the University of Edinburgh ATLAS team of ~20 PhD students, postdoctoral researchers and academics



- I also have an interest in future colliders e.g. a high energy e^+e^- collider
- Personal interest: looking for the decay of the Higgs boson into quarks, *e.g.* $H \to b\bar{b}$

References & Websites

- Course website: http://www2.ph.ed.ac.uk/~vjm/Lectures/SH_IM_Particle_Physics_2013.html

Introductory textbooks

- D.Griffiths – *Introduction to Elementary Particles* (Wiley 2008)
- C. Tully - *Elementary Particle Physics in a Nutshell* (Princeton 2011)
- B.R.Martin & G.Shaw – Particle Physics (Wiley 1997)
- D.H.Perkins – Introduction to High Energy Physics (CUP 2000)

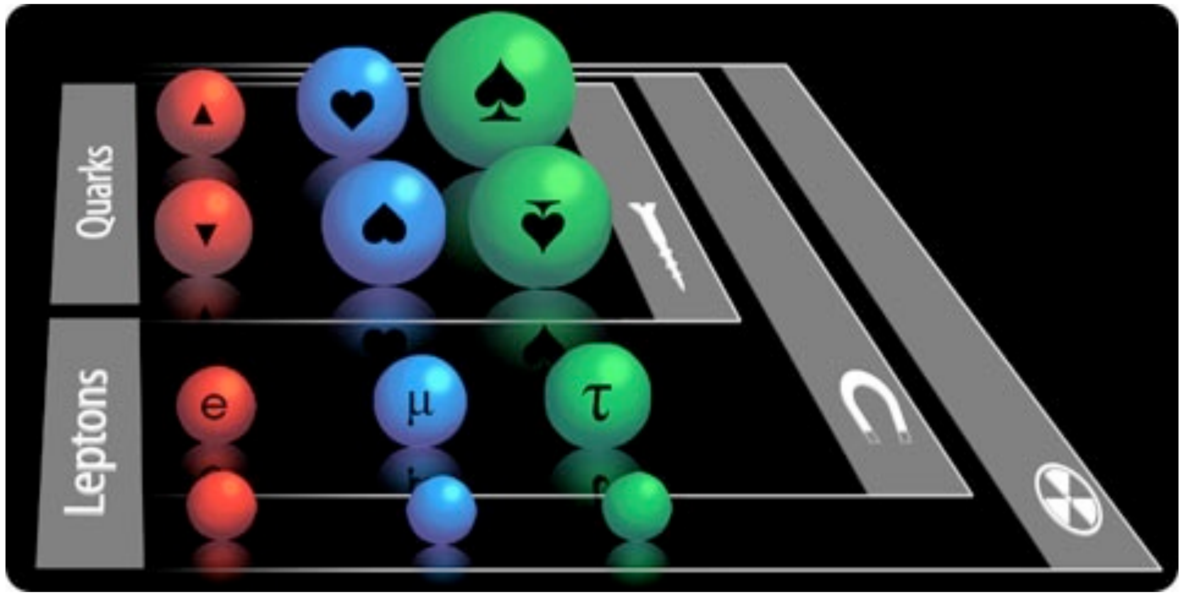
More advanced textbooks

- F.Halzen & A.D.Martin – Quarks & Leptons (Wiley 1984)
- A.Seiden – Particle Physics: A Comprehensive Introduction (Addison-Wesley 2005)
- I.J.R.Aitchison & A.J.G.Hey – Gauge Theories in Particle Physics (Hilger 1989)

Useful websites

- ✧ CERN/LHC http://public.web.cern.ch/public
- ✧ Particle Data Group (PDG) http://pdg.lbl.gov

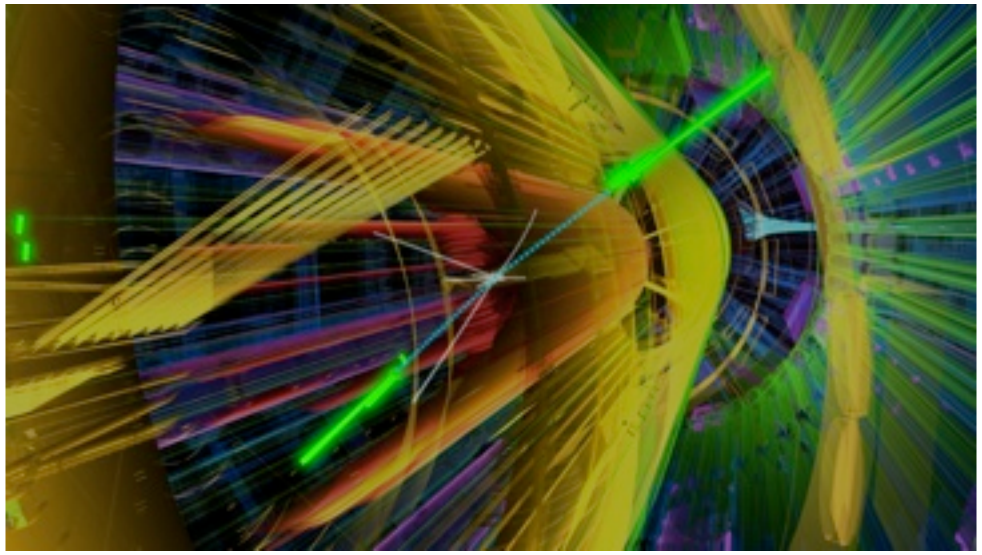
Particles & Interactions of the Standard Model

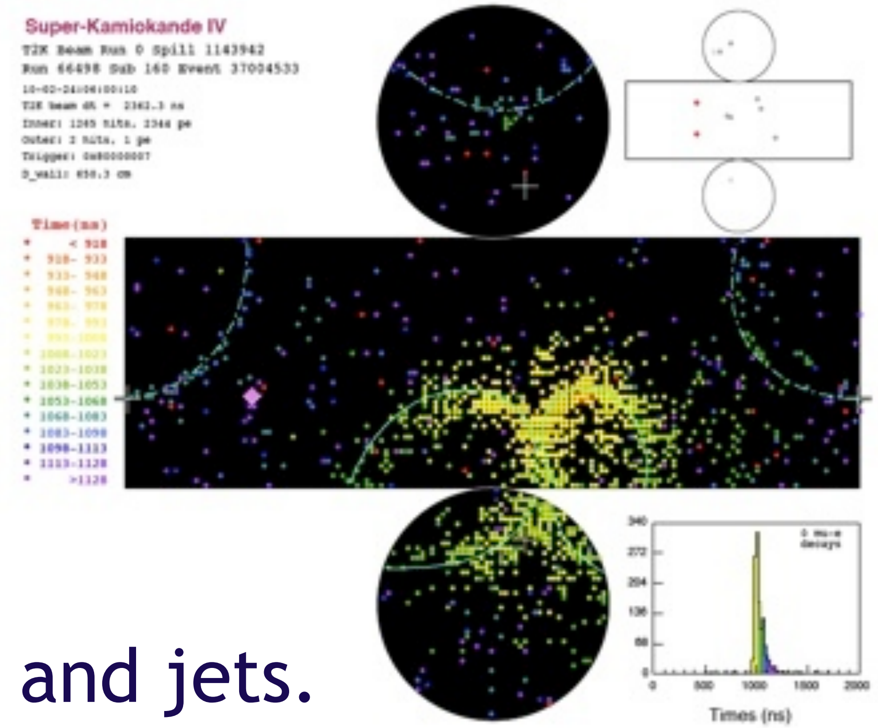


1. Introduction: *The Mysteries of the Standard Model*
2. Forces, feynman diagrams, scattering.
3. Dirac equation & spinors.
4. Electromagnetic interactions: Quantum Electrodynamics (QED).
5. Weak Interactions, Weak decays & Neutrino scattering.
6. Deep inelastic scattering, the parton model & parton density functions.
7. Strong interactions: Quantum Chromodynamics (QCD) and Gluons.
8. Quark model of hadrons. Isospin and Strangeness. Heavy quarks.

Current Topics in Particle Physics

Content to be finalised, but probably including...

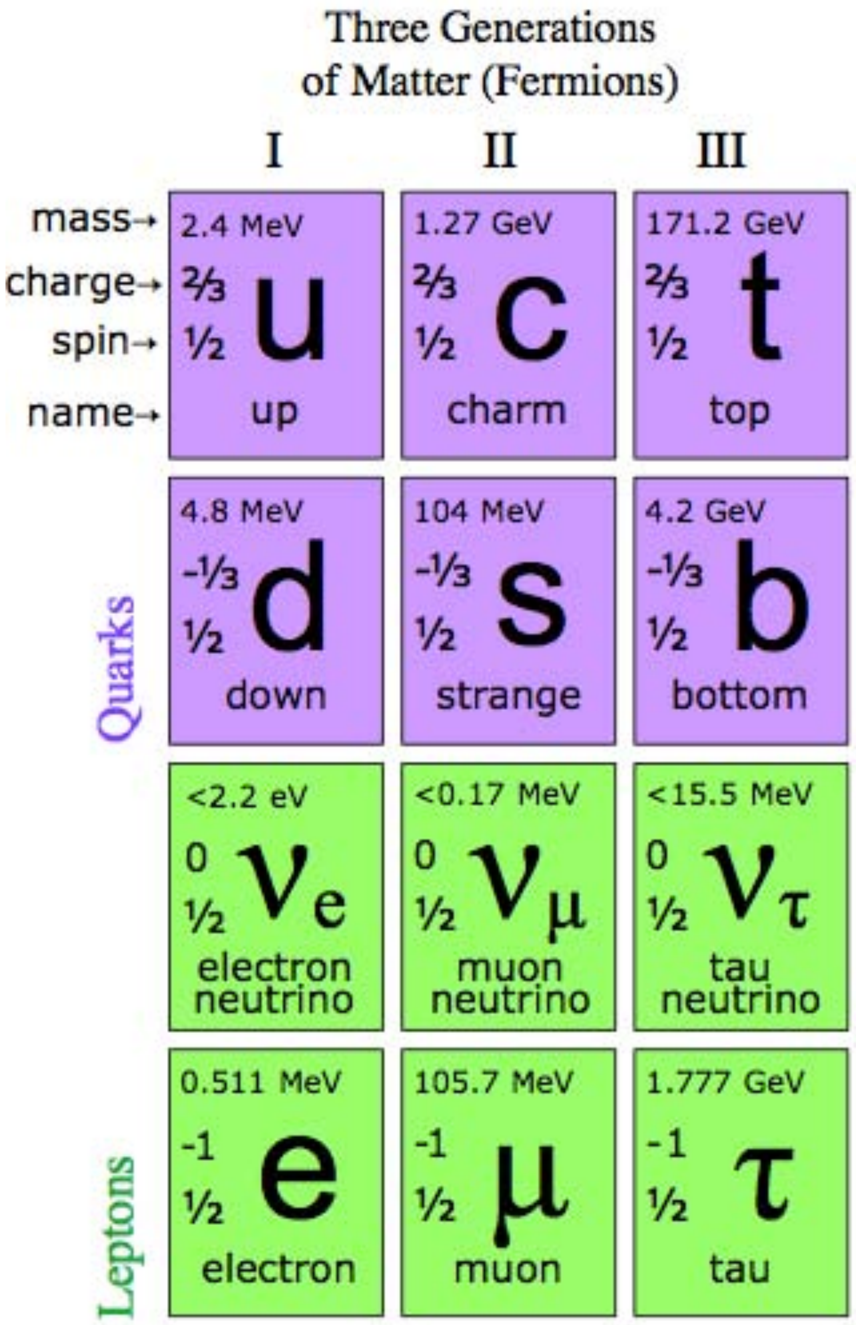




10. Hadron production at Colliders, Fragmentation and jets.
11. Weak decays of hadrons. CKM matrix.
12. Symmetries. Parity. Charge conjugation. Time reversal. CP and CPT.
13. Mixing and CP violation in K and B meson decays.
14. Neutrino oscillations. MNS matrix. Neutrino masses.
15. Electroweak Theory. W and Z boson masses.
16. Spontaneous symmetry breaking and the Higgs boson.
17. Beyond the Standard Model. Supersymmetry. Grand unification.
18. Recent physics results at the LHC.

Standard Model Matter Particles

- Matter particles are observed to be $s=½$ fermions.
- Two distinct types: **quarks** and **leptons**.
- Grouped into three, successively heavier, generations.
- Four key quantum numbers: charge (Q), isospin (I_Z), baryon number (B), lepton number (L)



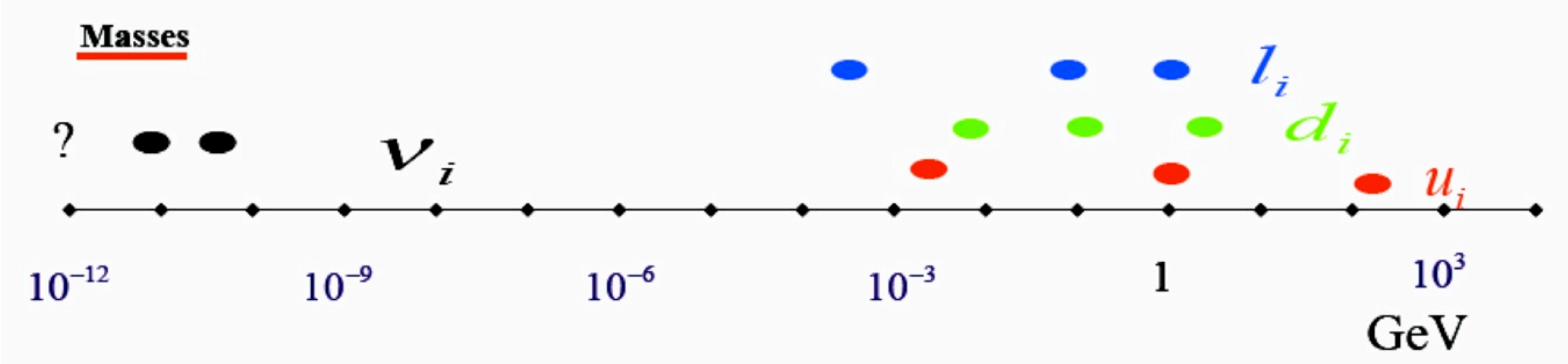
Three Generations of Matter (Fermions)

mass→
charge→
spin→
name→

Q Charge	I_Z Isospin		I	II	III	B Baryon Number	L Lepton Number
+2/3 e	+1/2	Quarks	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top	+1/3	0
-1/3 e	-1/2		4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	+1/3	0
0	+1/2	Leptons	<2.2 eV 0 ½ ν_e electron neutrino	<0.17 MeV 0 ½ ν_μ muon neutrino	<15.5 MeV 0 ½ ν_τ tau neutrino	0	+1
-1 e	-1/2		0.511 MeV -1 ½ e electron	105.7 MeV -1 ½ μ muon	1.777 GeV -1 ½ τ tau	0	+1

Mysteries of the Fermion Masses

- Masses are well measured (apart from the very low mass ν_i) but the hierarchy not understood:



- Logarithmic scale covers 15 orders of magnitude!

- Charged leptons ($\ell_i = e, \mu, \tau$), up-type quarks ($\mathbf{u}_i = \mathbf{u,c,t}$) and down-type quarks ($\mathbf{d}_i = \mathbf{d,s,b}$) quarks have similar masses but the patterns are not identical

- Absolute scale of neutrino (ν_i) masses is unknown apart from upper bound on $\mathbf{m}(\nu_e) < \mathbf{2eV}$
- Only two independent ν_i mass differences are known:

$$\Delta m_{12}^2 = (7.5 \pm 0.2) \text{ x } 10^{-5} \text{ eV}^2,\ \Delta m_{23}^2 = (2.32^{+0.12}{}_{-0.08}) \text{ x } 10^{-3} \text{ eV}^2$$

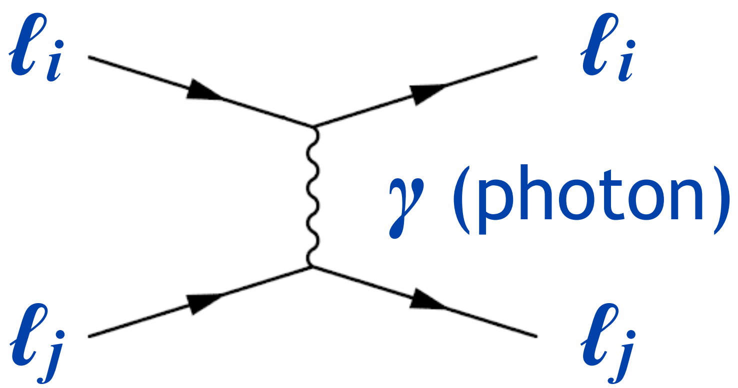
Standard Model Forces

- Four interactions observed in nature: electromagnetic, strong, weak and gravity.
- The Standard Model describes interactions due to electromagnetic, strong, weak.
- Interactions between the fermions are transmitted by "force carrying" gauge bosons with ***S*=*1***.
- Each force couples to a property of the fermions.
- The structure of the interactions of each force are described mathematically by a symmetry group (more on this later)

Interaction	Coupling Strength	Couples To	Gauge Bosons	Charge e	Mass GeV$/c^2$
Strong	$\alpha_s \approx 1$	colour-charge	Gluons (g)	0	0
Electromagnetic	$\alpha = 1/137$	electric charge	Photon (γ)	0	0
Weak	$G_F = 1 \times 10^{-5}$	weak hyercharge	$\left\{ \begin{array}{c} W^{\pm} \\ Z^0 \end{array} \right.$	± 1 0	80.385 ± 0.015 91.1876 ± 0.0021
Gravity	0.53×10^{-38}	mass	Graviton	0	0

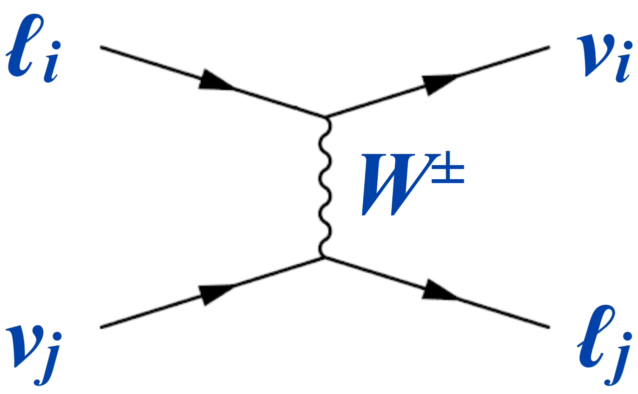
Interactions of the Leptons

Electromagnetic

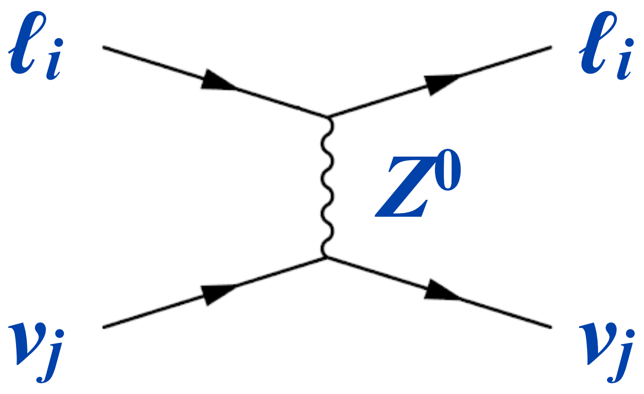


No strong interactions!

Weak (charged)



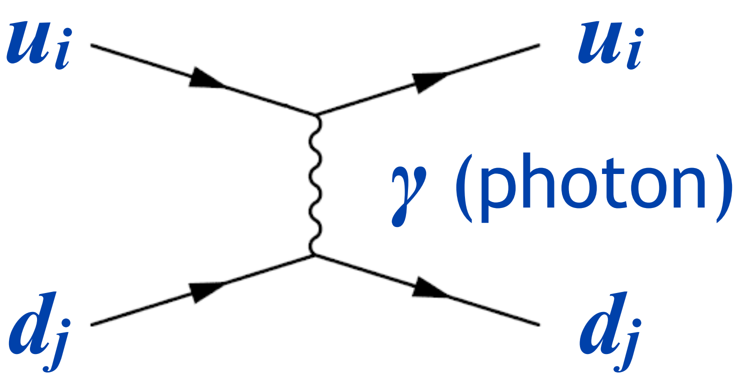
Weak (neutral)



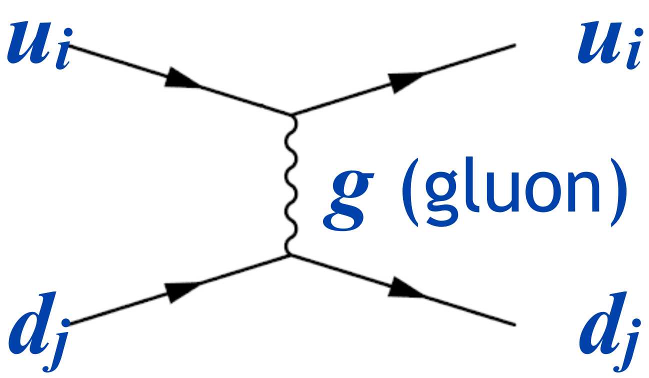
- Leptons interact due to all the electromagnetic and weak forces.
    - All charged leptons ($\ell_i = e, \mu, \tau$) have the same couplings (lepton universality)
    - Only W-boson interactions can cause the leptons to change flavour (from charged to neutral)

Interactions of the Quarks

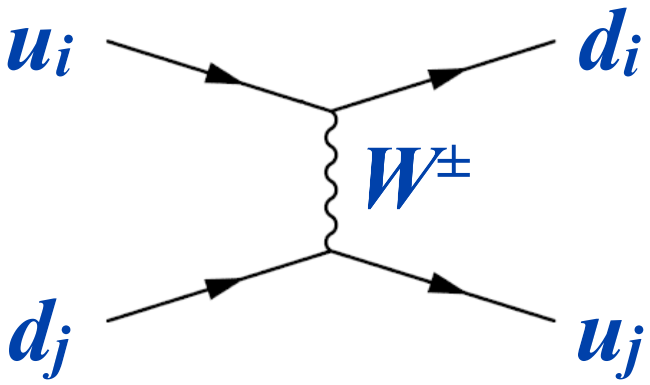
Electromagnetic



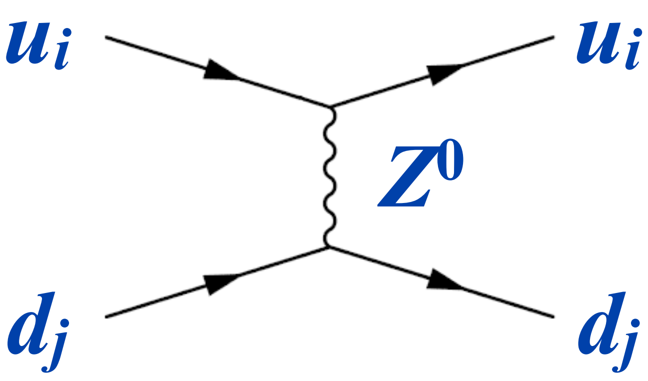
Strong



Weak (charged)



Weak (neutral)



- Quarks interact due to all the forces.
  - All flavours have the same strong force coupling
  - Only W-boson interactions can cause the quarks to change flavour (from up-type to down-type).

Mysteries of the Fermions

- Are the fermions really point-like objects ($r_e < 10^{-20}\mathrm{m}$)?
- Why are there exactly twelve (or 24) elementary fermions?
- Why are there three “generations” with different “flavours”?
- Why do quarks have strong interactions with three “colour charges”?
- Why do weak interactions change quark flavour, but not lepton flavour?
- Why do neutrinos have flavour oscillations?
- Why more matter than anti-matter (baryon asymmetry)?

Mysteries of the Bosons

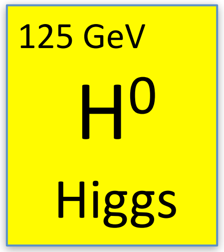
- Electromagnetic and weak interactions are unified at the Electroweak scale (**246 GeV**)
  - ➡ Is there a "grand unified" scale where the strong interaction is also included?

- What is the mechanism that breaks electroweak symmetry, and how does it explain the large masses of the W and Z bosons?

- Are there extra Higgs bosons?

- What are the couplings of Higgs boson(s)?

- How do we include gravity?



Quark Mixing

- Quark flavours are observed to change in ***W***-boson interactions

- Described in the Standard Model with the Cabibbo–Kobayashi–Maskawa quark mixing matrix

$$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{bmatrix}.$$

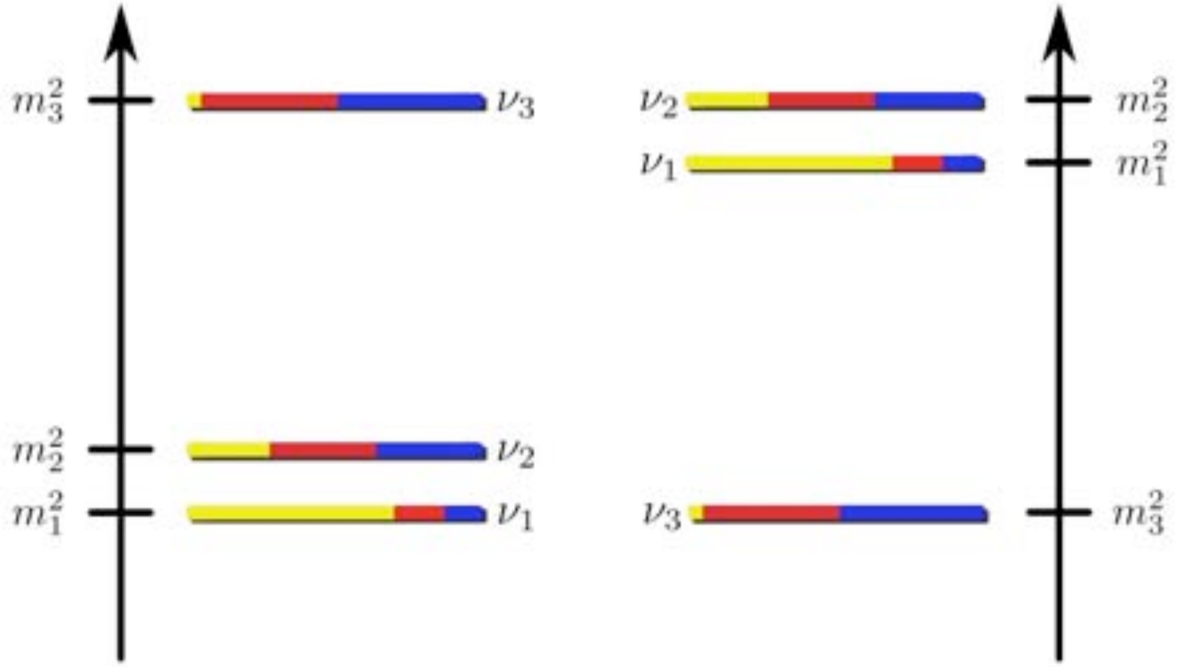
- The parameters of this matrix are experimentally measured, but why this structure!?

reference: Wikipedia http://en.wikipedia.org/wiki/Cabibbo-Kobayashi-Maskawa_matrix

Neutrino Mixing

- Neutrinos are also observed to change flavour e.g. muon neutrinos produced in the atmosphere from cosmic rays $\nu_\mu \rightarrow \nu_\tau$
- Implies neutrinos have mass. The mass eignenstates of the neutrinos are a mixture of ν_e, ν_μ and ν_τ. Two possible solutions for current measurements:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{MNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



$$\mathrm{U_{NH}} = \begin{bmatrix} 0.822 & 0.547 & -0.150+0.0381\mathrm{i} \\ -0.356+0.0198\mathrm{i} & 0.704+0.0131\mathrm{i} & 0.614 \\ 0.442+0.0248\mathrm{i} & -0.452+0.0166\mathrm{i} & 0.774 \end{bmatrix} \quad \mathrm{U_{IH}} = \begin{bmatrix} 0.822 & 0.547 & -0.150+0.0429\mathrm{i} \\ -0.354+0.0224\mathrm{i} & 0.701+0.0149\mathrm{i} & 0.618 \\ 0.444+0.0278\mathrm{i} & -0.456+0.0186\mathrm{i} & 0.770 \end{bmatrix}.$$

- Mixing fractions are experimentally measured. Why this pattern!?
- Neutrinos masses don't really fit into the Standard Model, they imply other particles/interactions we haven't observed yet.

reference: Wikipedia http://en.wikipedia.org/wiki/Pontecorvo-Maki-Nakagawa-Sakata_matrix

The Dark Side

- Only 4.6% of the current universe is normal matter (baryons + electrons = atoms)
- To account for rotation curves of galaxies, gravitational lensing and large scale structure need:

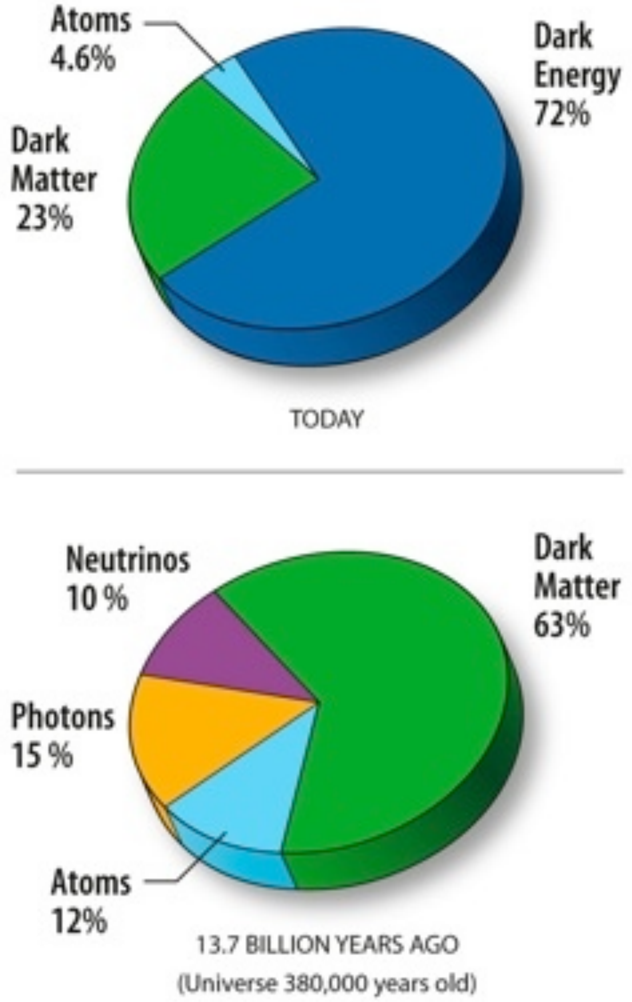
  23.3% "Dark Matter"

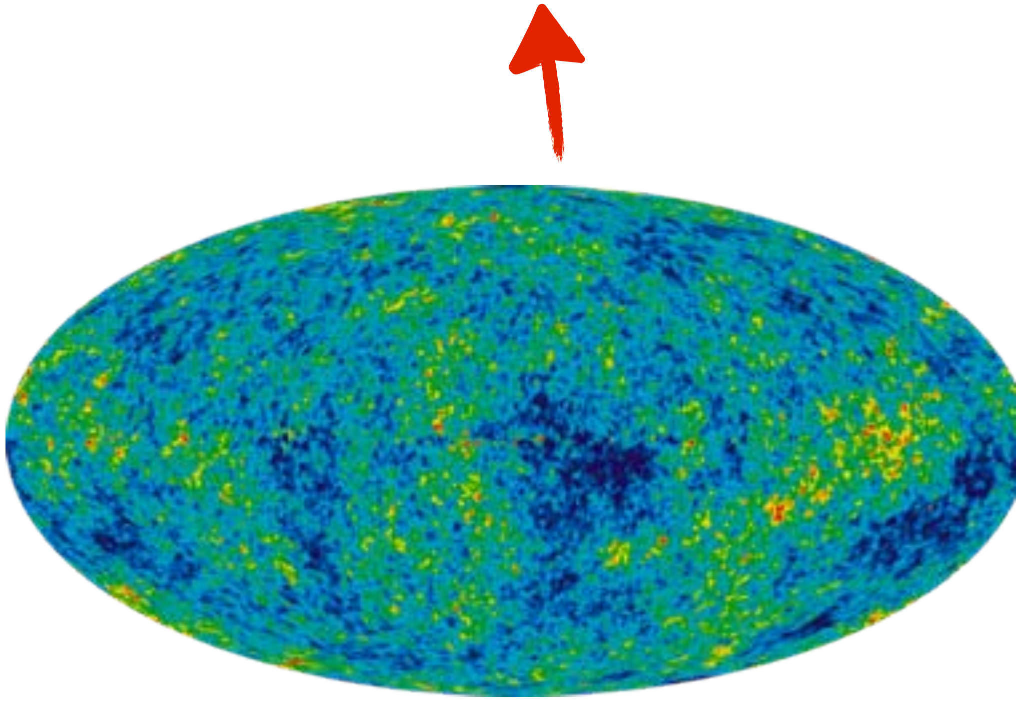
  Must be weakly interactive massive particles (not yet discovered) candidates are provided by a "supersymmetric" extension to the Standard Model
- To account for acceleration of expansion of the universe need:

  72.1% "Dark Energy"

  May be described by a cosmological constant Λ

Could particle physics describe either dark matter or dark energy?





Beyond the Standard Model

Many models proposed to explain some mysteries in the Standard Model, *e.g.*

★ **Supersymmetry (SUSY):** every SM particle has a supersymmetry partner:

- ➡ $S=0$ squarks and sleptons
- ➡ $S=½$ neutralinos, charginos, higgsinos
- ➡ automatically introduces extra Higgs bosons

We are searching for these new particles directly at the LHC. Neutralinos may be candidates for dark matter.

★ **Grand unified theories** merge strong & electroweak interaction at 10^{11} to 10^{16} GeV

- ➡ Proton decay? Lifetime >10^{29} to 10^{33} years (depending on model)

Search for evidence of proton decay

★ **Additional Heavy neutrino(s)** at GUT scale can explain neutrino oscillations and light neutrino masses.

★ **Extra dimension where only gravity interacts**

- ➡ Mini black holes, new resonances

Searches at the LHC.

Particle Physics in 2012

The two big results of the year were:

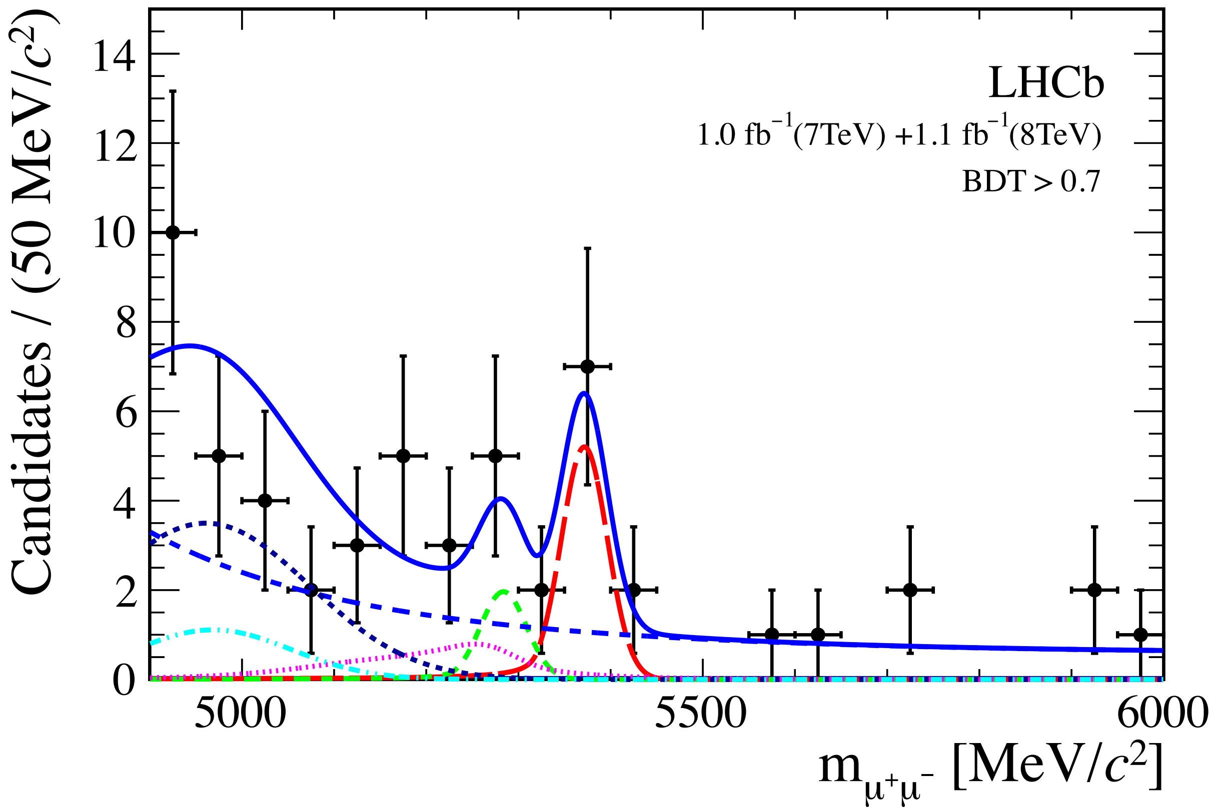
★Discovery of a new boson, very probably the Higgs boson!

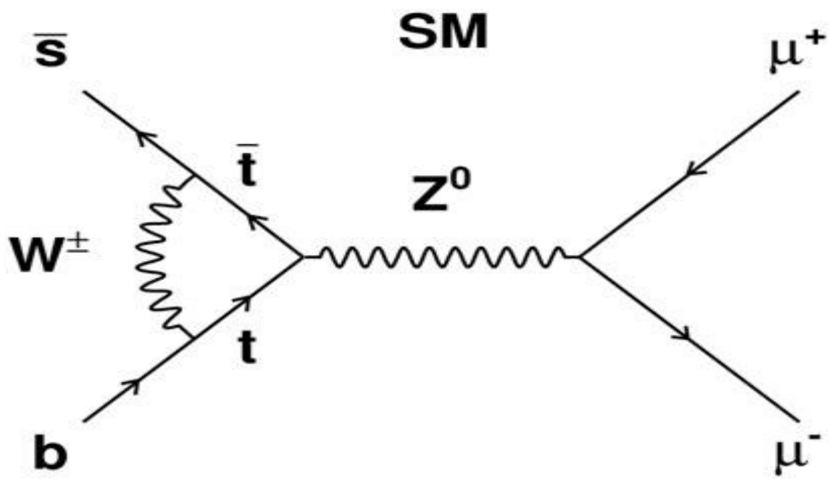
★A first measurement of $B_S \to \mu^+\mu^-$

★A measurement of neutrino mixing angle $\sin\theta_{13}$

Observation of $B_S \to \mu^+\mu^-$

- By LHCb experiment at CERN
- Measured Branching Ratio is
  BR($B_S \to \mu^+\mu^-$) = $(3.2 \pm^{1.5}{}_{1.2}) \times 10^{-9}$
- Compatible with the prediction of the Standard Model
- Better measurements could limit the contributions from non-Standard Model processes

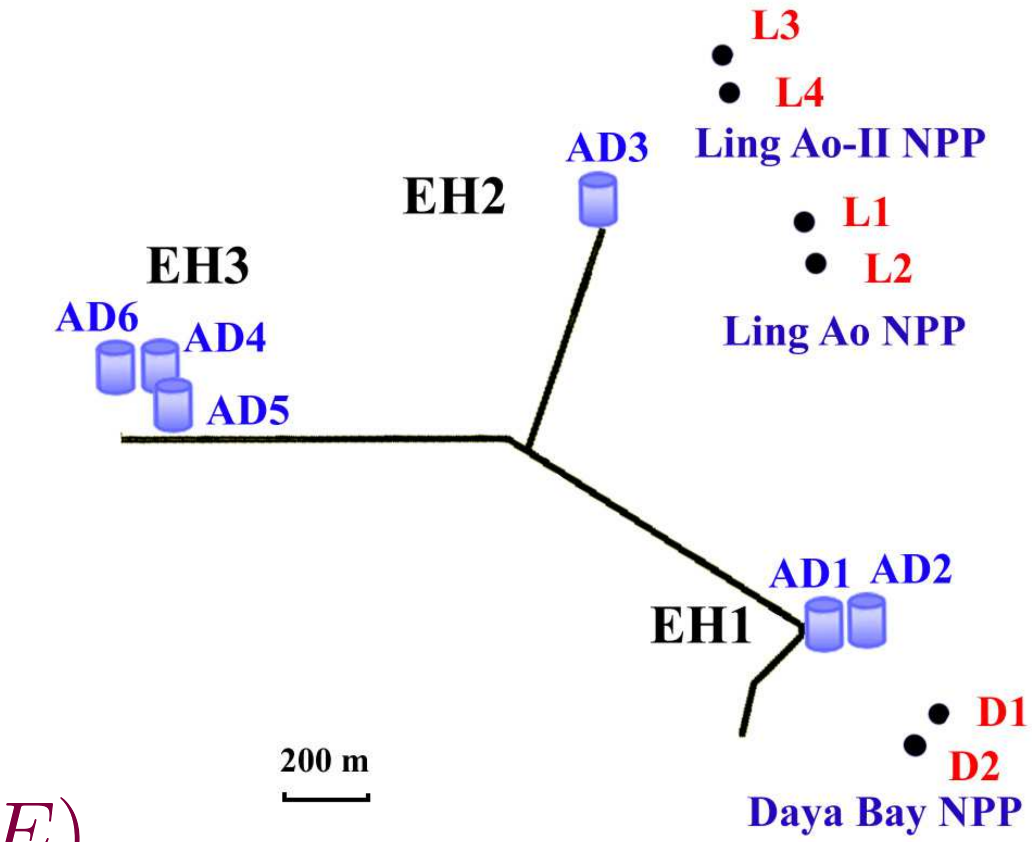




reference: http://arxiv.org/abs/1211.2674

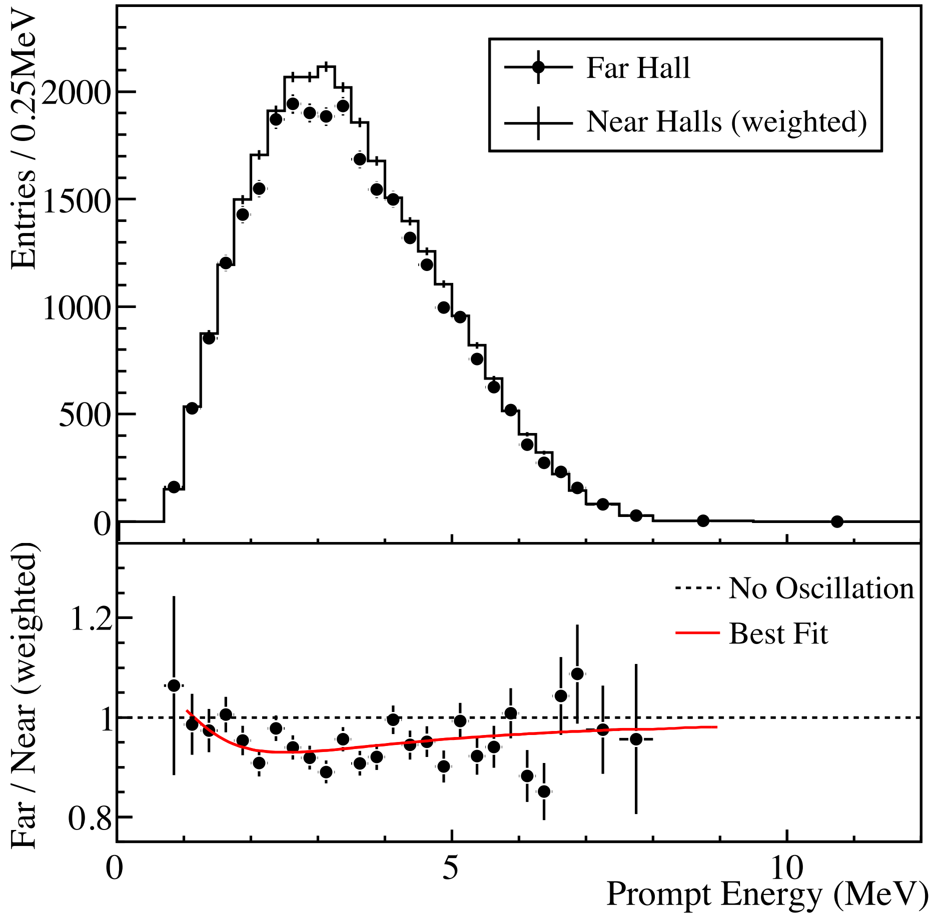
Electron Neutrino Disappearance

- Day Bay experiment in South China
- Sensitive to electron anti-neutrinos ($\bar{\nu}_e$) from six nuclear reactors (**D**, **L**) detected by six detectors (**AD**).
- Look at difference between detection rates between near (**EH1, EH2**) and far (**EH3**) detectors.



$$P_{\text{survival}} \approxeq 1 - \sin^2 2\theta_{13} \sin^2(1.267 \Delta m^2_{31} L/E)$$

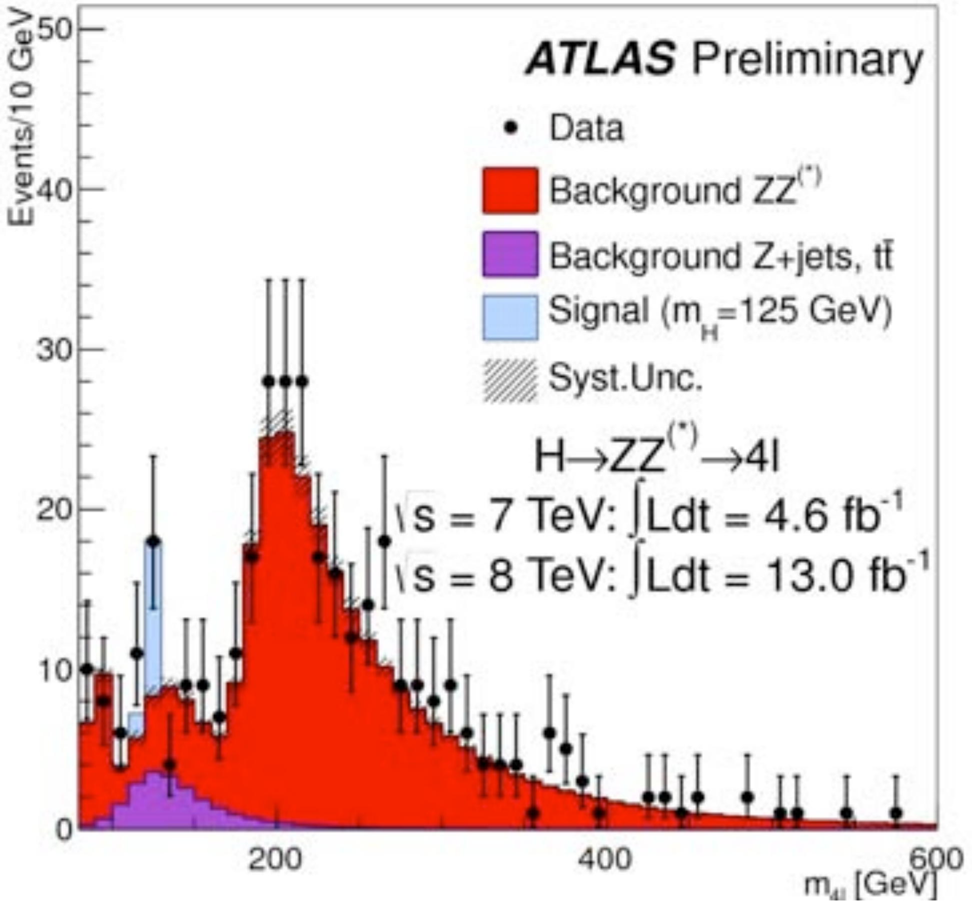
- $\Delta m_{31}{}^2 = 2.23 \pm {}^{0.12}_{0.08}\ \mathrm{meV}^2$ measured from the atmospheric reactions
- E is the energy of $\bar{\nu}_e$ in **MeV**
- L is the distance of between detectors in metres.
- Measurement is $\sin^2\theta_{13} = 0.0089 \pm 0.0011$

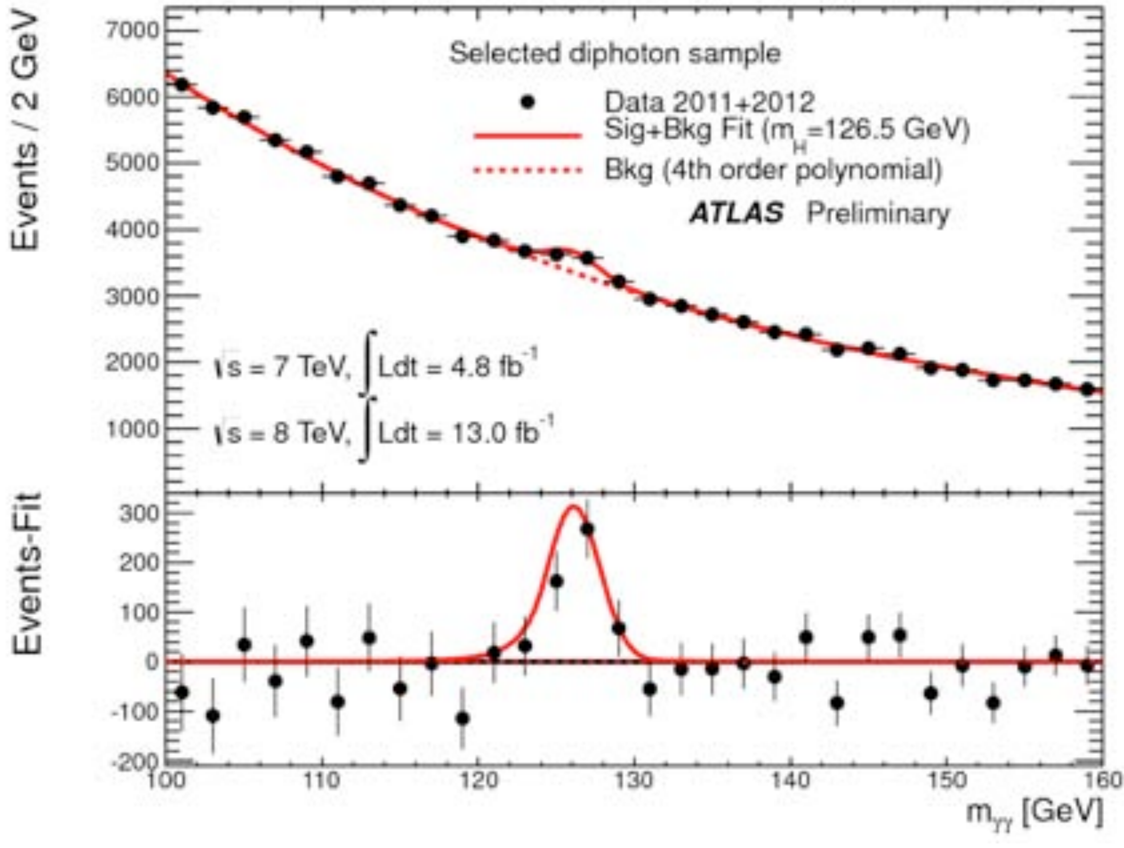


reference: http://arxiv.org/abs/1210.6327

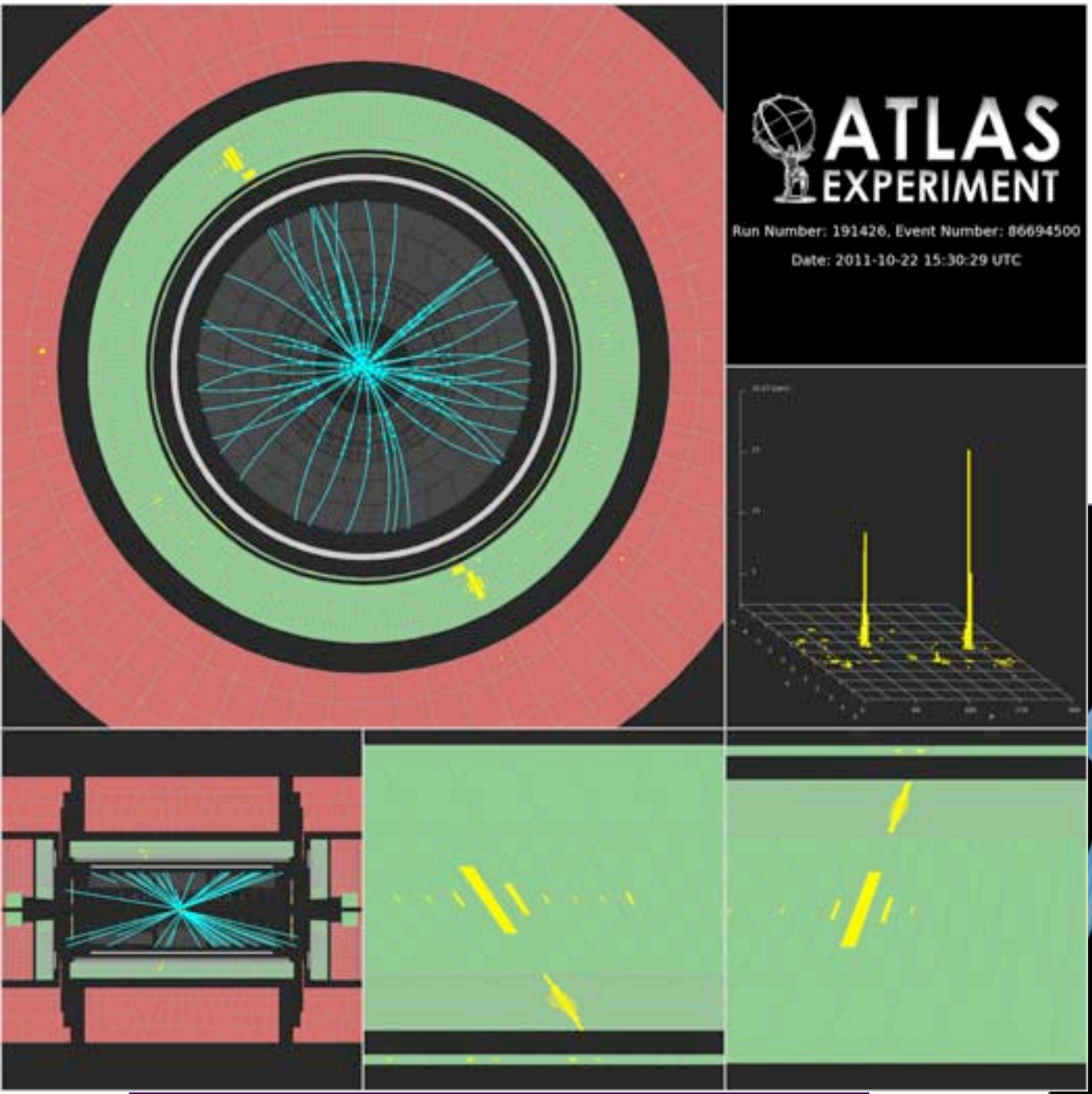
Discovery of the Higgs Boson

- ATLAS and CMS experiments at CERN
- "Bumps" observed in invariant mass at **$m \approx 125$ GeV** in:
  - ➡ $\gamma\gamma$
  - ➡ $\ell^+\ell^-\ell^+\ell^-$ $(\ell=\{e,\mu\})$
- Consistent with $H\to\gamma\gamma$ and $H\to ZZ\to 4\ell$ production
- Statistical significance of the excess is now 7 σ from ATLAS alone!



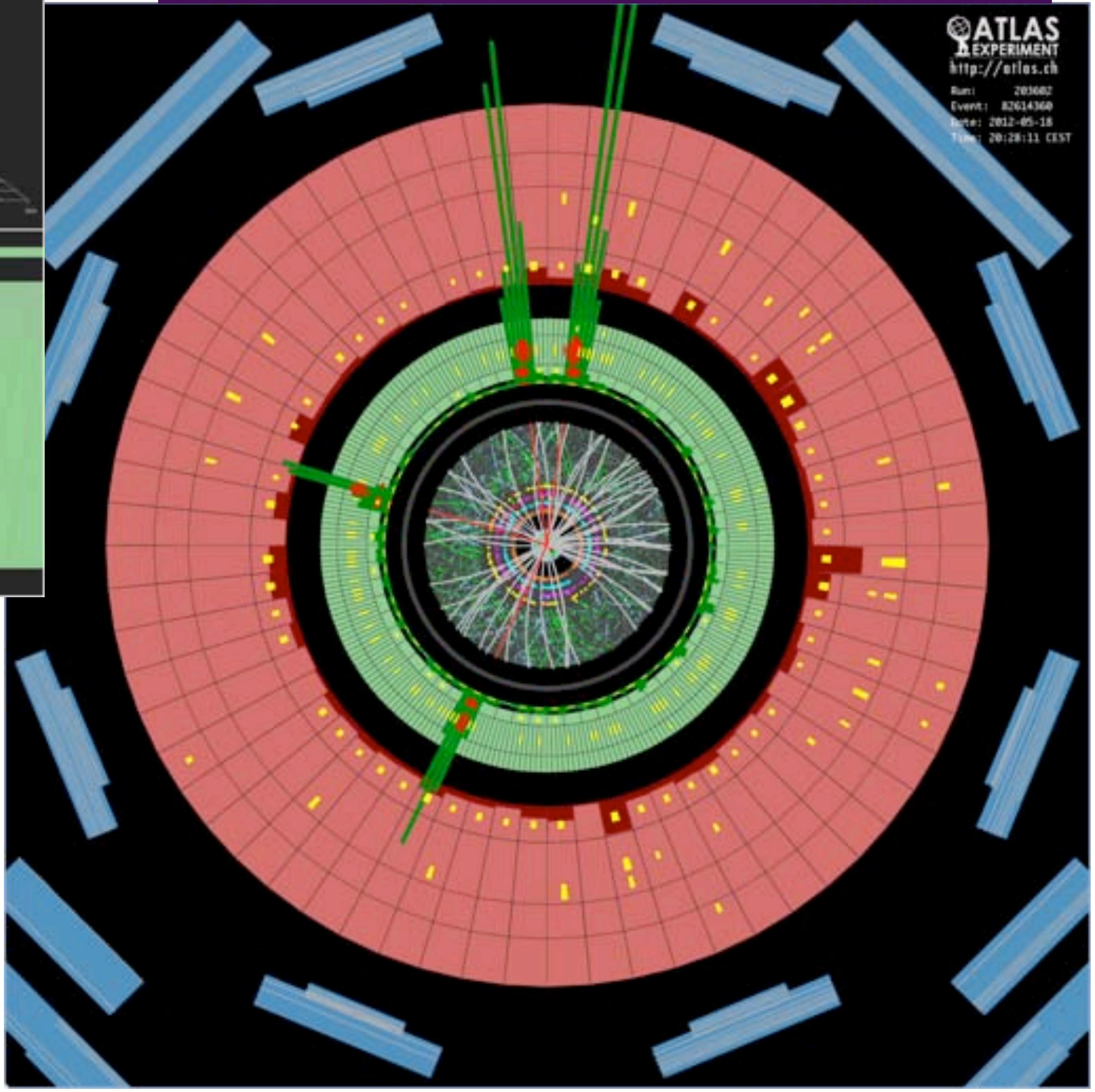


References: https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-170/, ATLAS-CONF-2012-169/, ATLAS-CONF-2012-168/



$H \rightarrow \gamma\gamma$ candidate event

$H \rightarrow ZZ \rightarrow 4e$ candidate event



December 2012

- Fabiola Gianotti is named Time magzine Person of the Year 2012, runner up
- Higgs boson is particle of year 2012.
- Professor Higgs awarded Membership of the Order of the Companions of Honour by Queen Elizabeth II
- Alan Walker is awarded an MBE for services to science engagement and science education in Scotland.

http://www.ph.ed.ac.uk/news/new-years-honours-2013-08-01-13

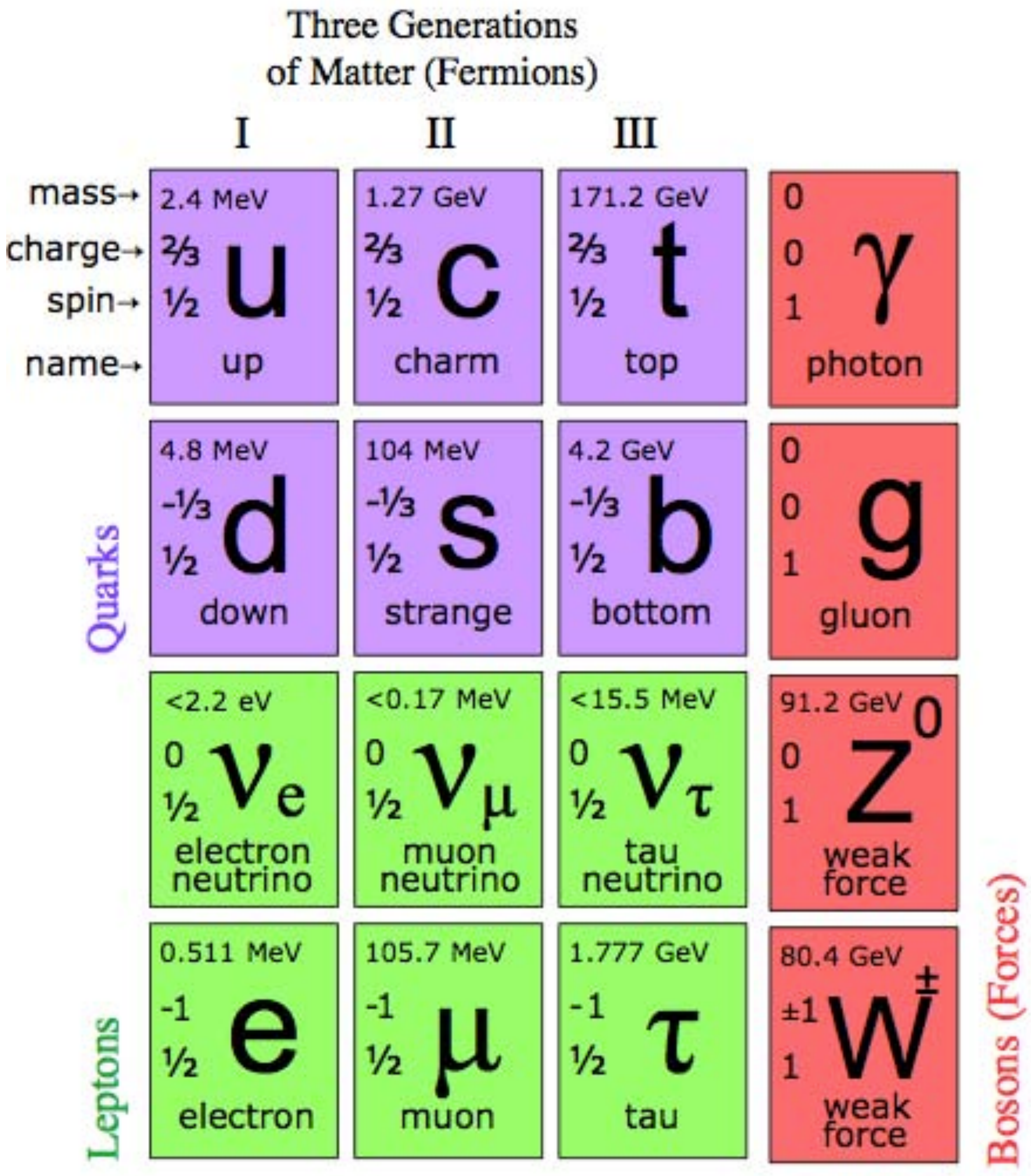
Prof Higgs visits ATLAS

Summary & Reading List

- Summary: the Standard Model is our current model for particle physics. But it doesn't explain all observations.

- Experiments are underway to try to make precise measurements and search for new phenomena.

- Key point from today: learn/review the Standard Model particles and forces.



Three Generations of Matter (Fermions)

		I	II	III	
Quarks	mass→ charge→ spin→ name→	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top	0 0 1 γ photon
		4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	0 0 1 g gluon
Leptons		<2.2 eV 0 ½ ν_e electron neutrino	<0.17 MeV 0 ½ ν_μ muon neutrino	<15.5 MeV 0 ½ ν_τ tau neutrino	91.2 GeV 0 1 Z^0 weak force
		0.511 MeV -1 ½ e electron	105.7 MeV -1 ½ μ muon	1.777 GeV -1 ½ τ tau	80.4 GeV ±1 1 $W^\pm$ weak force

Highly suggested reading:

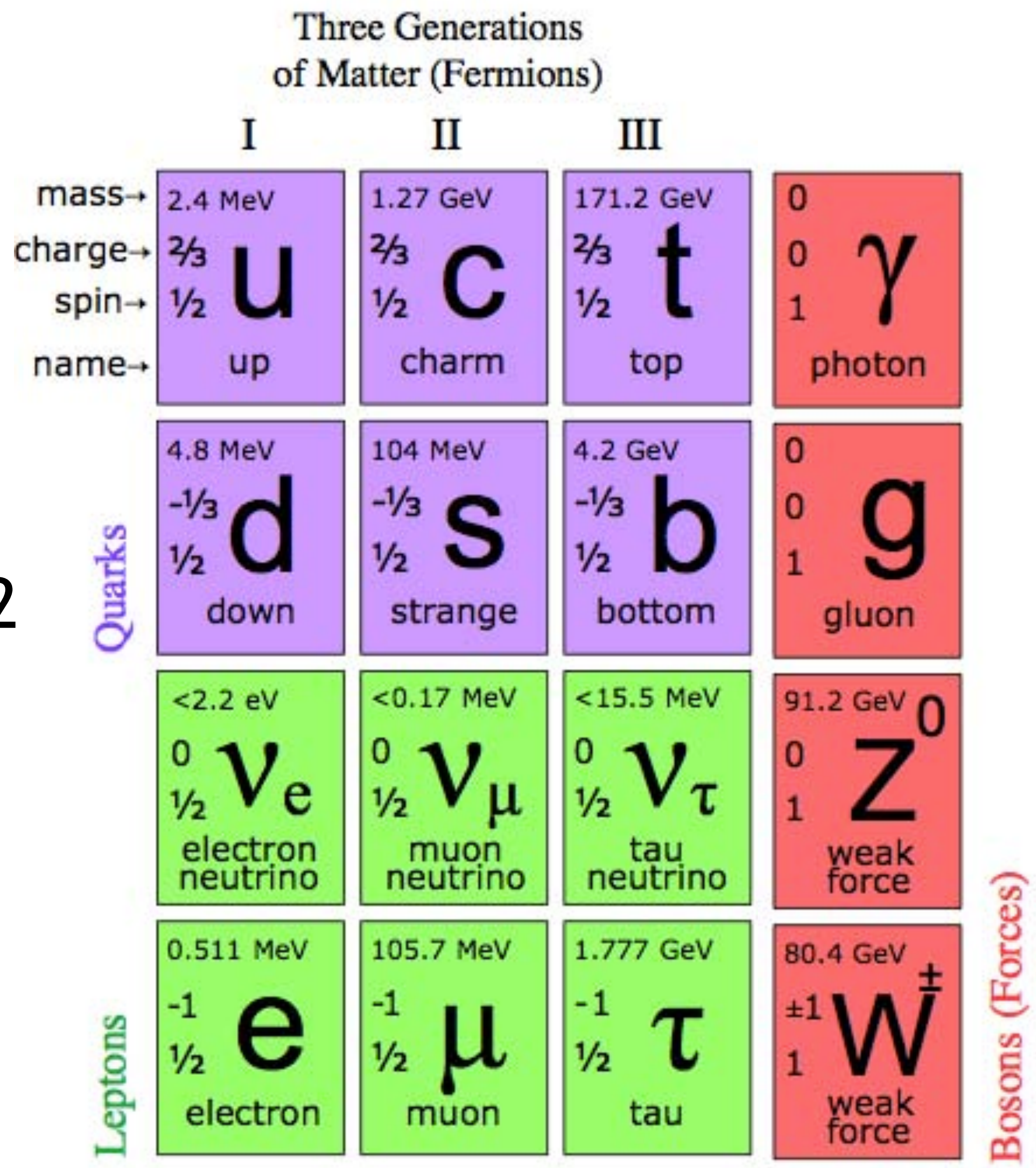
- Today's lecture: Griffiths 1.1 -1.5
- Friday's Lecture: Griffiths chapter 2

From Tuesday: Summary

- Summary: the Standard Model is our current model for particle physics. But it doesn't explain all observations.

Highly suggested reading:

- Tuesday's lecture: Griffiths 1.1 -1.5
- Today's Lecture: Griffiths chapters 2 & 6



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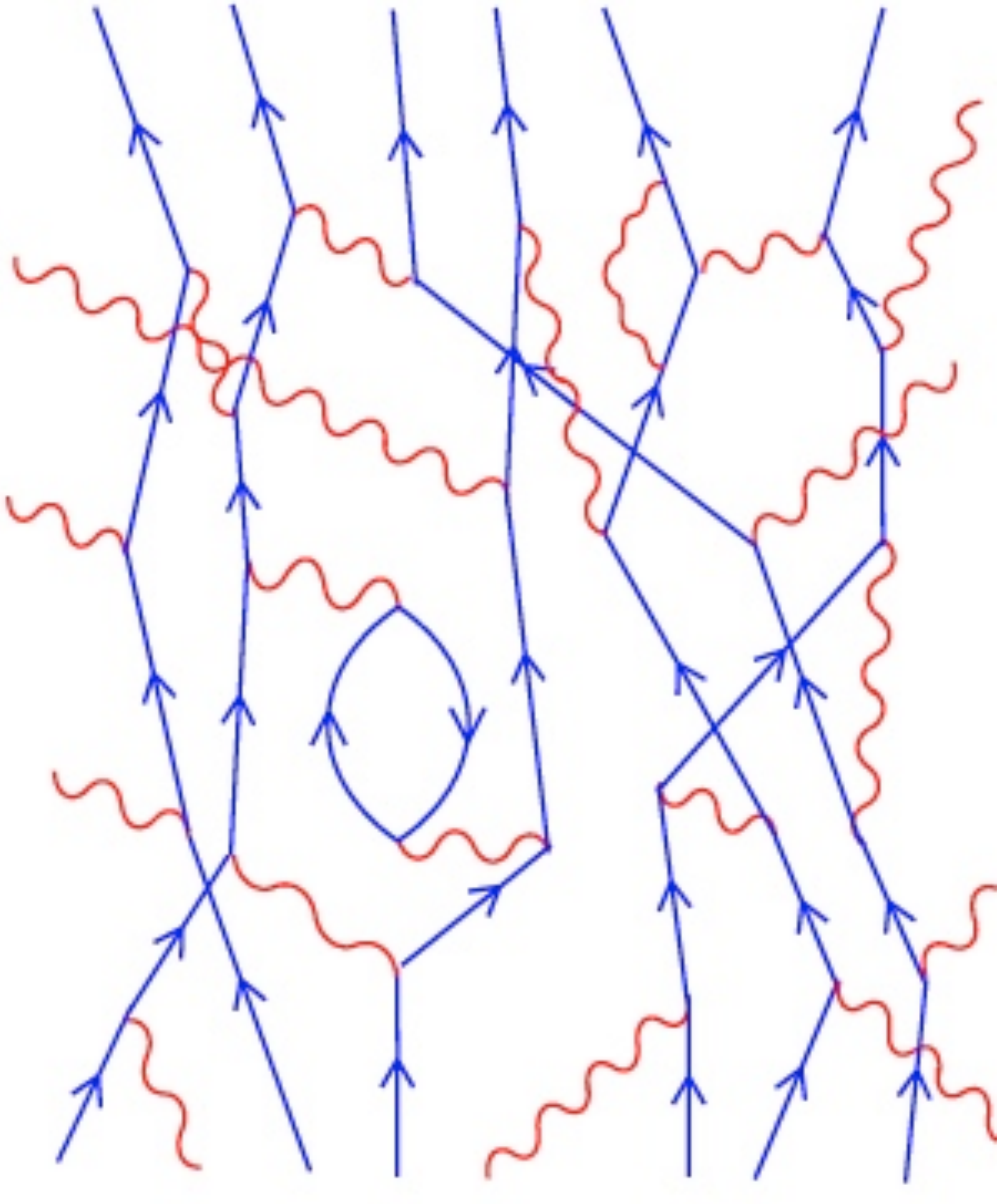
Lecture 2: Feynman Diagrams



The Plenum in Particle QED

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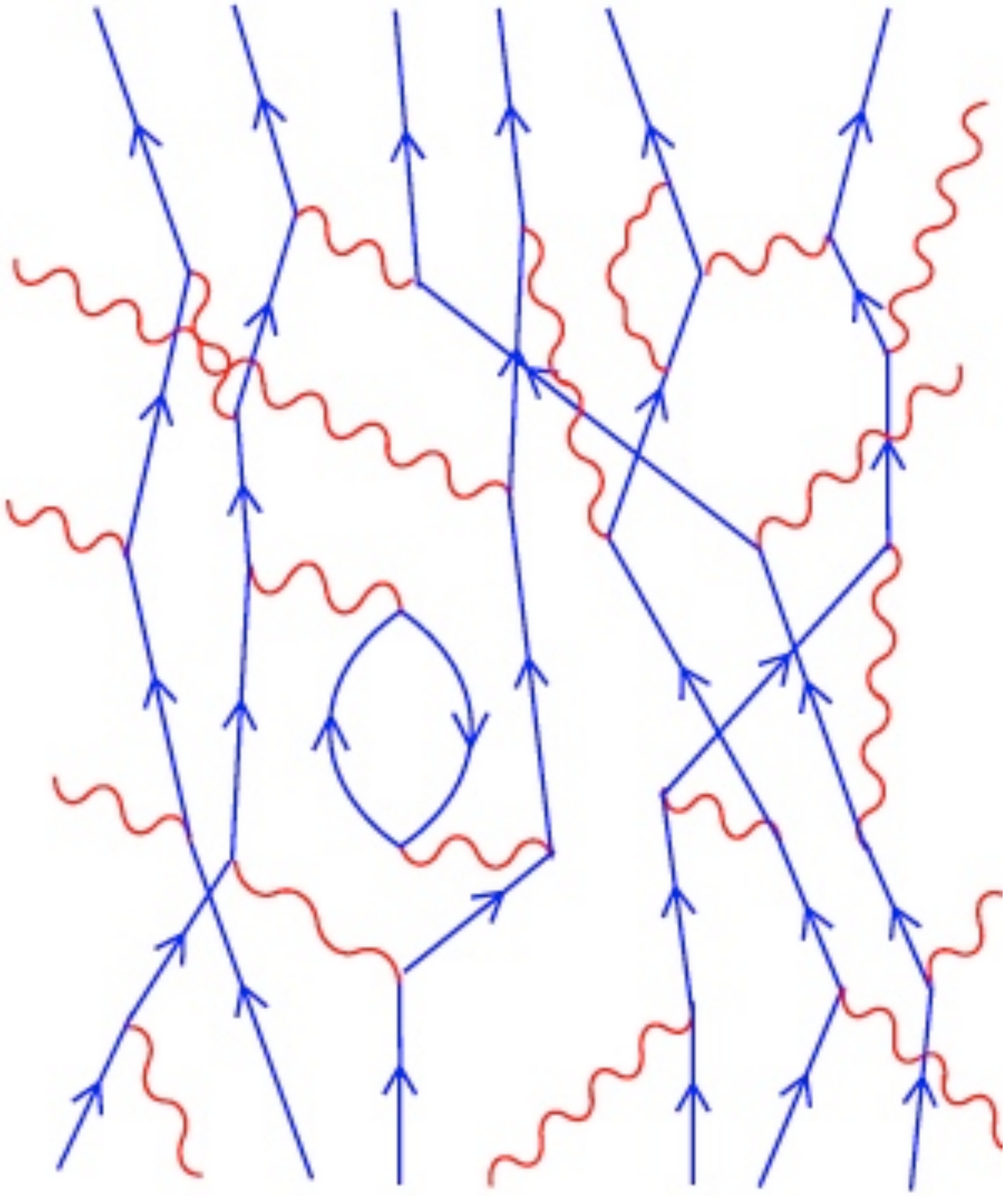

The Plenum in Particle QED

★2012 highlights

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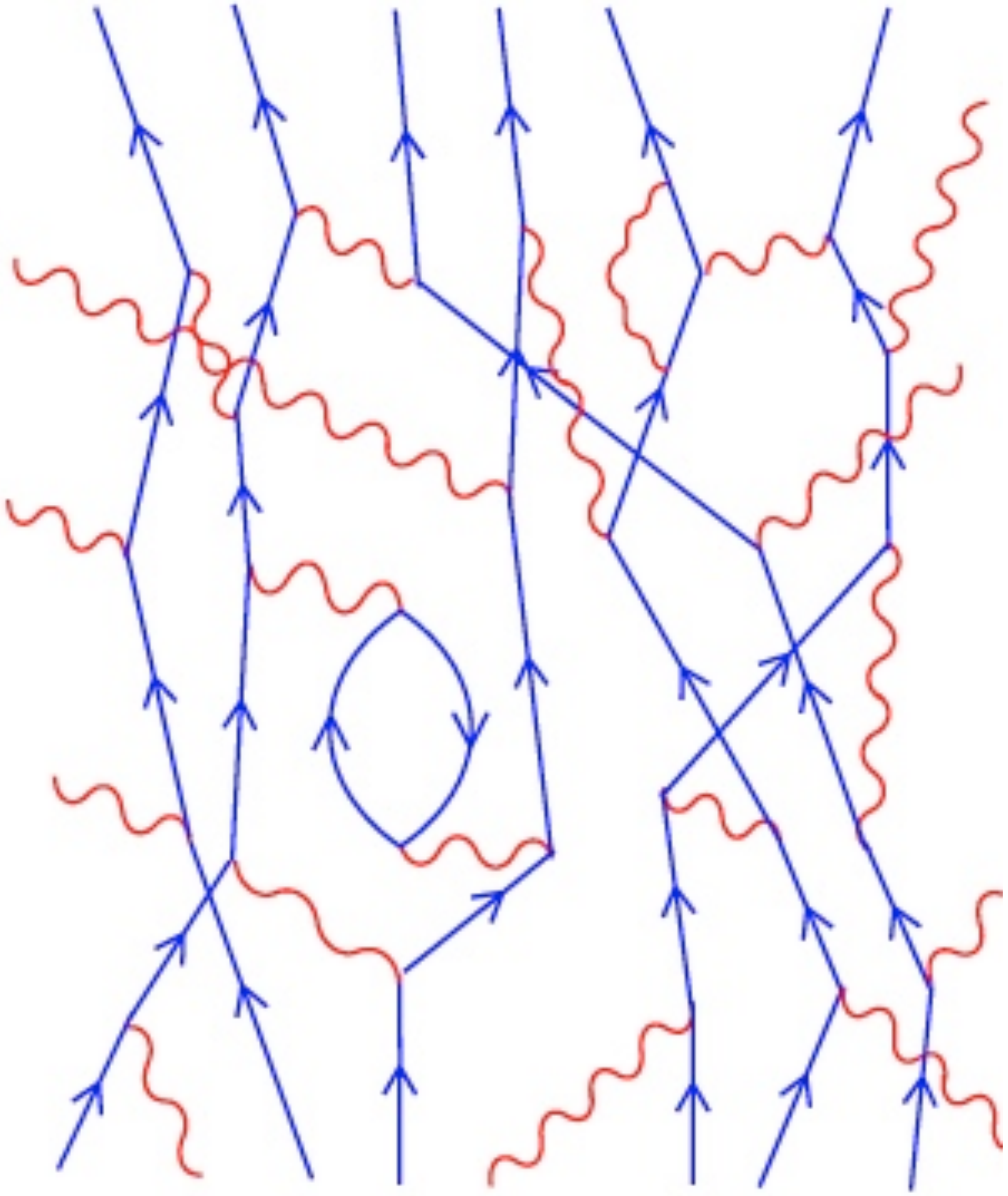
Dr Victoria Martin, Spring Semester 2012

Lecture 2: Feynman Diagrams



The Plenum in Particle QED

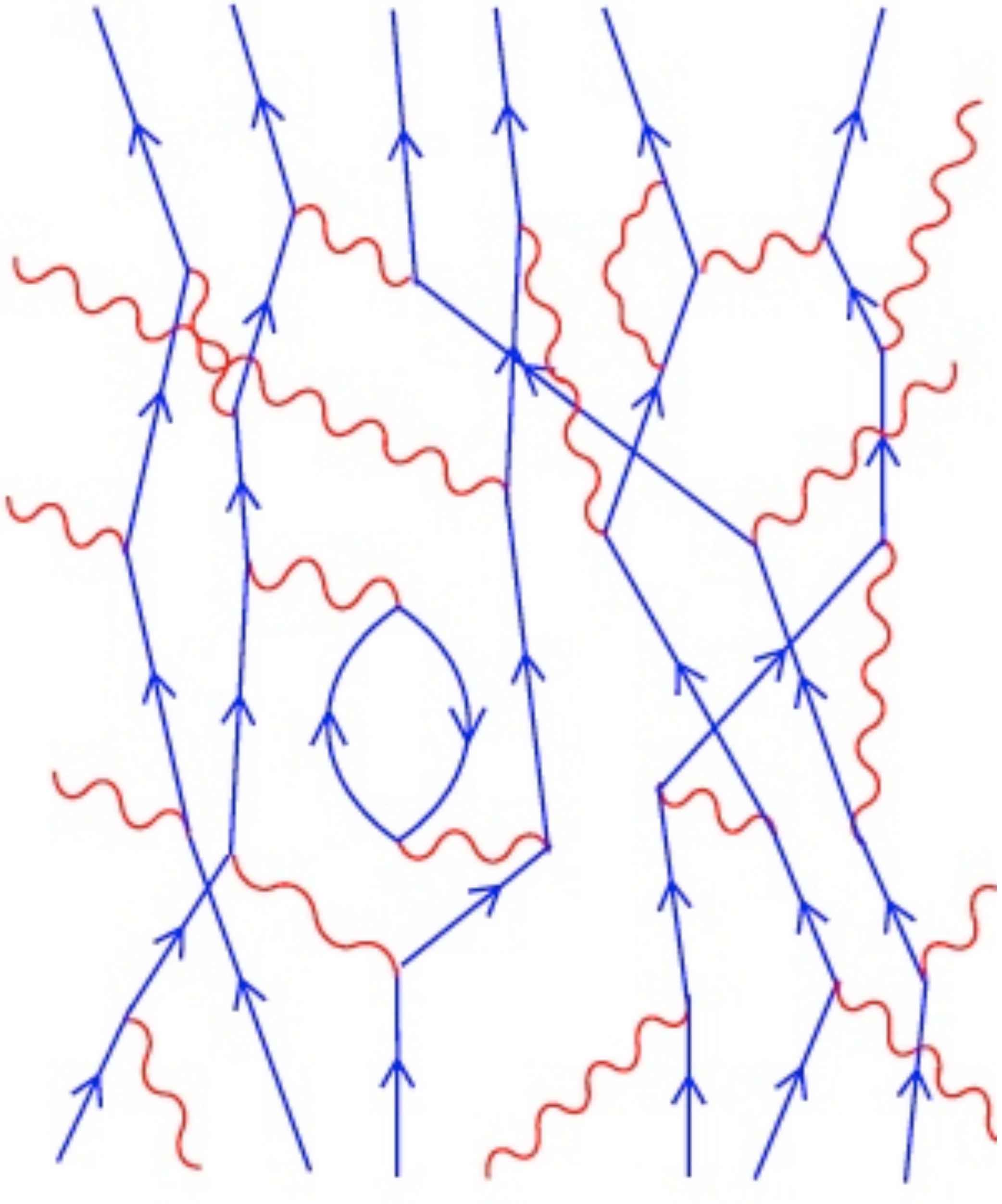
★2012 highlights

★Decays and Scatterings

Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 2: Feynman Diagrams



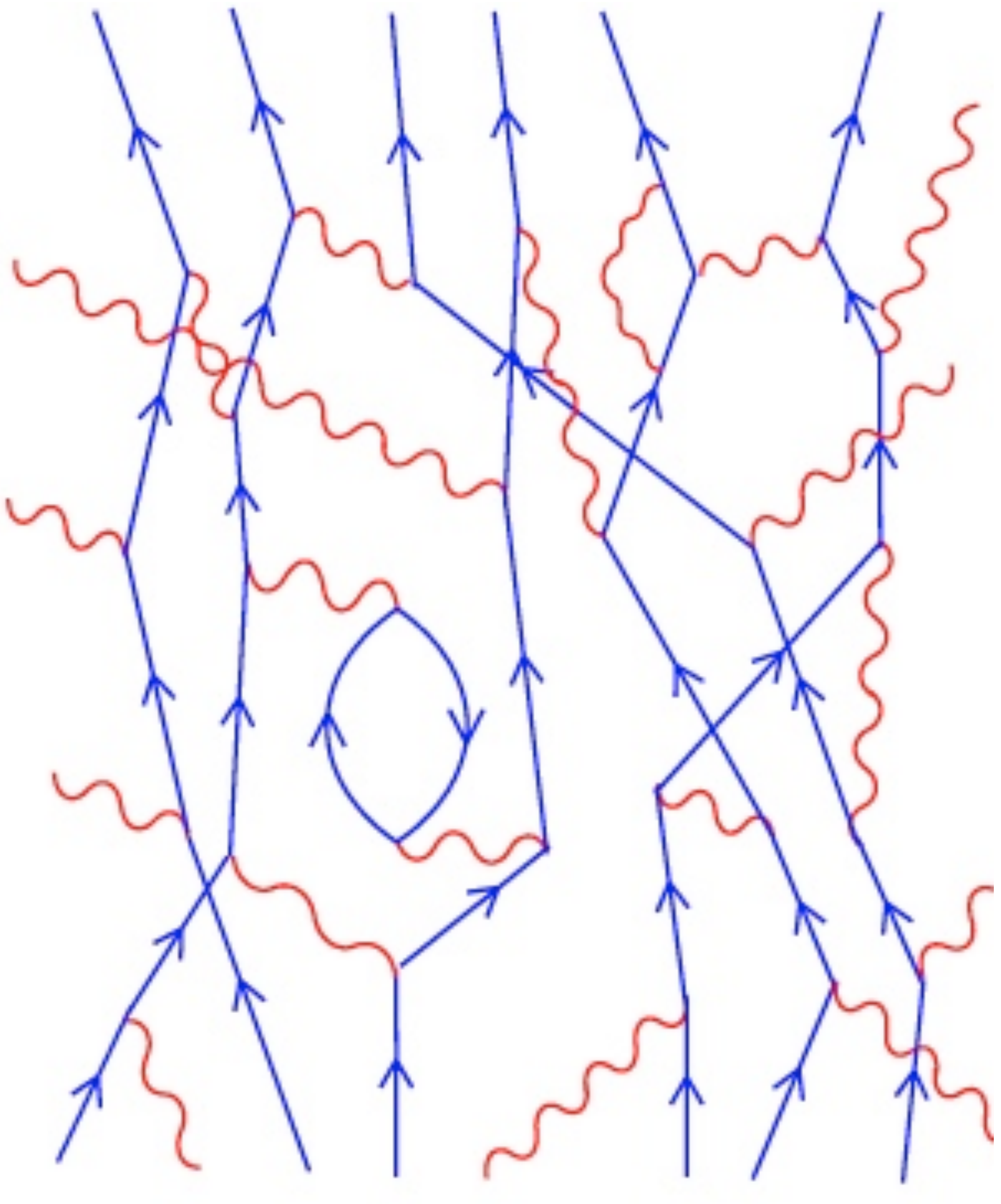
The Plenum in Particle QED

- ★2012 highlights
- ★Decays and Scatterings
- ★Feynman Diagrams

Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 2: Feynman Diagrams



The Plenum in Particle QED

- ★2012 highlights
- ★Decays and Scatterings
- ★Feynman Diagrams
- ★Fermi's Golden Rule

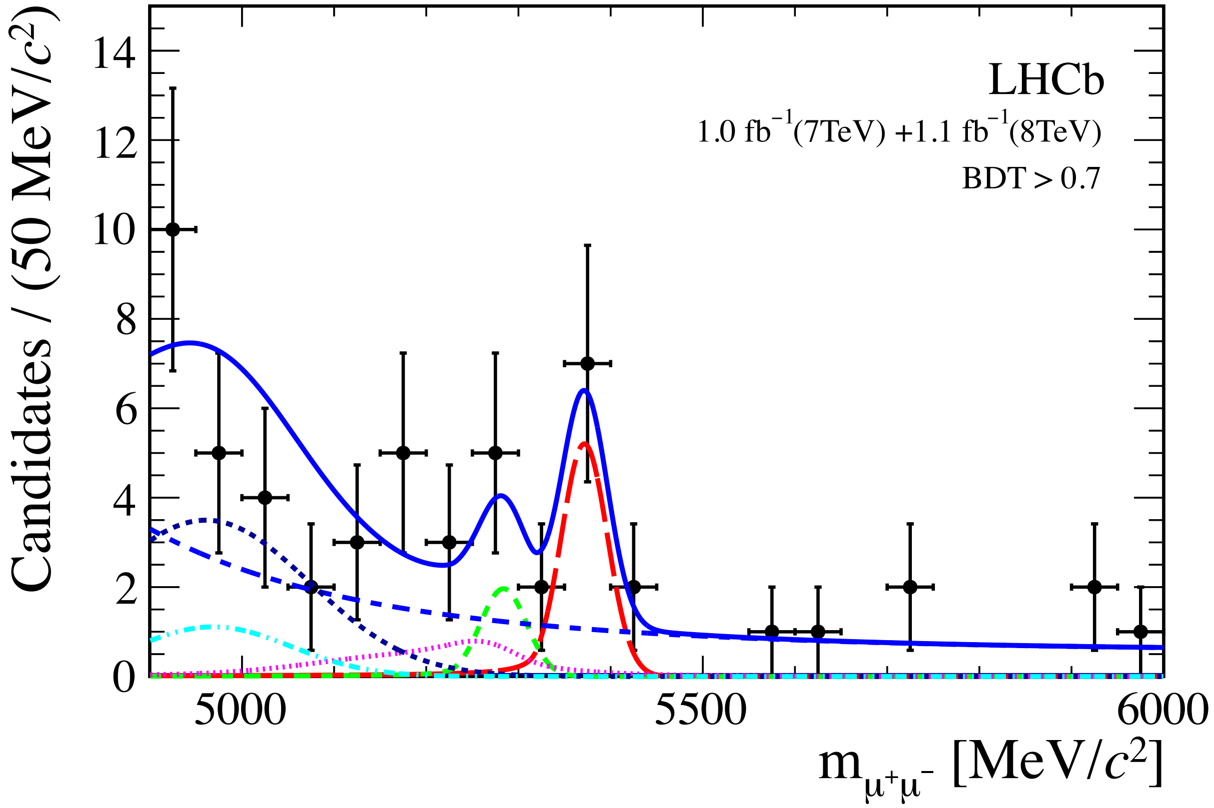
Particle Physics in 2012

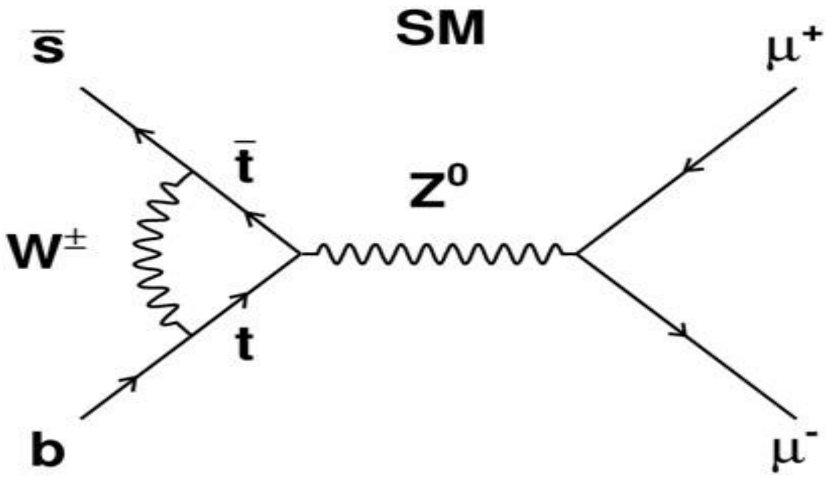
Three big results of the year were:

- ★Discovery of a new boson, very probably the Higgs boson!
- ★A first measurement of $B_S \rightarrow \mu^+\mu^-$
- ★A measurement of neutrino mixing angle $\sin\theta_{13}$

Observation of $B_S \rightarrow \mu^+\mu^-$

- By LHCb experiment at CERN
- Measured Branching Ratio is
  $\mathbf{BR}(B_S \rightarrow \mu^+\mu^-) = (3.2 \pm^{1.5}{}_{1.2}) \times 10^{-9}$
- Compatible with the prediction of the Standard Model
- Better measurements could limit the contributions from non-Standard Model processes

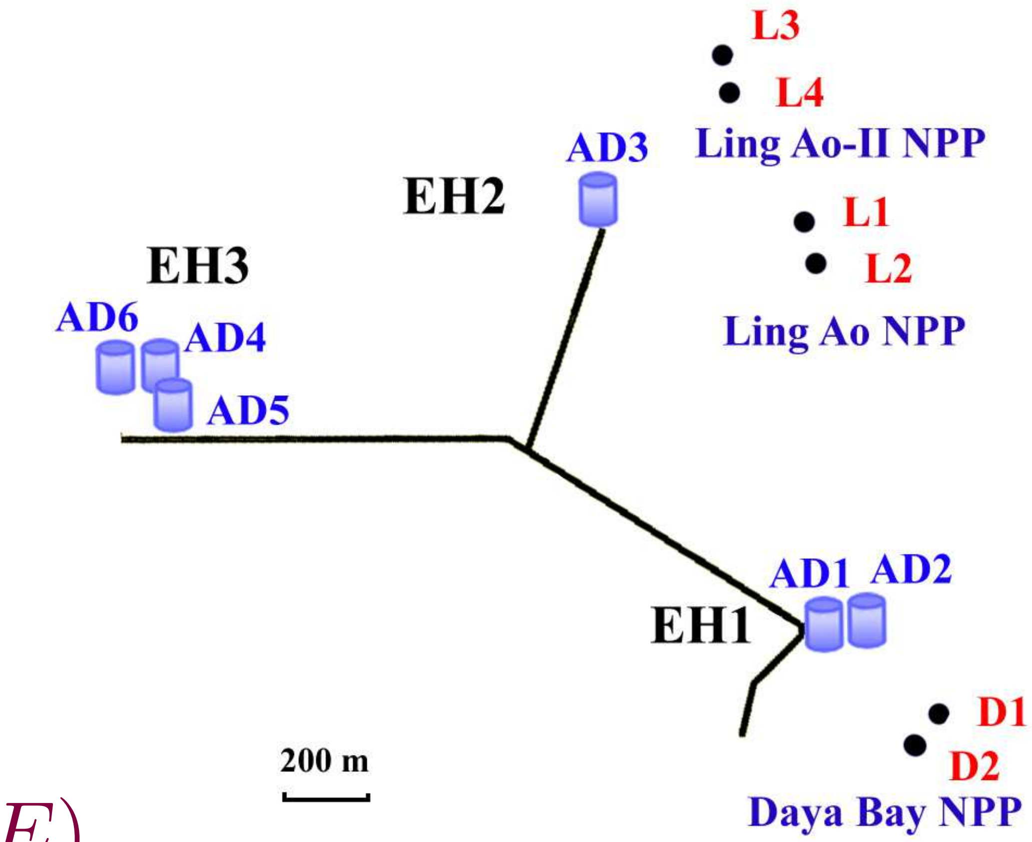




reference: http://arxiv.org/abs/1211.2674

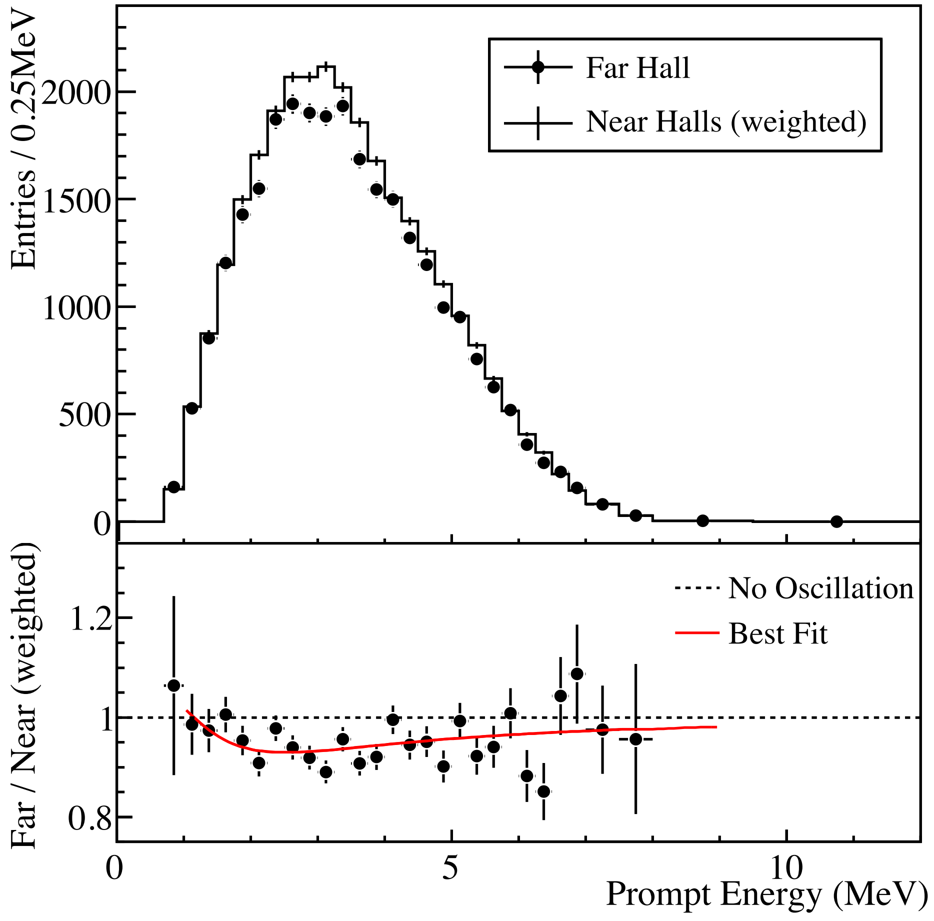
Electron Neutrino Disappearance

- Day Bay experiment in South China
- Sensitive to electron anti-neutrinos ($\bar{\nu}_e$) from six nuclear reactors (**D**, **L**) detected by six detectors (**AD**).
- Look at difference between detection rates between near (**EH1, EH2**) and far (**EH3**) detectors.



$$P_{\text{survival}} \approxeq 1 - \sin^2 2\theta_{13} \sin^2(1.267 \Delta m^2_{31} L/E)$$

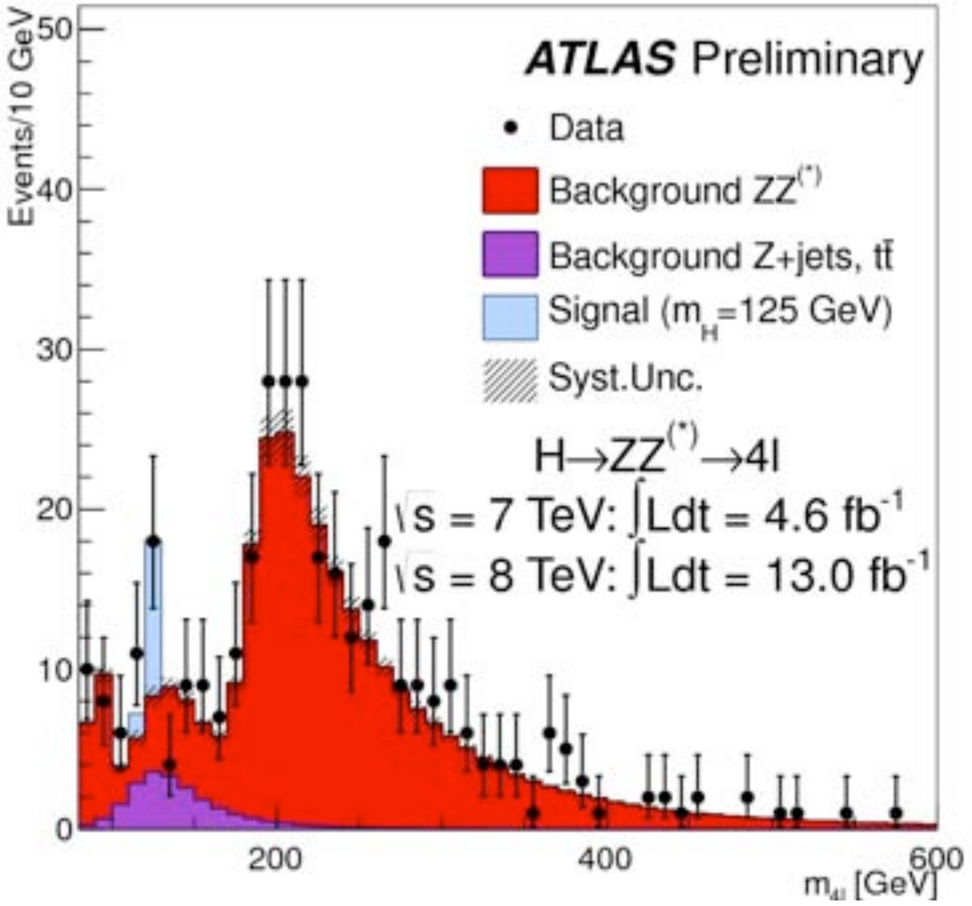
- $\Delta m_{31}^2 = 2.23 \pm {}^{0.12}_{0.08}$ **meV**^{2} measured from the atmospheric reactions
- E is the energy of $\bar{\nu}_e$ in **MeV**
- L is the distance of between detectors in metres.
- Measurement is $\sin^2\theta_{13} = 0.0089 \pm 0.0011$

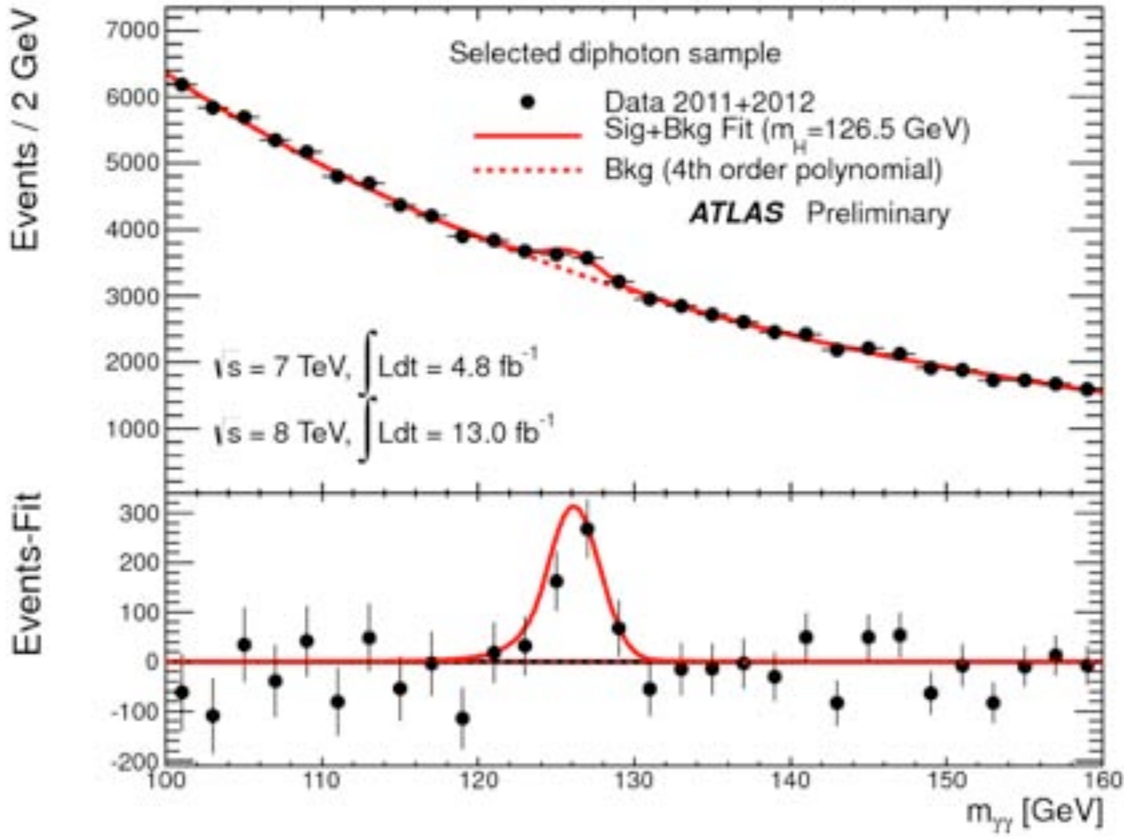


reference: http://arxiv.org/abs/1210.6327

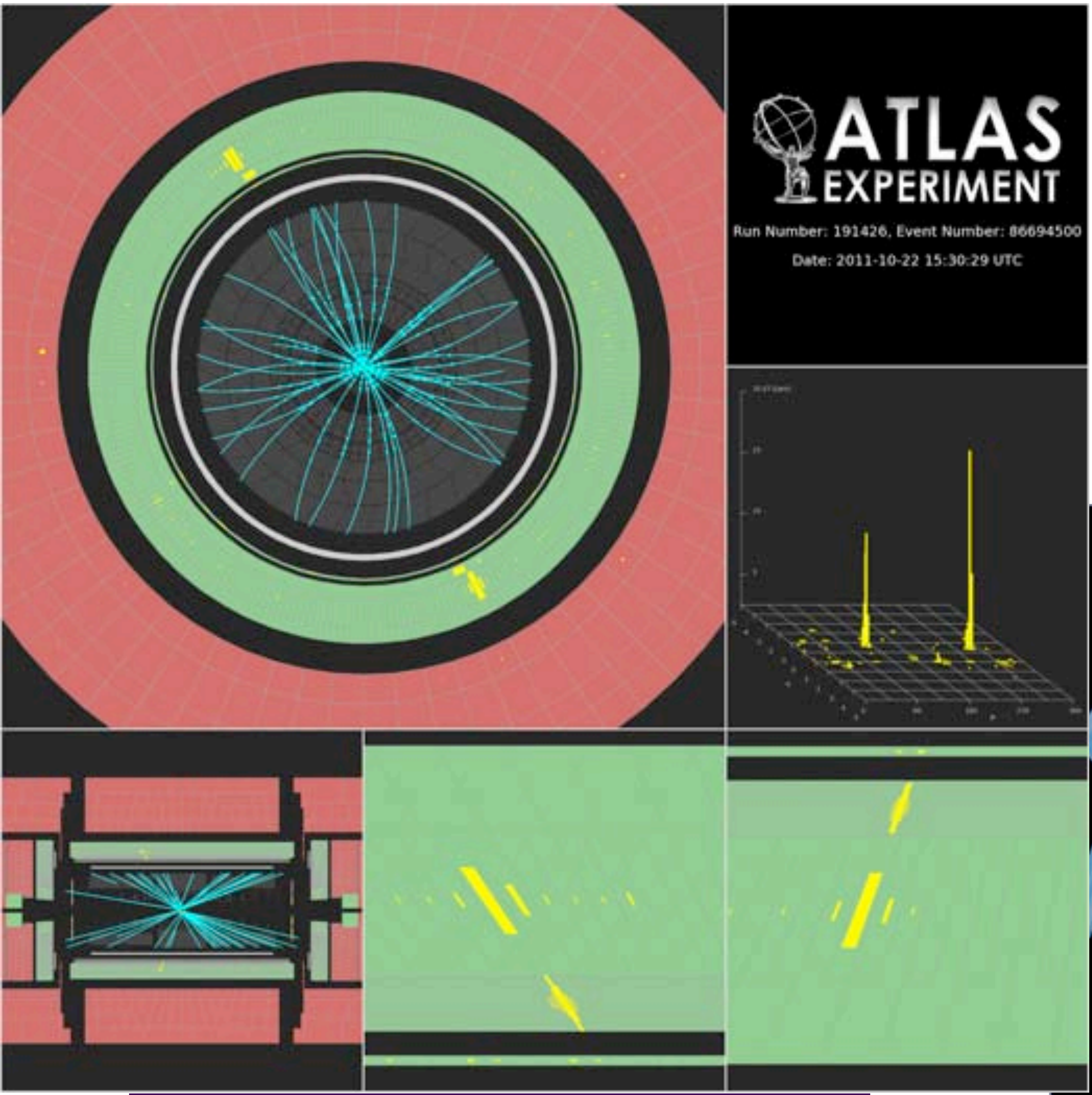
Discovery of the Higgs Boson

- ATLAS and CMS experiments at CERN
- “Bumps” observed in invariant mass at **m ≈ 125 GeV** in:
  - $\gamma\gamma$
  - $\ell^+\ell^-\ell^+\ell^-$ $(\ell=\{e,\mu\})$
- Consistent with $H\to\gamma\gamma$ and $H\to ZZ\to 4\ell$ production
- Statistical significance of the excess is now 7 σ from ATLAS alone!



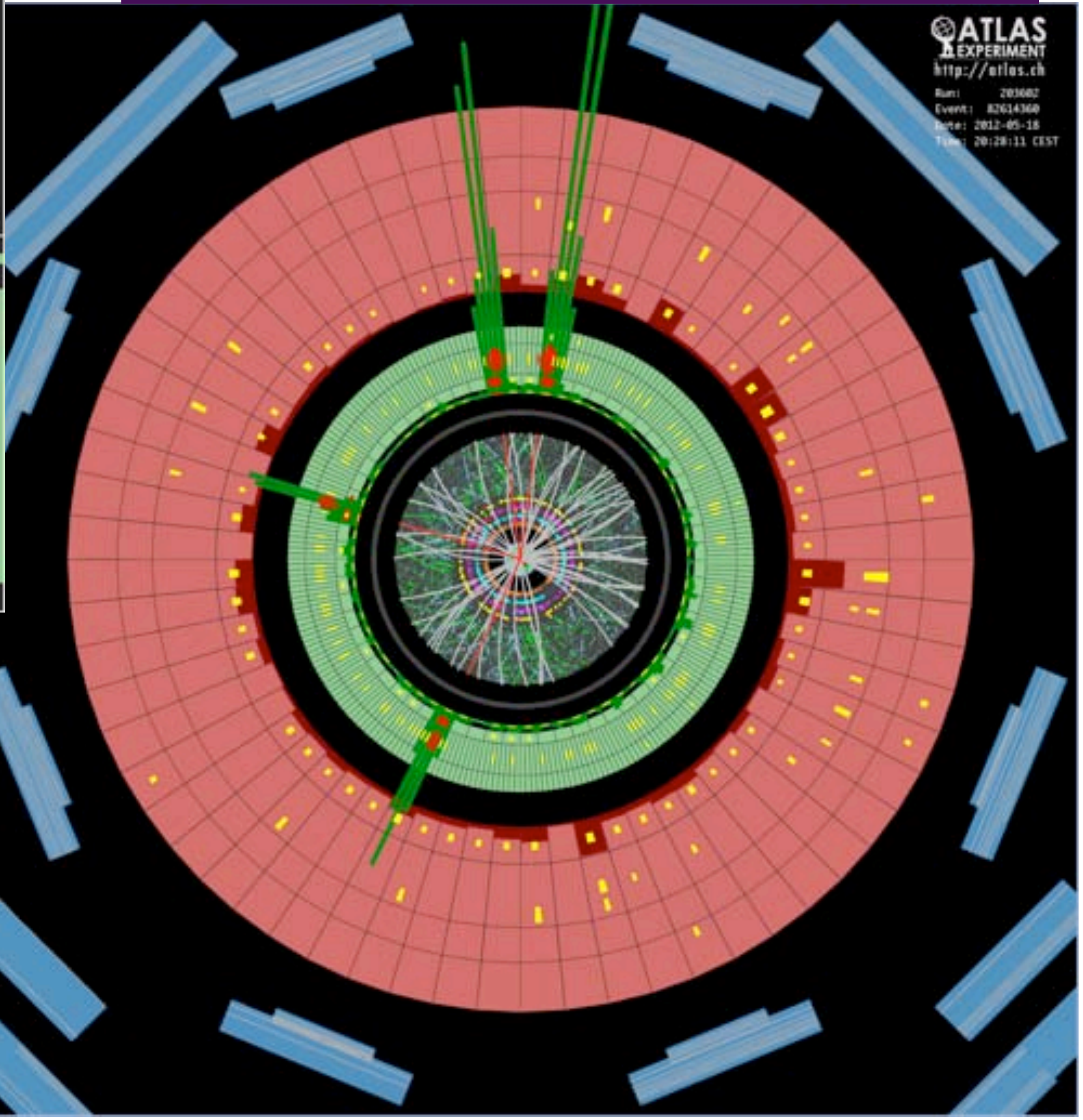


References: https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-170/, ATLAS-CONF-2012-169/, ATLAS-CONF-2012-168/



$H \rightarrow \gamma\gamma$ candidate event

$H \rightarrow ZZ \rightarrow 4e$ candidate event



December 2012

- Fabiola Gianotti is named Time magzine Person of the Year 2012, runner up

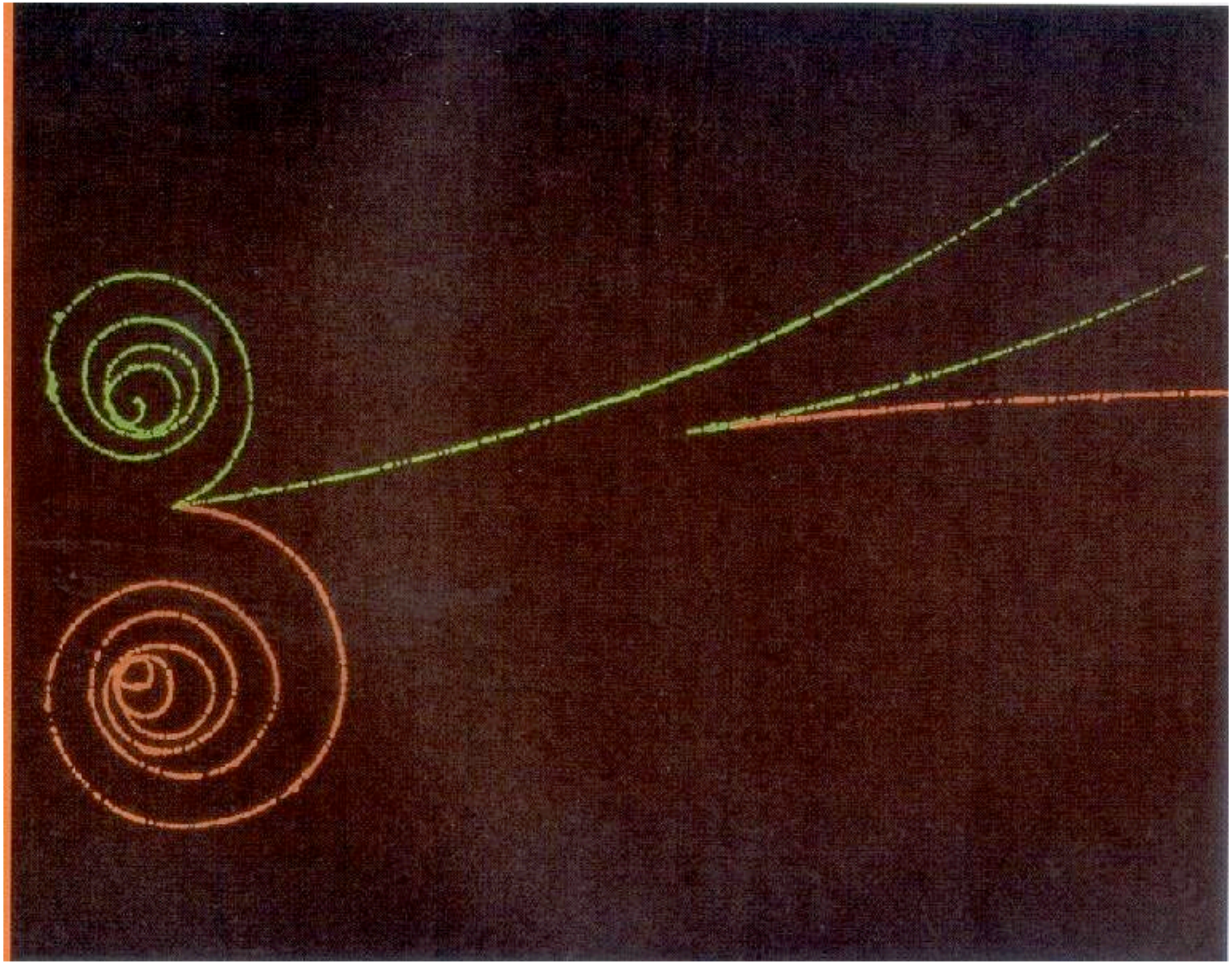
- Higgs boson is particle of year 2012.

- Professor Higgs awarded Membership of the Order of the Companions of Honour by Queen Elizabeth II

- Alan Walker is awarded an MBE for services to science engagement and science education in Scotland.

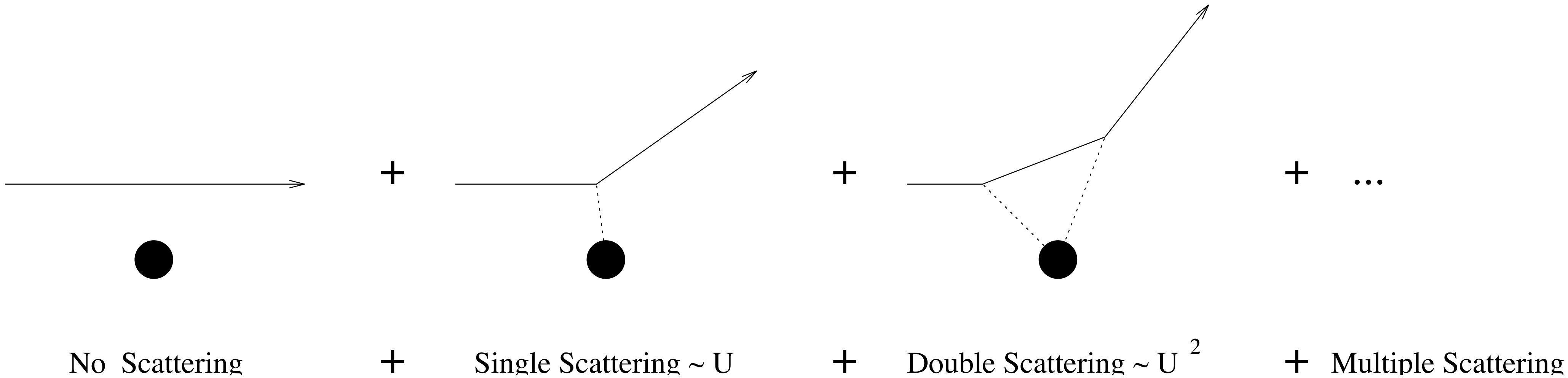
http://www.ph.ed.ac.uk/news/new-years-honours-2013-08-01-13

Prof Higgs visits ATLAS



Scattering Theory

- Consider the interactions between **elementary particles.**
- Review from Quantum Physics, Lecture 12, 13: Quantum Scattering Theory & the Born Approximation
- Born Series: we can think of a scattering in terms of series of terms

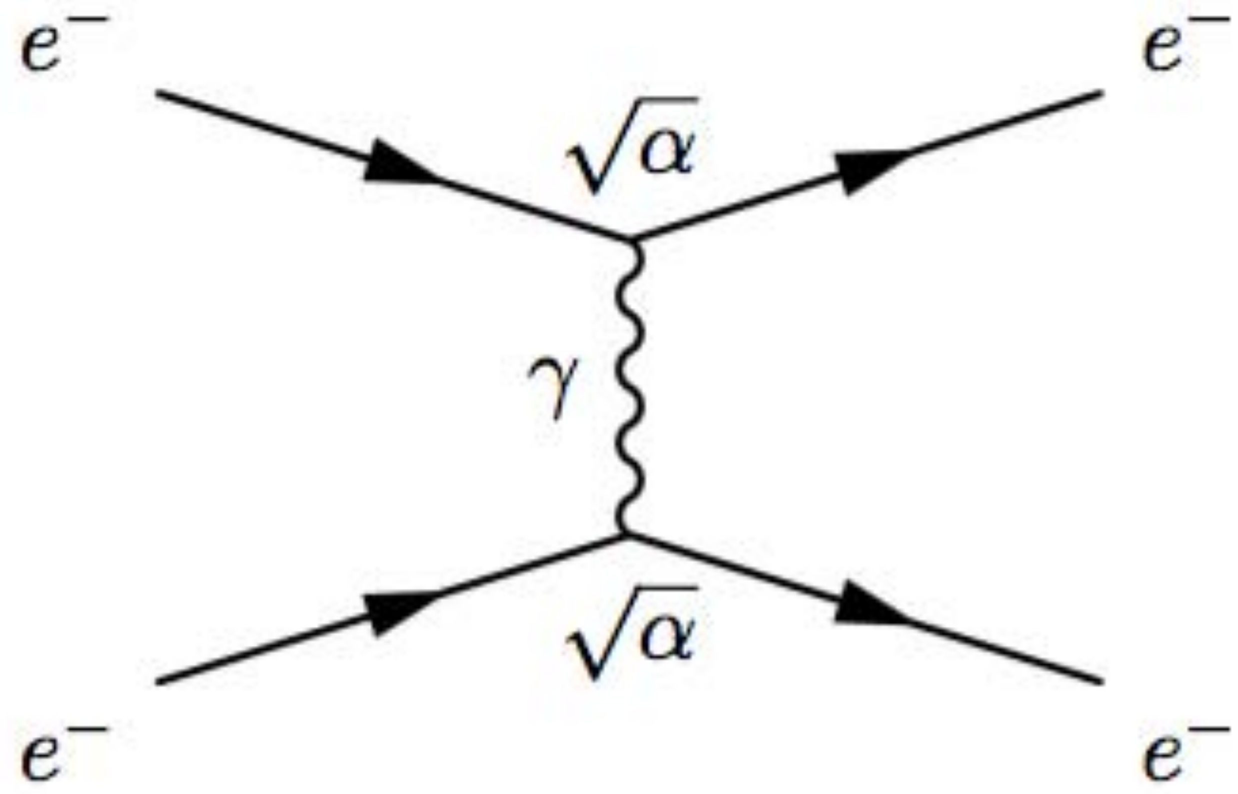


- 1 boson exchange is more probable than 2 boson exchange which is more probable than 3 boson exchange…
- The total probability is the sum of all possible numbers of boson exchange

$$\mathcal{M}_{\rm tot} = \mathcal{M}_1 + \mathcal{M}_2 + +\mathcal{M}_3 \ldots$$

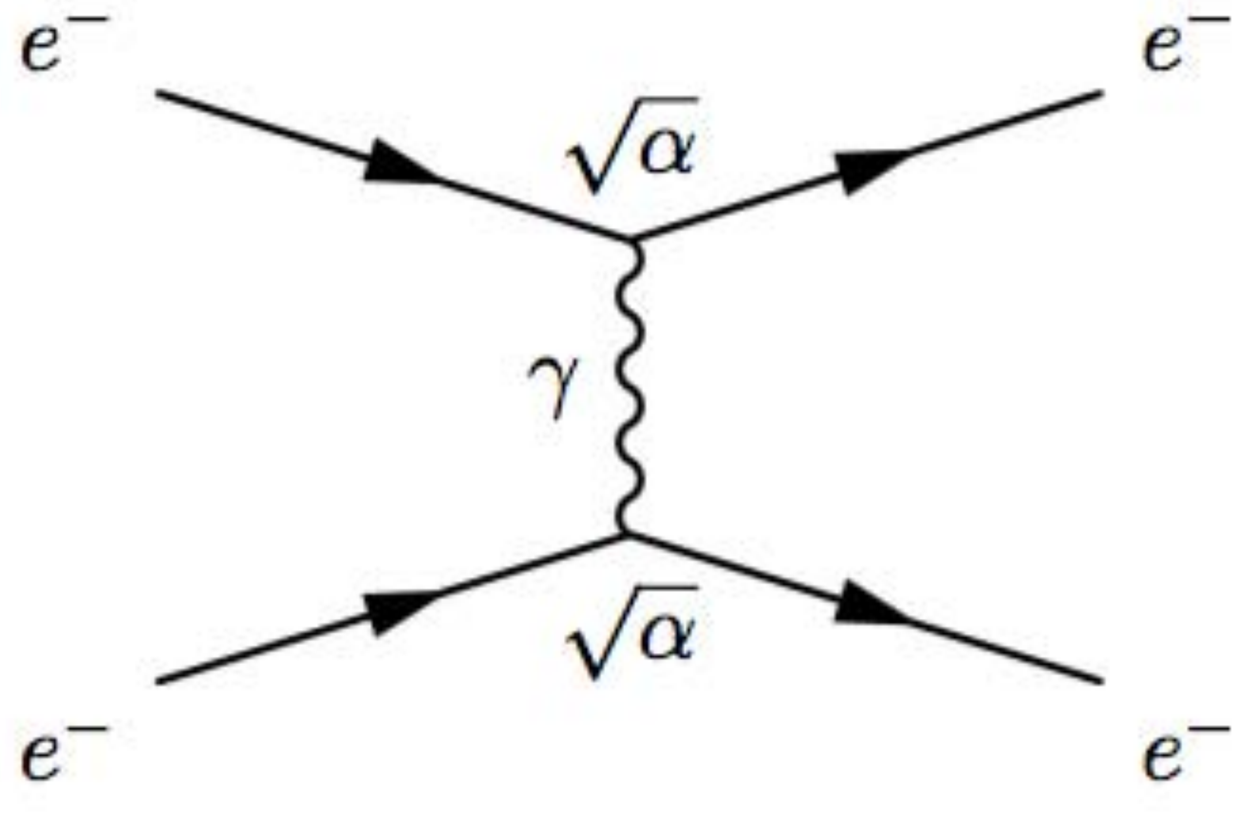
- Feynman diagrams make use of the Born series to calculate the individual **matrix elements** $\mathcal{M}_i$

Drawing Feynman Diagrams



Drawing Feynman Diagrams

Initial state particles on the left



Drawing Feynman Diagrams

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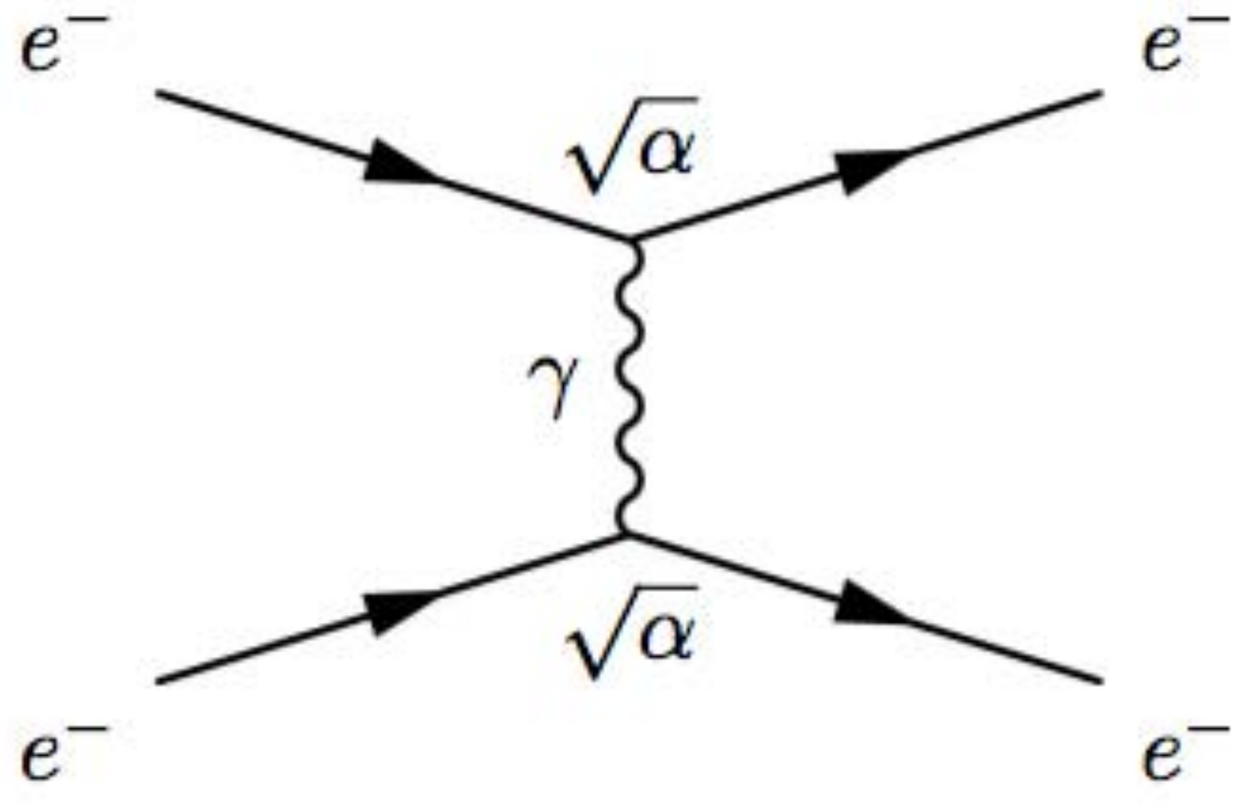
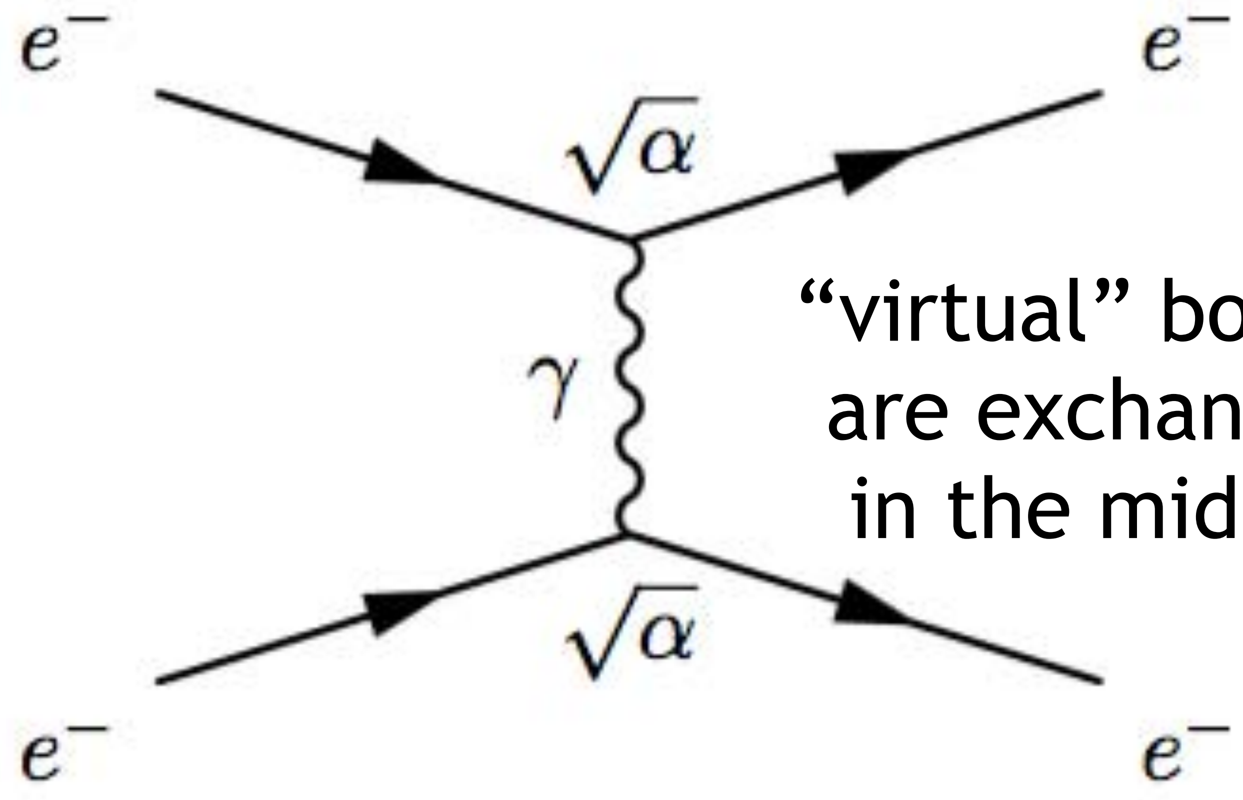


Final state particles on the right

Drawing Feynman Diagrams

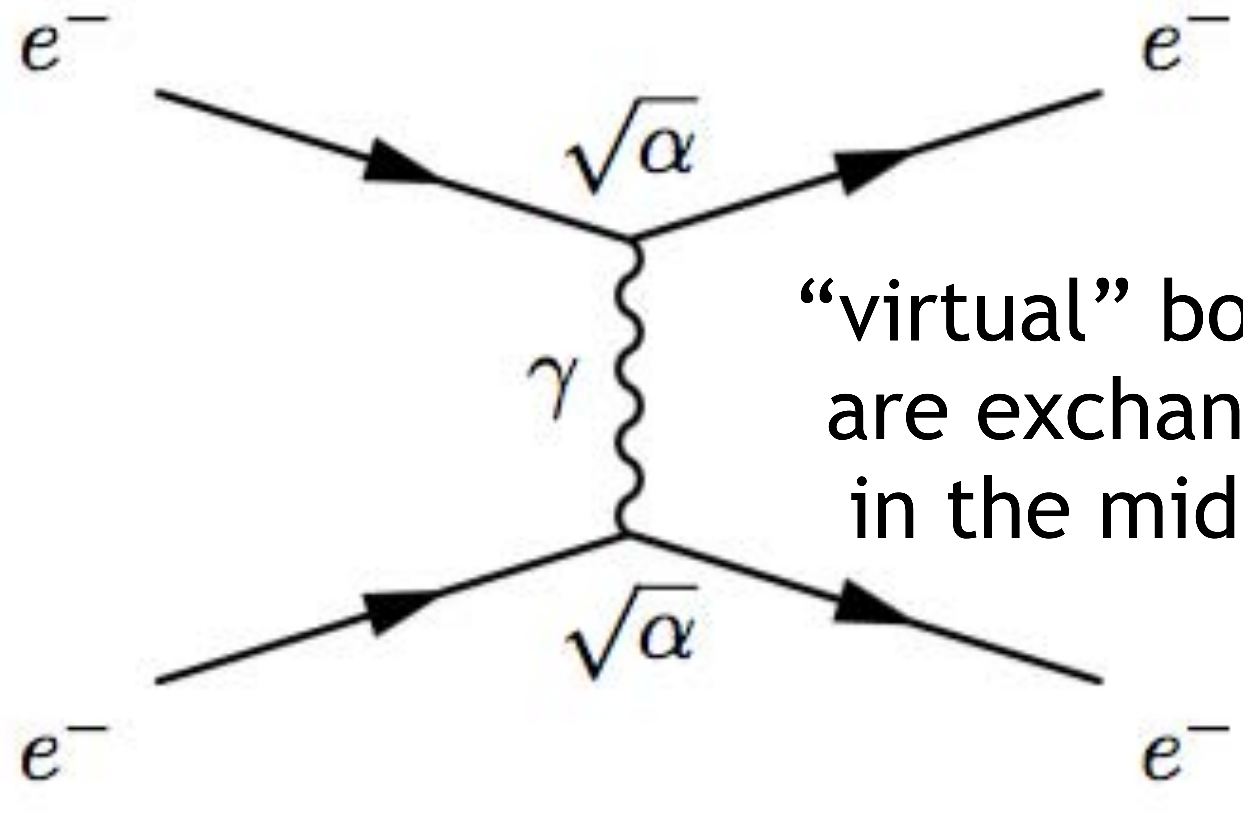


Initial state particles on the left

"virtual" bosons are exchanged in the middle

Final state particles on the right

Drawing Feynman Diagrams



Initial state particles on the left

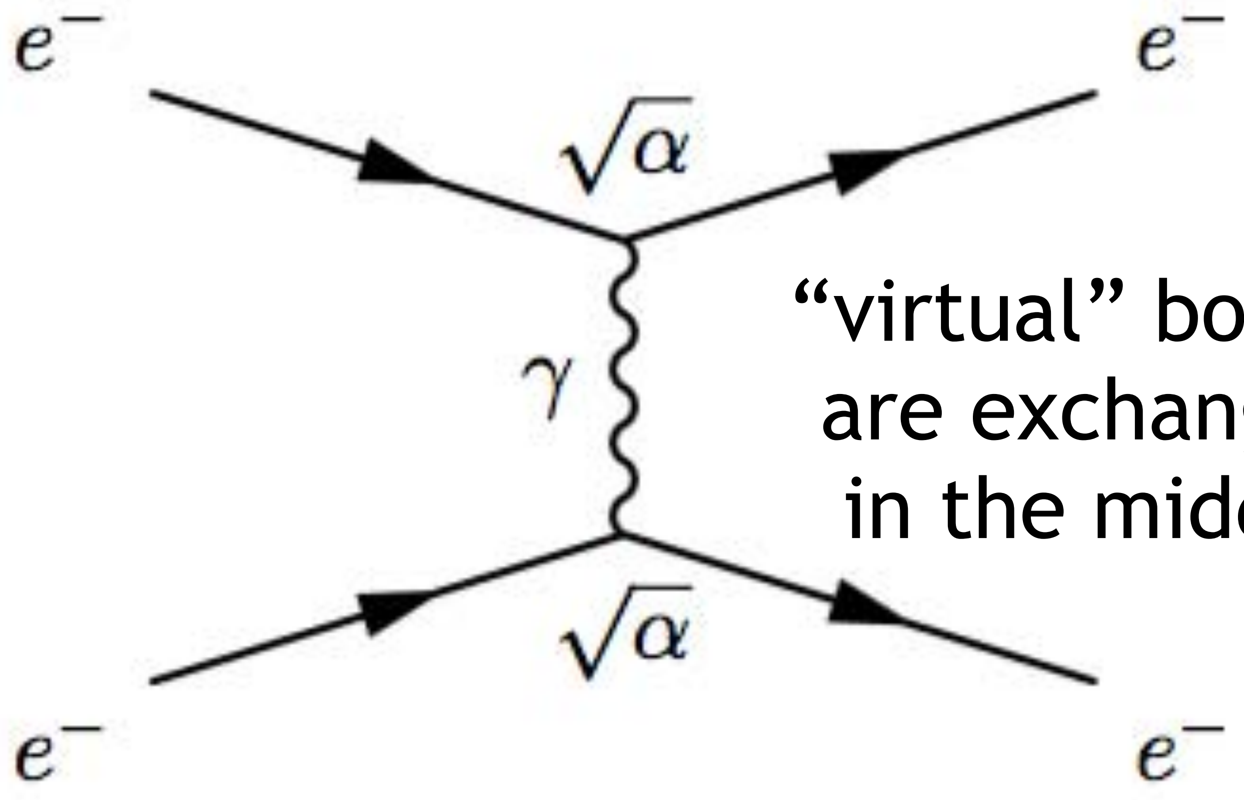
Final state particles on the right

“virtual” bosons are exchanged in the middle

Each interaction vertex has a coupling constant

Drawing Feynman Diagrams

Initial state particles on the left



Final state particles on the right

“virtual” bosons are exchanged in the middle

Each interaction vertex has a coupling constant

fermions

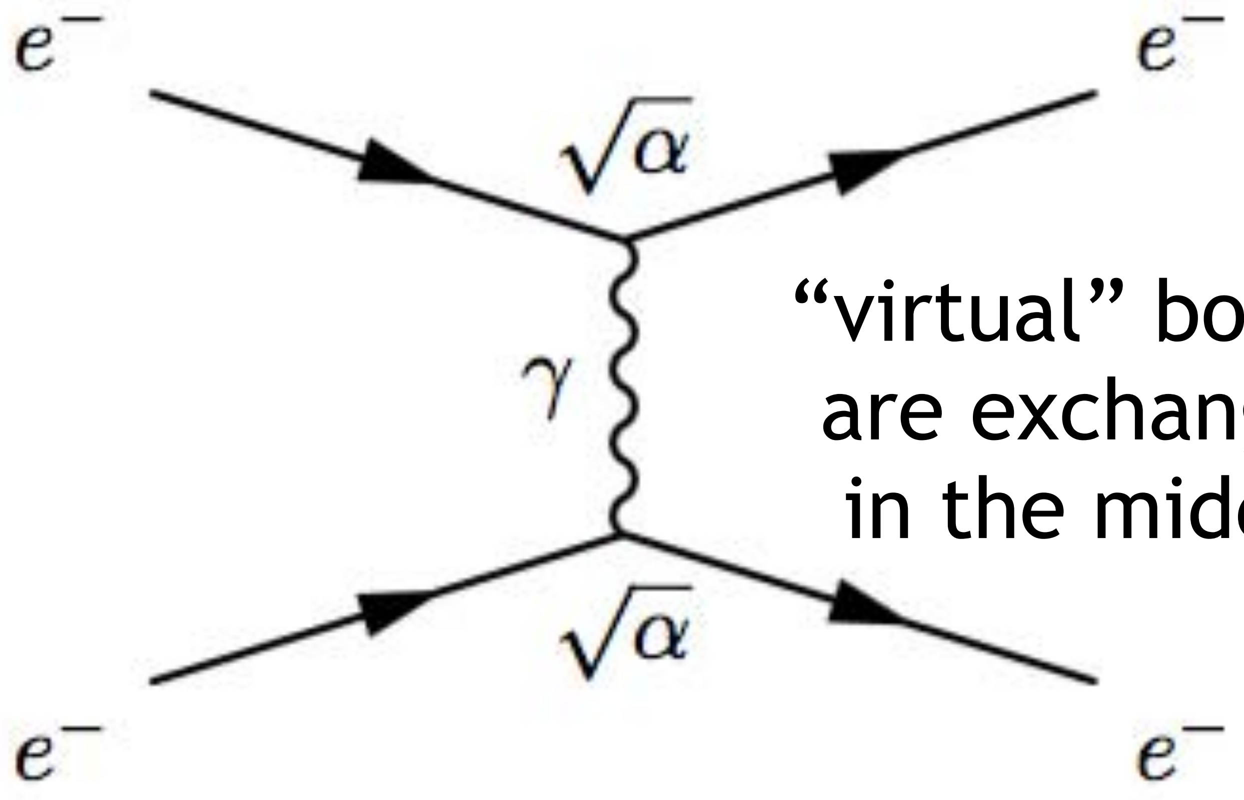
antifermions

photons, *W*, *Z* bosons

gluons

H bosons

Drawing Feynman Diagrams



Initial state particles on the left

“virtual” bosons are exchanged in the middle

Final state particles on the right

Times flows from left to right

Each interaction vertex has a coupling constant

fermions

antifermions

photons, *W*, *Z* bosons

gluons

H bosons

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 3: Feynman Diagrams, Decays and Scattering

★Feynman Diagrams continued

★Decays, Scattering and Fermi's Golden Rule

★Anti-matter?

Notation Review

- A μ sub- or super- script represents a four vector, e.g. x^μ, p^μ, p_μ
  - μ runs from 0 to 3

$$p^\mu = (p^0, p^1, p^2, p^3) = (E, p_x, p_y, p_z)$$

- This lecture also introduce other quantities with μ index, μ=0,1,2,3

- The scalar product of two four vectors

$$a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

- The three dimension differential operator

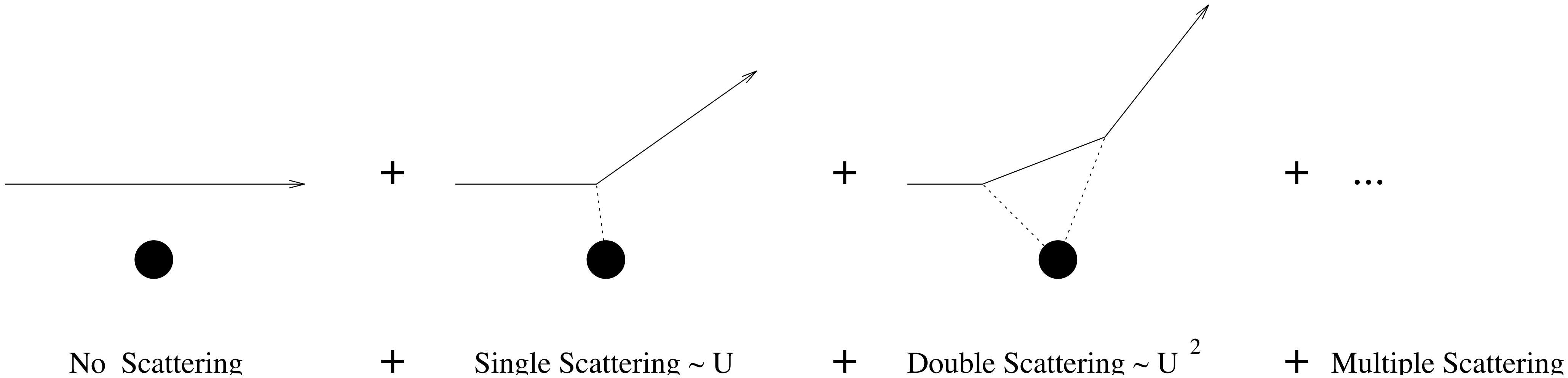
$$\vec{\bigtriangledown} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

- Four dimension differential operator

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Scattering Theory

- Consider the interactions between **elementary particles.**
- Review from Quantum Physics, Lecture 12, 13: Quantum Scattering Theory & the Born Approximation
- Born Series: we can think of a scattering in terms of series of terms

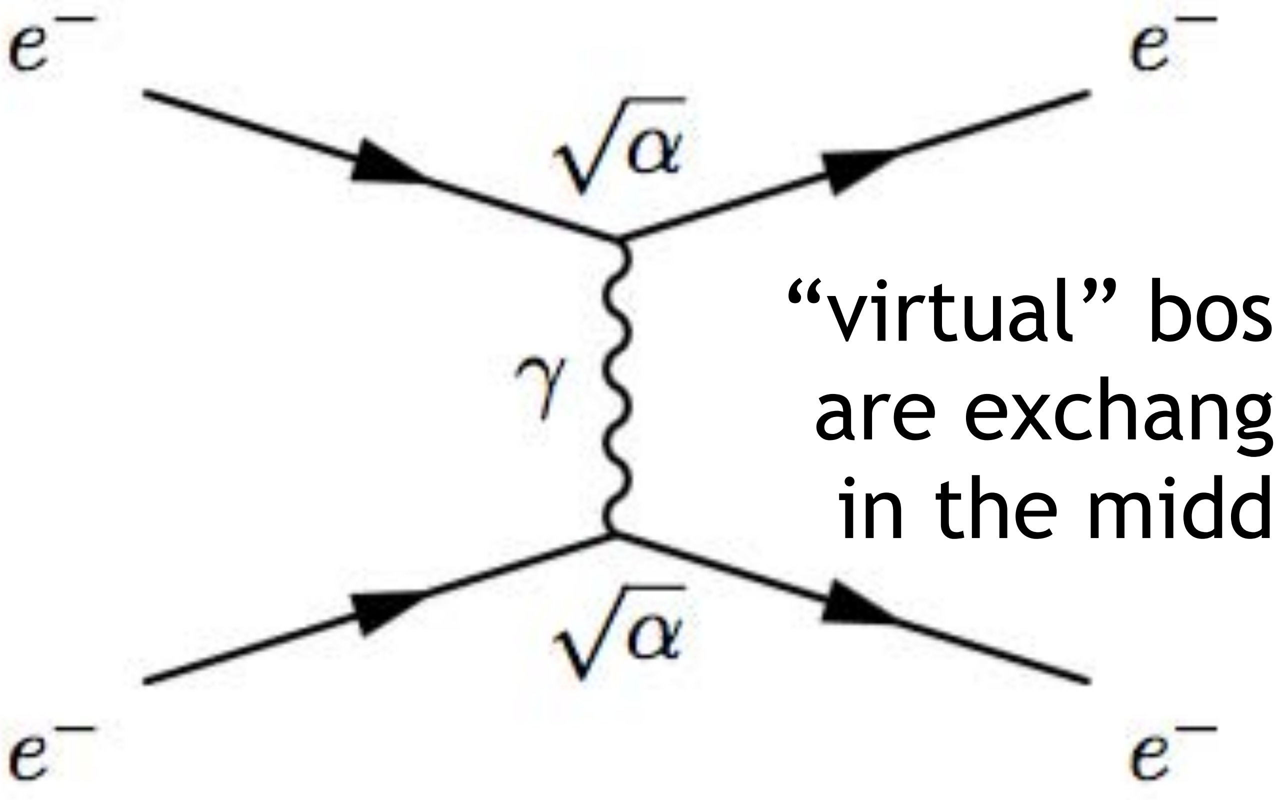


- 1 boson exchange is more probable than 2 boson exchange which is more probable than 3 boson exchange...
- The total probability for a scattering is the sum of all possible numbers of boson exchange:

$$\mathcal{M}_{\rm tot} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 \ldots$$

- Feynman diagrams make use of the Born series to calculate the individual terms in the **matrix elements** series $\mathcal{M}_i$

Drawing Feynman Diagrams



Initial state
particles on
the left

"virtual" bosons
are exchanged
in the middle

Final state
particles on
the right

Times flows from left to right

Each interaction vertex
has a coupling constant

fermions

antifermions

photons,
W, *Z* bosons

gluons

H bosons

The Feynman Rules

- Each part of the feynman diagram has a function associated with it. Multiply all parts together to get a term in the Born expansion

- Initial and final state particles have **wavefunctions**:
  - Spin-0 bosons are plane waves
  - Spin-½ fermions have Dirac spinors
  - Spin-1 bosons have polarization vectors ε^μ

- Vertices have dimensionless **coupling constants**:
  - Electromagnetism has $\sqrt{\alpha} = e$
  - Strong interaction has $\sqrt{\alpha_s} = g_s$
  - Weak interactions have g_L and g_R (or c_A and c_V)

- Virtual particles have **propagators**, q^μ is momentum transferred by boson
  - Virtual photon propagator is $1/q^2$
  - Virtual W/Z boson propagator is $1/(q^2 - M_W^2); 1/(q^2 - M_Z^2)$
  - Virtual fermion propagator is $(\gamma^\mu q_\mu + m)/(q^2 - m^2)$

γ^μ (Gamma matrices) and spinors … next lecture

Plane Waves

- Plane wave can be used to describe spinless, chargeless particles:

$$\psi = e^{-ip\cdot x} \qquad p\cdot x = p^\mu x_\mu = \hbar(\vec{k}\cdot\vec{x} - \omega t)$$

- Define the **probability current** for the particle

$$j_\mu = i\left[\psi^*\,\partial_\mu\psi - \psi\,\partial_\mu\psi^*\right]$$

- Note this is a four-dimensional quantity:

$$j^\mu = (j^0, \vec{j})$$

- For a scattered particle changing momentum $\boldsymbol{p_i \longrightarrow p_f}$:

Wave for initial state

$$\psi_i = e^{-ip_i\cdot x}$$

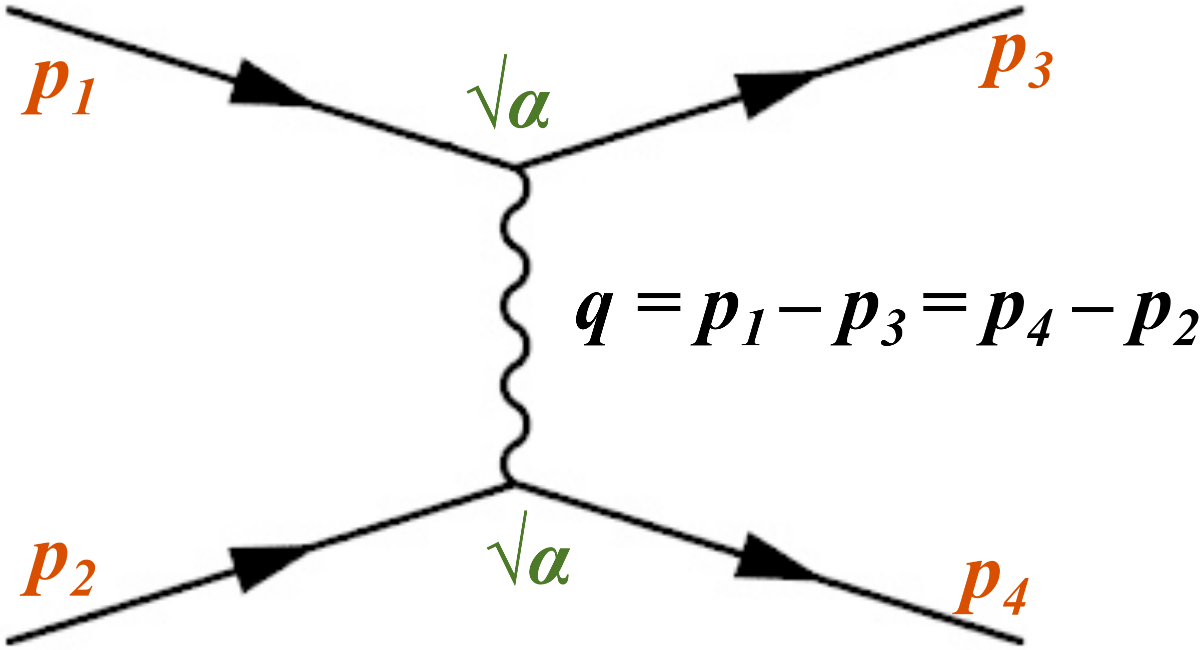
Wave for final state

$$\psi_f = e^{-ip_f\cdot x}$$

$$\begin{aligned} j_\mu(i\to f) &= i\left[\psi_f^*\,\partial_\mu\psi_i - \psi_i\,\partial_\mu\psi_f^*\right] \\ &= (p_i + p_f)e^{-i(p_i - p_f)\cdot x} \end{aligned}$$

Matrix Element for Spinless Scattering

Hypothetical interaction in which two ***spinless*** charged particles exchange one virtual photon



Vertex Couplings

$$\mathcal{M} = \frac{\alpha}{q^2}(p_1 + p_3)(p_2 + p_4)\,\delta^4(p_1 + p_2 - p_3 - p_4)$$

Photon Propagator

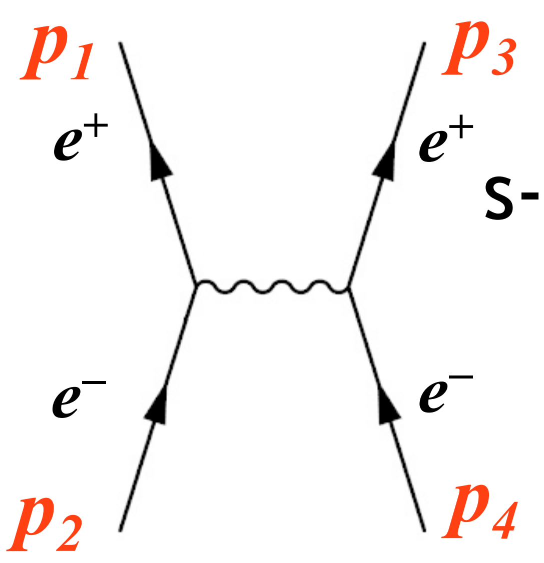
Plane Waves

Four momentum conservation

(in terms of Mandelstam variables) $\mathcal{M} = \dfrac{\alpha\,(s-u)}{t}$

Mandelstam Variables

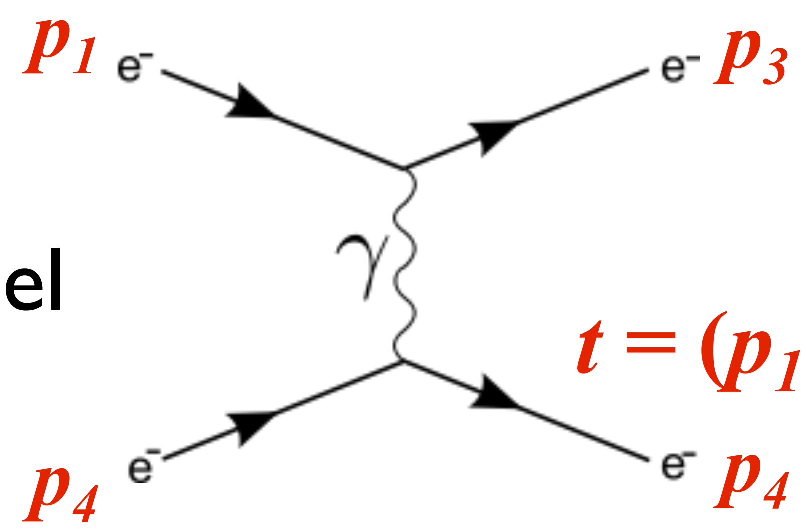
Introduce the Lorentz invariant scattering variables: s, t and u



s-channel

$$s = (p_1 + p_2)^2$$

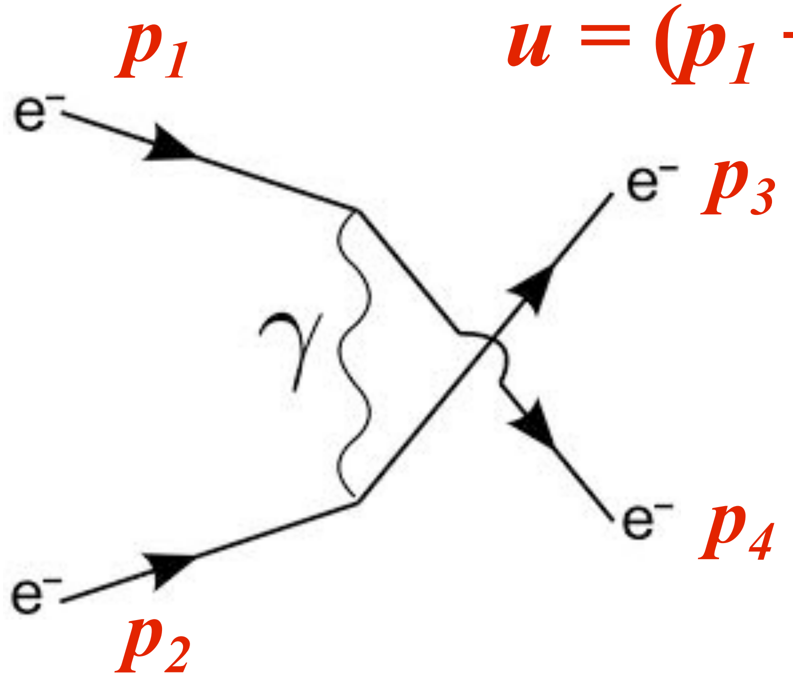
t-channel



$$t = (p_1 - p_3)^2$$

u-channel

$$u = (p_1 - p_4)^2$$



For highly relativistic elastic scattering

$p \sim E$, $m << E$:

$$s = 4\,p^{*2}$$

$$t = -2\,p^{*2}\,(1-\cos\theta^*)$$

$$u = -2\,p^{*2}\,(1+\cos\theta^*)$$

with $p^{*}=p_1=p_2$ is the CM momentum of the particles, and θ^* is the CM scattering angle

Measuring Interactions

- To test the Standard Model (or any other model) of particle physics, relate Feynman diagrams with measurable quantities.

- Two main measurable processes in particle physics:
  - ★ particle decay e.g. $A \rightarrow c\ d$
    - ➡ measure decay width, $\Gamma\ (A \rightarrow c\ d)$
  - ★ scattering e.g. $a\ b \rightarrow c\ d$
    - ➡ measure cross section, $\sigma\ (a\ b \rightarrow c\ d)$

- Related to $\mathcal{M}$ (calculated from Feynman diagrams) through **Fermi's Golden Rule:**

$$\Gamma \sim \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho \qquad\qquad \sigma \sim \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho$$

- ρ is the phase space - a purely kinematic quantity

(see tutorial sheet and/or Griffiths Appendix B)

Measuring Decays

- Measure the lifetime of a particle in its own rest frame.
- Define the decay rate, $\boldsymbol{\Gamma}$: the probability per unit time the particle will decay:

$$dN = -\Gamma N dt \qquad N(t) = N(0)e^{-\Gamma t}$$

- Mean lifetime is $\boldsymbol{\tau = 1 / \Gamma}$ (natural units).
  - For $\boldsymbol{\tau}$ in seconds can use $\boldsymbol{\tau = \hbar / \Gamma}$

- Most particles decay more than one different route: add up all decay rates to obtain the **total decay rate**:

$$\Gamma_{\text{tot}} = \sum_{i=1}^{n} \Gamma_i$$

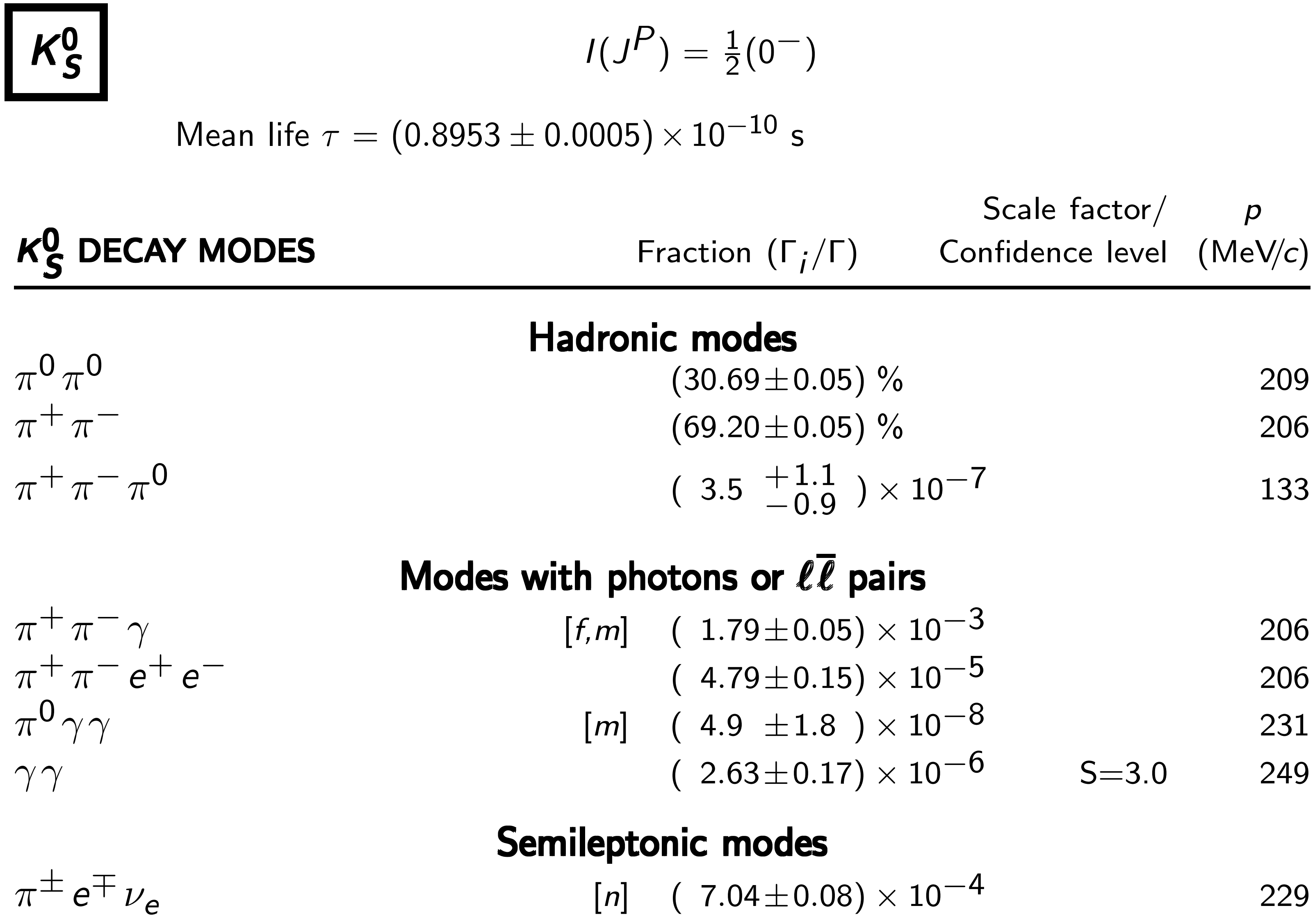
- The lifetime is the reciprocal of $\boldsymbol{\Gamma_{\text{tot}}}$:

$$\tau = \frac{1}{\Gamma_{\text{tot}}}$$

- The different final states of the particle are known as the **decay modes**.
- The **branching ratio** for the *i*th decay mode is: $\Gamma_i / \Gamma_{\text{tot}}$

Example: Decays of the $K^0{}_S$ meson

- Collated by the particle data group: http://pdglive.lbl.gov



$\boxed{K^0_S}$

$I(J^P) = \frac{1}{2}(0^-)$

Mean life $\tau = (0.8953 \pm 0.0005) \times 10^{-10}$ s

K^0_S DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
		Hadronic modes		
$\pi^0\pi^0$		(30.69 ± 0.05) %		209
$\pi^+\pi^-$		(69.20 ± 0.05) %		206
$\pi^+\pi^-\pi^0$		$(\ 3.5\ {}^{+1.1}_{-0.9}\ ) \times 10^{-7}$		133
		Modes with photons or $\ell\overline{\ell}$ pairs		
$\pi^+\pi^-\gamma$	[f,m]	$(\ 1.79\pm0.05) \times 10^{-3}$		206
$\pi^+\pi^-e^+e^-$		$(\ 4.79\pm0.15) \times 10^{-5}$		206
$\pi^0\gamma\gamma$	[m]	$(\ 4.9\ \ \pm1.8\ \ ) \times 10^{-8}$		231
$\gamma\gamma$		$(\ 2.63\pm0.17) \times 10^{-6}$	S=3.0	249
		Semileptonic modes		
$\pi^\pm e^\mp \nu_e$	[n]	$(\ 7.04\pm0.08) \times 10^{-4}$		229

- Which decay modes happen and which not provide information on symmetries and quantum numbers

Key Points

- Interactions in particle physics are caused by the exchange of bosons (photon, gluon, W, Z).
- Use perturbation theory to describe the interactions in terms of numbers of bosons.
    - ➡ 1 boson exchange is most probable
    - ➡ 2 boson exchange is next most probable
    - ➡ 3 “ “ “ “ “ “
    - ➡ 4 …
- Use Feynman diagrams to illustrate these terms in the perturbation series
- Use Feynman rules to calculate a value for each Feynman diagram, the matrix element, $\mathcal{M}$
- The matrix element is used to calculate cross sections and decay widths to compare to experimental results.

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 4: Dirac Spinors

- ★Schrödinger Equation
- ★Klein-Gordon Equation
- ★Dirac Equation
- ★Spinors
- ★Spin, helicity and chirality

Schrödinger Equation

- Classical energy-momentum relationship:

$$E = \frac{p^2}{2m} + V$$

- Substitute QM operators:

$$\hat{p} = -i\hbar\vec{\nabla} \qquad \hat{E} = i\hbar\frac{\partial}{\partial t}$$

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\hbar^2\frac{\nabla^2}{2m} + V\right)\psi = \hat{H}\psi$$

Schrödinger equation!

- 1st order in $\partial/\partial t$; 2nd order in $\partial/\partial x$. Space and time not treated equally.

Klein-Gordon Equation

- Relativistic energy-momentum relationship is:

$$E^2 = \vec{p}^{\,2}c^2 + m^2c^4$$

- Substitute the operators:

$$\hat{p} = -i\hbar\vec{\nabla} \qquad \hat{E} = i\hbar\frac{\partial}{\partial t}$$

- To give the **Klein-Gordon** equation:

$$-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \nabla^2\psi = \left(\frac{mc}{\hbar}\right)^2\psi$$

- **The Klein-Gordon equation describes spin-0 bosons.** Solutions are plane waves (see lecture 3):

$$\psi = e^{-ip\cdot x} \qquad p\cdot x = p^\mu x_\mu = \hbar(\vec{k}\cdot\vec{x} - \omega t)$$

- KG equation is 2nd order in $\partial/\partial t$ and $\partial/\partial x$

Negative Energy & the Dirac Equation

- The relativistic energy-momentum equation is quadratic, negative energy solutions are possible:

$$E^2 = \vec{p}^{\,2} + m^2 \qquad \Rightarrow \qquad E = \pm\sqrt{\vec{p}^{\,2} + m^2}$$

- Dirac searched for 1st order relationship between energy and momentum, using coefficients $\boldsymbol{\alpha}^1$ $\boldsymbol{\alpha}^2$ $\boldsymbol{\alpha}^3$ and $\boldsymbol{\beta}$

$$\hat{E}\,\psi = (\vec{\alpha}\cdot\vec{p} + \beta m)\psi = i\frac{\partial\psi}{\partial t}$$

- Need to find solutions for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$

Dirac Equation

- Solution is more elegant defining $\gamma^0 \equiv \beta, \gamma^1 \equiv \beta\alpha^1, \gamma^2 \equiv \beta\alpha^2, \gamma^3 \equiv \beta\alpha^3$
- The Dirac equation can be written (with $c = \hbar = 1$) as:

$$i\left(\gamma^0 \frac{\partial\psi}{\partial t} + \vec{\gamma}\cdot\vec{\nabla}\right)\psi = m\psi$$

in covariant notation: $i\gamma^\mu \partial_\mu \psi = m\psi$

- Multiplying the Dirac equation by its complex conjugate must give KG:

$$\left(-i\gamma^0 \frac{\partial}{\partial t} - i\vec{\gamma}\cdot\vec{\nabla} - m\right)\left(i\gamma^0 \frac{\partial}{\partial t} + i\vec{\gamma}\cdot\vec{\nabla} - m\right) = 0$$

- This leads to a set of conditions on the four coefficients γ^μ:

$$(\gamma^0)^2 = 1 \quad (\gamma^1)^2 = -1 \quad (\gamma^2)^2 = -1 \quad (\gamma^3)^2 = -1$$

$$\{\gamma^i, \gamma^j\} = \gamma^i\gamma^j + \gamma^j\gamma^i = 0$$

γ^μ are unitary and anticommute

The Gamma Matrices - 1

- To satisfy unitarity and anti-commutation the γ^μ must be at least 4 × 4 matrices.
- More than one representation. The usual one is:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

- γ^μ are not tensors or four vectors! They do remain constant under Lorentz transformations

The Gamma Matrices - 2

- Gamma Matrices are also often written in a 2x2 form:

$$\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}$$

- where $\mathbf{I}$ and $\mathbf{0}$ are the 2 × 2 identity and null matrices:

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

- and the $\boldsymbol{\sigma}^i$ are the 2 × 2 Pauli spin matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Dirac Equation and Solution

- In matrix notation:

$$\begin{pmatrix} i\frac{\partial}{\partial t}-m & 0 & i\frac{\partial}{\partial z} & i\frac{\partial}{\partial x}+\frac{\partial}{\partial y} \\ 0 & i\frac{\partial}{\partial t}-m & i\frac{\partial}{\partial x}-\frac{\partial}{\partial y} & -i\frac{\partial}{\partial z} \\ -i\frac{\partial}{\partial z} & -i\frac{\partial}{\partial x}-\frac{\partial}{\partial y} & -i\frac{\partial}{\partial t}-m & 0 \\ -i\frac{\partial}{\partial x}+\frac{\partial}{\partial y} & i\frac{\partial}{\partial z} & 0 & -i\frac{\partial}{\partial t}-m \end{pmatrix} \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \\ \psi^4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- In co-variant notation:

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

- Solutions ψ to the Dirac Equation have a:
  - phase term: $e^{-ip\cdot x}$
  - Dirac spinor term, a function of the four-momentum: $u(p^\mu)$

$\psi = u(p^\mu)e^{-ip\cdot x}$ with u solution to $(\gamma^\mu p_\mu - m)u = 0$

Solutions to the Dirac Equation

- Dirac equation: $(i\gamma^\mu \partial_\mu - m)\,\psi = 0$

$$(i\gamma^0 \frac{\partial}{\partial t} - i\gamma^1 \frac{\partial}{\partial x} - i\gamma^2 \frac{\partial}{\partial y} - i\gamma^3 \frac{\partial}{\partial z} - m)\,\psi = 0$$

- Solve for a particle at rest, $p^\mu = (m, \mathbf{0})$, to illustrate main features of the solutions

$$\psi = u(p^\mu) e^{-ip\cdot x} = u(p^\mu) e^{-imt}$$

- Dirac equation becomes: $(\gamma^0 E - m)\,u(p^\mu) = 0$

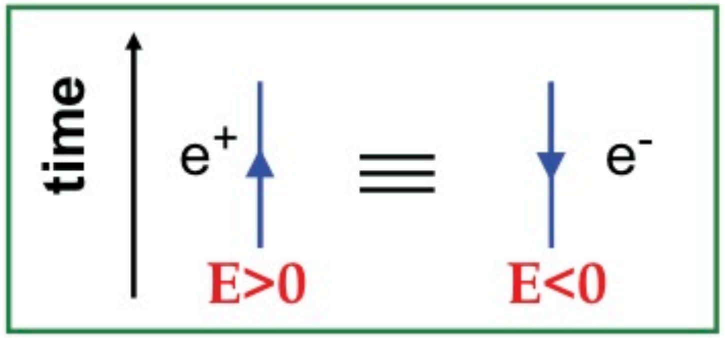
$$\begin{pmatrix} E-m & 0 & 0 & 0 \\ 0 & E-m & 0 & 0 \\ 0 & 0 & -E-m & 0 \\ 0 & 0 & 0 & -E-m \end{pmatrix} \begin{pmatrix} u^1(p^\mu) \\ u^2(p^\mu) \\ u^3(p^\mu) \\ u^4(p^\mu) \end{pmatrix} = 0$$

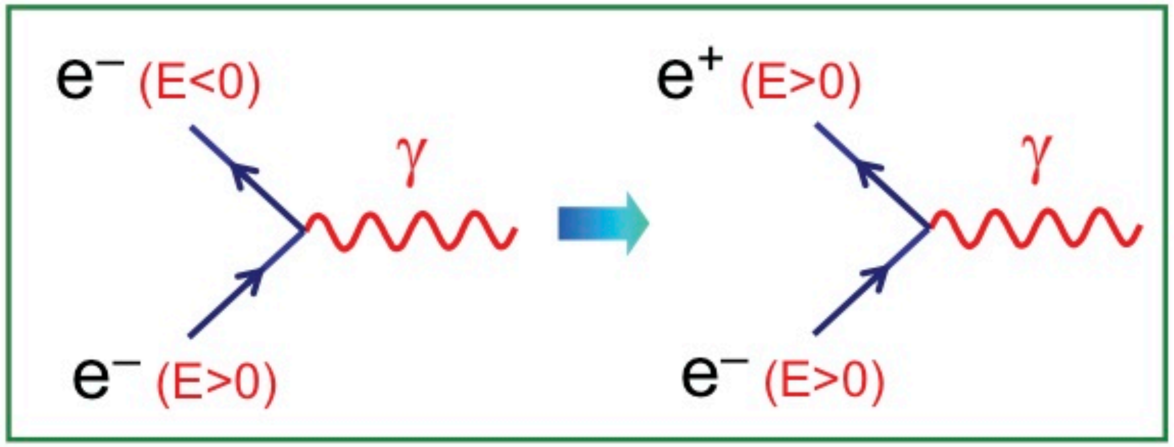
- Four energy eigenstates:
  - u^1 and u^2 with $E = +m$
  - u^3 and u^4 with $E = -m$

Negative Energy Solutions

- We can’t escape negative energy solutions. How should we interpret them?
- **Modern Feynman-Stückelberg Interpretation:**

  A negative energy solution is a negative energy particle which propagates backwards in time or equivalently a positive energy anti-particle which propagates forwards in time.





$$e^{-i(-E)(-t)} \longrightarrow e^{-iEt}$$

- This is why in Feynman diagrams the backwards pointing lines represent anti-particles.

Discovery of Positron

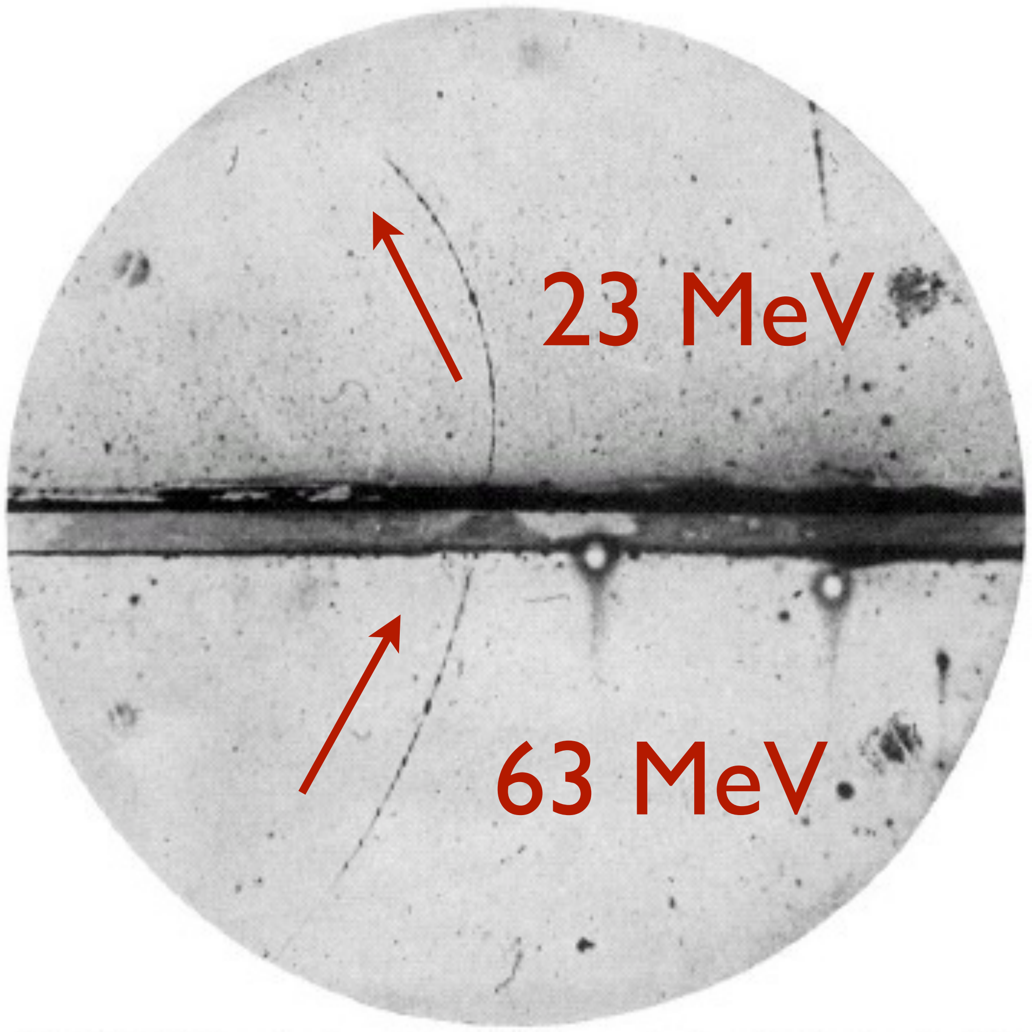
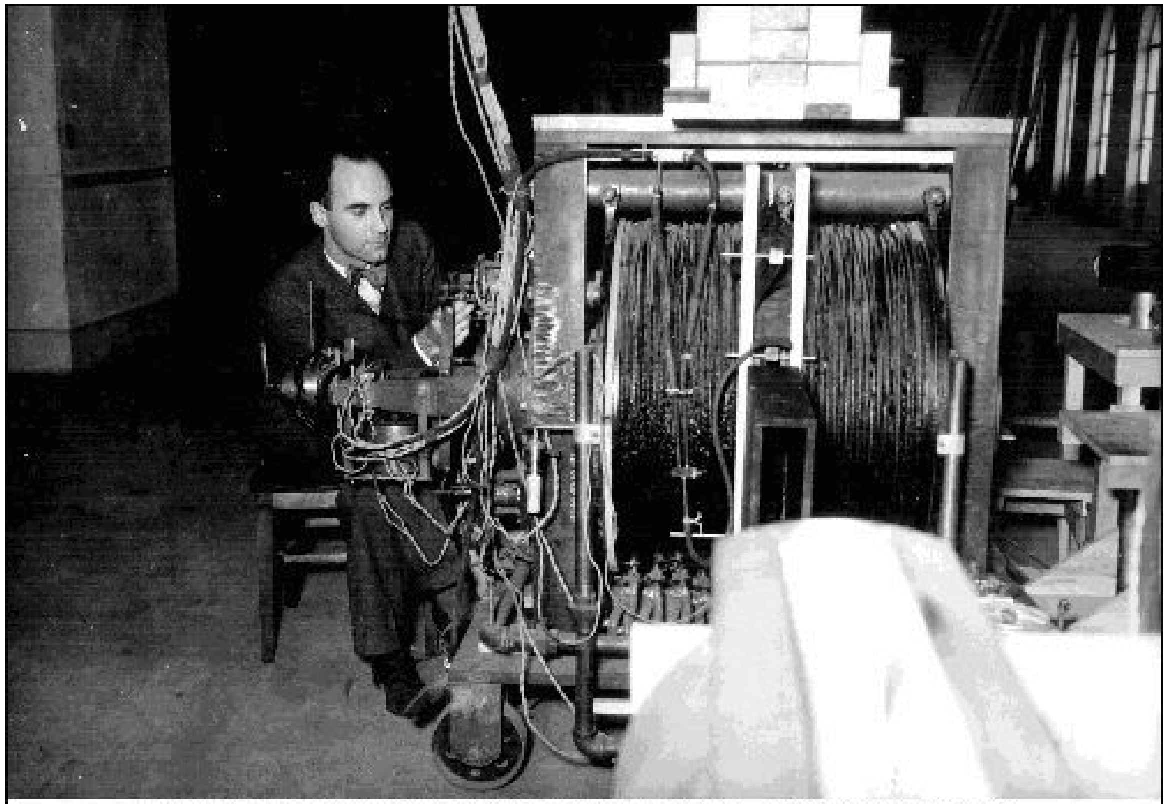


Fig. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

C.D.Anderson, Phys Rev 43 (1933) 491



- e^+ enters at bottom, slows down in the lead plate - know direction
- Curvature in **B-field** shows that it is a **positive particle**
- Can’t be a proton as would have stopped in the lead

Solutions

- Making the equation first order in all derivatives introduces new degrees of freedom!
- The four solutions represent the **four** possible states of a fermion.
- The $\boldsymbol{u}$ are 1 x 4 matrices - **spinors** or **Dirac spinors** (not four-vectors)!

- Using the electron as an example:
  - $\boldsymbol{u^1}$ represents an electron ($\boldsymbol{E = m}$) with spin-up
  - $\boldsymbol{u^2}$ represents an electron ($\boldsymbol{E = m}$) with spin-down
  - $\boldsymbol{u^3}$ represents a positron ($\boldsymbol{E = -m}$) with spin-down
  - $\boldsymbol{u^4}$ represents a positron ($\boldsymbol{E = -m}$) with spin-up

- $\boldsymbol{u^3(p)}$ and $\boldsymbol{u^4(p)}$ are often written as $\boldsymbol{v^1(p^\mu)=u^4(-p^\mu)}$ and $\boldsymbol{v^2(p^\mu)=u^3(-p^\mu)}$

$$\psi = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \\ \psi^4 \end{pmatrix} = \begin{pmatrix} u^1(p^\mu) \\ u^2(p^\mu) \\ u^3(p^\mu) \\ u^4(p^\mu) \end{pmatrix} e^{-ip\cdot x} = \begin{pmatrix} u^1(p^\mu) \\ u^2(p^\mu) \\ v^2(-p^\mu) \\ v^1(-p^\mu) \end{pmatrix} e^{-ip\cdot x}$$

Spinors moving and at rest

- For a particle at rest spinors take the trivial form:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad u^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u^4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

- For the moving particles (derivation see Griffiths Pp. 231-234):

Fermions:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x+ip_y)/(E+m) \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ (p_x-ip_y)/(E+m) \\ -p_z/(E+m) \end{pmatrix}$$

Antifermions:

$$v^2 = \begin{pmatrix} p_z/(E+m) \\ (p_x+ip_y)/(E+m) \\ 1 \\ 0 \end{pmatrix} \quad v^1 = \begin{pmatrix} (p_x-ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{pmatrix}$$

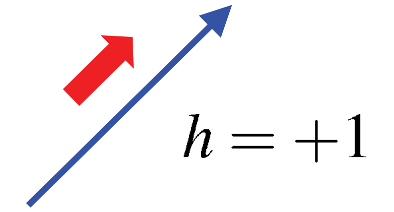
Where we have changed notation for antiparticles from $u^3(p) \rightarrow v^2(-p)$ and $u^4(p) \rightarrow v^1(-p)$

Helicity

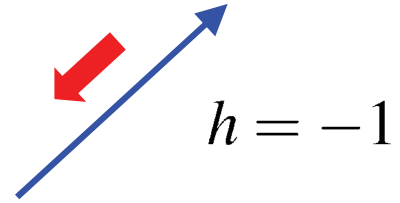
- Spin is usually defined w.r.t the z-axis → not Lorentz invariant.
- Define helicity, $\hat{h}$, the component of the spin along direction of flight.

$$\hat{h} = \frac{\vec{S}\cdot\vec{p}}{|\vec{S}||\vec{p}|} = \frac{2\vec{S}\cdot\vec{p}}{|\vec{p}|}$$

- For a S=½ fermion, the project of spin along any axis can only be ±½.
- For a S=½ fermion, eigenvalues of $\hat{h}$ are ±**1**.
- We call h=+1, “right-handed”, h=−1 “left handed”.



“right-handed”



“left-handed”

- Massless fermions with (p=E) are purely left-handed (only u^2)
- Massless antifermions are purely right-handed (only v^1)
- For massive particles helicity is still not Lorentz invariant: we can boost to frame such that particle direction of flight reverses

Chirality and Handedness

- Chirality is a Lorentz invariant quantify: identical to helicity for massless particles.
- S=½ fermions have two chiral states: left-handed and right-handed.

- Defined using chiral **projection operators** P_L and P_R:
- **LH projection** operator $P_L = (1 - \gamma^5)/2$ projects out **left-handed chiral** state
- **RH projection** operator $P_R = (1 + \gamma^5)/2$ projects out **right-handed chiral** state

where $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ is 4×4 matrix:

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Chiral Projection Operators and γ^5

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

- Properties:
  - ➡ Unitary $(\gamma^5)^2 = 1$
  - ➡ Anti commutes with all other γ matrices: $\{\gamma^5 , \gamma^i\} = \gamma^5\gamma^i + \gamma^i\gamma^5 = 0$.

- Left and right handed component of a fermion state are $\psi_L = P_L\,\psi$, $\psi_R = P_R\,\psi$

- $P_L + P_R = 1 \Rightarrow \psi = P_L\,\psi + P_R\,\psi$
  - A state can always be written as the sum of LH and RH components

- $P_L^2 = P_L \qquad P_R^2 = P_R \qquad P_L\,P_R = 0$
  - No overlap between the LH and RH components

Summary and Reading List

- The Dirac Equation describes spin-½ particles.

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

- Solutions include four component **spinors**, u and v.

$$(\gamma^\mu p_\mu - m)u = 0 \qquad (\gamma^\mu p_\mu + m)v = 0$$

$$\psi = u(p)e^{-ip\cdot x} \qquad \psi = v(p)e^{-ip\cdot x}$$

- With γ^μ, $\mu = 0,1,2,3$ the 4 × 4 Gamma matrices
- The four solutions describe the different states of the electron *e.g.* left-handed electrons, right-handed electrons, right-handed positrons, left-handed positrons

- We use chiral projection operators to define left-handed and right-handed states
- Any particle can be written in terms of left handed and right handed components: $\psi = (1 - \gamma^5)\psi + (1 + \gamma^5)\psi = \psi_L + \psi_R$

- Next Lecture: The Electromagnetic Force. Griffiths 7.5 & 7.6

Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 5: Quantum Electrodynamics
(The Electromagnetic force, quantised)

- ★Fermion currents
- ★Spin-1 Bosons
- ★$e^- \mu^- \to e^- \mu^-$

Announcements

- Lectures as usual this week.

- Next week:
  - lecture on Tuesday only.
  - No lecture on Friday 8th February.
  - Tutorial next week mainly for catch up.

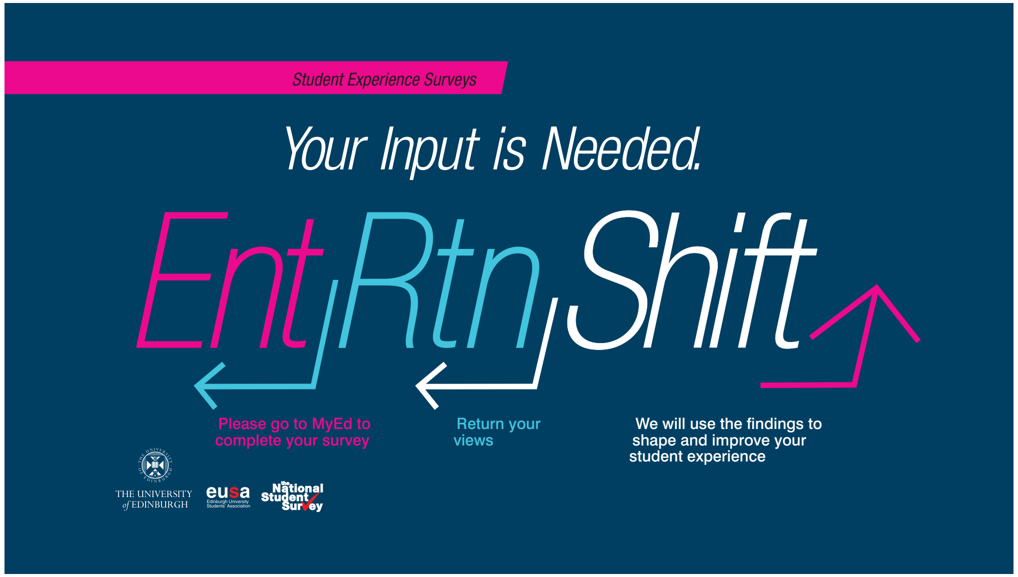
- Tutorial sheet solutions and notes are in preparation.

- Notes from last year are online.
  - ➡ Linked from course web page.
  - ➡ Last year’s notes cover the same material, but are a bit verbose...

Student Surveys

Do you want your opinions on your student experience to be heard? Take part in the National Student Survey

www.thestudentsurvey.com



Your Input is Needed.

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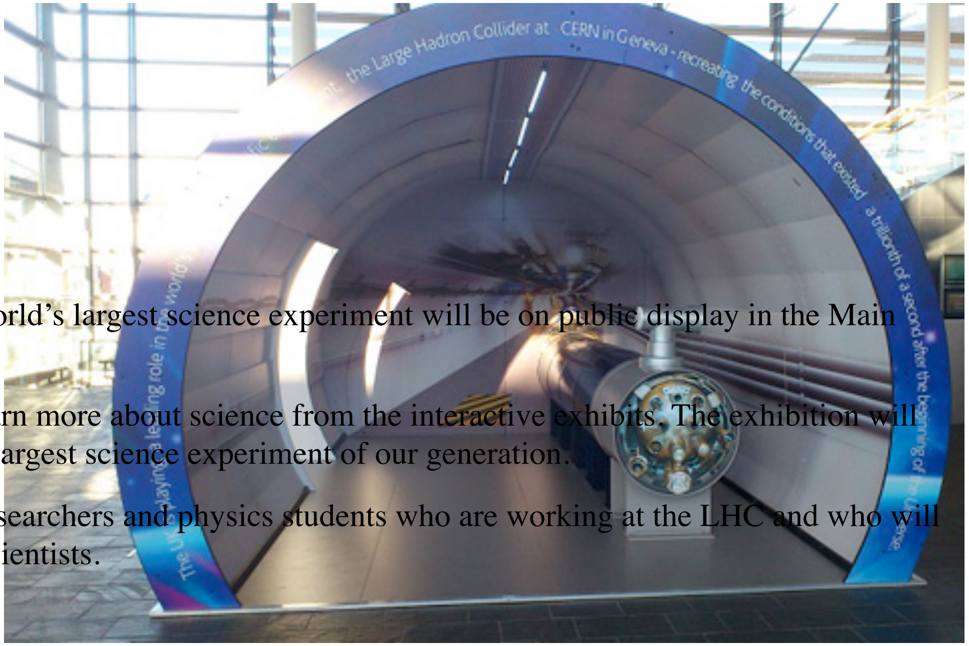
Return your views

We will use the findings to shape and improve your student experience

LHC in the Parliment

http://www.scottish.parliament.uk/visitandlearn/58283.aspx



Saturday 2 February 2013 - Friday 8 February 2013

Main Hall, Free Exhibition

A travelling exhibition showcasing the Large Hadron Collider (LHC), the world's largest science experiment will be on public display in the Main Hall of the Scottish Parliament.

Visitors can walk through a life size model of part of the LHC tunnel and learn more about science from the interactive exhibits. The exhibition will help create a sense of what it's like to be a particle physicist working on the largest science experiment of our generation.

Visitors will also have the opportunity to meet some of the UK's top LHC researchers and physics students who are working at the LHC and who will be available to answer questions and help to inspire the next generation of scientists.

Associated Events

Explore Your Universe - Saturday 2 February

Come along and get involved with the fantastic new family show brought to you by Dynamic Earth as part of the Explore Your Universe project in partnership with the Science and Technologies Facilities Council (STFC) and the Association of Science and Discovery Centres (ASDC). Find out what our world is made of, discover what can be found inside an atom and experiment with making your own electricity! You will also have the chance to see a cloud chamber in action and even make your very own particle to take home with you. Admission is free and will run within normal opening hours.

CERN Lecture - Thursday 7 February

Dr Aidan Robson from the University of Glasgow will be giving a free lecture on CERN, the Large Hadron Collider and his part working on the world's largest scientific experiment. The lecture will begin at 7.00pm. If you wish to take a look at the 'LHC on Tour' exhibition beforehand, please arrive approximately 30 minutes prior to the start.

Notation

- Today we can't avoid using the metric tensor $g_{\mu\nu}$.

$$g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- If take the scalar product of two four-vectors, we actually implicitly use the metric tensor:

$$a \cdot b = a_\mu b^\mu = g_{\mu\nu} a^\nu b^\mu = +1 \times (a^0 b^0) - 1 \times (a^1 b^1) - 1 \times (a^2 b^2) - 1 \times (a^3 b^3)$$

- The factors of $+1$ and -1 are due to the metric tensor.

- The important parts today: if we have two four-momentum vectors with different indices $g_{\mu\nu}$ makes the scalar product: e.g. $a^\nu b^\mu \; g_{\mu\nu} = a^\nu b_\nu = a \cdot b$

The Adjoint Spinor

- Need to define a **fermion current, *j*,** for use in Feynman diagrams.

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \qquad \psi^\dagger = (\psi^*)^T = (\psi_1^*, \psi_2^*, \psi_3^*, \psi_4^*)$$

- $\psi^\dagger \psi$ is not invariant under Lorentz transformations (Spinors are not vectors!)

- Define an **adjoint spinor**, $\bar{\psi} \equiv \psi^\dagger \gamma^0$ $\quad \bar{\psi} \equiv \psi^\dagger \gamma^0 = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*)$

- $\bar{\psi}\psi$ is invariant under Lorentz transformation

$$\bar{\psi}\psi = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*) \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \psi_1^*\psi_1 + \psi_2^*\psi_2 - \psi_3^*\psi_3 - \psi_4^*\psi_4$$

Fermion Currents

- 16 Lorentz invariant quantities can be defined from spinors.

- Each describes a different kind of fermion currents (fermion lines of Feynman diagrams)

scalar: $\bar{\psi}\psi$

pseudo-scalar: $\bar{\psi}\gamma^5\psi$

vector: $\bar{\psi}\gamma^\mu\psi$

axial-vector: $\bar{\psi}\gamma^\mu\gamma^5\psi$

tensor: $\bar{\psi}\frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi$

- Look forward: we will see “vector minus axial vector” currents are important in the Standard Model

vector – axial-vector: $\frac{1}{2}(\bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^5\gamma^\mu\psi) = \bar{\psi}\gamma^\mu\frac{1}{2}(1-\gamma^5)\psi$

The Photon

- The photon is described by Maxwell's Equations:

$$\vec{\nabla} \cdot \vec{E} = \rho \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}$$

- Maxwell's equations can be re-written as (see Griffiths section 7.4)

$$\partial^2 A^\mu = \frac{4\pi}{c} j^\mu \qquad \begin{array}{l} A^\mu = (V, \vec{A}) \text{ with } \vec{B} = \vec{\nabla} \times \vec{A} \\ j^\mu = (c\rho, \vec{j}) \end{array}$$

- A^μ is the electromagnetic four-potential and j^μ is the electromagnetic four-current

- The solutions are $A^\mu = \varepsilon^\mu(s)\, e^{-ip\cdot x}$ where $\varepsilon^\mu(s)$ is the **polarisation vector** (s is helicity)

- Three possible spin orientations along photon direction of travel $s = +1, 0, -1$
  - $s = +1$ corresponds to a right-handed helicity
  - $s = -1$ corresponds to a left-handed helicity

  } **transverse** polarisation states (left and right circular polarisation)

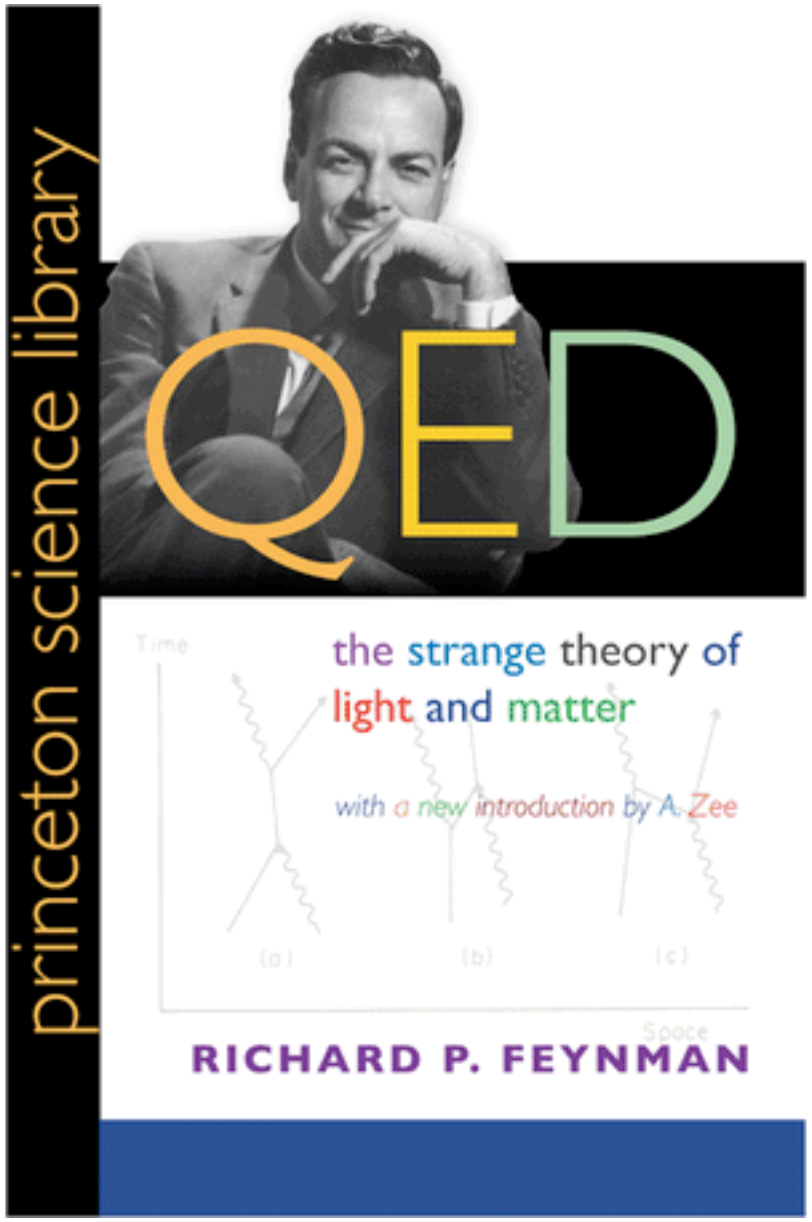
  - $s = 0$ is the **longitudinal** polarisation state (does not exist for a real photon; does exists for W and Z and virtual photons)

Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 6: Quantum Electrodynamics

The Electromagnetic force, quantised)



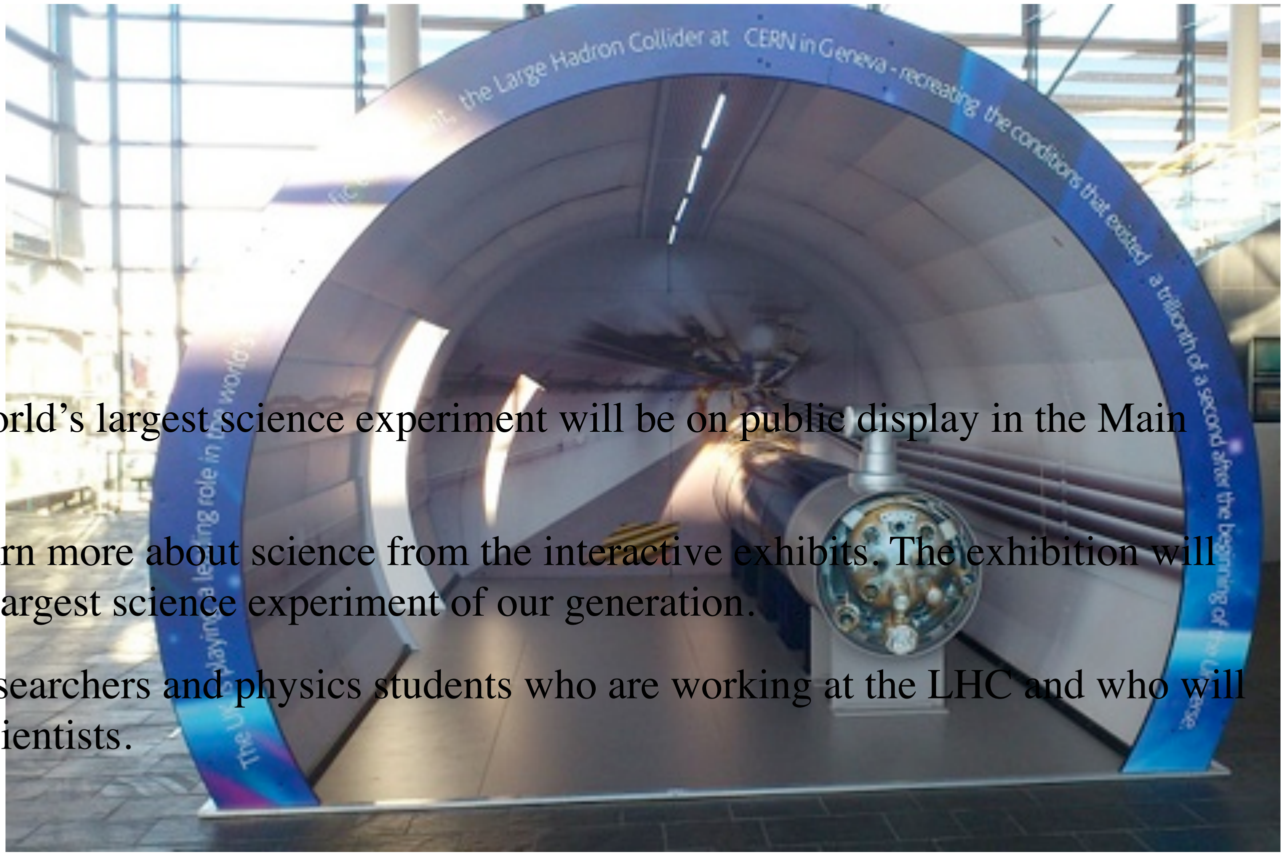
- ★$e^-\mu^-\rightarrow e^-\mu^-$ scattering
- ★helicity configurations
- ★higher order diagrams
- ★running coupling and renormalisation

Next Week

- Lecture on Tuesday only.
- No lecture on Friday 8th February.
- Tutorial next week for catch up: plus a few group activities.

LHC in the Parliament: from tomorrow

http://www.scottish.parliament.uk/visitandlearn/58283.aspx



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Review: Feynman Rules for QED

- **External Lines**

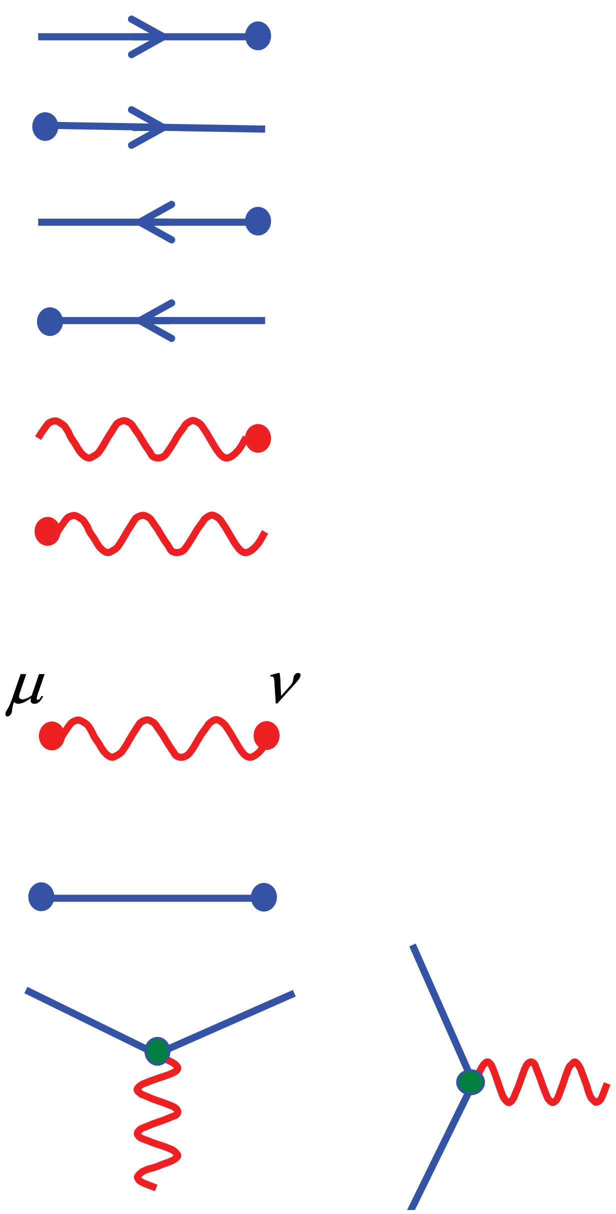
spin 1/2	incoming particle	$u(p)$
	outgoing particle	$\overline{u}(p)$
	incoming antiparticle	$\overline{v}(p)$
	outgoing antiparticle	$v(p)$
spin 1	incoming photon	$\varepsilon^\mu(p)$
	outgoing photon	$\varepsilon^\mu(p)^*$

- **Internal Lines (propagators)**

spin 1	photon	$\dfrac{g_{\mu\nu}}{q^2}$
spin 1/2	fermion	$\dfrac{(\gamma^\mu q_\mu + m)}{q^2 - m^2}$

- **Vertex Factors**

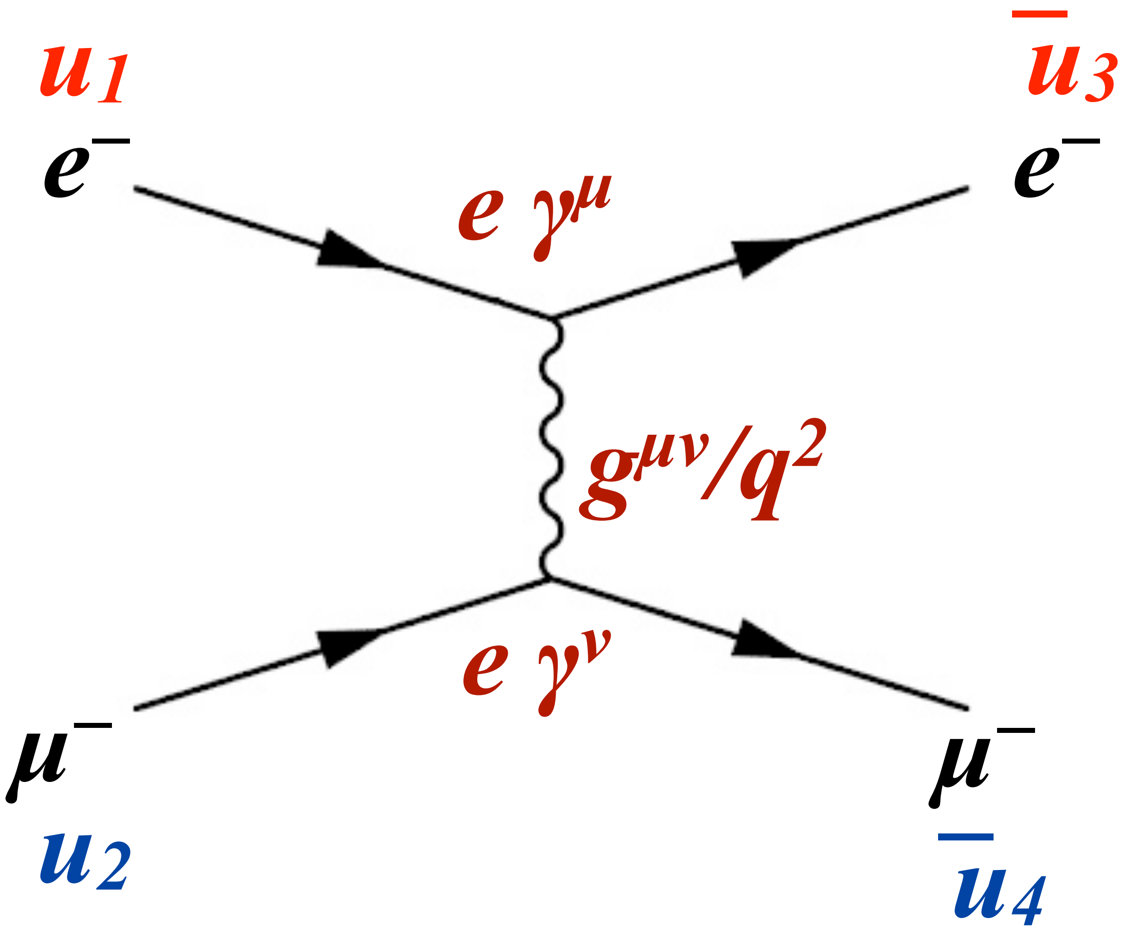
spin 1/2	fermion (charge -|e|)	$e\gamma^\mu$



- Matrix element $\mathcal{M}$ is product of all factors
- Integrate over all allowed internal momenta and spins, consistent with momentum conservation

Rewview: Electron-Muon Scattering

- Just one lowest order diagram
- In lecture 3, we considered a similar diagram (but with spinless particles)



$$\mathcal{M} = e^2 \frac{g^{\mu\nu}}{q^2} j_{13}^\mu \, j_{24}^\nu$$

Vertex Couplings

$$\mathcal{M} = e^2 \frac{g^{\mu\nu}}{q^2} \underbrace{(\bar{u}_3 \gamma^\mu u_1)}_{\text{Electron current}} \underbrace{(\bar{u}_4 \gamma^\nu u_2)}_{\text{Muon current}}$$

Photon propagator,

Squaring: take complex conjugate, use $g^{\mu\nu}$ term to set ν to μ

$$|\mathcal{M}|^2 = \frac{e^4}{q^4} (\bar{u}_3 \gamma^\mu u_1)\,(\bar{u}_3 \gamma^\mu u_1)^* (\bar{u}_4 \gamma^\mu u_2)(\bar{u}_4 \gamma^\mu u_2)^*$$

Spin & Scattering

- Spinors describe fermions with a given helicity (h=+1 (R) or h=−1 (L)).

  ⇒ The value of $\mathcal{M}$ depends on the initial and final state helicities

- In reality:
  - ➡ Measure the same process many times (often millions or billions).
  - ➡ The initial helicity (spins) of the fermions are known, either:
    - unpolarised (50% h=+1; 50% h=−1)
    - polarised (known fractions of h=±1)
  - ➡ Often measure the final state in all outgoing helicity configurations

- To calculate unpolarised cross sections (*i.e.* when initial state is unpolarised):
  - ➡ **average over initial state helicities** and **sum over final state helicities**

High Energy e-μ scattering

- 16 possible helicity configurations:

$$\begin{array}{cccc}
e_L\mu_L \longrightarrow e_L\mu_L & e_L\mu_L \longrightarrow e_L\mu_R & e_L\mu_L \longrightarrow e_R\mu_L & e_L\mu_L \longrightarrow e_R\mu_R \\
e_L\mu_R \longrightarrow e_L\mu_L & e_L\mu_R \longrightarrow e_L\mu_R & e_L\mu_R \longrightarrow e_R\mu_L & e_L\mu_R \longrightarrow e_R\mu_R \\
e_R\mu_L \longrightarrow e_L\mu_L & e_R\mu_L \longrightarrow e_L\mu_R & e_R\mu_L \longrightarrow e_R\mu_L & e_R\mu_L \longrightarrow e_R\mu_R \\
e_R\mu_R \longrightarrow e_L\mu_L & e_R\mu_R \longrightarrow e_L\mu_R & e_R\mu_R \longrightarrow e_R\mu_L & e_R\mu_R \longrightarrow e_R\mu_R
\end{array}$$

- At high energy helicity is conserved.
- For $e^-\mu^-\to e^-\mu^-$ scattering only four configurations contribute:

$$\mathcal{M}(e_L\mu_L\to e_L\mu_L)\ ,\ \mathcal{M}(e_L\mu_R\to e_L\mu_R)\ ,\ \mathcal{M}(e_R\mu_L\to e_R\mu_L)\ ,\ \mathcal{M}(e_R\mu_R\to e_R\mu_R)$$

  - ➡ Average over initial states: $(2S+1)$ initial states for spin, S
  - ➡ Sum over final states

$$|\mathcal{M}|^2 = \frac{e^4}{q^4}\frac{1}{(2S_1+1)(2S_2+1)}\sum_{S_3,S_4}(\bar{u}_3\gamma^\mu u_1)\,(\bar{u}_3\gamma^\nu u_1)^*(\bar{u}_4\gamma^\mu u_2)(\bar{u}_4\gamma^\nu u_2)^*$$

$$= \frac{e^4}{q^4}\left(\frac{1}{(2S_1+1)}\sum_{S_3}(\bar{u}_3\gamma^\mu u_1)\,(\bar{u}_3\gamma^\nu u_1)^*\right)\left(\frac{1}{(2S_2+1)}\sum_{S_4}(\bar{u}_4\gamma^\mu u_2)(\bar{u}_4\gamma^\nu u_2)^*\right)$$

$$= \frac{e^4}{q^4}\,L_e\,L_\mu$$

Trace Theorems

- The spinor-gamma matrices products in the sum can be evaluated using **trace theorems.** (See details in Griffiths 7.7, equation 7.128)

$$\begin{aligned} L_e &= \frac{1}{(2S_1+1)} \sum_{S_3} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_3 \gamma^\nu u_1)^* \\ &= 2\left[\underbrace{p_3^\mu p_1^\nu}_{\text{matrix}} + \underbrace{p_3^\nu p_1^\mu}_{\text{matrix}} - (p_3 \cdot p_1 - m_e^2)\underbrace{g^{\mu\nu}}_{\text{matrix}}\right] \end{aligned}$$

- No spinors left! Just matrices, likewise:

$$L_\mu = 2\left[p_4^\mu p_2^\nu + p_4^\nu p_2^\mu - (p_4 \cdot p_2 - m_\mu^2)g^{\mu\nu}\right]$$

- If the electron and muon are energetic, $E>>m$, can ignore m_e and m_μ terms

$$|\mathcal{M}|^2 = \frac{e^4}{q^4} L_e L_\mu = \frac{8e^4}{q^4}\left[(p_3 \cdot p_4)(p_1 \cdot p_2) + (p_3 \cdot p_2)(p_1 \cdot p_4)\right]$$

- Substitute in Mandelstam variables:

$$|\mathcal{M}|^2 = 2e^4 \frac{s^2+u^2}{t^2} = 2e^4 \frac{1+4\cos^4(\theta^*/2)}{\sin^4(\theta^*/2)}$$

Cross section for $e^-\mu^-\rightarrow e^-\mu^-$ scattering

- Cross section = $|\mathcal{M}|^2\,\rho$, substituting for ρ (see problem sheet 1):

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2}\frac{|\vec{p}^{\,*}_f|}{|\vec{p}^{\,*}_i|}$$

- With:
  - ➡ centre of mass energy, $(E_1+E_2)^2=s$
  - ➡ $|p^*_f|=|p^*_i|$
  - ➡ $S=1$ as no identical particles in final state
  - ➡ $\alpha=e^2/4\pi$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\pi s}\left(\frac{s^2+u^2}{t^2}\right)$$

- The different spin configurations give scattering distributions:

$$\mathcal{M}(LL\rightarrow LL)=\mathcal{M}(RR\rightarrow RR)=e^2\frac{u}{t}=e^2\frac{1+\cos\theta^*}{1-\cos\theta^*}$$

$$\mathcal{M}(LR\rightarrow LR)=\mathcal{M}(RL\rightarrow RL)=e^2\frac{s}{t}=e^2\frac{2}{1-\cos\theta^*}$$

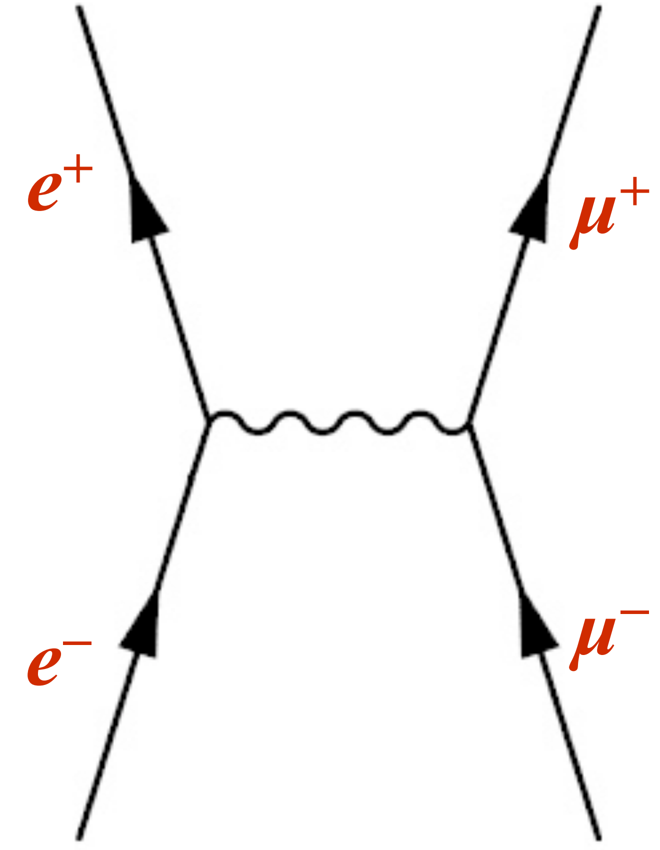
- θ^* is the scattering angle between the incoming and outgoing electron

$e^-e^+\rightarrow\mu^-\mu^+$ Scattering

- Related to $e\mu\rightarrow e\mu$ scattering by exchanging $t \leftrightarrow s$

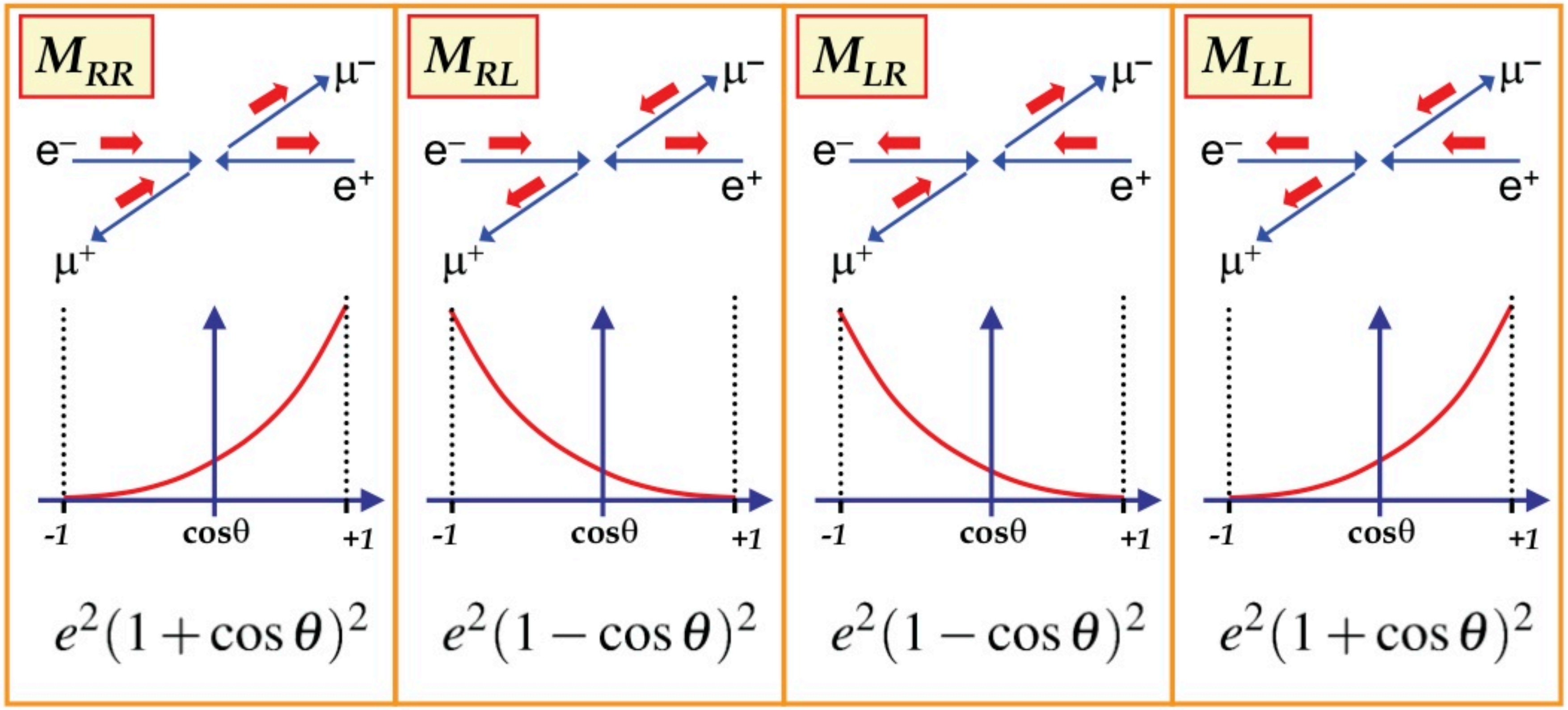
$$|\mathcal{M}|^2 = 2e^4\frac{(t^2+u^2)}{s^2} = e^4(1+\cos^2\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4\pi s}(1+\cos^2\theta)$$



In the high energy limit, there are 4 contributions:
RL→RL ($\mathcal{M}_{RR}$), RL→LR ($\mathcal{M}_{RL}$), LR→RL ($\mathcal{M}_{LR}$), LR→LR ($\mathcal{M}_{LL}$)

We have averaged over initial states and summed over final states to get unpolarised cross section:



$e^-e^+ \rightarrow \mu^-\mu^+$ Total Cross Section

- Total cross section, integrate over solid angle:

$$\begin{aligned}
\sigma &= \int \frac{d\sigma}{d\Omega}\,d\Omega \\
&= \frac{\alpha^2}{4\pi s}\int (1+\cos^2\theta)\,d\cos\theta\, d\phi \\
&= \frac{\alpha^2}{4\pi s}\,[\phi]_{-\pi}^{\pi}\left[\cos\theta + \frac{1}{3}\cos^3\theta\right]_{\cos\theta=-1}^{\cos\theta=+1} \\
&= \frac{4\alpha^2}{3s}
\end{aligned}$$

- Comparison prediction to measurement. Pretty good for a 1st order calculation!

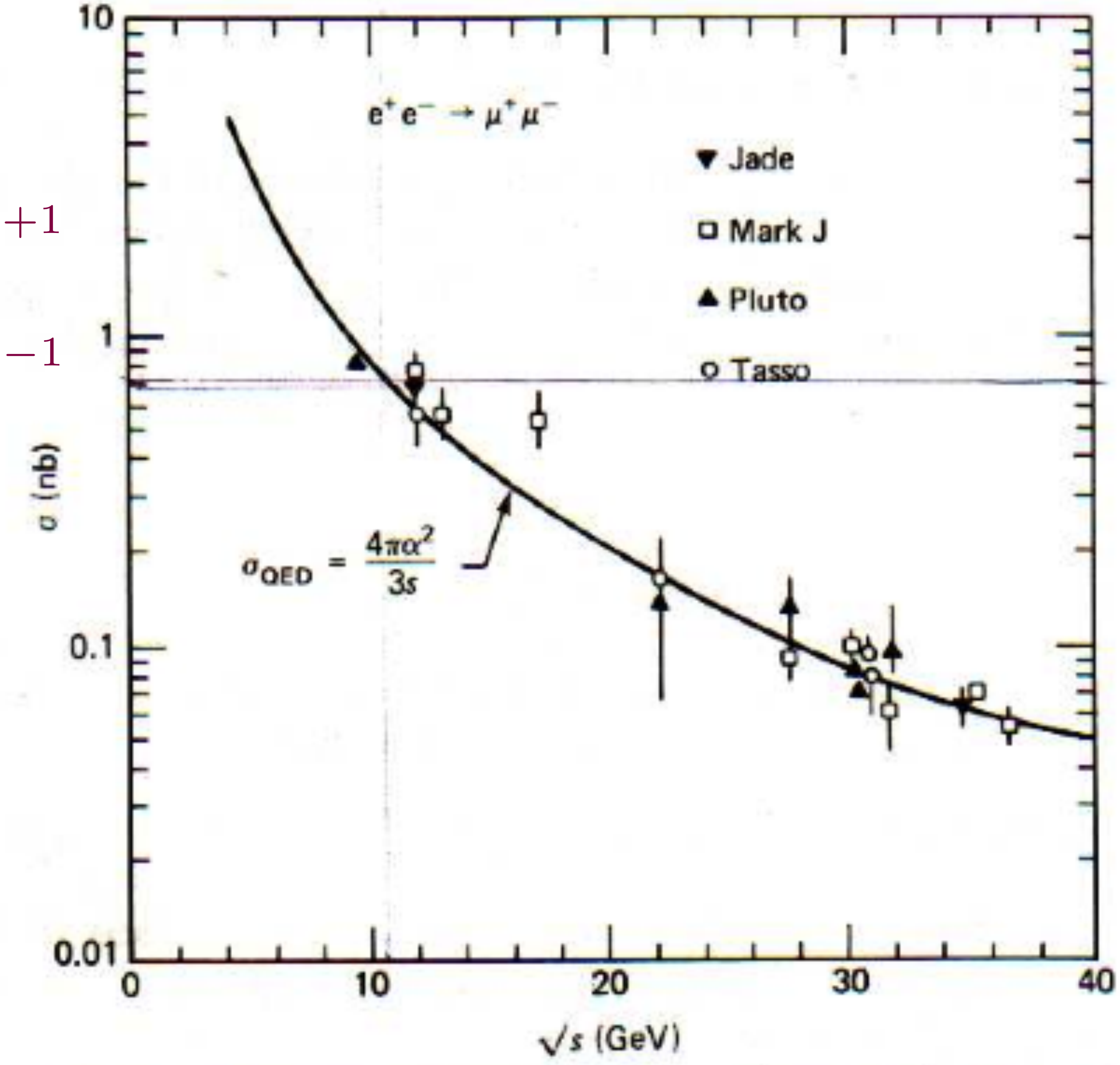


Fig. 6.6 The total cross section for $e^-e^+ \rightarrow \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

Higher Order QED

- We have been drawing and calculating 1st order Feynman diagrams with one boson exchanged
- There are more diagrams with higher numbers of vertices.
- We should sum them all to obtain the total value for $\mathcal{M}$

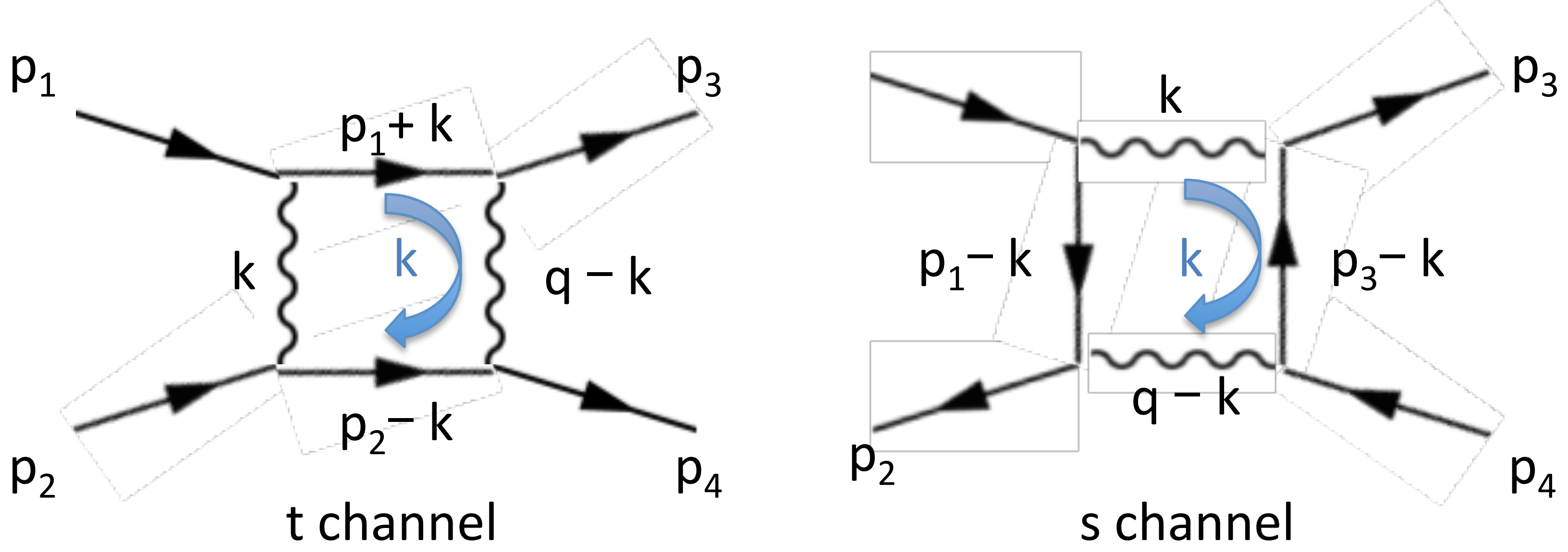
$$\mathcal{M}_{\rm tot} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 \ldots$$

➡ ... but for every two vertices you have a suppression factor of ***α=1/137***

- The most precise QED calculations go up to ***O(α⁵)*** diagrams

Higher Order QED

e.g. Two photon "box" diagrams also contribute to QED scattering



Two extra vertices ⇒ contribution is suppress by a factor of $\alpha = 1/137$

- The four momentum must be conserved at each vertex.
- However, four momentum k flowing round the loop can be anything!

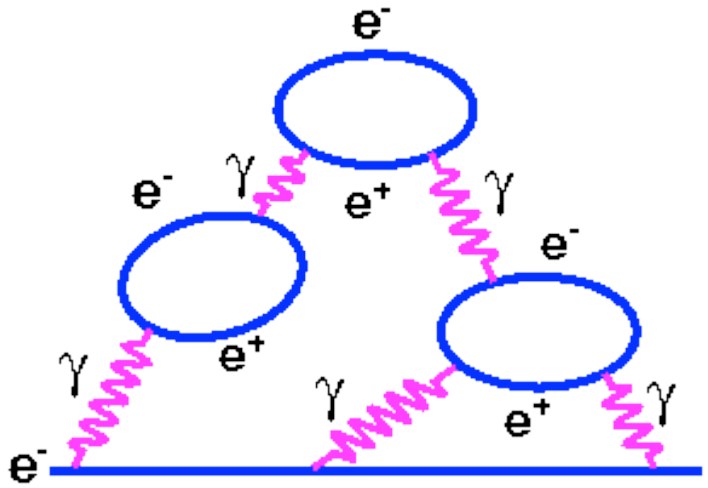
- In calculating $\mathcal{M}$ integrate over all possible allowed momentum configurations: $\int f(k)\, d^4k \sim \ln(k)$ leads to a divergent integral!
- This is solved by **renormalisation** in which the infinities are "miraculously swept up into redefinitions of mass and charge" (Aitchison & Hey P.51)

Renormalisation

- Impose a "cutoff" mass M, do not allow the loop four momentum to be larger than M. Use $M^2 >> q^2$, the momentum transferred between initial and final state.
  - ➡ This can be interpreted as a limit on the shortest range of the interaction
  - ➡ Or interpreted as possible substructure in pointlike fermions
  - ➡ Physical amplitudes should not depend on choice of M
- Find that $\ln(M^2)$ terms appear in the $\mathcal{M}$
- Absorb $\ln(M^2)$ into **redefining fermion masses and vertex couplings**
  - ➡ Masses $m(q^2)$ and couplings $\alpha(q^2)$ are now functions of q^2

- *e.g.* Renormalisation of electric charge (considering only effects from one type of fermion):

$$e_R = e\sqrt{1 - \frac{e^2}{12\pi^2}\ln\left(\frac{M^2}{q^2}\right)}$$



- Can be interpreted as a "screening" correction due to the production of electron/positron pairs in a region round the primary vertex
- e_R is the effective charge we actually measure!

Running Coupling Constant

- Renormalise α, and correct for all possible fermion types in the loop:

$$\alpha(q^2) = \alpha(0)\left(1 + \frac{\alpha(0)}{3\pi} z_f \ln(\frac{-q^2}{M^2})\right)$$

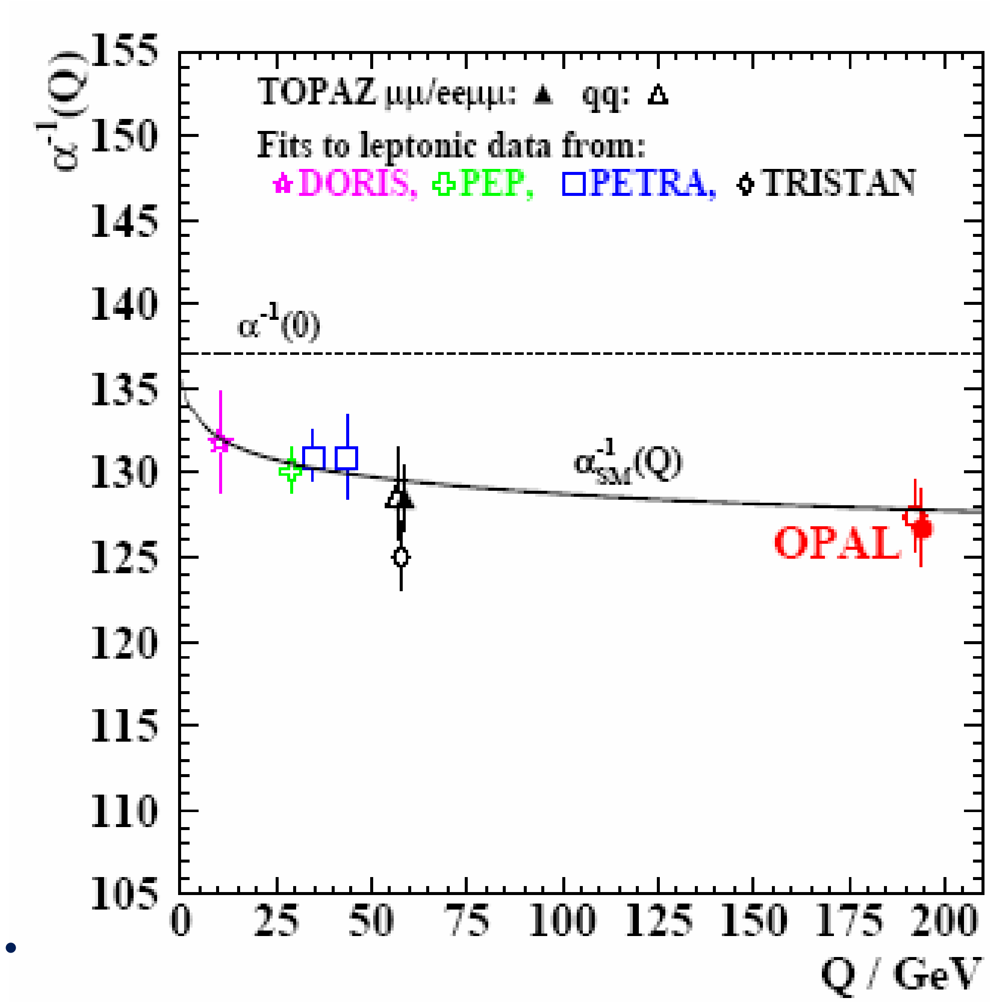
- z_f is the sum of charges over all possible fermions in the loop
  - ➡ At $q^2 \sim 1$ **MeV** only electron, $z_f = 1$
  - ➡ At $q^2 \sim 100$ **GeV**, f=e,μ,τ,u,d,s,c,b $z_f = 60/3$

$$z_f = \sum_f Q_f^2$$

- Instead of using M^2 dependence, replace with a reference value μ^2:

$$\alpha(q^2) = \alpha(\mu^2)\left(1 - \frac{\alpha(\mu^2)}{3\pi} z_f \ln(\frac{q^2}{\mu^2})\right)^{-1}$$

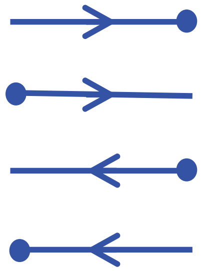
- Usual choices for μ are **1 MeV** or $m_Z \sim 91$ **GeV.**
  - ➡ $\alpha(\mu^2 = 1\ \mathrm{MeV}^2) = 1/137$
  - ➡ $\alpha(\mu^2 = (91\ \mathrm{GeV})^2) = 1/128$
- We choose a value of μ where make a initial measurement of α, but once we do the evolution of the values of α are determined by the above eqn.



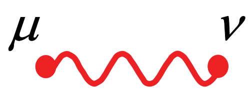
QED Summary

- For full QED calculations use spinors to define fermion current in Feynman diagrams.
- Fermion current are $\bar{\psi}\gamma^{\mu}\psi$ where $\bar{\psi}$ is the adjoint spinor: $\bar{\psi} \equiv \psi^{\dagger}\gamma^{0}$
- Spin-1 bosons are described by polarisation vectors, $\varepsilon^{\mu}(s)$
- To calculate the cross section for an unpolarised process need to average over initial helicities and sum over all possible final states.
- Fermion masses, charges and the coupling constant α evolve as a function of momentum transfer.

spin 1/2	incoming particle	$u(p)$
	outgoing particle	$\overline{u}(p)$
	incoming antiparticle	$\overline{v}(p)$
	outgoing antiparticle	$v(p)$

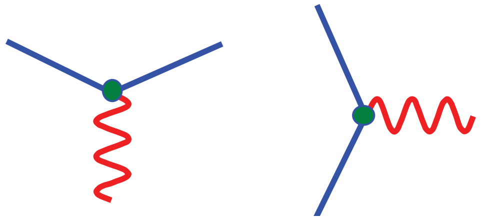


spin 1 photon $\frac{g_{\mu\nu}}{q^2}$



- **Vertex Factors**

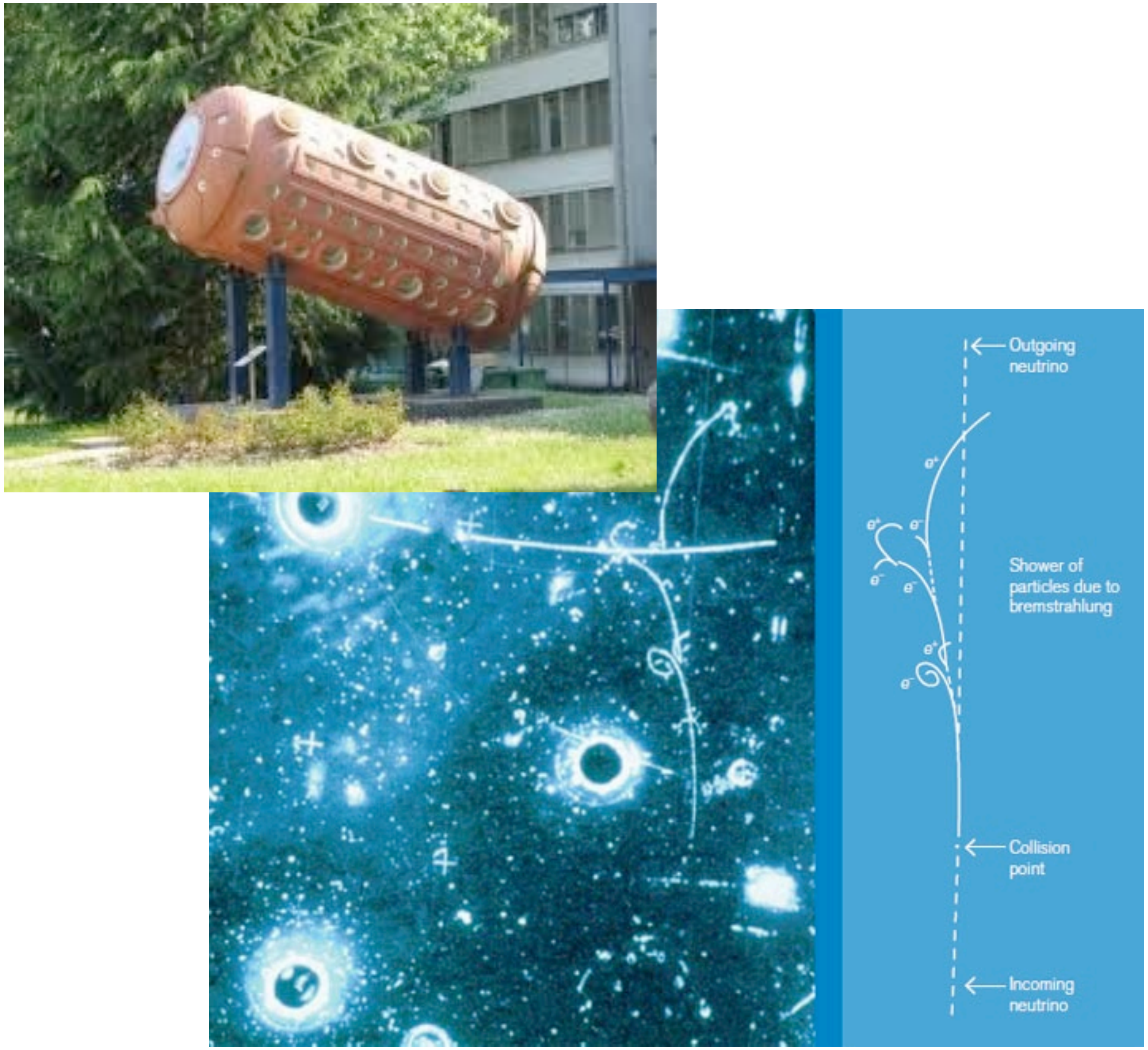
spin 1/2 fermion (charge -|e|) $e\gamma^{\mu}$



Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 7: Renormalisation and Weak Force



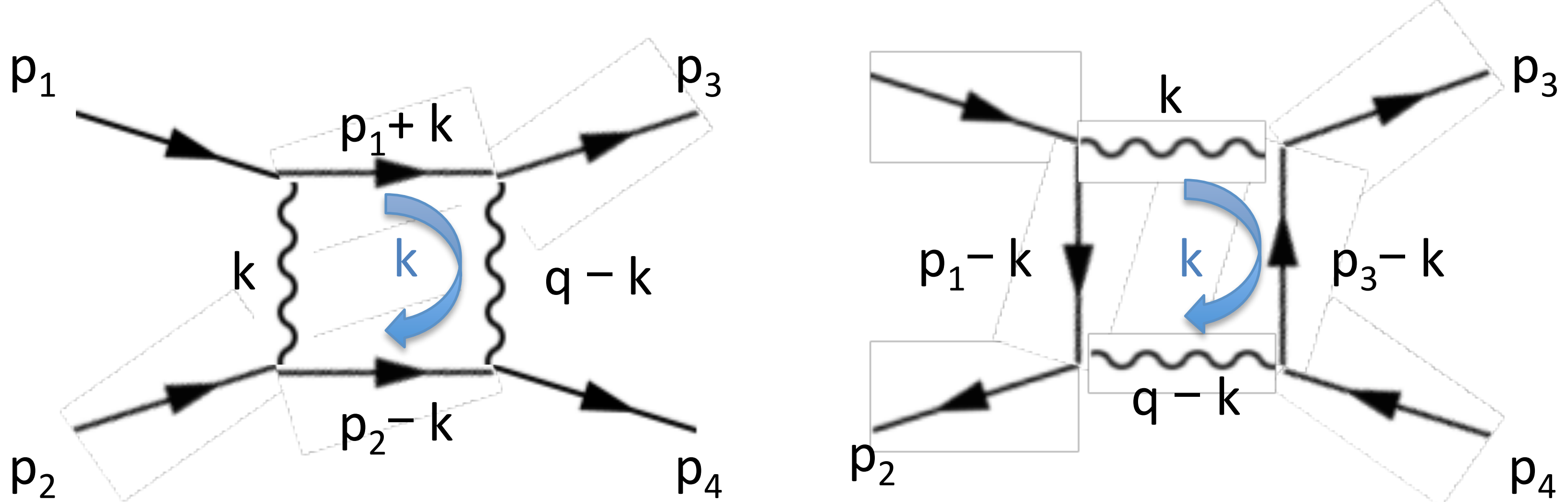
- ★ Renormalisation in QED
- ★ Weak Charged Current
- ★ $V\!-\!A$ structure of W-boson interactions
- ★ Muon decay
- ★ Beta decay
- ★ Weak Neutral Current
- ★ Neutrino scattering

Reminders

- No lecture on Friday.
- Next tutorial: Monday 11th February.
  - New tutorial sheet will be posted to webpage. Paper copies available at tutorial
- Next lecture: Tuesday 12th February!

Higher Order QED

Two photon "box" diagrams also contribute to QED scattering



t channel

s channel

Two extra vertices ⇒ contribution is suppress by a factor of $\alpha = 1/137$

- The four momentum must be conserved at each vertex.
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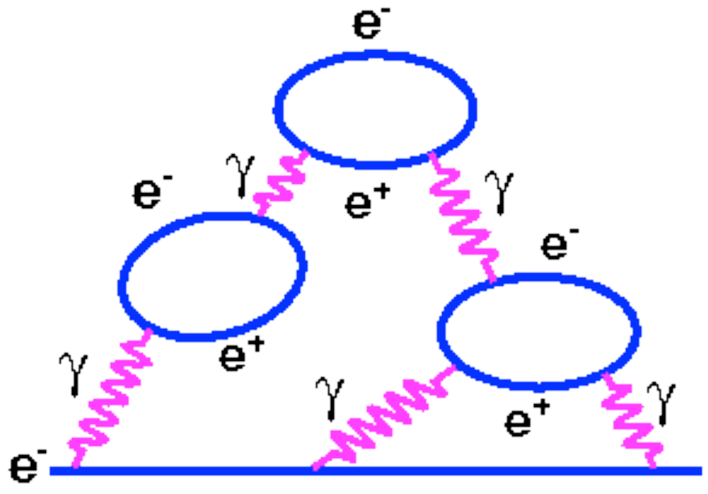

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- e_R is the effective charge we actually measure!

Running Coupling Constant

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  - At $q^2 \sim$ **1 MeV** only electron, $z_f = 1$
  - At $q^2 \sim$ **100 GeV**, $f = e, \mu, \tau, u, d, s, c, b$ $z_f = 38/9$

$$z_f = \sum_f Q_f^2$$

- Instead of using M^2 dependence, replace with a reference value μ^2:

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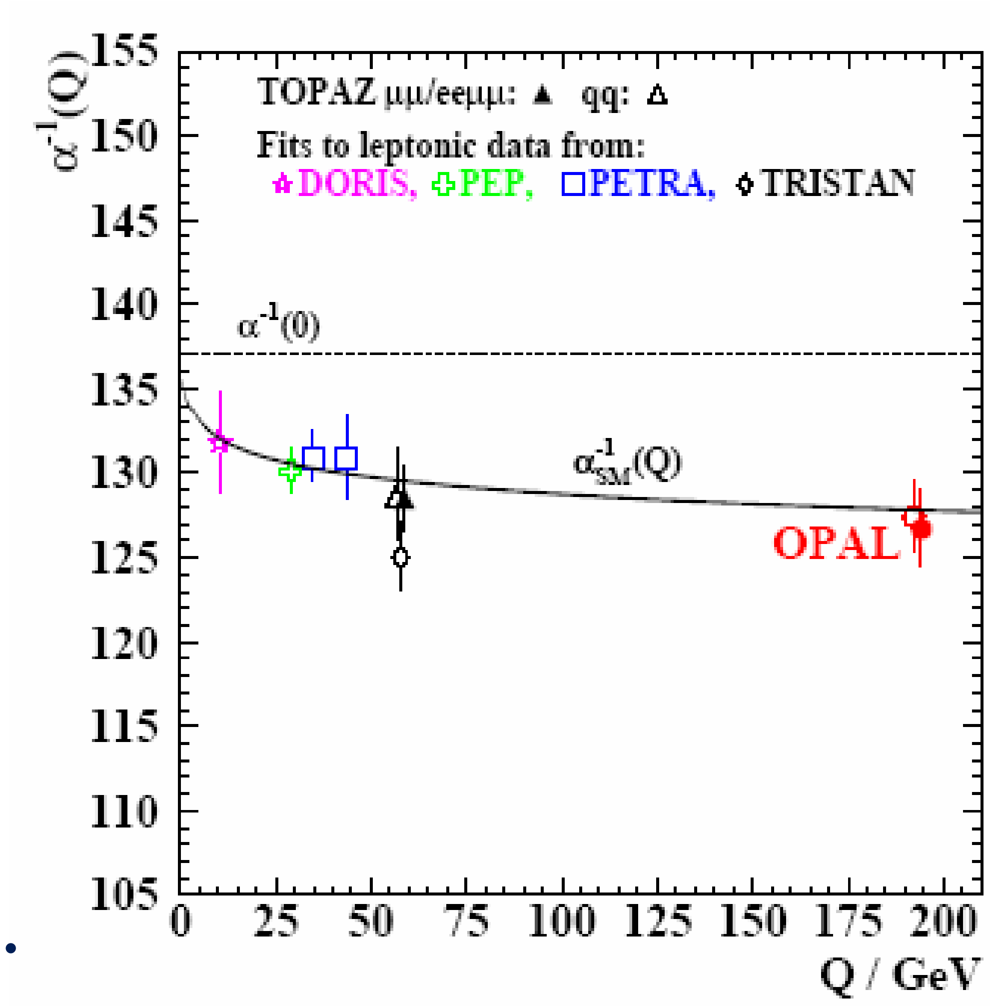

QED Summary

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	outgoing particle	$\bar{u}(p)$
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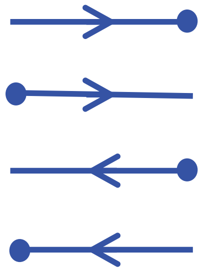
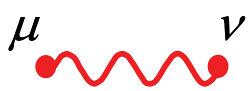
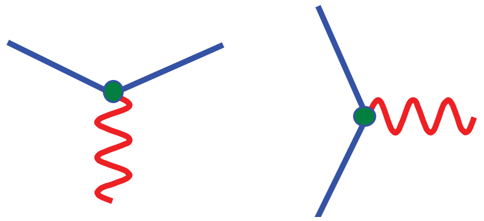


spin 1	**photon**	$\frac{g_{\mu\nu}}{q^2}$



- **Vertex Factors**

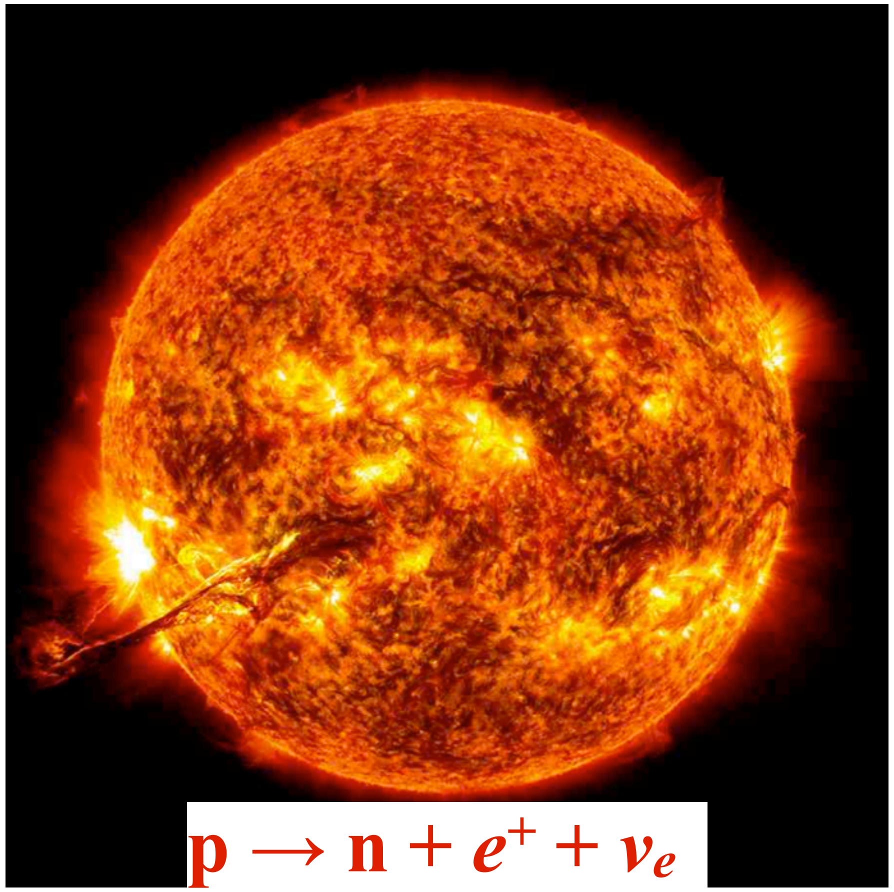
spin 1/2	**fermion (charge -|e|)**	$e\gamma^{\mu}$



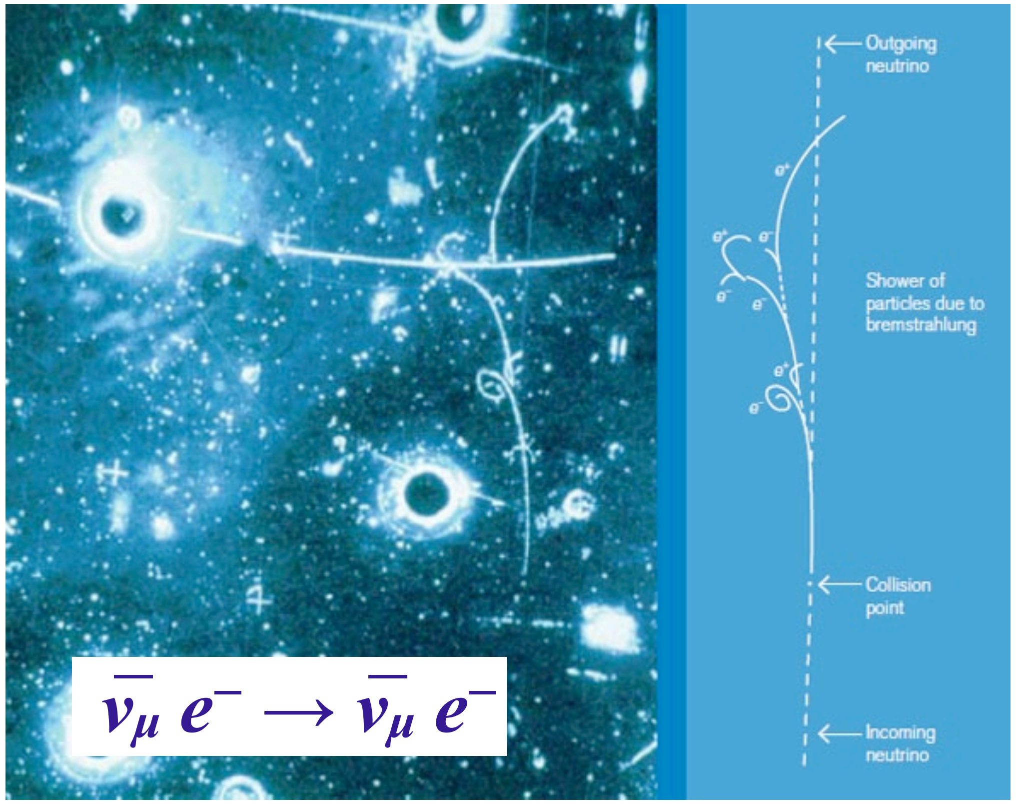
The Weak Force

- Exchange of **massive** W and Z bosons.
  - $m_W = 80.385 \pm 0.015$ GeV
  - $m_Z = 91.1876 \pm 0.0021$ GeV

- Responsible for:
  - Beta decay
  - Fusion
  - Neutrino interactions



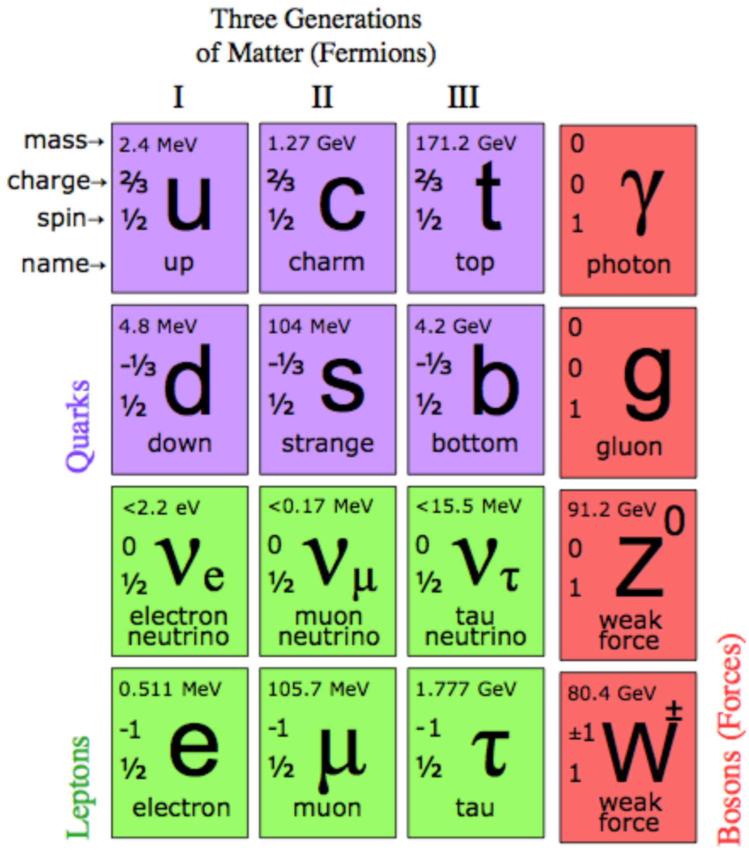
$p \rightarrow n + e^+ + \nu_e$



$\overline{\nu}_\mu e^- \rightarrow \overline{\nu}_\mu e^-$

Review: Baryon and Lepton Number; Chirality

- **Lepton number** is the number of leptons minus the number of anti-leptons:
  - ➡ Electron number: $L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\overline{\nu_e})$
  - ➡ Muon number: $L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\overline{\nu_\mu})$
  - ➡ Tau-lepton number: $L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\overline{\nu_\tau})$

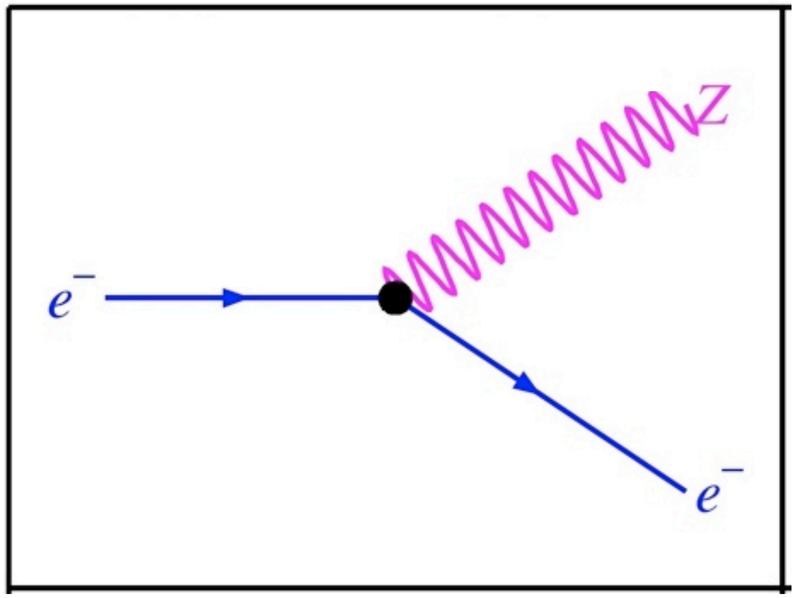


- **Baryon number** is a measure of the net number of quarks: $\mathcal{B} = ⅓(N(q) - N(\overline{q}))$

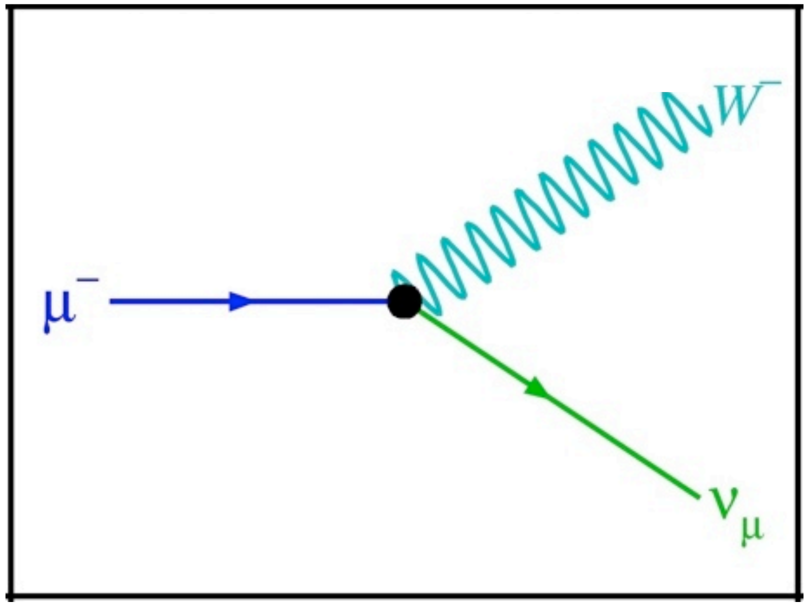
- LH projection operator $P_L = (1 - \gamma^5)/2$ projects out left-handed chiral state
- RH projection operator $P_R = (1 + \gamma^5)/2$ projects out right-handed chiral state
  - Massive fermions have both left-handed and right-handed chiral components
  - Massless fermions would have only left-handed components
  - Massless anti-fermions would have only right-handed components
  - To date, only **left-handed neutrinos** and **right-handed anti-neutrinos** have been observed

W and *Z* boson interactions

- Any fermion (quark, lepton) may emit or absorb a *Z*-boson.
  - ➡ That fermion will remain the same flavour.
  - ➡ Very similar to QED, but neutrinos can interact with a *Z* boson too.

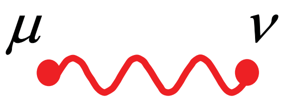
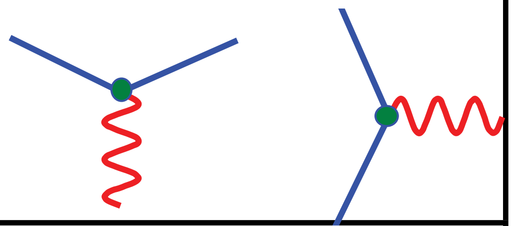


- Any fermion (quark, lepton) may emit or absorb a *W*-boson.
  - ➡ To conserve electric charge that fermion **must** change flavour!
  - ➡ To conserve lepton number $e\leftrightarrow\nu_e$, $\mu\leftrightarrow\nu_\mu$, $\tau\leftrightarrow\nu_\tau$
  - ➡ To conserve baryon number **(d, s, b)** ↔ **(u, c, t)**



down-type quark ↔ up-type quark

Feynman Rules for Charged Current

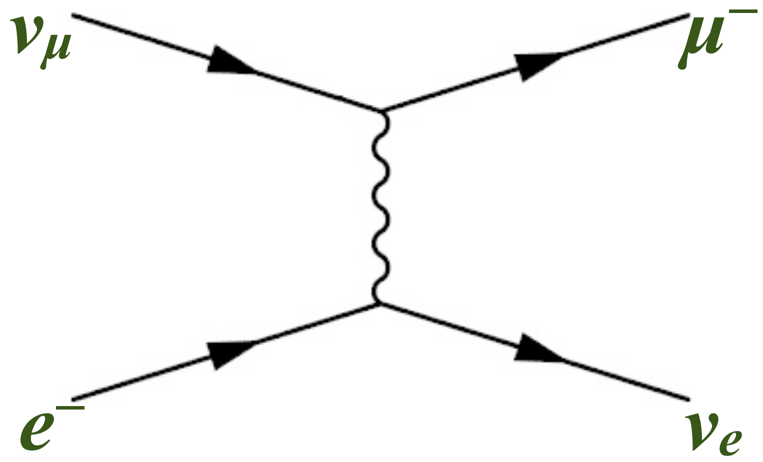
	propagator 	interaction vertex 
***W*-boson**	$\frac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \, \gamma^\mu (1-\gamma^5)$
photon, γ	$\frac{g_{\mu\nu}}{q^2}$	$e \, \gamma^\mu$

- Left-handed interactions are also known as $V\!-\!A$ theory
  - ➡ γ^μ gives a vector current (V)
  - ➡ $\gamma^\mu\gamma^5$ gives an axial vector current (A)
- Photon interactions are purely vector

- Key differences w.r.t QED.
  - ➡ $q^2 - m_W^2$ as denominator of propagator
  - ➡ The $\frac{1}{2}(1-\gamma^5)$ term: this is observed experimentally.
- The overall factor of $1/\sqrt{8}$ is conventional
- Recall $P_L=(1-\gamma^5)/2$ is the Left Handed projection operator
  - ➡ *W*-boson interactions only act on **left-handed chiral components** of fermions

- For low energy interactions with $q << m_W$: effective propagator is $g_{\mu\nu}/m_W^2$

"Inverse Muon Decay"



- Start with a calculation of the process $\nu_\mu\, e^- \rightarrow \mu^-\, \nu_e$
- Not an easy process to measure experimentally, but easy to calculate!

$$\mathcal{M} = \frac{g_W^2}{8}\, \bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\, \frac{g_{\mu\nu}}{q^2 - m_W^2}\, u(\nu_\mu)\gamma^\nu(1-\gamma^5)u(\mu)$$

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 \left[\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\right]^2 \left[\bar{u}(\mu)\gamma^\mu(1-\gamma^5)u(\nu_\mu)\right]^2$$

- Usually we would average over initial spin and sum over final spin states:
  - However the neutrinos are only left handed
  - The equation can be solved as (see Griffiths section 9.1):

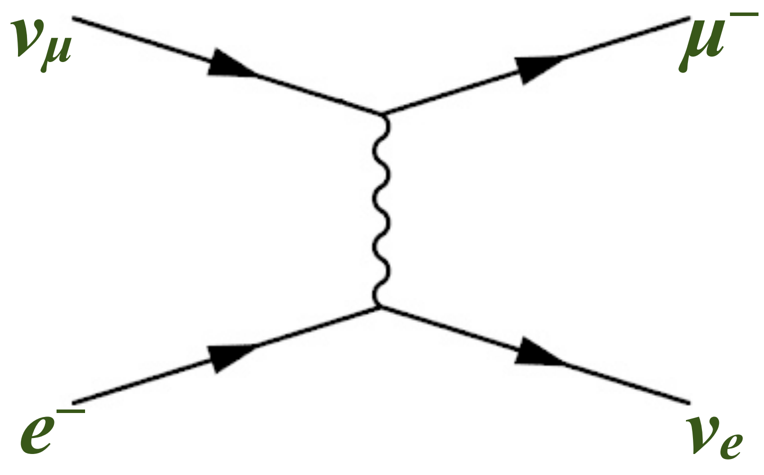
$$|\mathcal{M}|^2 = 2\left(\frac{g_W^2}{m_W^2}\right)^2 \left(p^\mu(e)\cdot p^\mu(\nu_\mu)\right)\left(p^\mu(\mu)\cdot p^\mu(\nu_e)\right)$$

  - In the CM frame, where E is energy of initial electron or neutrino, and m_e neglected as $m_e << E$:

$$|\mathcal{M}|^2 = 8E^4\left(\frac{g_W^2}{m_W^2}\right)^2\left(1 - \frac{m_\mu^2}{2E^2}\right)^2$$

“Inverse Muon Decay” Cross Section

- Cross section = $|\mathcal{M}|^2\,\rho$, substituting for ρ (see problem sheet 1):



$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2}\frac{|\vec{p}^{\,*}_f|}{|\vec{p}^{\,*}_i|}$$

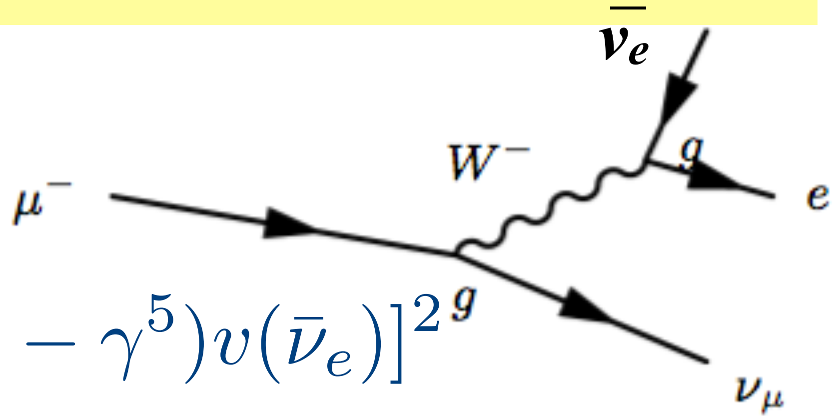
- Substitute:
  - ➡ centre of mass energy, $(E_1+E_2)^2=4E^2$
  - ➡ For elastic scattering particle $|p^*_f|=|p^*_i|$
  - ➡ $S=1$ as no identical particles in final state

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{32\pi^2}\left(\frac{g_W^2}{m_W^2}\right)^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

  - ➡ Fermi coupling constant $G_F = \sqrt{2}g_W^2/8m_W^2$
  - ➡ Unlike electromagnetic interaction, no angular dependence
  - ➡ Integral over 4π solid angle

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \frac{4}{\pi}E^2G_F^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

Muon Decay



- Muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ (Griffiths 9.2):

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 [\bar{u}(\nu_\mu)\gamma^\mu(1-\gamma^5)u(\mu)]^2[\bar{u}(e)\gamma^\mu(1-\gamma^5)v(\bar{\nu}_e)]^2$$

$$= 2\left(\frac{g_W^2}{m_W^2}\right)^2 (p^\mu(e)\cdot p^\mu(\nu_\mu))\,(p^\mu(\mu)\cdot p^\mu(\nu_e))$$

- The phase space, ρ, for a 1 → 3 decay is, (Griffiths equation 6.21):

$$\frac{d\Gamma}{dE_e} = \frac{1}{4\pi^3}\left(\frac{\sqrt{2}g_W^2}{8M_W^2}\right)^2 m_\mu^2 E_e^2 \left(1 - \frac{4E_e}{3m_\mu^2}\right)$$

- Integrate over allowed values of E_e:

$$\Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE_e} dE_e = \frac{G_F^2 m_\mu^2}{4\pi^3}\int_0^{m_\mu/2} E_e^2\left(1 - \frac{4E_e}{3m_\mu^2}\right) dE_e = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- Only muon decay mode for muons **BR($\mu^- \to e^- \bar{\nu}_e \nu_\mu$) ≈ 100%**, only one decay mode contributes to lifetime

$$\tau \equiv \frac{1}{\Gamma} = \frac{192\pi^3}{G_F^2 m_\mu^5} = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$

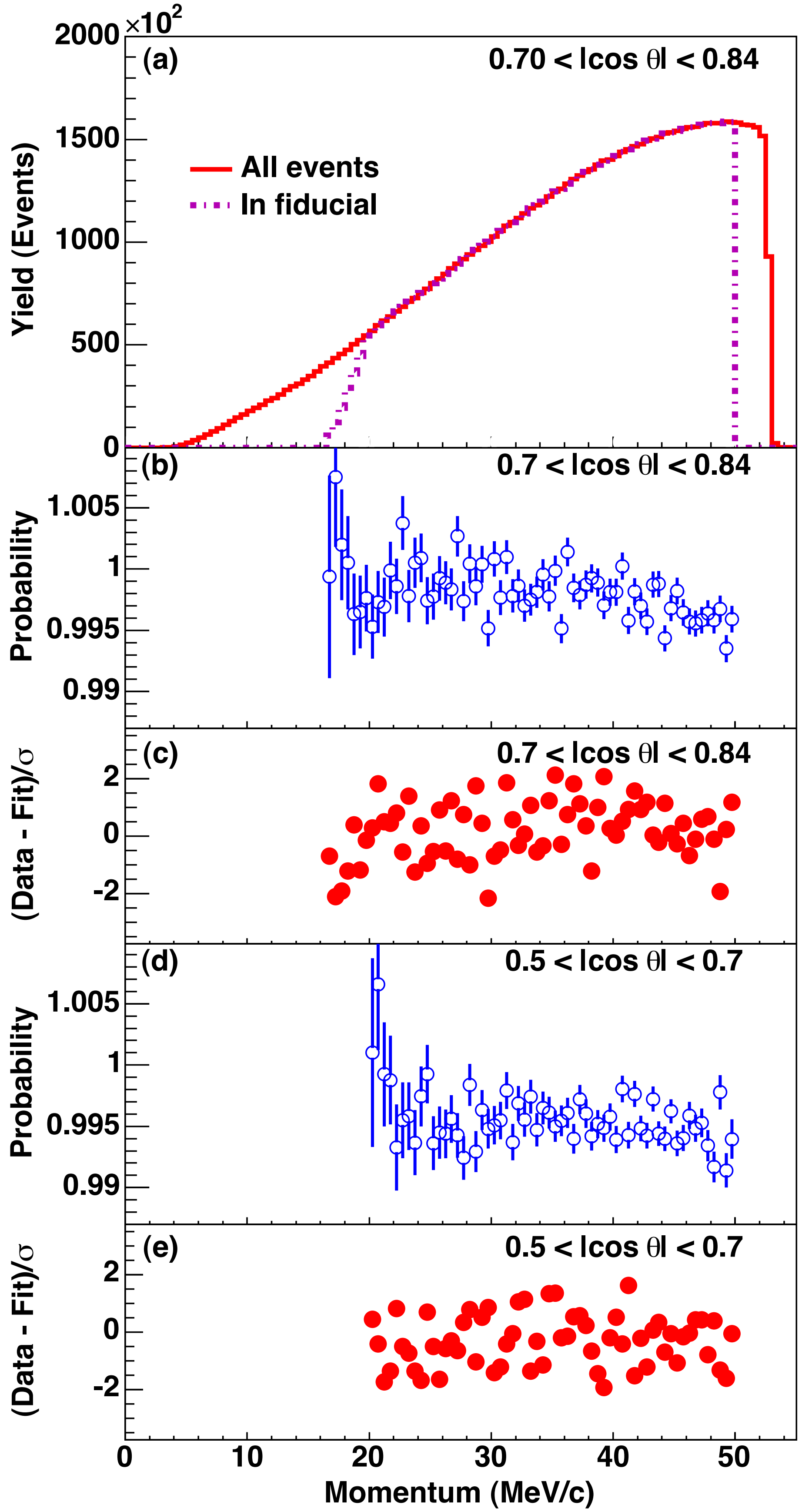
Muon Decay Measurements

- TWIST experiment at TRIMF in Canada measures $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decay spectrum.
- Excellent agreement between data and prediction!

- Measurements of muon lifetime and mass used to define a value for G_F (values from PDG 2010)
  - ➡ $\tau = (2.19703 \pm 0.00002) \times 10^6$ s
  - ➡ m = 105.658367 ± 0.000004 MeV

- Applying small corrections for finite electron mass and second order effects
  - ➡ $G_F = 1.166364(5) \times 10^{-5}\ \mathrm{GeV}^{-2}$

- Implies $g_W = 0.653$, $\alpha_W = g_W^2/4\pi = 1/29.5$
- $\alpha_W >> \alpha_{EM}$, the weak force not intrinsically weak, just appears so due to mass of W-boson



J.R. Musser et al., Physical Review Letters **94**, 101805 (2005)

Summary

Renormalisation

- QED: Photons and fermions can appear in loops. Effectively modifying coupling (effective charge) and masses of fermions.
- This leads to running coupling α as a function of energy of scattering q.
- This happens in QED, Weak and QCD

Weak Charged Current

- Carried by the massive W-boson: acts on all quarks and leptons.

$$\frac{g_{\mu\nu}}{q^2 - m_W^2}$$

- A W-boson interaction changes the flavour of the fermion.
- Acts only on the left-handed components of the fermions: $V{-}A$ structure.

$$\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)$$

- At low energy, responsible for muon & tau decay, beta decay…

Weak Neutral Current

- Carried by the massive Z-boson: acts on all quarks and leptons.

$$\frac{g_{\mu\nu}}{q^2 - m_Z^2}$$

- No flavour changes observed.

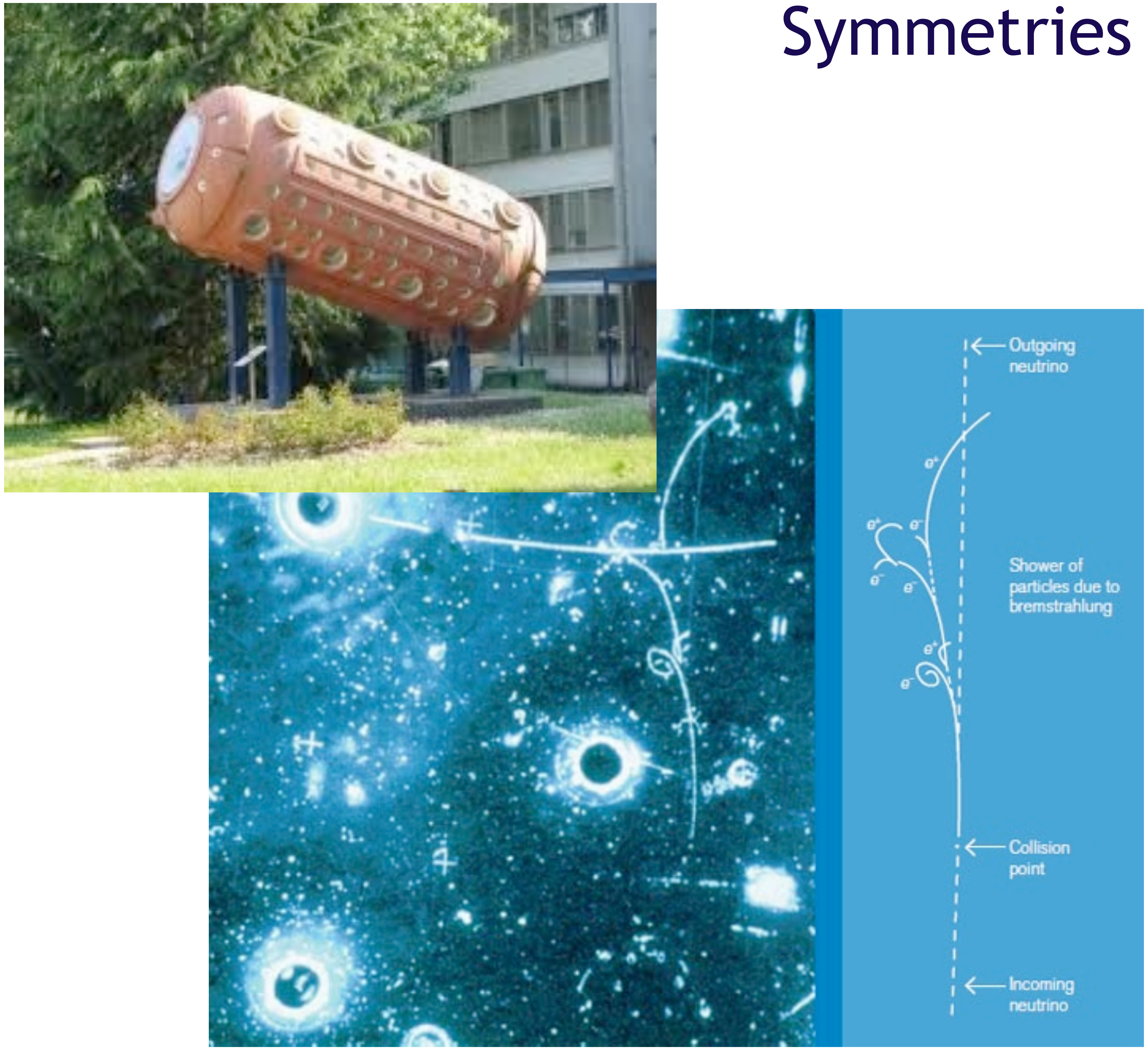
$$\bar{u}(e)\gamma^\mu(c_V^e - c_A^e\gamma^5)u(e)$$

- At low energies, responsible for neutrino scattering

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 8: Calculating the Weak Force and Symmetries

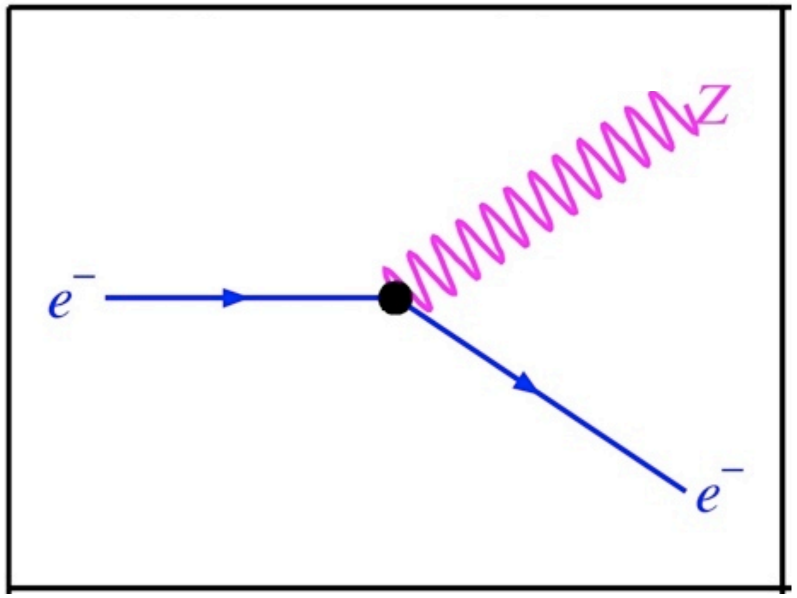


- ★Muon decay
- ★Beta decay
- ★Weak Neutral Current
- ★Neutrino scattering

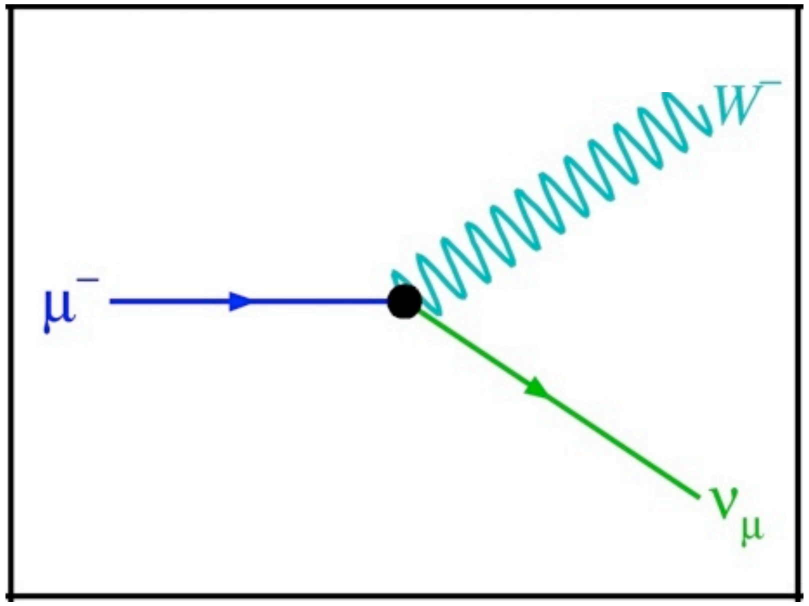
- ★Symmetries in PP

W and *Z* boson interactions

- Any fermion (quark, lepton) may emit or absorb a *Z*-boson.
  - ➡ That fermion will remain the same flavour.
  - ➡ Very similar to QED, but neutrinos can interact with a *Z* boson too.

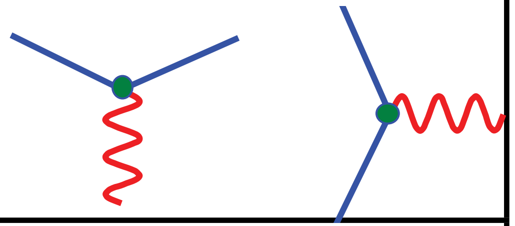


- Any fermion (quark, lepton) may emit or absorb a *W*-boson.
  - ➡ To conserve electric charge that fermion **must** change flavour!
  - ➡ To conserve lepton number $e\leftrightarrow\nu_e$, $\mu\leftrightarrow\nu_\mu$, $\tau\leftrightarrow\nu_\tau$
  - ➡ To conserve baryon number $(\mathrm{d, s, b}) \leftrightarrow (\mathrm{u, c, t})$



down-type quark ↔ up-type quark

Feynman Rules for Charged Current

	propagator  	interaction vertex 
***W*-boson**	$\frac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \, \gamma^\mu (1 - \gamma^5)$
photon, γ	$\frac{g_{\mu\nu}}{q^2}$	$e\,\gamma^\mu$

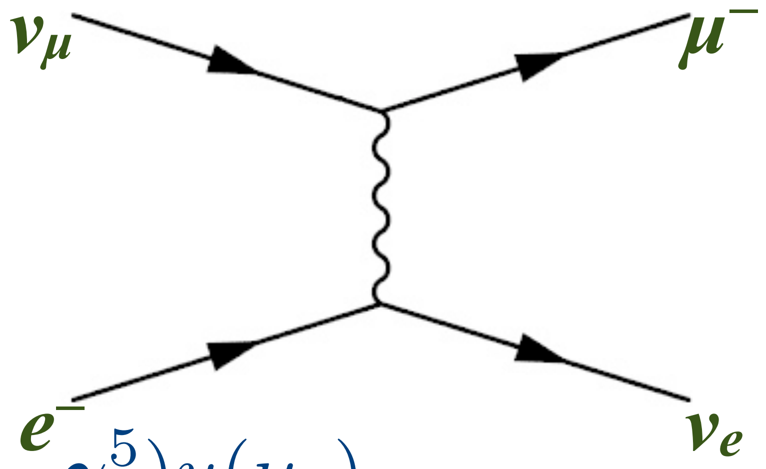
- Left-handed interactions are also known as $V\!-\!A$ theory
  - ➡ γ^μ gives a vector current (V)
  - ➡ $\gamma^\mu\gamma^5$ gives an axial vector current (A)
- Photon interactions are purely vector

- Key differences w.r.t QED.
  - ➡ $q^2 - m_W^2$ as denominator of propagator
  - ➡ The $\frac{1}{2}(1-\gamma^5)$ term: this is observed experimentally.
- The overall factor of $1/\sqrt{8}$ is conventional
- Recall $P_L=(1-\gamma^5)/2$ is the Left Handed projection operator
  - ➡ *W*-boson interactions only act on left-handed chiral components of fermions

- For low energy interactions with $q << m_W$: effective propagator is $g_{\mu\nu}/m_W^2$

“Inverse Muon Decay”

- Start with a calculation of the process $\nu_\mu \, e^- \rightarrow \mu^- \, \nu_e$
- Not an easy process to measure experimentally, but easy to calculate!



$$\mathcal{M} = \frac{g_W^2}{8}\,\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\,\frac{g_{\mu\nu}}{q^2 - m_W^2}\,\bar{u}(\mu)\gamma^\nu(1-\gamma^5)u(\nu_\mu)$$

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 \left[\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\right]^2 \left[\bar{u}(\mu)\gamma^\mu(1-\gamma^5)u(\nu_\mu)\right]^2$$

- Usually we would average over initial spin and sum over final spin states:
  - However the neutrinos are only left handed
  - The equation can be solved as (see Griffiths section 9.1):

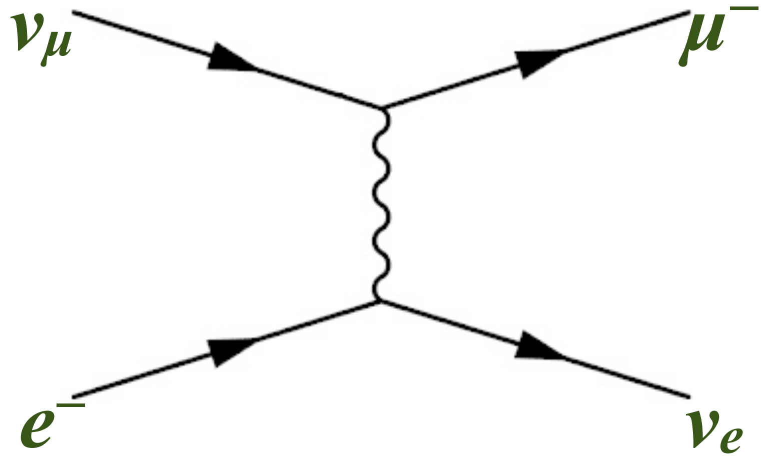
$$|\mathcal{M}|^2 = 2\left(\frac{g_W^2}{m_W^2}\right)^2 \left(p^\mu(e)\cdot p^\mu(\nu_\mu)\right)\left(p^\mu(\mu)\cdot p^\mu(\nu_e)\right)$$

  - In the CM frame, where E is energy of initial electron or neutrino, and m_e neglected as $m_e << E$:

$$|\mathcal{M}|^2 = 8E^4\left(\frac{g_W^2}{m_W^2}\right)^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

"Inverse Muon Decay" Cross Section

- Cross section = $|\mathcal{M}|^2 \rho$, substituting for ρ (see problem sheet 1):



$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\vec{p}^{\,*}_f|}{|\vec{p}^{\,*}_i|}$$

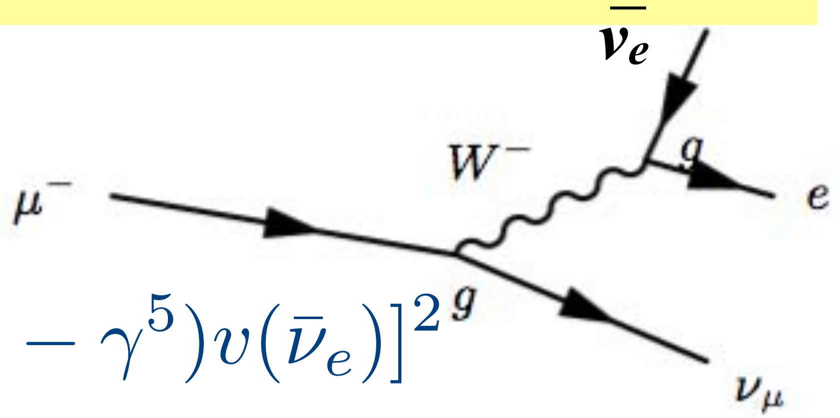
- Substitute:
  - ➡centre of mass energy, $(E_1+E_2)^2=4E^2$
  - ➡For elastic scattering particle $|p^*_f|=|p^*_i|$
  - ➡$S=1$ as no identical particles in final state

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{32\pi^2}\left(\frac{g_W^2}{m_W^2}\right)^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

- ➡Fermi coupling constant $G_F = \sqrt{2}g_W^2/8m_W^2$
- ➡Unlike electromagnetic interaction, no angular dependence
- ➡Integral over 4π solid angle

$$\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \frac{4}{\pi}E^2G_F^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

Muon Decay



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$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 [\bar{u}(\nu_\mu)\gamma^\mu(1-\gamma^5)u(\mu)]^2[\bar{u}(e)\gamma^\mu(1-\gamma^5)v(\bar{\nu}_e)]^2$$

$$= 2\left(\frac{g_W^2}{m_W^2}\right)^2 (p^\mu(e)\cdot p^\mu(\nu_\mu))\,(p^\mu(\mu)\cdot p^\mu(\nu_e))$$

- The phase space, ρ, for a 1 → 3 decay is, (Griffiths equation 6.21):

$$\frac{d\Gamma}{dE_e} = \frac{1}{4\pi^3}\left(\frac{\sqrt{2}g_W^2}{8M_W^2}\right)^2 m_\mu^2 E_e^2\left(1-\frac{4E_e}{3m_\mu^2}\right)$$

- Integrate over allowed values of E_e:

$$\Gamma = \int_0^{m_\mu/2}\frac{d\Gamma}{dE_e}dE_e = \frac{G_F^2 m_\mu^2}{4\pi^3}\int_0^{m_\mu/2}E_e^2\left(1-\frac{4E_e}{3m_\mu^2}\right)dE_e = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- Only muon decay mode for muons **BR($\mu^- \to e^- \bar{\nu}_e \nu_\mu$) ≈ 100%**, only one decay mode contributes to lifetime

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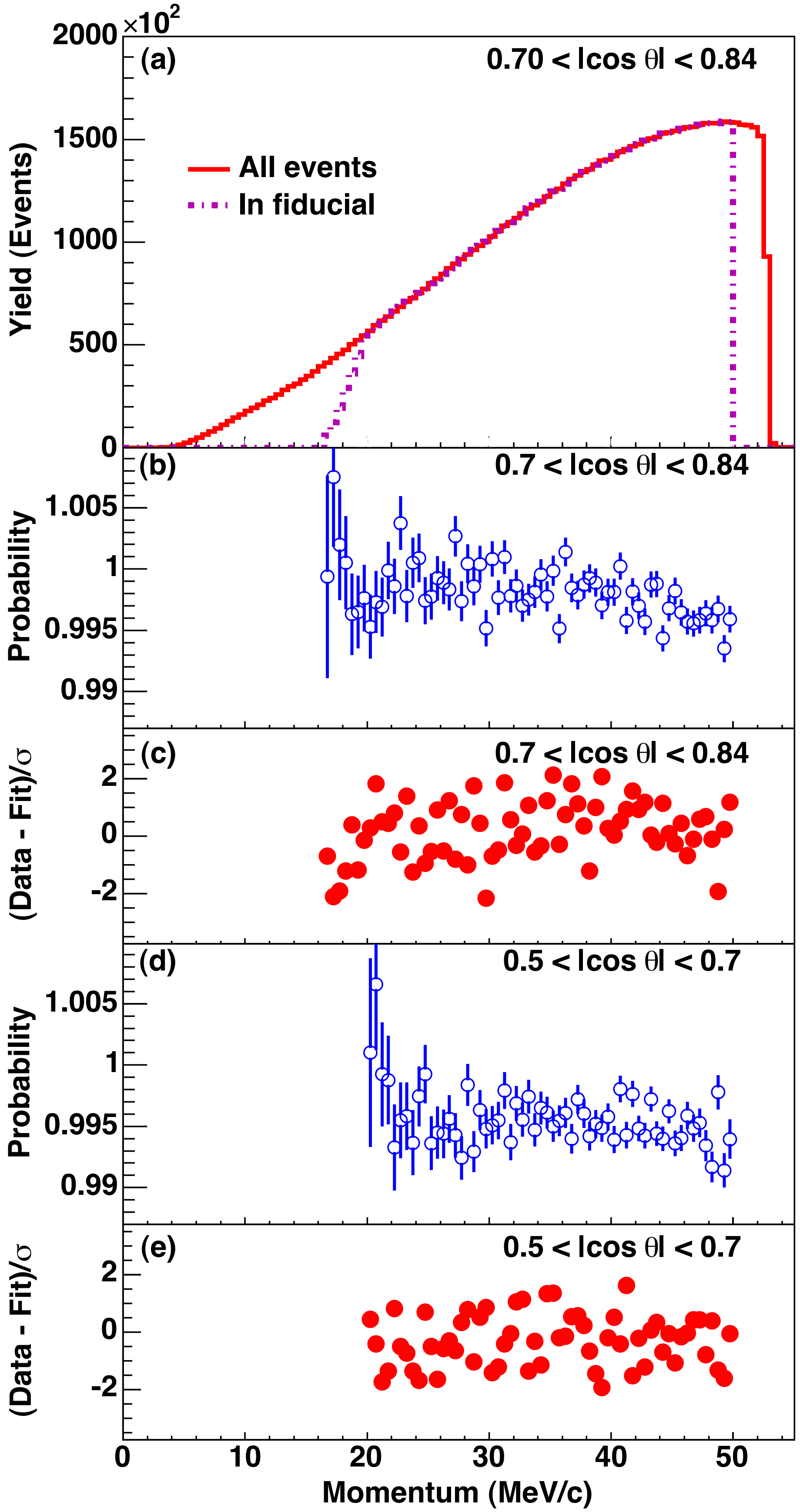
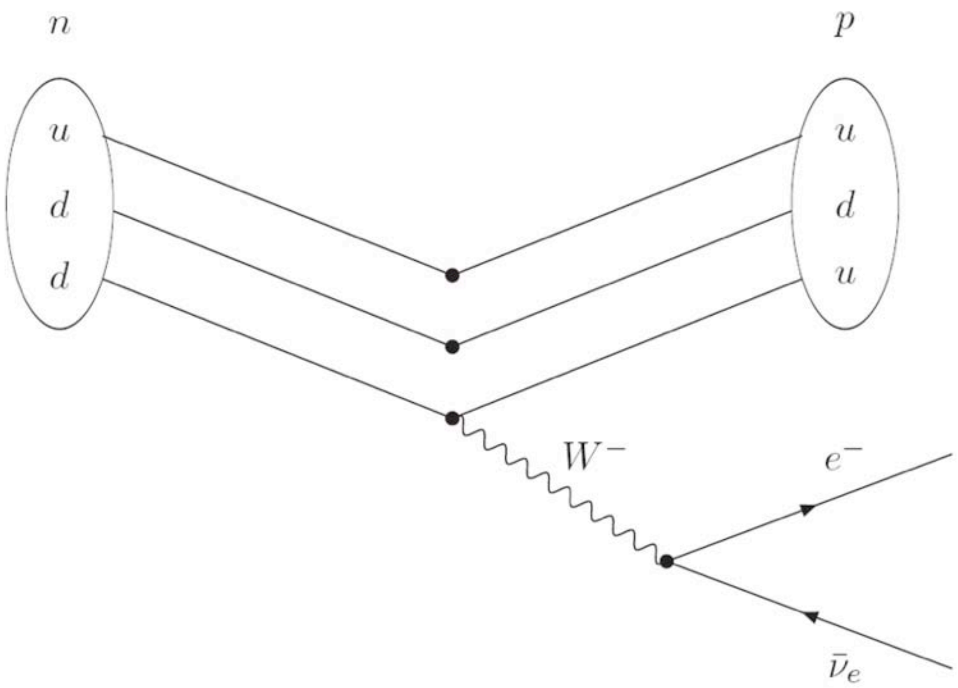


J.R. Musser et al., Physical Review Letters 94, 101805 (2005)

Beta Decay



- W boson is responsible for beta decay.

Quark level

- $\mathbf{u} \rightarrow \mathbf{d}\ e^+\ \nu_e$ or $\mathbf{d} \rightarrow \mathbf{u}\ e^-\ \overline{\nu}_e$ with coupling $g_W\ V_{\mathbf{ud}}$, $V_{\mathbf{ud}} = 0.974$
- not directly observable because no free quarks

$$\mathcal{M} = \frac{V_{\mathrm{ud}} g_W^2}{8}\, \bar{u}(\mathrm{d})\gamma^\nu(1-\gamma^5)u(\mathrm{u})\, \frac{g_{\mu\nu}}{q^2 - m_W^2}\, \bar{v}(\bar{\nu}_e)\gamma^\mu(1-\gamma^5)u(e)$$

Hadron level

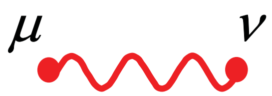
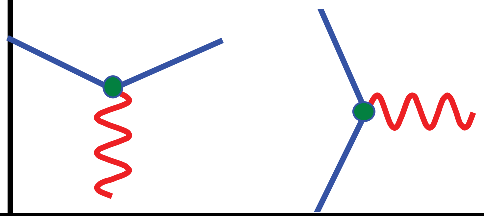
- $\mathbf{n} \rightarrow \mathbf{p}\ e^-\ \overline{\nu}_e$ is allowed (free neutron lifetime $\tau_n = 886\mathrm{s}$)
- $\mathbf{p} \rightarrow \mathbf{n}\ e^+\ \nu_e$ is forbidden $m_p < m_n$ (free proton stable)
- Hadronic interactions (form factors) play a role in decay rate/lifetime

Nuclear level

- $\beta+$ decay e.g. $^{22}\mathrm{Na} \rightarrow {}^{22}\mathrm{Ne}^*\ e^-\ \overline{\nu}_e$
- $\beta-$ decay e.g. $^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}^*\ e^-\ \overline{\nu}_e$
- which type occurs depends on the energy available (Q)

Neutral Current Interactions

- Exchange of massive Z-bosons, $m_Z = 91.1897(21)$ GeV
- Couples to all quarks and all leptons (including neutrinos)
- No allowed flavour changes!

	propagator 	interaction vertex 
W-boson	$\frac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \, \gamma^\mu (1 - \gamma^5)$
Z-boson	$\frac{g_{\mu\nu}}{q^2 - m_Z^2}$	$\frac{1}{2} g_Z \, \gamma^\mu (c_V^f - c_A^f \gamma^5)$
photon, γ	$\frac{g_{\mu\nu}}{q^2}$	$e \, \gamma^\mu$

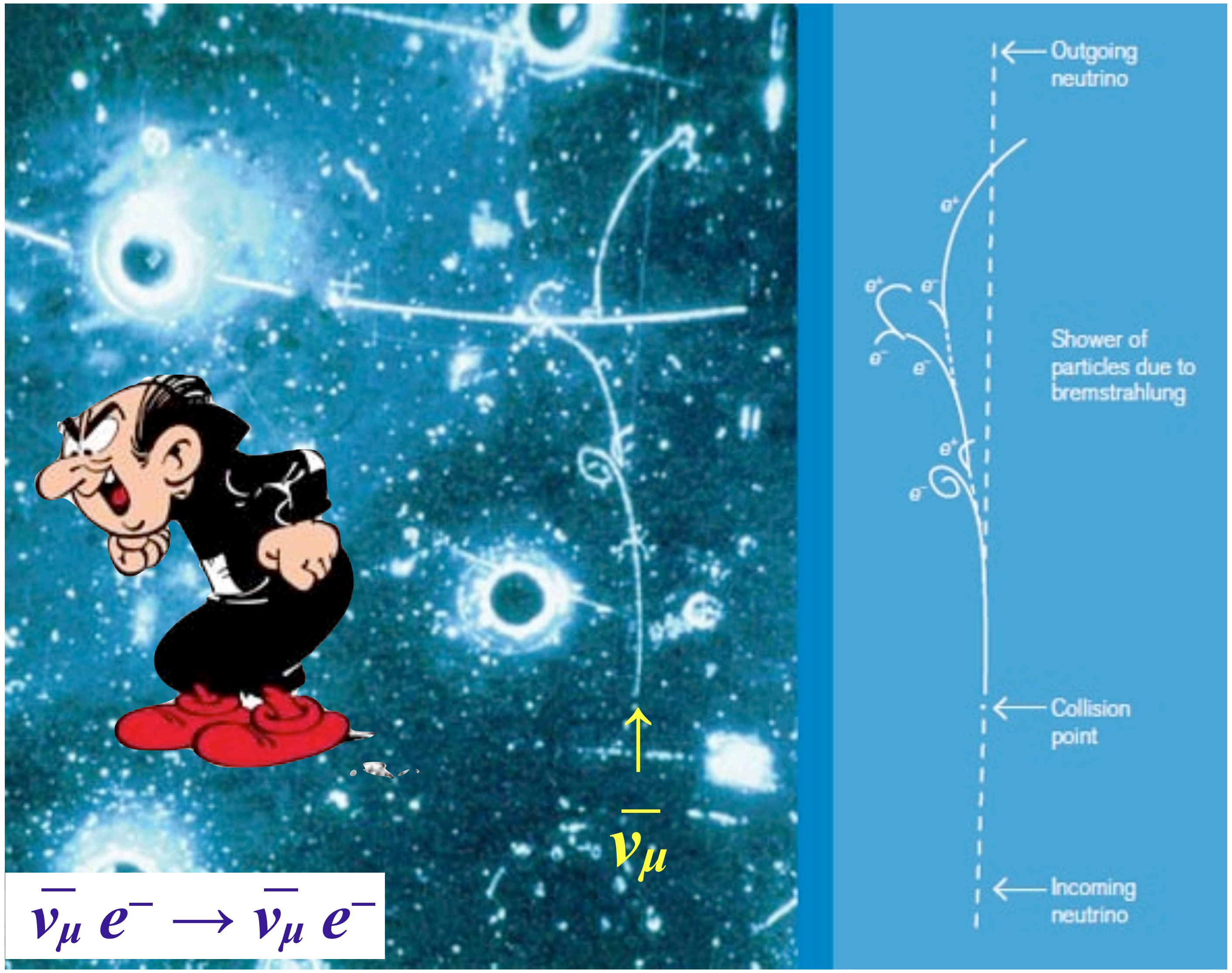
- Coupling to Z-boson depends on the flavour of the fermion (f): c^f_V, c^f_A
- Both vector ($c^f_V \gamma^\mu$) and axial vector contributions ($c^f_A \gamma^\mu \gamma^5$).

Lepton	c^f_V	c^f_A	Quark	c^f_V	c^f_A
ν_e, ν_μ, ν_τ	½	½	**u, c, t**	0.19	½
e, μ, τ	-0.03	−½	**d, s, b**	-0.34	− ½

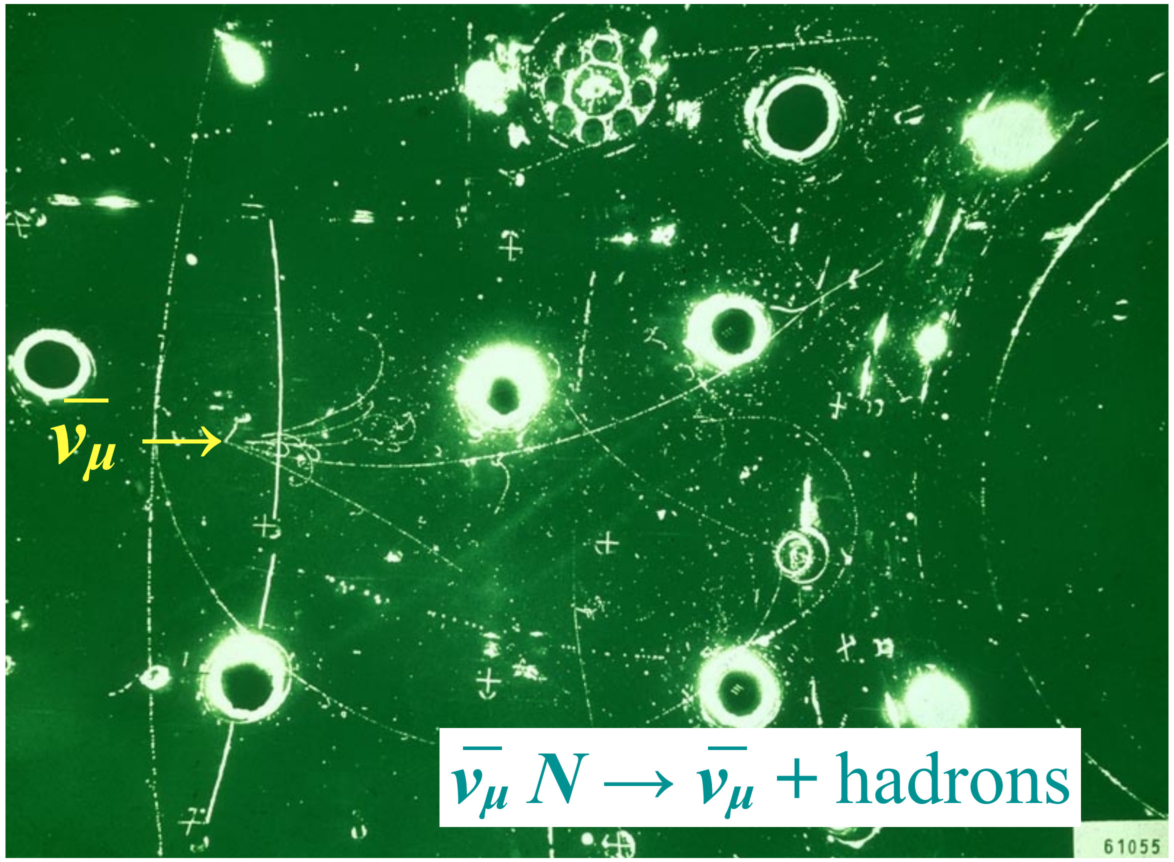
- g_Z coupling is related to g_W: $g_Z = g_W \, m_Z / m_W$
- Neutral weak current for electron: $\bar{u}(e)\gamma^\mu(c_V^e - c_A^e \gamma^5)u(e)$

Weak Neutral Current

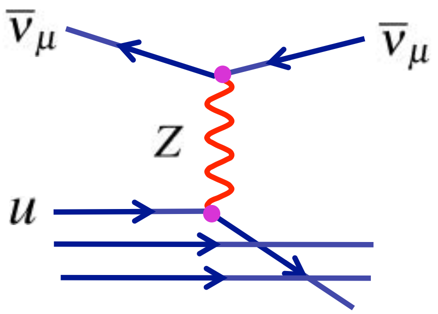
- At low energy, the main effect of Z-boson exchange is neutrino scattering. (All other Z-boson phenomena can also due to γ exchange.)
- Z-boson exchange first observed in the Gargamelle bubble chamber in 1973.
- Interaction of muon neutrinos produce a final state muon.

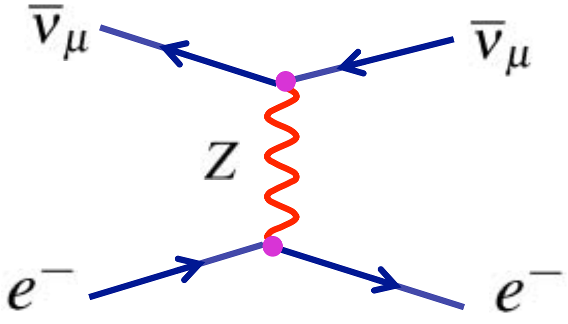


$$\bar{\nu}_\mu\, e^- \longrightarrow \bar{\nu}_\mu\, e^-$$



$$\bar{\nu}_\mu\, N \longrightarrow \bar{\nu}_\mu + \text{hadrons}$$





Symmetries in Particle Physics

- The EM, Weak and Strong forces all display a property known as Gauge Symmetry.
- In QM, a symmetry is present if **physical observables** (e.g. cross section, decay widths) are invariant under the following change in the wavefunction:

$$\psi \rightarrow \psi' = \hat{U}\psi$$

- e.g. in electromagnetism, the physical observable fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are independent of the value of the EM potential, $\boldsymbol{A_\mu}$:

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \chi \qquad A_\mu = (V, \vec{A}) \text{ with } \vec{B} = \vec{\nabla} \times \vec{A}$$

- The conditions on $\boldsymbol{U}$ are that $\boldsymbol{U}$ is unitary, and commutes with the Hamiltonian:

$$\hat{U}^\dagger \hat{U} = \mathbf{1} \qquad [\hat{U}, \hat{H}] = 0$$

- e.g. for EM, $\hat{U} = e^{i\phi}$ where ϕ is an arbitrary phase: $\psi \rightarrow \psi' = e^{i\phi}\psi$

Symmetries in QED

- Instead of a global phase transformation $e^{i\phi}$ imagine a local phase transformation, where the phase $\phi \sim q\,\chi$ is a function of x^μ: $\chi(x^\mu)$.
  - q is a constant (will be electric charge)

$$\psi \to \psi' = \hat{U}\psi = e^{iq\chi(x^\mu)}\psi$$

- Substitute into Dirac Equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$

$$\begin{aligned}(i\gamma^\mu\partial_\mu - m)\psi' &= 0\\ (i\gamma^\mu\partial_\mu - m)e^{iq\chi(x)}\psi &= 0\\ i\gamma^\mu(e^{iq\chi(x)}\psi + iq\partial_\mu\chi - m)e^{iq\chi(x)}\psi &= 0\end{aligned}$$

- An interaction term $-q\gamma^\mu\partial_\mu\chi\psi$ term appears in the Dirac Equation.
- To cancel this, modify the Dirac Equation for interacting fermions:

$$(i\gamma^\mu\partial_\mu + iqA_\mu - m)\psi = 0$$

- With A^μ transforming as:
- $A_\mu \to A'_\mu = A_\mu - \partial_\mu\chi$ to cancel interaction term

Gauge Symmetry in QED

- Demanding that QED is invariant by a local phase shift:

$$\psi \rightarrow \psi' = \hat{U}\psi = e^{iq\chi(x^\mu)}\psi$$

- Tells us that fermions interact with the photon field as:

$$q\gamma^\mu A_\mu \psi$$

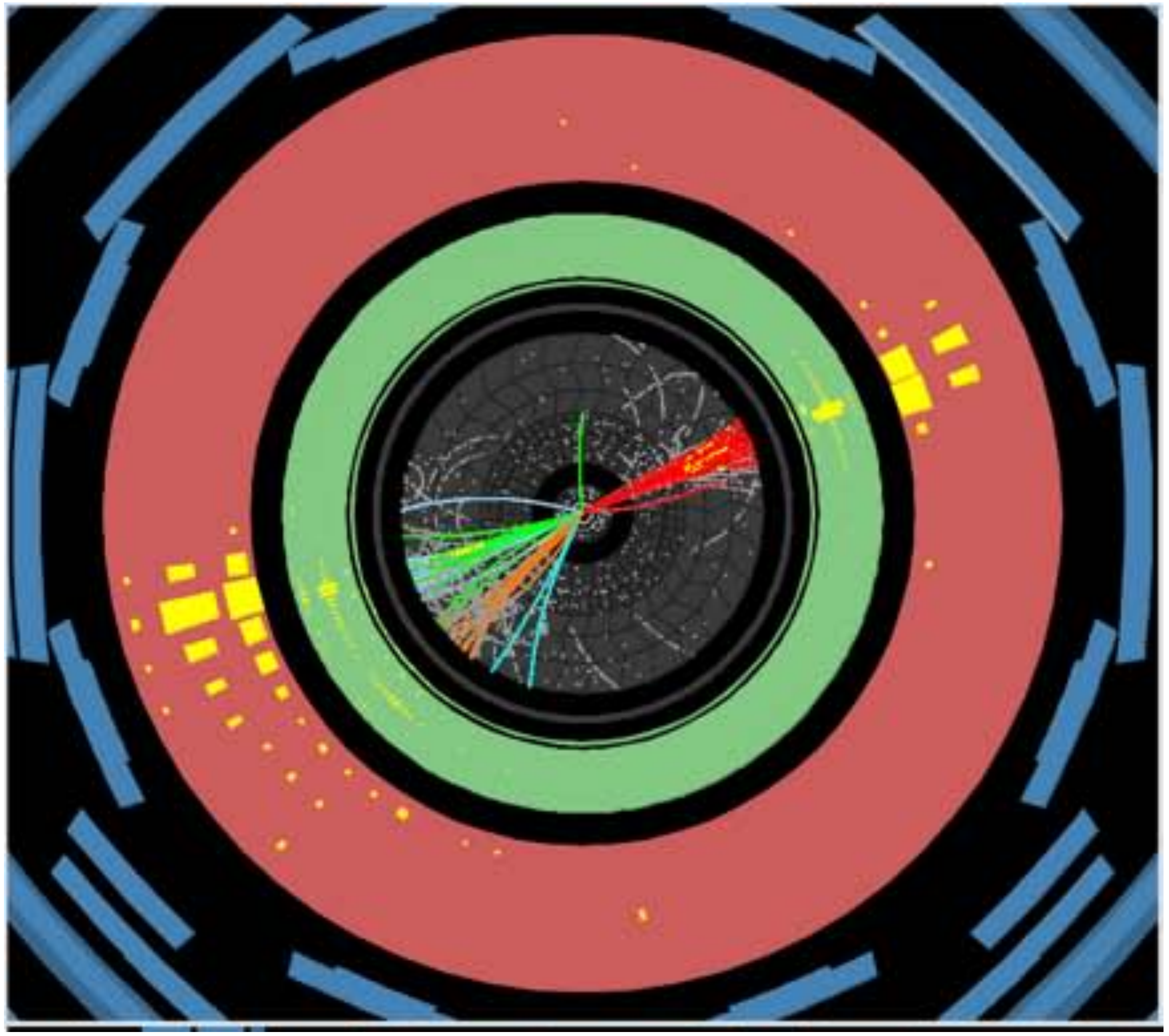
- This local phase shift is know as a **local U(1) gauge symmetry**.

- Next lecture we will see a similar effect in QCD.

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 9: Quantum Chromodynamics (QCD)



- ★Colour charge and symmetry
- ★Gluons
- ★QCD Feynman Rules
- ★$q\,\bar{q} \rightarrow q\,\bar{q}$ scattering
- ★QCD potential

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- In QM, a symmetry is present if **physical observables** (e.g. cross section, decay widths) are invariant under the following change in the wavefunction:

$$\psi \rightarrow \psi' = \hat{U}\psi$$

- e.g. in electromagnetism, the physical observable fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are independent of the value of the EM potential, $\boldsymbol{A_\mu}$:

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$$\hat{U}^\dagger \hat{U} = \mathbf{1} \qquad [\hat{U}, \hat{H}] = 0$$

- e.g. for EM, $\hat{U} = e^{i\phi}$ where $\boldsymbol{\phi}$ is an arbitrary phase: $\psi \rightarrow \psi' = e^{i\phi}\psi$

Symmetries in QED

- Instead of a global phase transformation $e^{i\phi}$ imagine a local phase transformation, where the phase $\phi \sim q\,\chi$ is a function of x^μ: $\chi(x^\mu)$.
    - q is a constant (will be electric charge)

$$\psi \rightarrow \psi' = \hat{U}\psi = e^{iq\chi(x^\mu)}\psi$$

- Substitute into Dirac Equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$

$$\begin{aligned}(i\gamma^\mu\partial_\mu - m)\psi' &= 0\\ (i\gamma^\mu\partial_\mu - m)e^{iq\chi(x)}\psi &= 0\\ i\gamma^\mu(e^{iq\chi(x)}\partial_\mu\psi + iq\partial_\mu\chi\,\psi) - me^{iq\chi(x)}\psi &= 0\\ (i\gamma^\mu\partial_\mu - m)e^{iq\chi(x)}\psi - q\gamma^\mu\partial_\mu\chi\,\psi &= 0\end{aligned}$$

- An interaction term $-q\gamma^\mu\partial_\mu\chi\psi$ term appears in the Dirac Equation.
- To cancel this, modify the Dirac Equation for **interacting** fermions:

$$(i\gamma^\mu\partial_\mu + iqA_\mu - m)\psi = 0$$

- With A^μ transforming as:
- $A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\chi$ to cancel interaction term

Gauge Symmetry in QED & QCD

- Demanding that QED is invariant by a local phase shift:

$$\psi \rightarrow \psi' = \hat{U}\psi = e^{iq\chi(x^{\mu})}\psi$$

- Tells us that fermions interact with the photon field as:

$$q\gamma^{\mu}A_{\mu}\psi$$

- This invariance of QED under the local phase shift status is know as a **local U(1) gauge symmetry**.

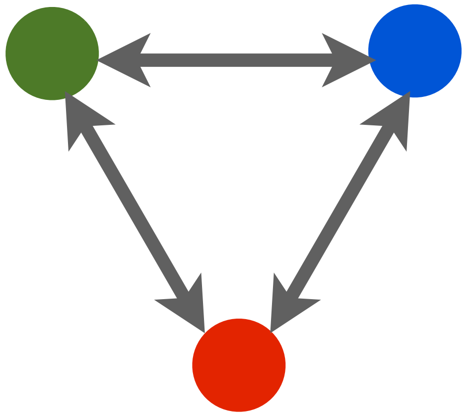
- Today we will see the consequences of a symmetry in QCD, but with a different symmetry, known as **SU(3).**
  - ➡ QCD exhibits a local SU(3) gauge symmetry.

Colour Charge

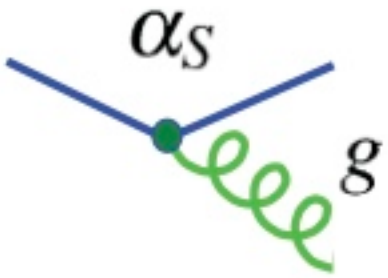
- Each quark carries a colour charge: red, blue or green.
- The coupling strength is the same for all three colours colours.
- To describe a quark, use a spinor **plus** a colour column vector:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Mathematically, this is described by an SU(3) symmetry. QCD interactions are invariant under SU(3) rotations in colour space:



- Gluons responsible for exchanging momentum and colour between quarks.



- Each gluon contains colour and anti-colour.
- Naively expect nine gluons:
  $r\bar{r}$ $r\bar{b}$ $r\bar{g}$ $b\bar{r}$ $b\bar{b}$ $b\bar{g}$ $g\bar{r}$ $g\bar{b}$ $g\bar{g}$
- However gluons are described by the generators of the SU(3) group, giving eight linear colour–anti-colour combinations of these

Eight Gluons

- The Gell-Mann matrices describe the allowed colour configurations of gluons. (The Gell-Mann matrices are the *generators* of the SU(3) symmetry.)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Each gluon is described by:

$$g^i = \begin{pmatrix} \mathrm{r} & \mathrm{b} & \mathrm{g} \end{pmatrix} \lambda^i \begin{pmatrix} \bar{\mathrm{r}} \\ \bar{\mathrm{b}} \\ \bar{\mathrm{g}} \end{pmatrix}$$

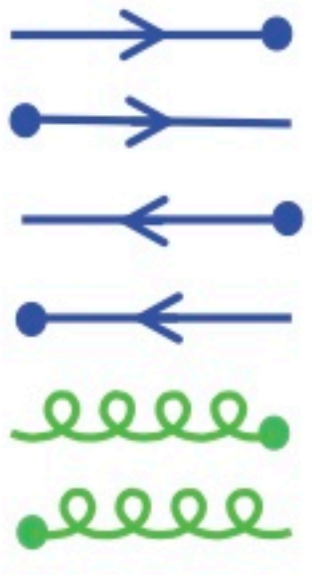
$$g^1 = \frac{1}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{b}} + \mathrm{b}\bar{\mathrm{r}}) \quad g^2 = \frac{i}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{b}} - \mathrm{b}\bar{\mathrm{r}}) \quad g^3 = \frac{1}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{r}} - \mathrm{b}\bar{\mathrm{b}})$$

$$g^4 = \frac{1}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{g}} + \mathrm{g}\bar{\mathrm{r}}) \quad g^5 = \frac{i}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{g}} - \mathrm{g}\bar{\mathrm{r}}) \quad g^6 = \frac{1}{\sqrt{2}}(\mathrm{b}\bar{\mathrm{g}} + \mathrm{g}\bar{\mathrm{b}})$$

$$g^7 = \frac{i}{\sqrt{2}}(\mathrm{b}\bar{\mathrm{g}} - \mathrm{g}\bar{\mathrm{b}}) \quad g^8 = \frac{1}{\sqrt{6}}(\mathrm{r}\bar{\mathrm{r}} + \mathrm{b}\bar{\mathrm{b}} - 2\mathrm{g}\bar{\mathrm{g}})$$

Feynman Rules for QCD

- **External Lines**



spin 1/2	incoming quark	$u(p)$
	outgoing quark	$\overline{u}(p)$
	incoming anti-quark	$\overline{v}(p)$
	outgoing anti-quark	$v(p)$
spin 1	incoming gluon	$\varepsilon^{\mu}(p)$
	outgoing gluon	$\varepsilon^{\mu}(p)^{*}$

- **Internal Lines (propagators)**

spin 1 gluon

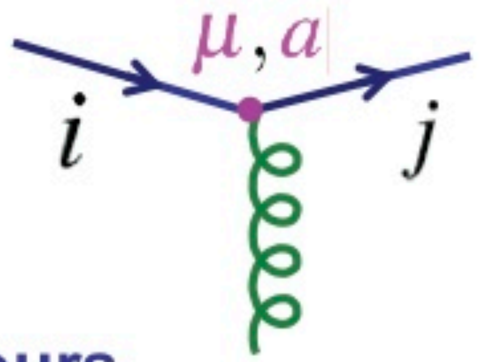
$$\boxed{\frac{g_{\mu\nu}}{q^2}\delta^{ab}}$$

a, b = 1,2,…,8 are gluon colour indices

- **Vertex Factors**

spin 1/2 quark

$$\boxed{g_s\frac{1}{2}\lambda^{a}_{ji}\gamma^{\mu}}$$



$$\alpha_S = \frac{g_S^2}{4\pi}$$

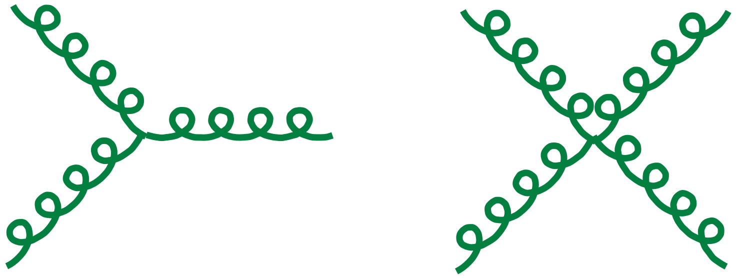
i, j = 1,2,3 are quark colours,

λ^a a = 1,2,..8 are the Gell-Mann SU(3) matrices

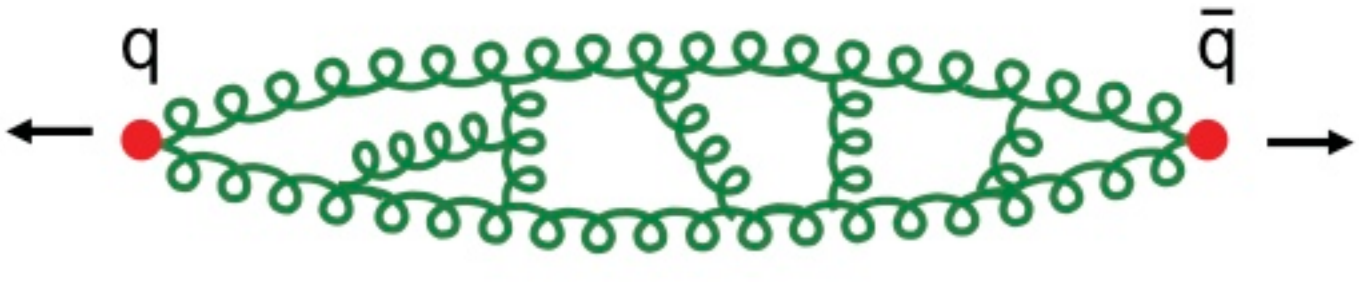
The $\lambda^a{}_{ij}$ terms account for the quark colour

Gluon-Gluon Interactions

- Gluons also carry colour charge and can therefore self-interact.
- Two allowed possibilities:



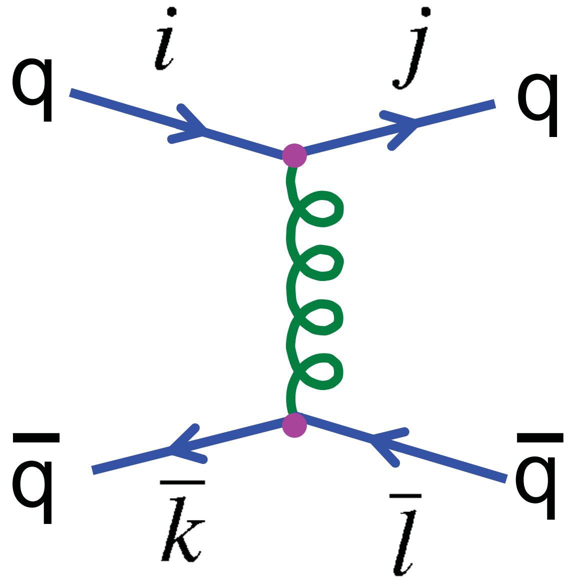
- Gluon interactions are believed to give rise to **colour confinement**
  - Try to separate an electron-positron pair $V_{\mathrm{QED}}(r) = -\frac{q_2\, q_1}{4\pi\epsilon_0 r} = -\frac{\alpha}{r}$
  - Try to separate an quark anti-quark pair $V_{\mathrm{QCD}}(r) \sim \lambda r$



- A gluon **flux tube** of interacting gluons is formed. Energy **~1 GeV/fm.**
- Gluon-gluon interactions are responsible for holding quarks in mesons and baryons.

$q\bar{q}\rightarrow q\bar{q}$ scattering

- To write down the matrix element, follow the fermion arrows backwards.
  - ➡ For the quark line $j\rightarrow i$: λ_{ji} term at vertex
  - ➡ For the antiquark line $k\rightarrow l$: λ_{kl} term at vertex



$$\mathcal{M} = \left[\bar{u}_j \frac{g_S}{2}\lambda^a_{ji}\gamma^\mu u_i\right] \frac{g^{\mu\nu}}{q^2}\delta^{ab} \left[\bar{v}_k \frac{g_S}{2}\lambda^b_{kl}\gamma^\nu v_l\right]$$

$$\mathcal{M} = \frac{g_S^2}{q^2}\frac{\lambda^a_{ji}\lambda^a_{kl}}{4}\left[\bar{u}_j\gamma^\mu u_i\right]\left[\bar{v}_k\gamma^\mu v_l\right]$$

- The matrix element looks very similar to electromagnetic scattering except $e\rightarrow g_S$, and the addition of the terms $\lambda^a_{ji}\lambda^a_{kl}/4$

- In the lowest order approximation, the dynamics of the $q\bar{q}\rightarrow q\bar{q}$ scattering is the same as electromagnetic $e^+e^-\rightarrow e^+e^-$ scattering.

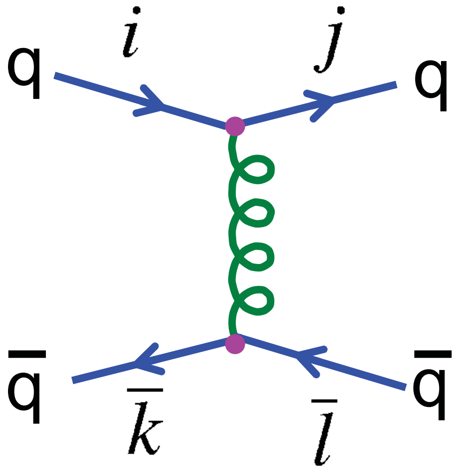
- Describe in terms of Coloumb-like potential $V_{q\bar{q}} = -\frac{f\alpha_S}{r}$

- The **colour factor** $f = \frac{1}{4}\lambda^a_{ji}\lambda^a_{kl} = \frac{1}{4}\sum_a \lambda^a_{ji}\lambda^a_{kl}$ is a sum over elements in the λ matrices.

Colour Factor for $q\,\overline{q} \to q\,\overline{q}$

- Need to calculate the **colour factor**

$$f = \frac{1}{4}\lambda^a_{ji}\lambda^a_{kl} = \frac{1}{4}\sum_a \lambda^a_{ji}\lambda^a_{kl}$$



- For the calculation we choose colours for q and $\overline{q}$. As the theory is invariant under rotations in colour space any choice of colours will give the same answer.

- Three colour options:
  1. i=1 $k=\overline{1}$ → j=1 $l=\overline{1}$ e.g. $r\,\overline{r} \to r\,\overline{r}$ $\quad f_1 = \frac{1}{4}\sum_a \lambda^a_{11}\lambda^a_{11}$
  2. i=1 $k=\overline{2}$ → j=1 $l=\overline{2}$ e.g. $r\,\overline{b} \to r\,\overline{b}$ $\quad f_2 = \frac{1}{4}\sum_a \lambda^a_{11}\lambda^a_{22}$
  3. i=1 $k=\overline{1}$ → j=2 $l=\overline{2}$ e.g. $r\,\overline{r} \to b\,\overline{b}$ $\quad f_3 = \frac{1}{4}\sum_a \lambda^a_{21}\lambda^a_{12}$

- Calculate option 2. The only matrices with non-zero element in the red-red (11) and blue-blue (22) elements are λ^3 and λ^8.

$$f_2 = \frac{1}{4}\sum_a \lambda^a_{11}\lambda^a_{22} = \frac{1}{4}[\lambda^3_{11}\lambda^3_{22} + \lambda^8_{11}\lambda^8_{22}] = \frac{1}{4}[(1)(-1) + (\frac{1}{\sqrt{3}})(\frac{1}{\sqrt{3}})] = -\frac{1}{6}$$

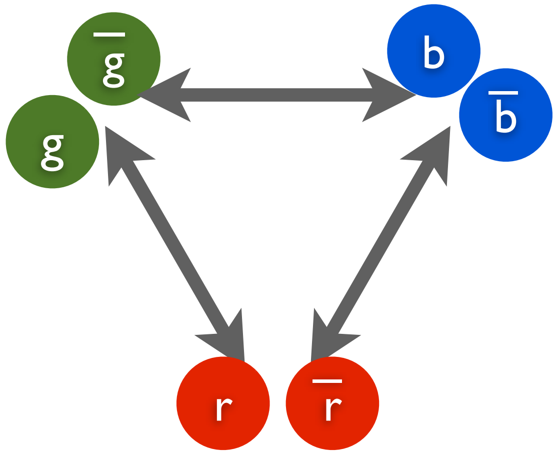
- Similarly,

$$f_1 = \frac{1}{4}\sum_a \lambda^a_{11}\lambda^a_{11} = \frac{1}{4}[\lambda^3_{11}\lambda^3_{11} + \lambda^8_{11}\lambda^8_{11}] = \frac{1}{4}[(1)(1) + (\frac{1}{\sqrt{3}})(\frac{1}{\sqrt{3}})] = \frac{1}{3}$$

$$f_3 = \frac{1}{4}\sum_a \lambda^a_{21}\lambda^a_{12} = \frac{1}{4}(\lambda^1_{21}\lambda^1_{12} + \lambda^2_{12}\lambda^2_{21}) = \frac{1}{4}[(-i)(i) + (1)(1)] = \frac{1}{2}$$

Colour Factor for Mesons

- Mesons are colourless $q\bar{q}$ states in a “colour singlet”: $r\bar{r} + g\bar{g} + b\bar{b}$
- Calculate colour factor for $q\bar{q} \to q\bar{q}$ scattering in a meson.



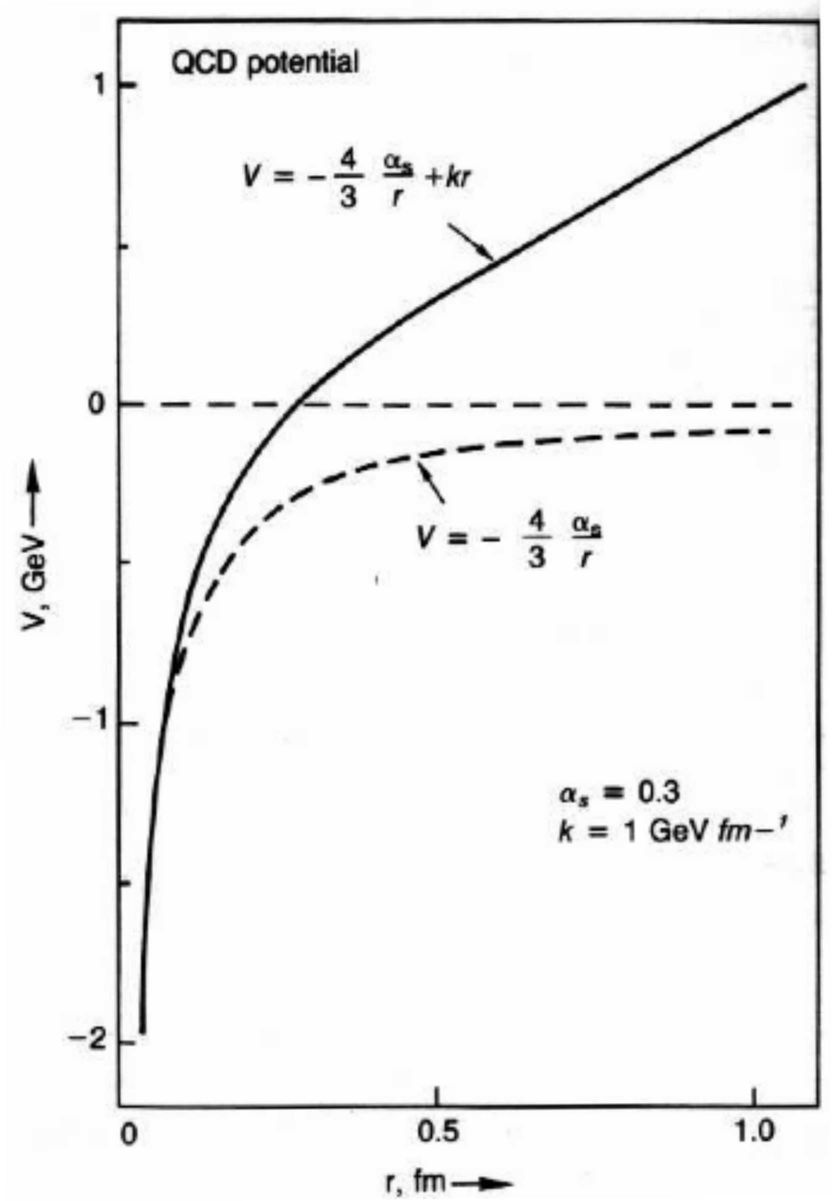
- Two possibilities for colour combinations:
  - ➡Quarks stay the same colour e.g. $r\bar{r} \to r\bar{r}$ f_1=⅓
  - ➡Quarks change colour e.g. $r\bar{r} \to b\bar{b}$ and $r\bar{r} \to g\bar{g}$ each contributes f_3=½

- Sum over all possible final states for $r\bar{r} \to q\bar{q}$ gives $f_r = ⅓ + ½ + ½ = 4/3$
- Average over all possible initial states, $r\bar{r}$, $g\bar{g}$, $b\bar{b}$:

$$f = ⅓\,(\,r\bar{r} \to q\bar{q} \;+\; g\bar{g} \to q\bar{q} \;+\; b\bar{b} \to q\bar{q}\,) = ⅓\,(\,4/3 + 4/3 + 4/3) = 4/3$$

- The colour factor for the $q\bar{q}$ interactions within a meson is 4/3
  - The potential within a meson (to lowest order) is: $V_{q\bar{q}} = -\frac{4}{3}\frac{\alpha_S}{r}$

QCD Potential



- At large distances: gluon-gluon interactions

$$V_{\mathrm{QCD}}(r) \sim \lambda r$$

- At short distances: $q\bar{q}\to q\bar{q}$ scattering

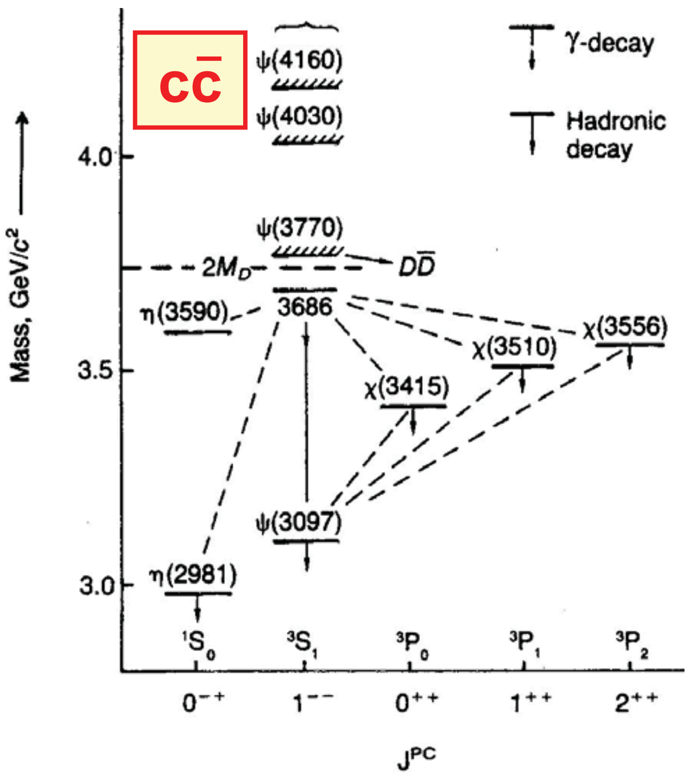
$$V_{q\bar{q}} = -\frac{4}{3}\frac{\alpha_S}{r}$$

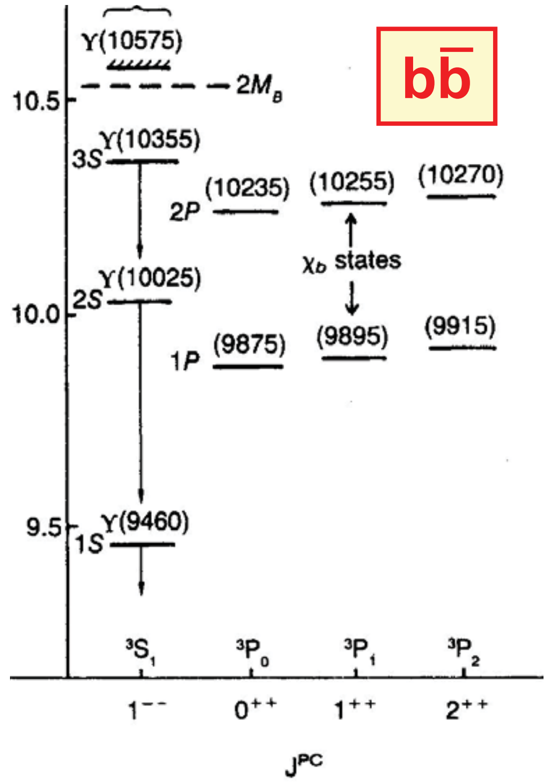
- Overall potential is:

$$V_{\mathrm{QCD}}(r) = -\frac{4}{3}\frac{\alpha_S}{r} + \lambda r$$

This model provides a good description of the bound states of heavy quarks:

- charmonium ($c\,\bar{c}$)
- bottomonium ($b\,\bar{b}$)

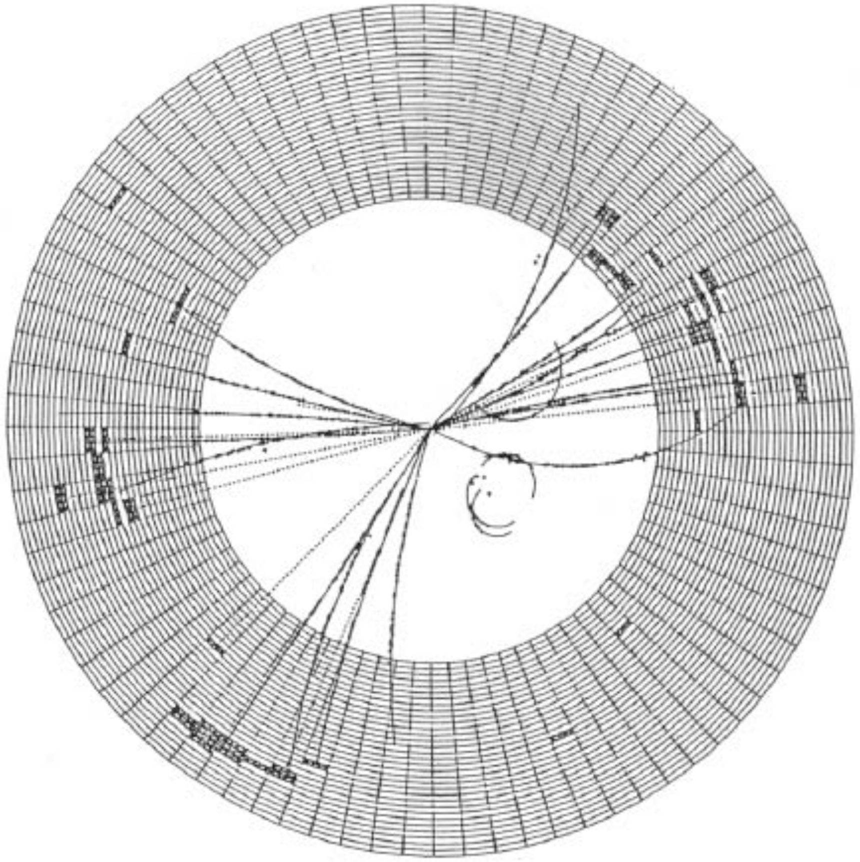




Particle Physics

Dr Victoria Martin, Spring Semester 2013

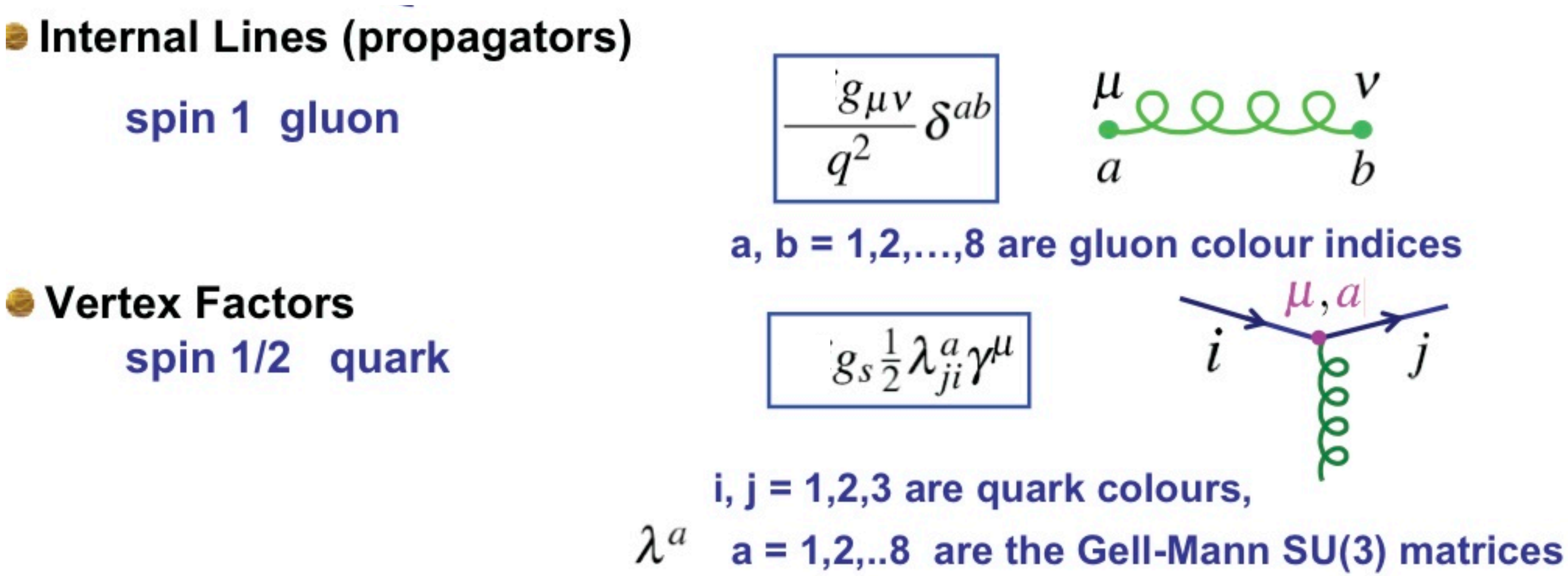
Lecture 10: QCD at Colliders



- ★Jets
- ★Renormalisation in QCD
- ★Asymptotic Freedom and Confinement in QCD
- ★Lepton and Hadron Colliders
- ★**R** = $(e^+e^-\rightarrow\text{hadrons})/(e^+e^-\rightarrow\mu^+\mu^-)$
- ★Measuring Jets
- ★Fragmentation

QCD Summary

- QCD: Quantum Chromodymanics is the quantum description of the strong force.
- Quarks are colour charged: red, green or blue
- Anti-quarks are colour charged: anti-red, anti-green, anti-blue
- Gluons are the propagators of the QCD and carry colour and anti-colour, described by 8 Gell-Mann matrices, λ.



Internal Lines (propagators)

spin 1 gluon

$$\boxed{\frac{g_{\mu\nu}}{q^2}\delta^{ab}}$$

a, b = 1,2,…,8 are gluon colour indices

Vertex Factors

spin 1/2 quark

$$\boxed{g_s\frac{1}{2}\lambda^a_{ji}\gamma^\mu}$$

i, j = 1,2,3 are quark colours,

λ^a **a = 1,2,..8 are the Gell-Mann SU(3) matrices**

- $\mathcal{M}$ includes a colour factor, f, calculable from the λ matrices.
- The QCD coupling constant is $\alpha_S = g_S^2/4\pi$

From lecture 6: Running Coupling Constant in QED

- Renormalise α, and correct for all possible fermion types in the loop:

$$\alpha(q^2) = \alpha(0)\left(1 + \frac{\alpha(0)}{3\pi} z_f \ln(\frac{-q^2}{M^2})\right)$$

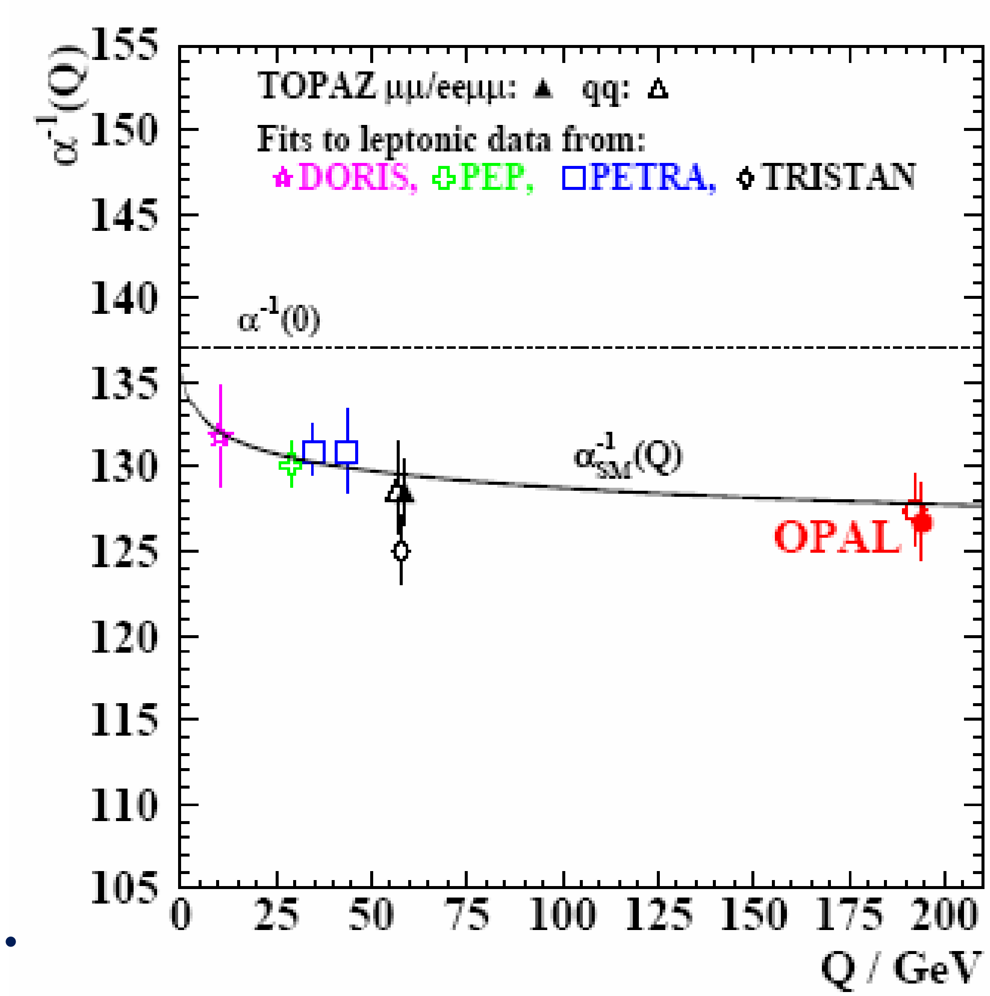
- z_f is the sum of charges over all possible fermions in the loop
  - ➡ At $q^2 \sim 1$ MeV only electron, $z_f = 1$
  - ➡ At $q^2 \sim 100$ GeV, $f = e, \mu, \tau, u, d, s, c, b$ $z_f = 60/3$

$$z_f = \sum_f Q_f^2$$

- Instead of using M^2 dependence, replace with a reference value μ^2:

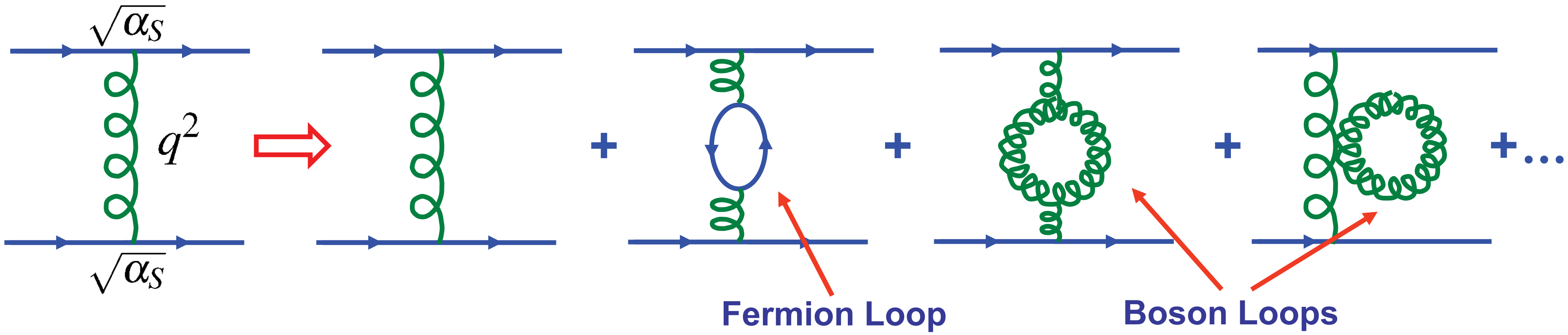
$$\alpha(q^2) = \alpha(\mu^2)\left(1 - \frac{\alpha(\mu^2)}{3\pi} z_f \ln(\frac{q^2}{\mu^2})\right)^{-1}$$

- Usual choices for μ are **1 MeV** or $m_Z \sim$ **91 GeV.**
  - ➡ $\alpha(\mu^2 = 1\ \mathrm{MeV}^2) = 1/137$
  - ➡ $\alpha(\mu^2 = (91\ \mathrm{GeV})^2) = 1/128$
- We choose a value of μ where make a initial measurement of α, but once we do the evolution of the values of α are determined by the above eqn.



Running Coupling in QCD

- The observed (renormalised) value of the coupling constant α_S depends on diagrams such as:



Similar to QED

No equivalent in QED, as no photon-photon interactions

  - The measured value of α_S at a energy scale q^2 can be written as:

$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{1 + B\alpha_S(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)}$$

  - B can be calculated to be $B = \dfrac{11N_c - 2N_f}{12\pi}$
    - with n_C=3 number of colours, n_f=6 number of quark flavours
  - Measuring α_S at a known energy scale μ^2 determine uniquely the value of α_S for all other energies, q^2.

Running of α_S

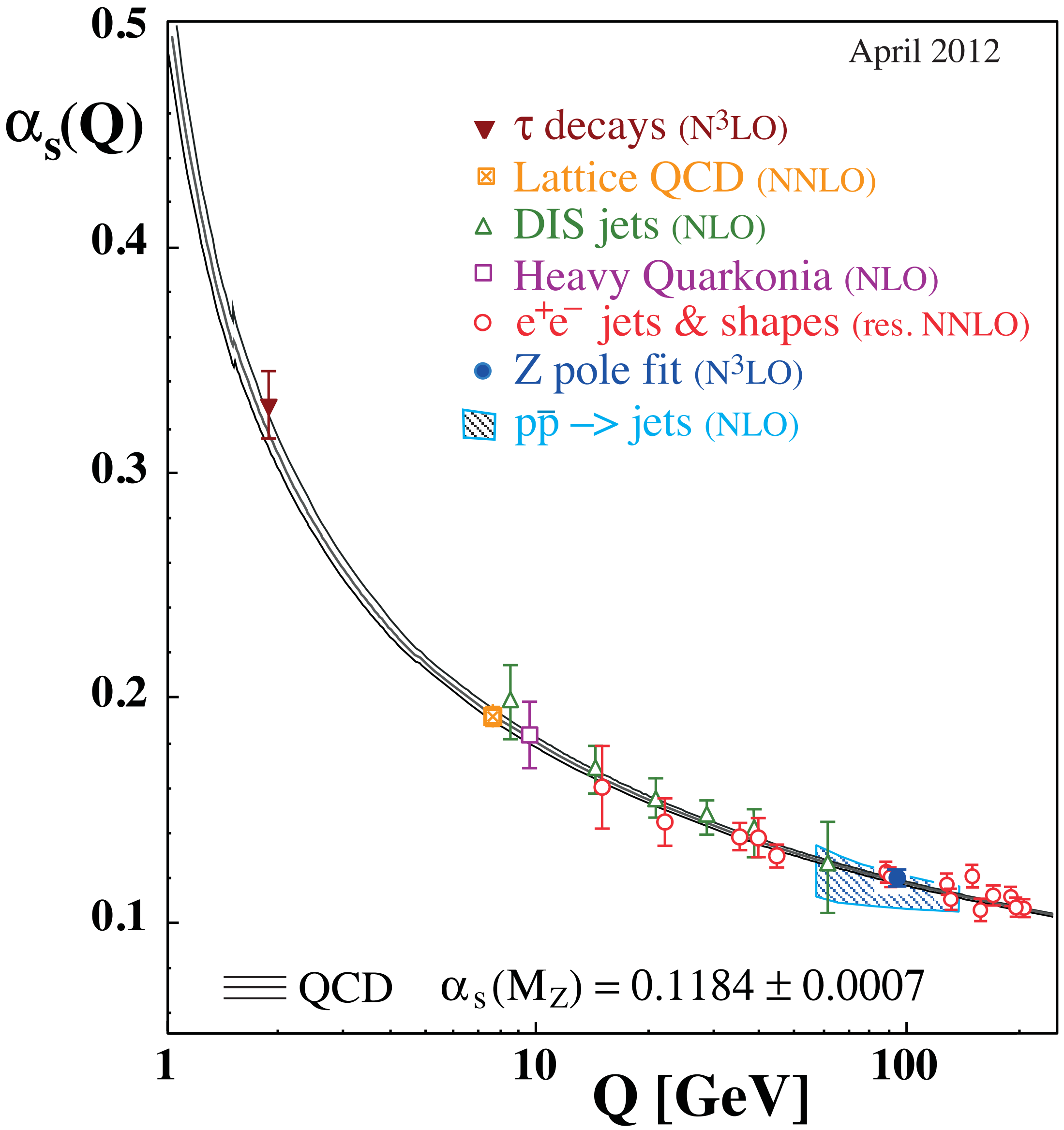
$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{1 + B\alpha_S(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)}$$

This calculation won the Nobel Prize for Physics 2004 for Gross, Politzer and Wilczek

- α_S decrease with increasing q^2
  - ➡The more energetic the interaction (high q^2), the weaker α_S.

    $\alpha_S\ (q^2=m_Z^2) =\sim 0.12$
  - ➡The less energetic the interaction (low q^2), the stronger α_S.

    $\alpha_S\ (q^2=1\ \text{GeV}^2) \sim O(1)$



Measured values of α_S as a function of q

(from pdg.lbl.gov)

Asymptotic Freedom and Confinement

- At high energy, $q^2 >> 1$ **GeV**, α_S is small, *e.g.* $\alpha_S\ (q^2{=}m_Z^2) \sim \mathbf{0.12.}$
  - ➡Quarks and gluons behave like free objects at high energy or short distances.
  - ➡This is known as **asymptotic freedom**.
  - ➡*e.g.*At high q^2 consider the scattering from the individual quarks.
  - ➡Use perturbation theory to calculate processes. However due to moderately large α_S need to calculate the more than just the simplest diagrams.
    - ➡Leading order (α_S^2), Next-to-leading order (α_S^4), Next-to-next-to-leading order (α_S^6)

- At low energy, $q^2 \sim 1$ **GeV**, α_S is large, *e.g.* $\alpha_S\ (q{=}1\ \mathrm{GeV}) \sim \mathbf{1.}$
  - ➡Quarks and gluons are locked (**confined**) inside mesons and baryons.
  - ➡Cannot use perturbation theory to obtain sensible results.
  - ➡Many approaches to calculating QCD non-perturbatively, e.g. lattice QCD, MC techniques.

Jets

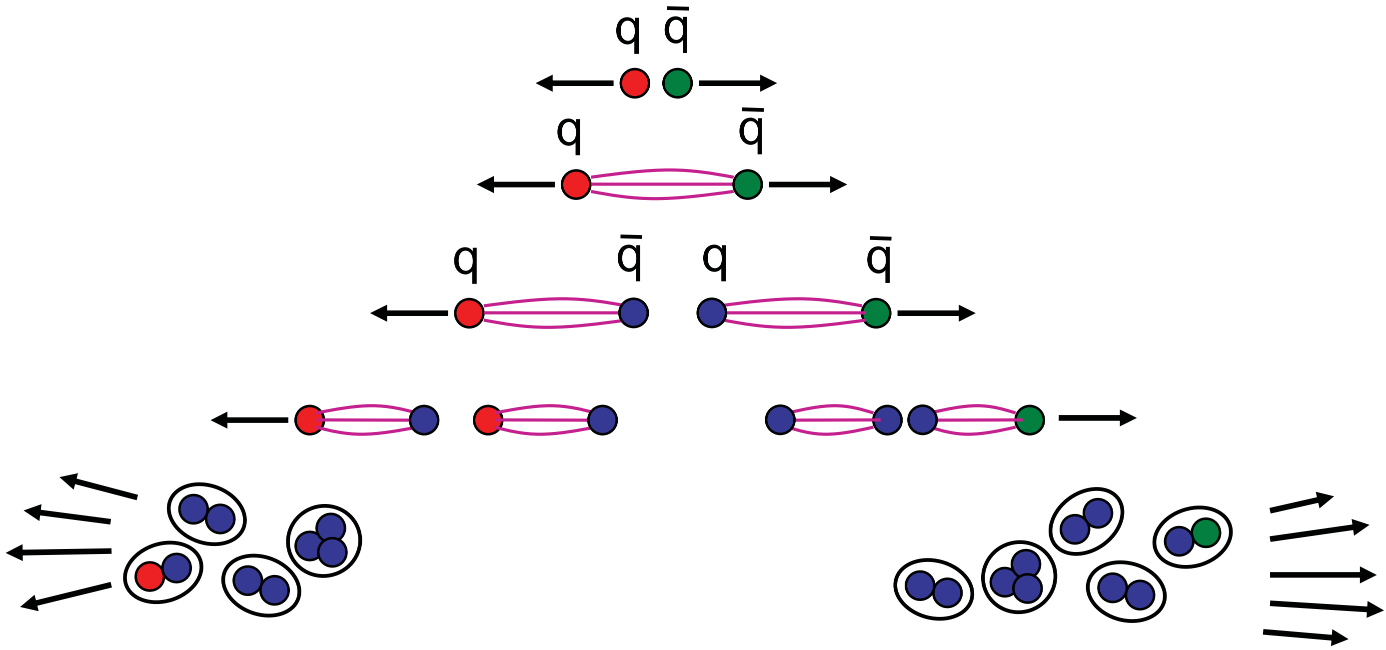
- Consider a quark and anti-quark produced in electron positron annihilation

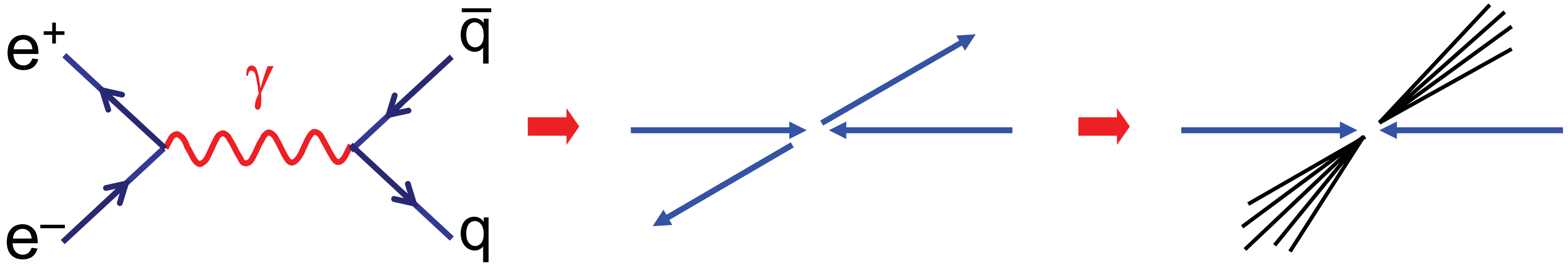
(i) Initially Quarks separate at high velocity

(ii) Colour flux tube forms between quarks

(iii) Energy stored in the flux tube sufficient to produce $q\overline{q}$ pairs

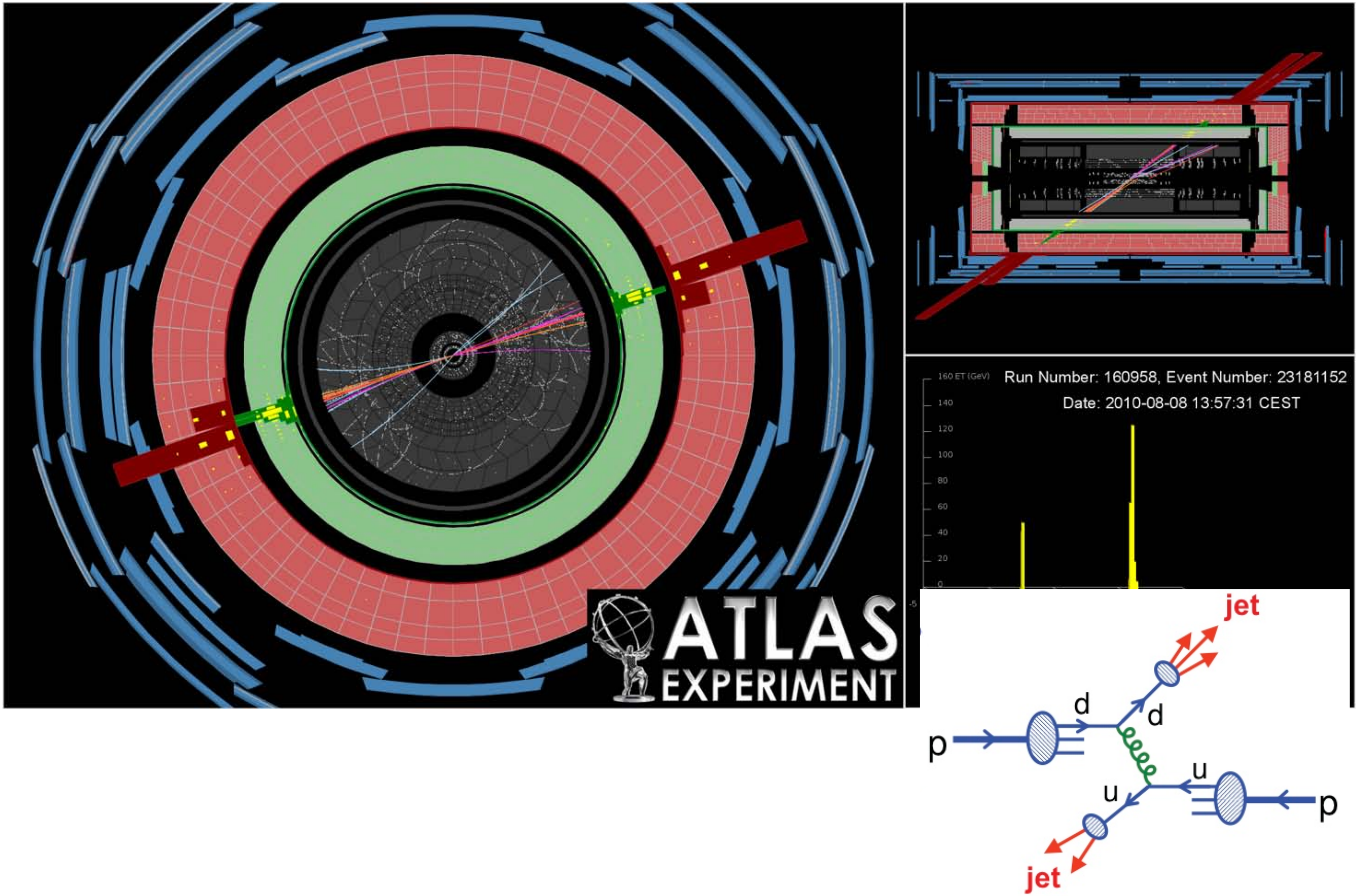
(iv) Process continues until quarks pair up into jets of colourless hadrons



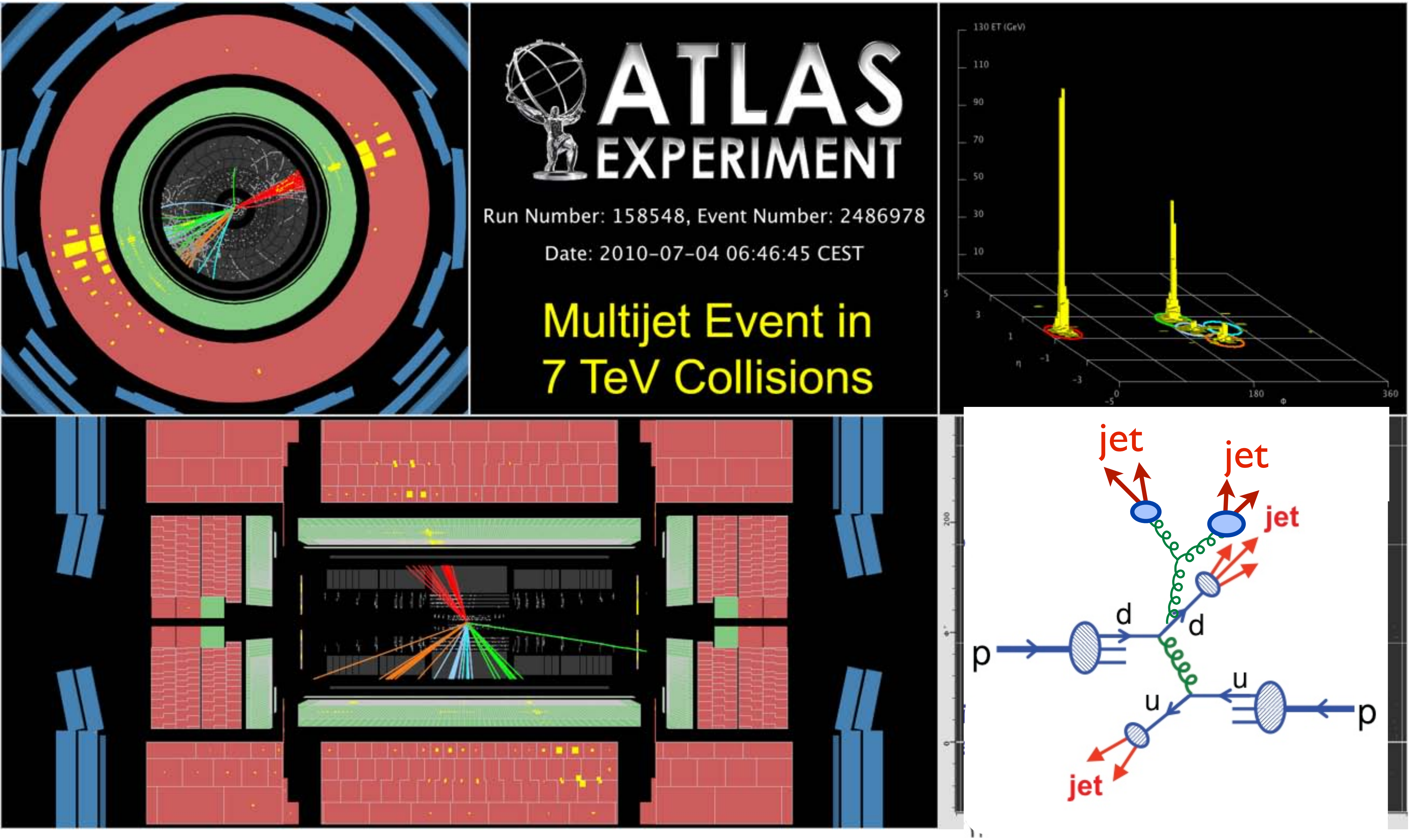


- This process is called **hadronisation**. It is not (yet) calculable.
- The main consequence is that at collider experiments quarks and gluons observed as **jets** of particles

LHC Event with Two Jets



LHC Event with Multi Jets

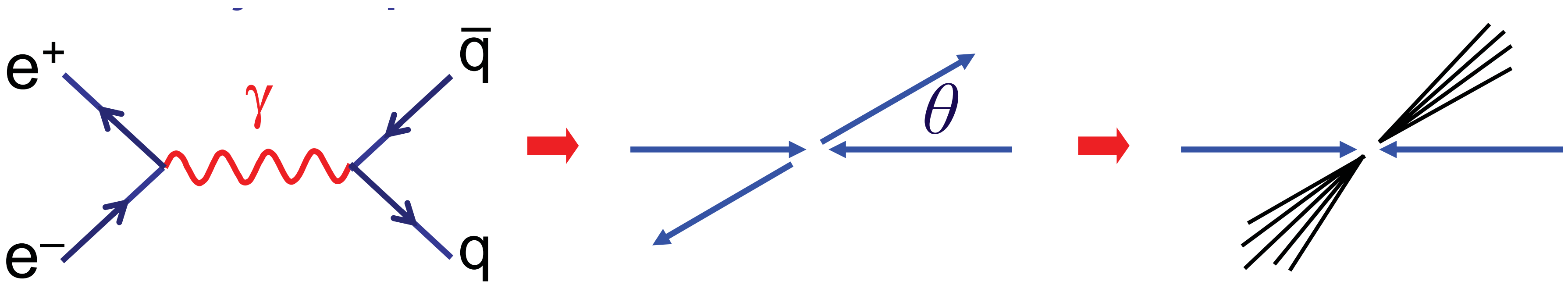


Colliders

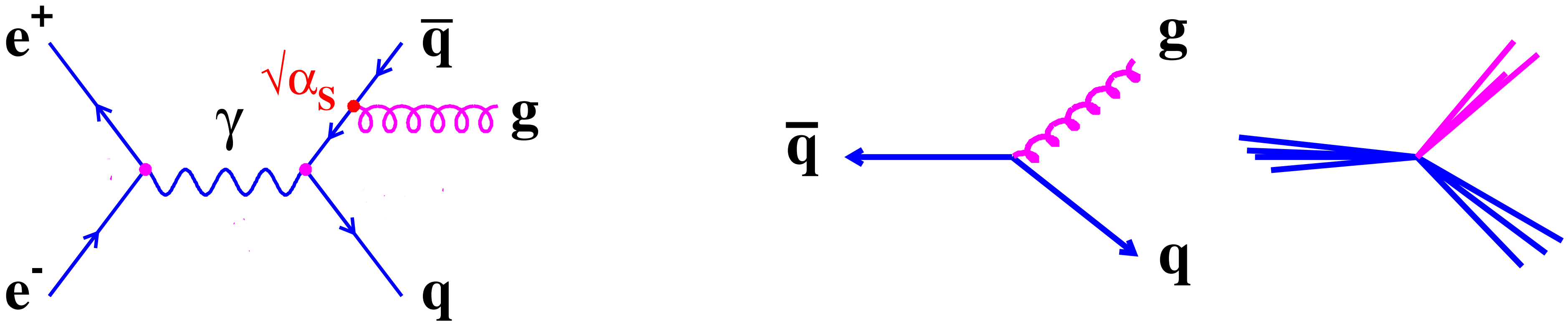
- Collider experiments collide beams of particles e.g. e^+e^-, $p\bar{p}$, e^-p, pp

- Key parameters:
  - centre of mass energy: $\sqrt{s} = \sqrt{(p_a + p_b)^2}$
  - Integrated luminosity $\int \mathscr{L}\,dt = \mathscr{L} \times$ time to run experiment

$e^+e^-\rightarrow$hadrons



- Electromagnetic production of quark pair: $\mathbf{q\overline{q}}$
  - $\mathbf{q}$ and $\mathbf{\overline{q}}$ hadronise into **two jets**
- In CM frame jets are produced back-to-back.
- Angular distribution $(\mathbf{1+cos^2\theta})$, same as $e^+e^-\rightarrow\mu^+\mu^-$



- Emission of a hard gluon in final state gives three jets (rate measures α_s)
- Observation of three jet events is direct evidence for gluons

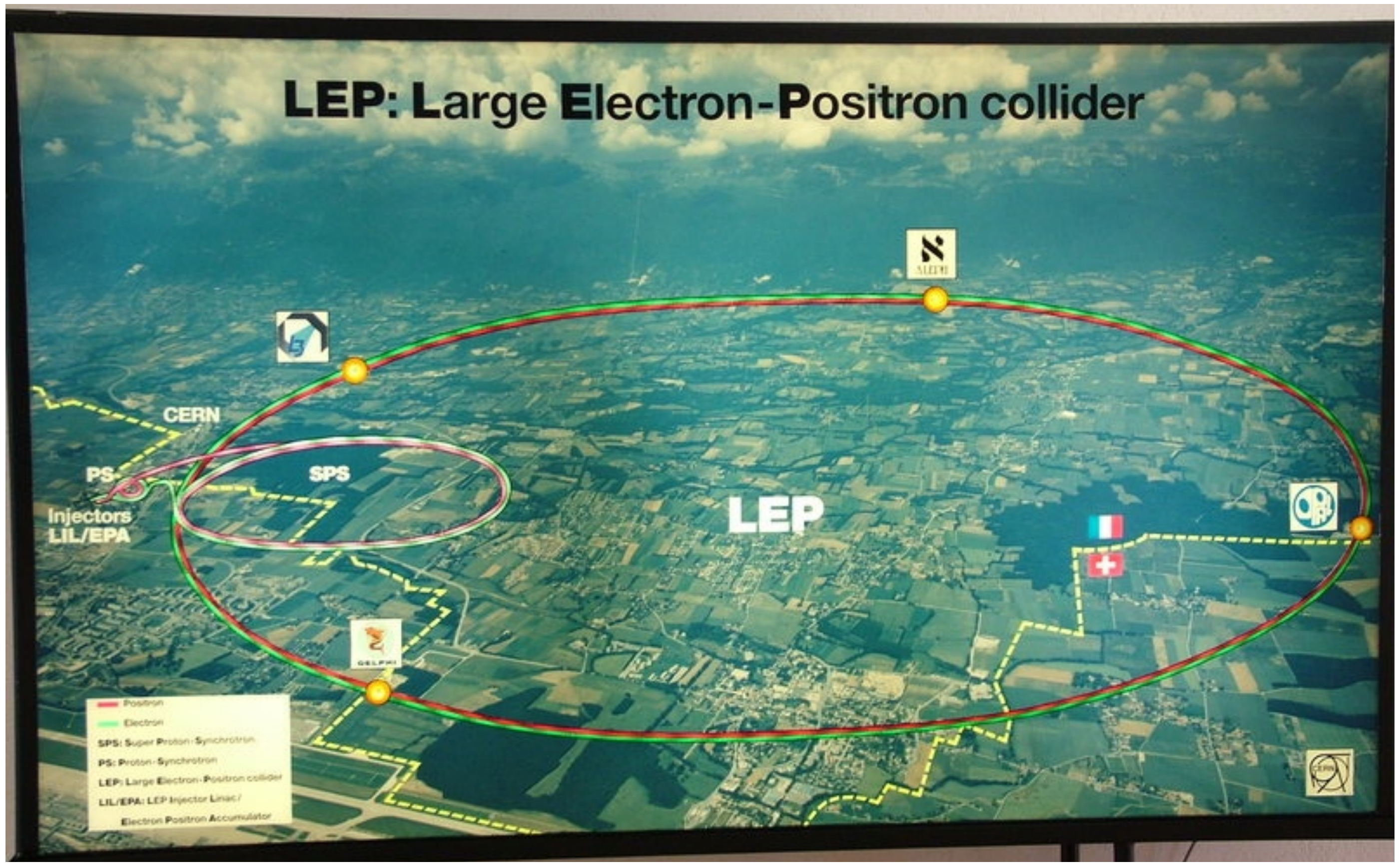
Lepton Colliders

PETRA: Positron-Elektron-Tandem-Ring-Anlage

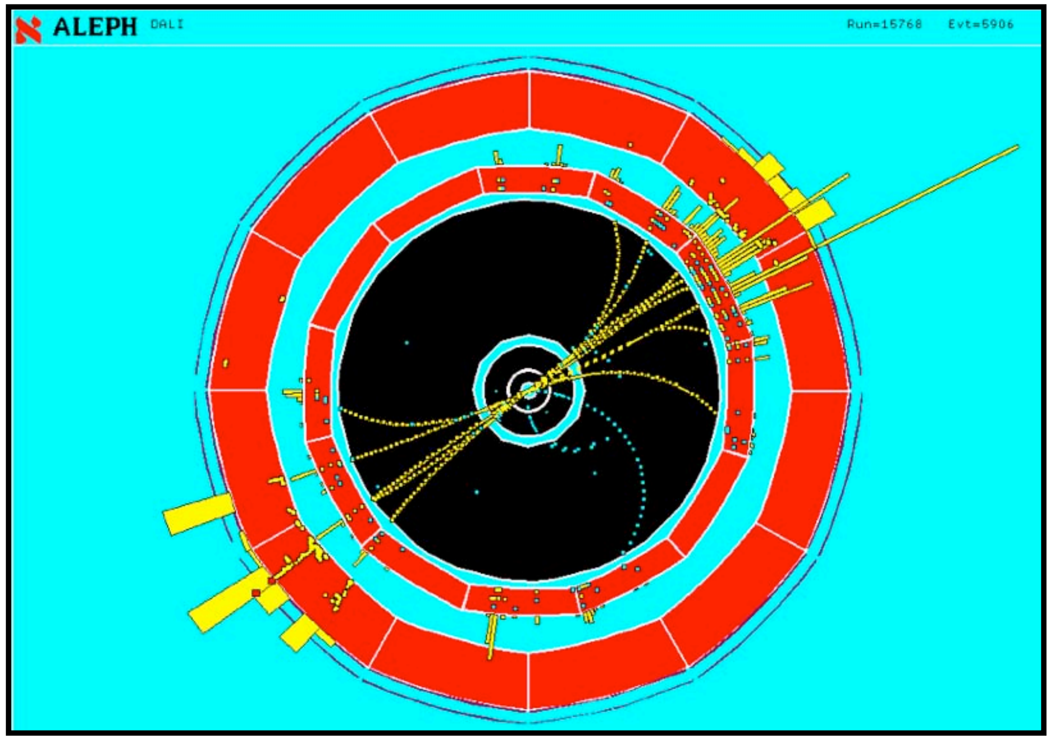
- At DESY, Hamburg
- ran 1978 to 1986
- e^+e^- collider, 2.3 km
- $\sqrt{s}$ = 14 to 46 GeV.
- Two experimental collision points: TASSO and JADE.
- Highlight: discovery of the gluon!

LEP: Large Electron Positron Collider

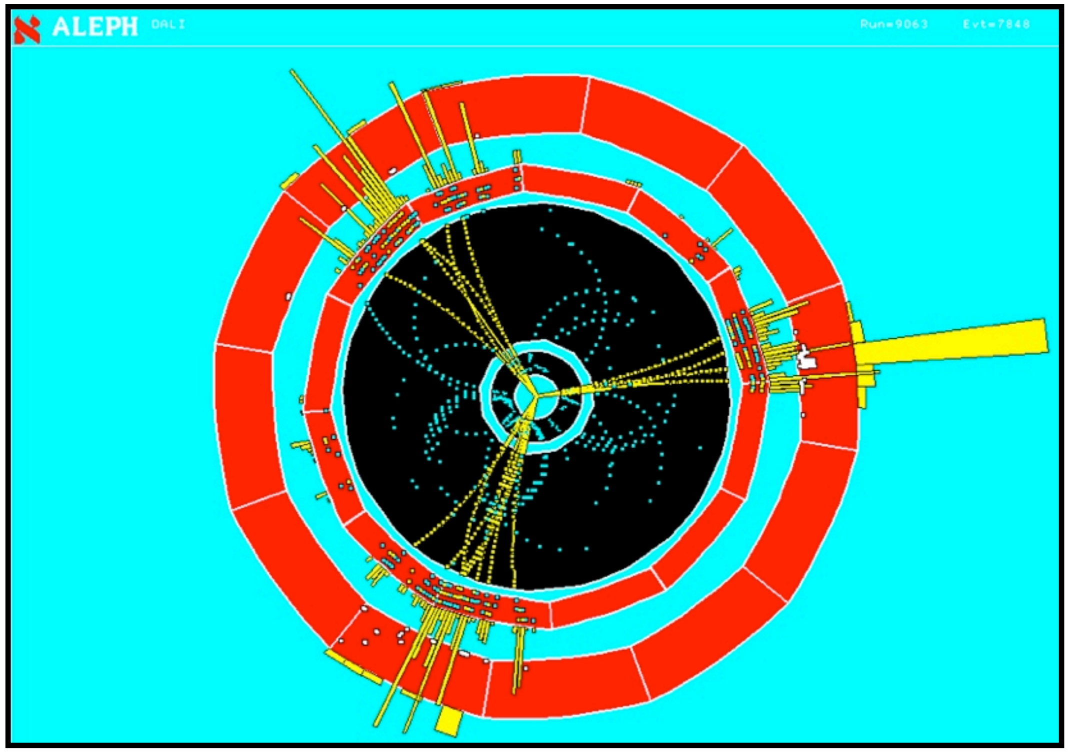
- At CERN
- The world's highest energy e^+e^- collider, 27 km circumference.
- LHC was built in LEP tunnel
- Ran from 1989 to 2000
- Centre of mass energy, $\sqrt{s}$=89 to 206 GeV
- Four experimental collision points: Aleph, Delphi, L3, Opal
- Highlight: beautiful confirmation of the electroweak model



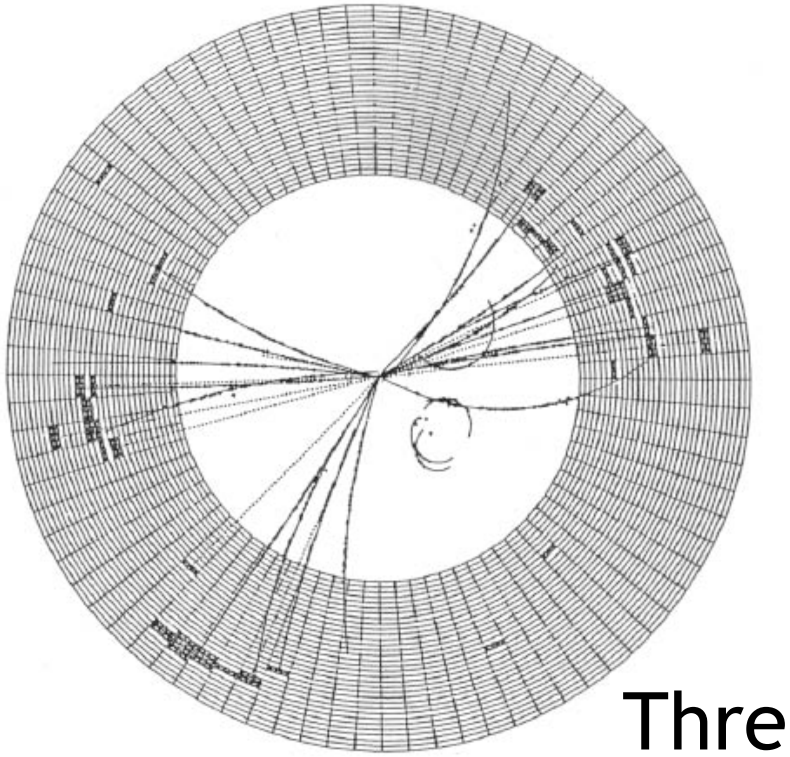
Jet Events at Lepton Colliders



Two jet event from LEP

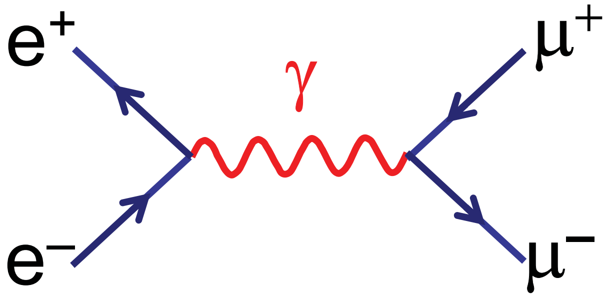


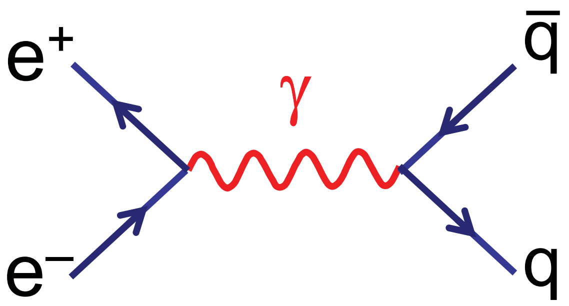
Three jet event from LEP



Three jet event from Petra

Rate for $e^+e^-\rightarrow$hadrons





$$\mathcal{M}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{e^2}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\mu^+)\gamma^\mu \bar{u}(\mu^-)]$$

$$\mathcal{M}(e^+e^- \rightarrow \mathrm{q}\bar{\mathrm{q}}) = \frac{e\,e_q}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\bar{\mathrm{q}})\gamma^\mu \bar{u}(\mathrm{q})]$$

- Ignoring differences in the phase space, ratio, **R** between hadron production and muon production:

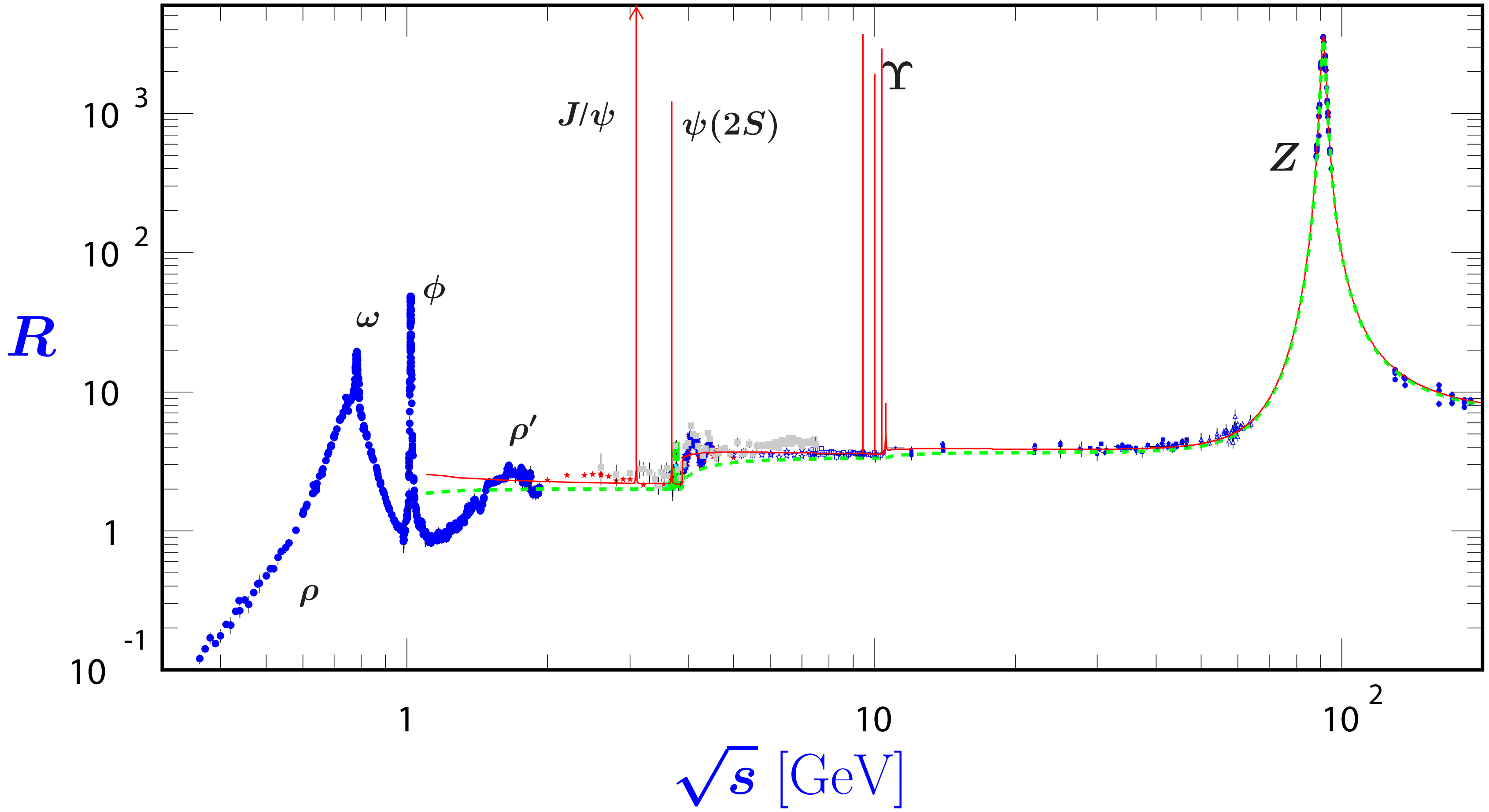
$$\mathbf{R} = \frac{\sigma(e^+e^- \rightarrow \mathrm{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c\frac{e_q^2}{e^2}$$

- $N_c=3$ is the number of quark colours
- $e_q = +⅔, -⅓$ is the charge of the quark
- The number of available quark flavours depends on the available $s=q^2$
- $\sqrt{s} > 2\ m_q$ for a quark flavour **q** to be produced.

CM energy (GeV)	Available quark pairs	R
$1 < \sqrt{s} < 3$	u, d, s	2
$4 < \sqrt{s} < 9$	u, d, s, c	10/3
$\sqrt{s} > 10$	u, d, s, c, b	11/3

Measurement of R

- Compendium of measurements from many lepton colliders.

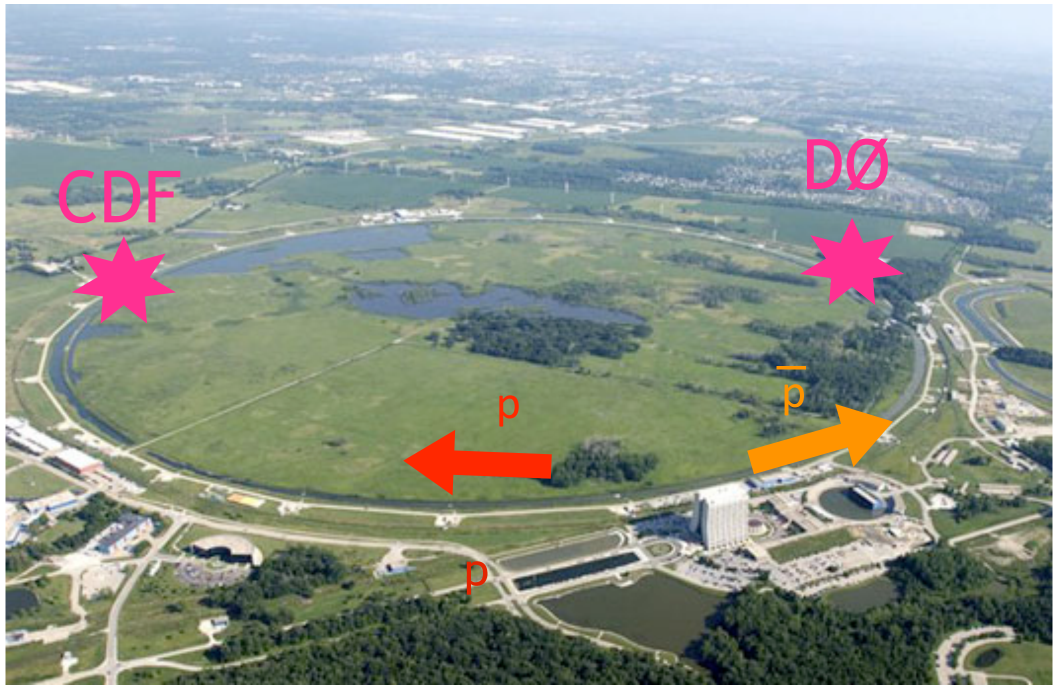


- Consistent with N_C=3, this is one of the key pieces of evidence for three quark colours.
- At quark thresholds, $\sqrt{s} \sim 2m_q$ "resonances" occur as bound states of $q\bar{q}$ more easily produced.
- Steps at ~4 and ~10 GeV due to charm and bottom quark threshold
- At $\sqrt{s} \sim 100$ GeV, Z-boson exchange takes over.

Hadron Colliders

- **Sp$\bar{p}$S**: Super Proton anti-Proton Synchrotron at CERN
- 1981 - 1984, 6.9 km, $\sqrt{s} = 400$ GeV
- Two experiments: UA1 and UA2
- Tunnel now used for pre-acceleration for LHC

Nobel Prize for Physics 1984
Carlo Rubbia and Simon van der Meer, from CERN
“For their decisive contributions to large projects, which led to the discovery of the field particles *W* and *Z*, communicators of the weak interaction.”



- **TeVatron** at Fermilab, near Chicago
- Proton anti-proton collider, 6.3 km
- Run 1: 1987 - 1995 $\sqrt{s} = 1.80$ TeV
- Run 2: 2000 - 2011 $\sqrt{s} = 1.96$ TeV
- Two experiments: CDF and DØ
- Highlight: discovery of the top quark!

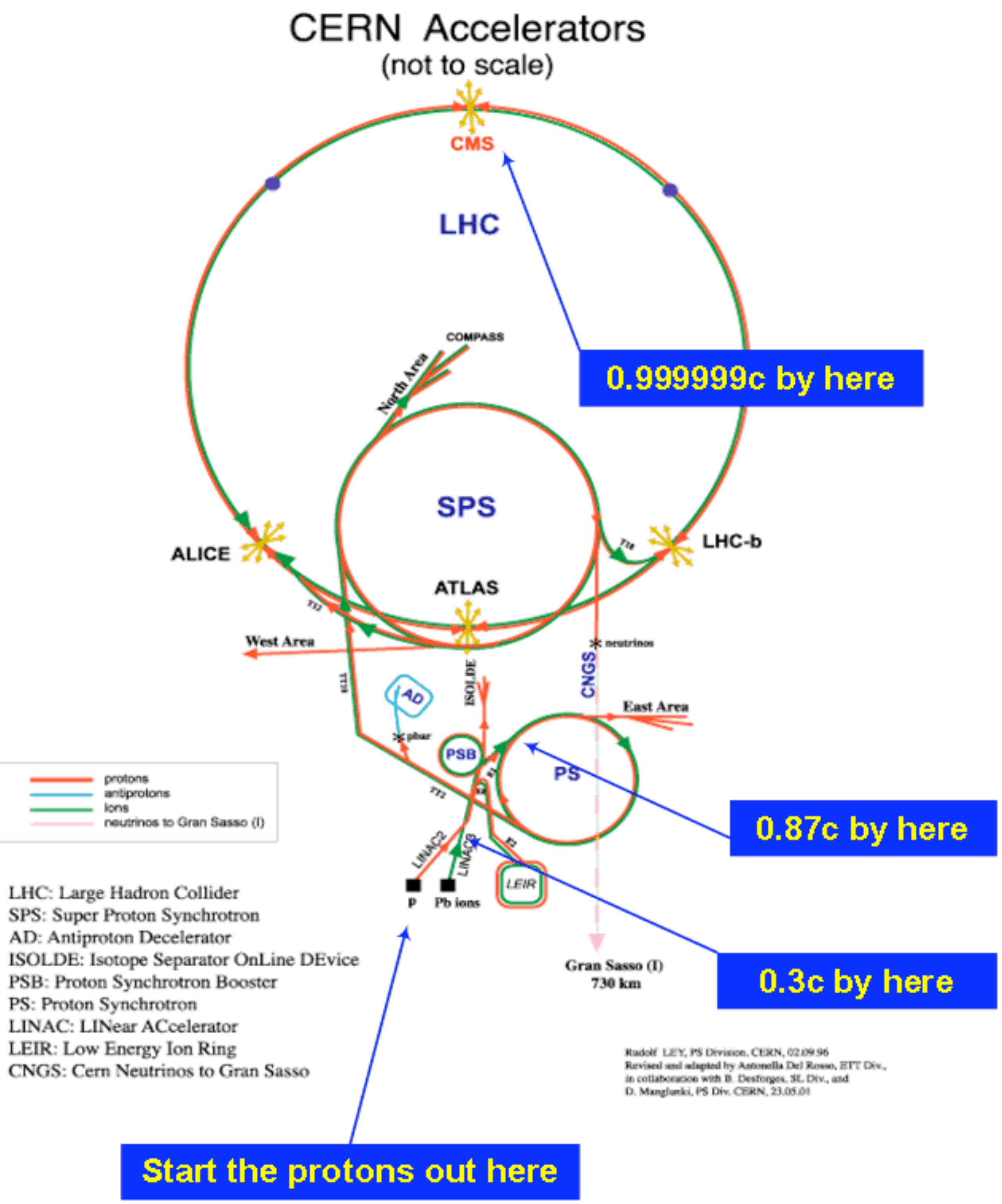
The Large Hadron Collider

- At CERN
- Proton-proton collider, $\sqrt{s} = 7$ to 14 TeV
- 2009 – 202X

- Relies on network of accelerators

- Four collision points: ATLAS, CMS, LHCb, ALICE

- CMS & ATLAS: general purpose detectors: observation of highest energy collisions
- LHCb: specialist experiment looking at b-hadrons
- ALICE: specialist experiment looking at Pb ion collisions



CERN Accelerators
(not to scale)

LHC: Large Hadron Collider
SPS: Super Proton Synchrotron
AD: Antiproton Decelerator
ISOLDE: Isotope Separator OnLine DEvice
PSB: Proton Synchrotron Booster
PS: Proton Synchrotron
LINAC: LINear ACcelerator
LEIR: Low Energy Ion Ring
CNGS: Cern Neutrinos to Gran Sasso

Rudolf LEY, PS Division, CERN, 02.09.96
Revised and adapted by Antonella Del Rosso, ETT Div.,
in collaboration with B. Desforges, SL Div., and
D. Manglunki, PS Div. CERN, 23.05.01

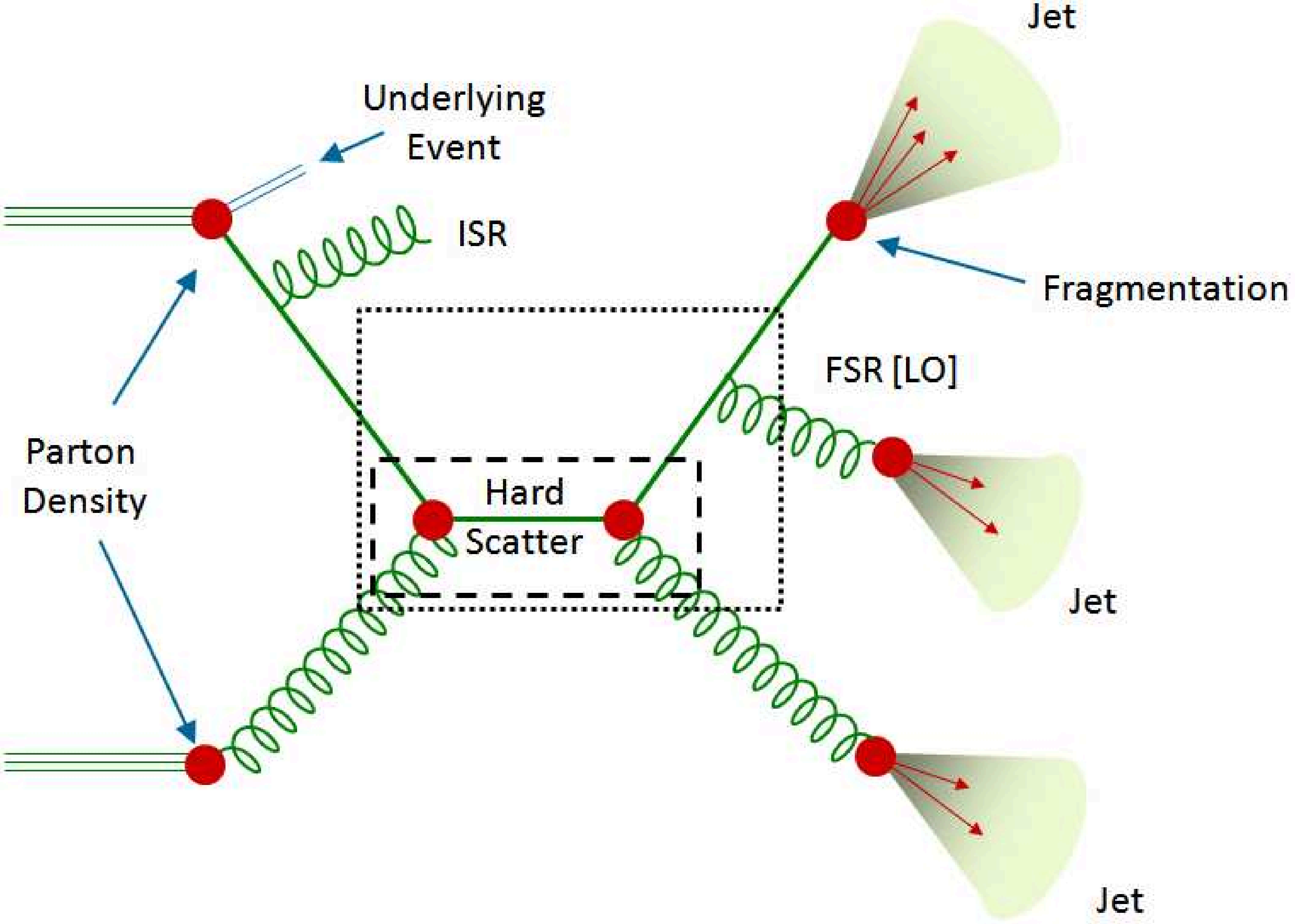
0.999999c by here

0.87c by here

0.3c by here

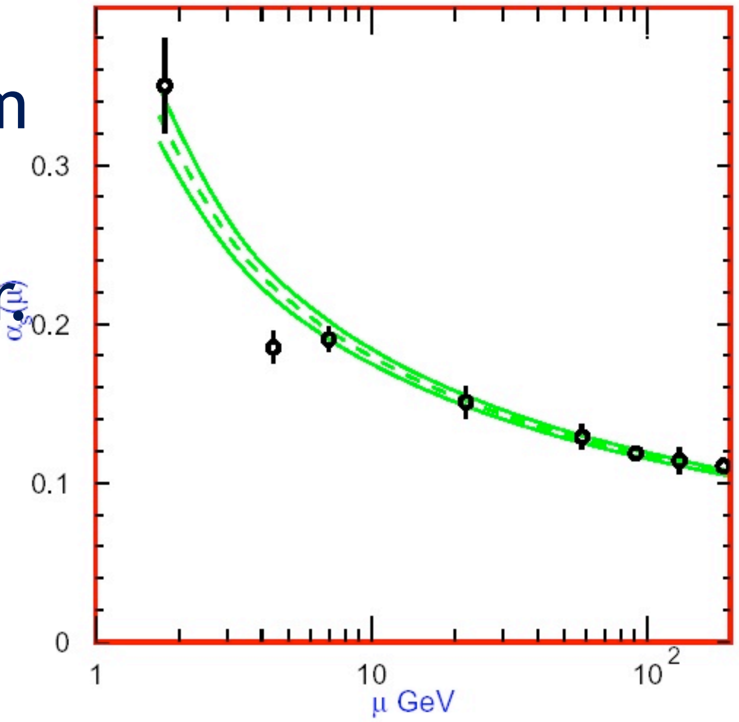
Start the protons out here

QCD production at Hadron Colliders



- Much more complicated due initial state hadrons not being fundamental particles
- Every object is colour charged: all object can interact with each other.
- QCD is very strong
- Not able to use perturbation theory to describe the interactions with low four momentum transfer q.

Summary



- In QCD, the coupling strength α_S decreases at high momentum transfer (q^2) increases at low momentum transfer.
- Perturbation theory is only useful at high momentum transfer.
- Non-perturbative techniques required at low momentum transfer.

- At colliders, hard scatter produces quark, anti-quarks and gluons.
- Fragmentation (hadronisation) describes how partons produced in hard scatter become final state hadrons. Need non-perturbative techniques.
- Final state hadrons observed in experiments as jets. Measure jet p_T, η, ϕ
- Key measurement at lepton collider, evidence for N_C=3 colours of quarks.

$$\mathbf{R} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \frac{e_q^2}{e^2}$$

Hadron Collider Dictionary

- The **hard scatter** is an initial scattering at high q^2 between partons (gluons, quarks, antiquarks).
- The **underlying event** is the interactions of what is left of the protons after parton scattering.
- **Initial and final state radiation** (ISR and FSR) are high energy gluon emissions from the scattering partons.
- **Fragmentation** is the process of producing final state particles from the parton produced in the hard scatter.
- A hadronic **jet** is a collimated cone of particles associated with a final state parton, produced through fragmentation.
- Transverse quantities are measured transverse to the beam direction.
- An event with high **transverse momentum** (p_T) jets or isolated leptons, is a signature for the production of high mass particles (W,Z,H,t).
- An event with **missing transverse energy** (E_T) is a signature for neutrinos, or other missing neutral particles.
- A **minimum bias** event has no missing energy, and no high mass final states particles ($W,Z,H,\mathrm{b},\mathrm{t}$). At the LHC these are treated as background.

Measuring Jets

- A jet has a four-momentum $E = \sum_i E_i \quad \vec{p} = \sum_i \vec{p}_i$
  - ➡ Where the constituents (*i*) are hadrons detected as charged tracks and neutral energy deposits.

- Transverse momentum of jet:

$$p_T^{\mathrm{JET}} = \sqrt{p_x^2 + p_y^2}$$

- Position in the detector in two coordinates:
  - ➡ ***Pseudorapidity*** of jet (η) $\eta^{\mathrm{JET}} = -\ln\left(\tan\frac{\theta}{2}\right)$

    with polar angle, θ $\cos\theta = \frac{\sqrt{p_x^2+p_y^2}}{p_z}$
  - ➡ **Azimuthal angle** of jet (ϕ) $\phi^{\mathrm{JET}} = \tan^{-1}\left(\frac{p_y}{p_x}\right)$

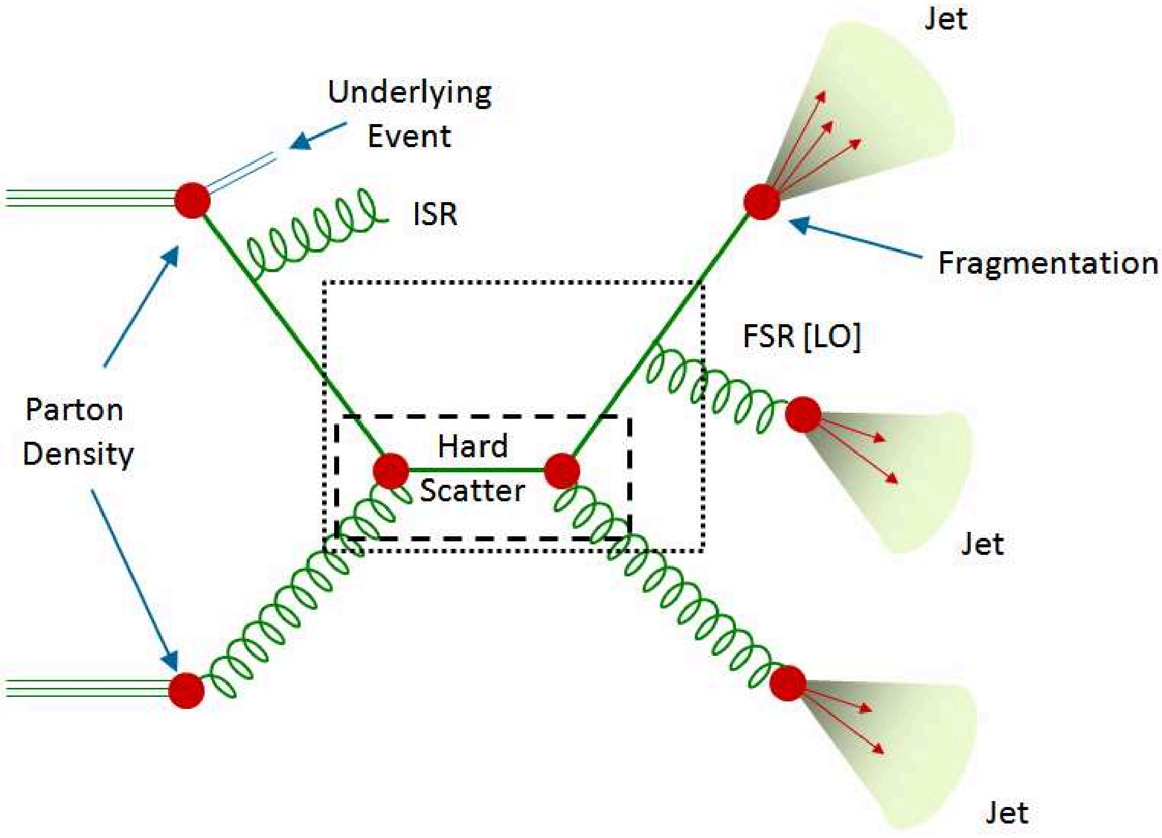
- To assign individual constituents to the jet, simplest algorithm is to define a **cone** around a central value: η^{JET}, ϕ^{JET}.

$$R^2 = (\eta_i - \eta^{\mathrm{JET}})^2 + (\phi_i - \phi^{\mathrm{JET}})^2$$

- All objects with R less than a given value (typically 0.4 or 0.7) are assigned to the jet
- Many sophisticated jet clustering algorithms exist which take into account QCD effects.

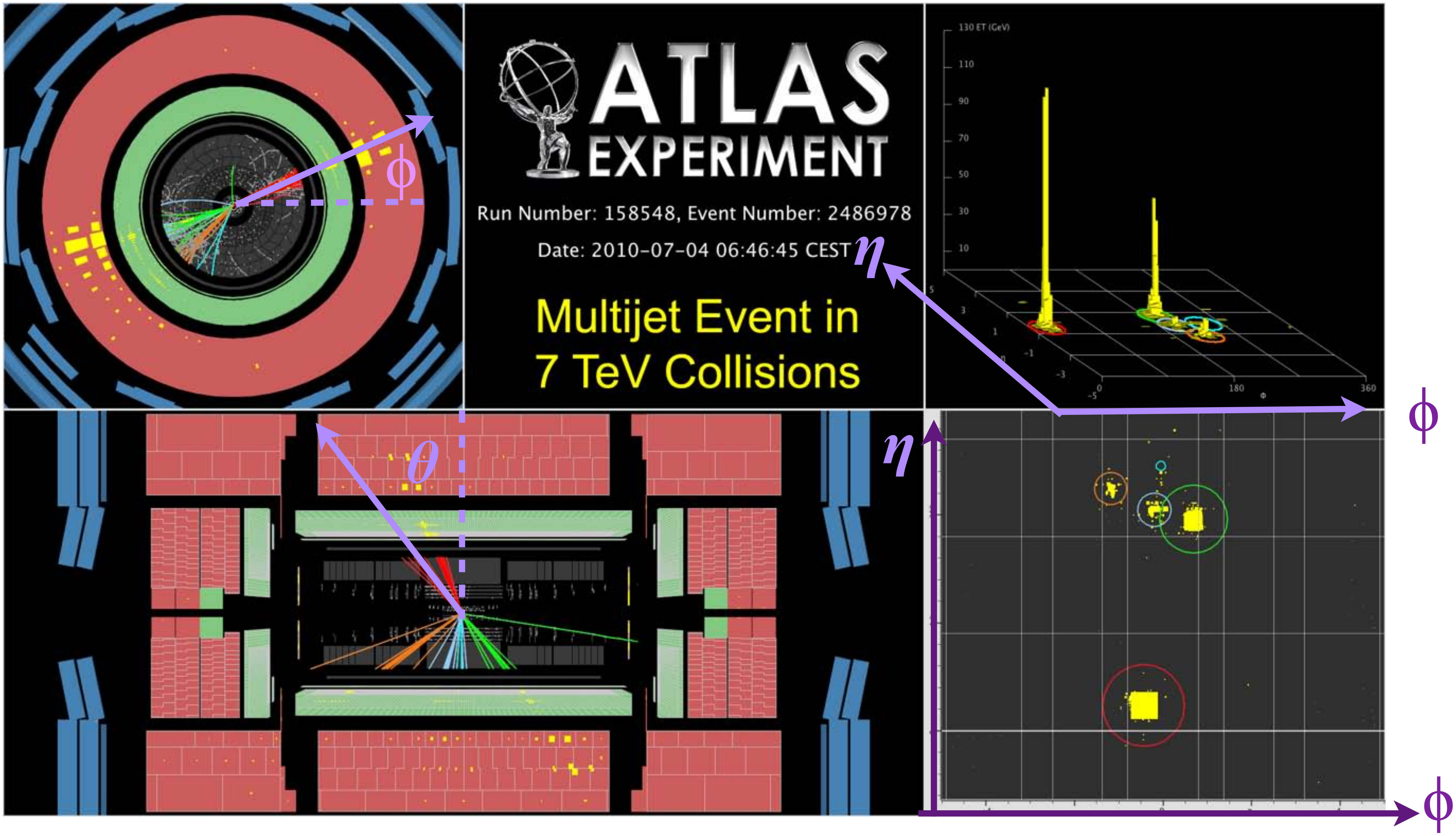
Jet Fragmentation

- **Fragmentation** or **hadronisation** is the process of producing final state particles from the parton produced in the hard scatter.
- Gluon momentum transfer, q, varies
  - ➡ both perturbative and non-perturbative methods are needed to describe/model fragmentation



- Initial parton radiates gluon, which can form quark anti-quark pairs, modeled using perturbative QCD
- Hadrons are formed using non-perturbative models of colour confinement. Several stochastic models exist, with many parameters:
  - PYTHIA, HERWIG, SHERPA
- The jet fragmentation algorithms are tuned to match data

ATLAS Multijet Event

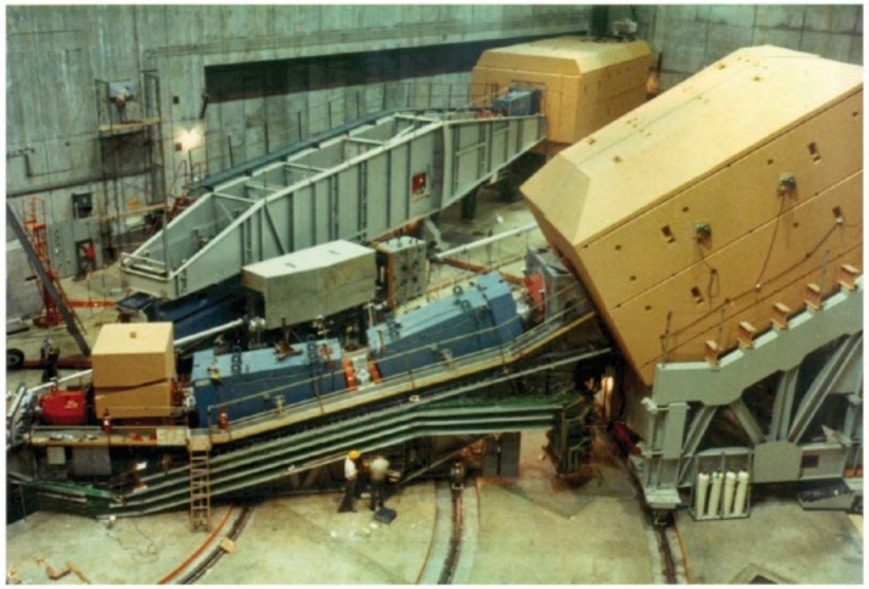


- η and ϕ act as map of activity in the detector

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 11: *Probing the proton*, or Deep Inelastic scattering



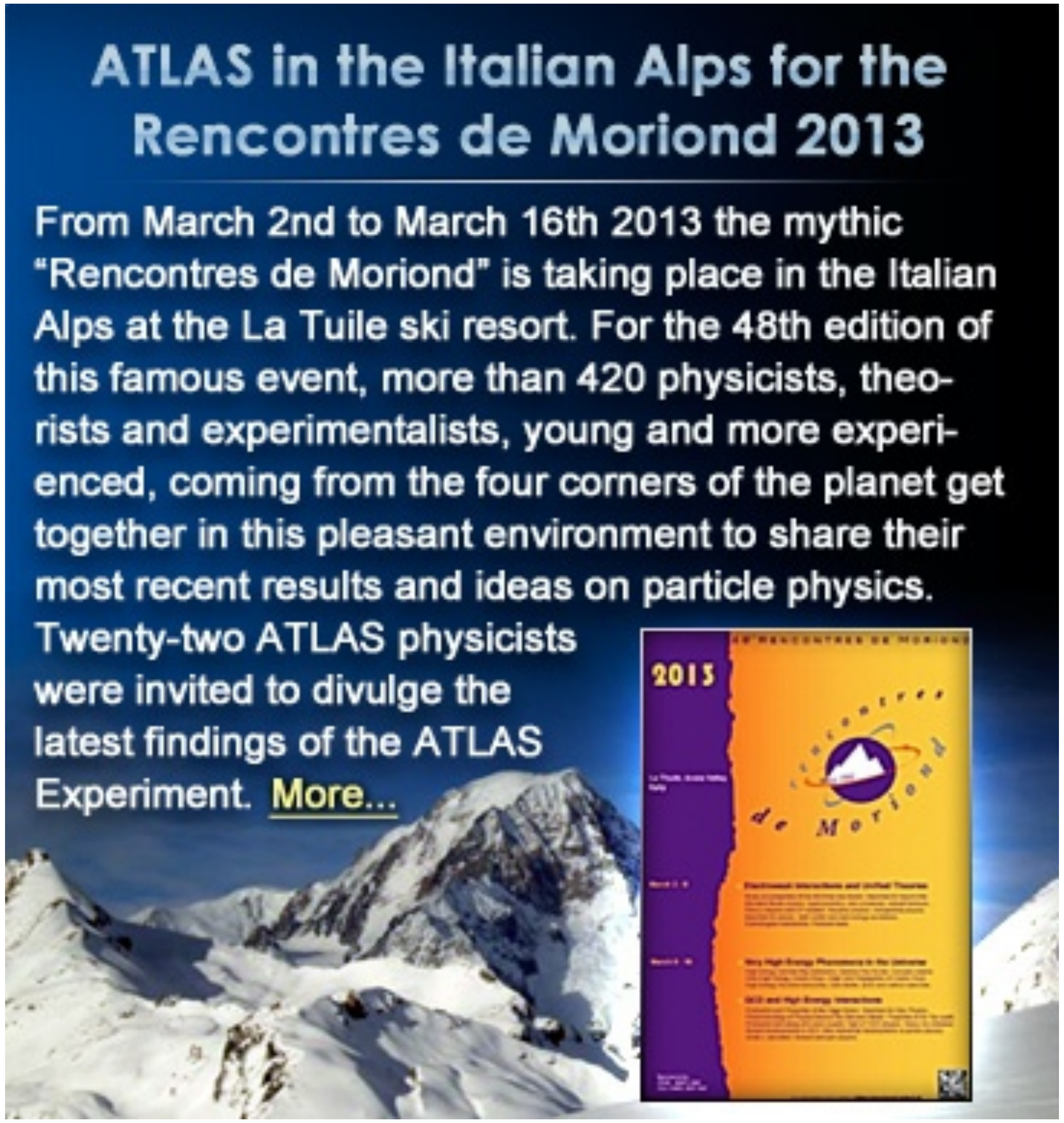
$$\mathbf{R} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \frac{e_q^2}{e^2}$$

- ★Hadron Colliders
- ★Electron-proton scattering
- ★Deep Inelastic scattering
- ★(Dolly) Partons in the proton

Announcements

- I’m going to a conference next week.
  - Steve Playfer will give lectures in my place.
  - Topics are hadrons, hadron decays and the CKM matrix.

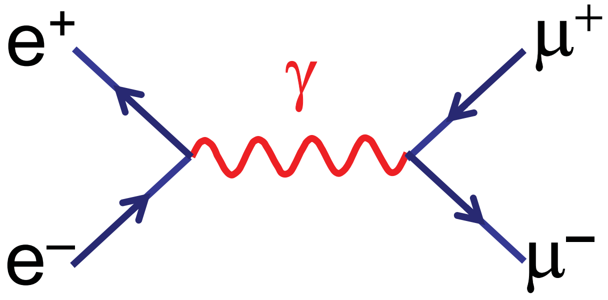
- Tutors will be at the tutorial 3-5pm on Monday, but no new tutorial sheet.

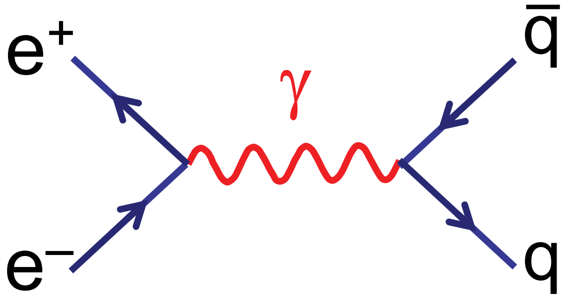


ATLAS in the Italian Alps for the Rencontres de Moriond 2013

From March 2nd to March 16th 2013 the mythic “Rencontres de Moriond” is taking place in the Italian Alps at the La Tuile ski resort. For the 48th edition of this famous event, more than 420 physicists, theorists and experimentalists, young and more experienced, coming from the four corners of the planet get together in this pleasant environment to share their most recent results and ideas on particle physics. Twenty-two ATLAS physicists were invited to divulge the latest findings of the ATLAS Experiment. More...

Review from Tuesday: Rate for $e^+e^-\rightarrow$hadrons





$$\mathcal{M}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{e^2}{q^2}[\bar{v}(e^+)\gamma^\mu u(e^-)][v(\mu^+)\gamma^\mu \bar{u}(\mu^-)]$$

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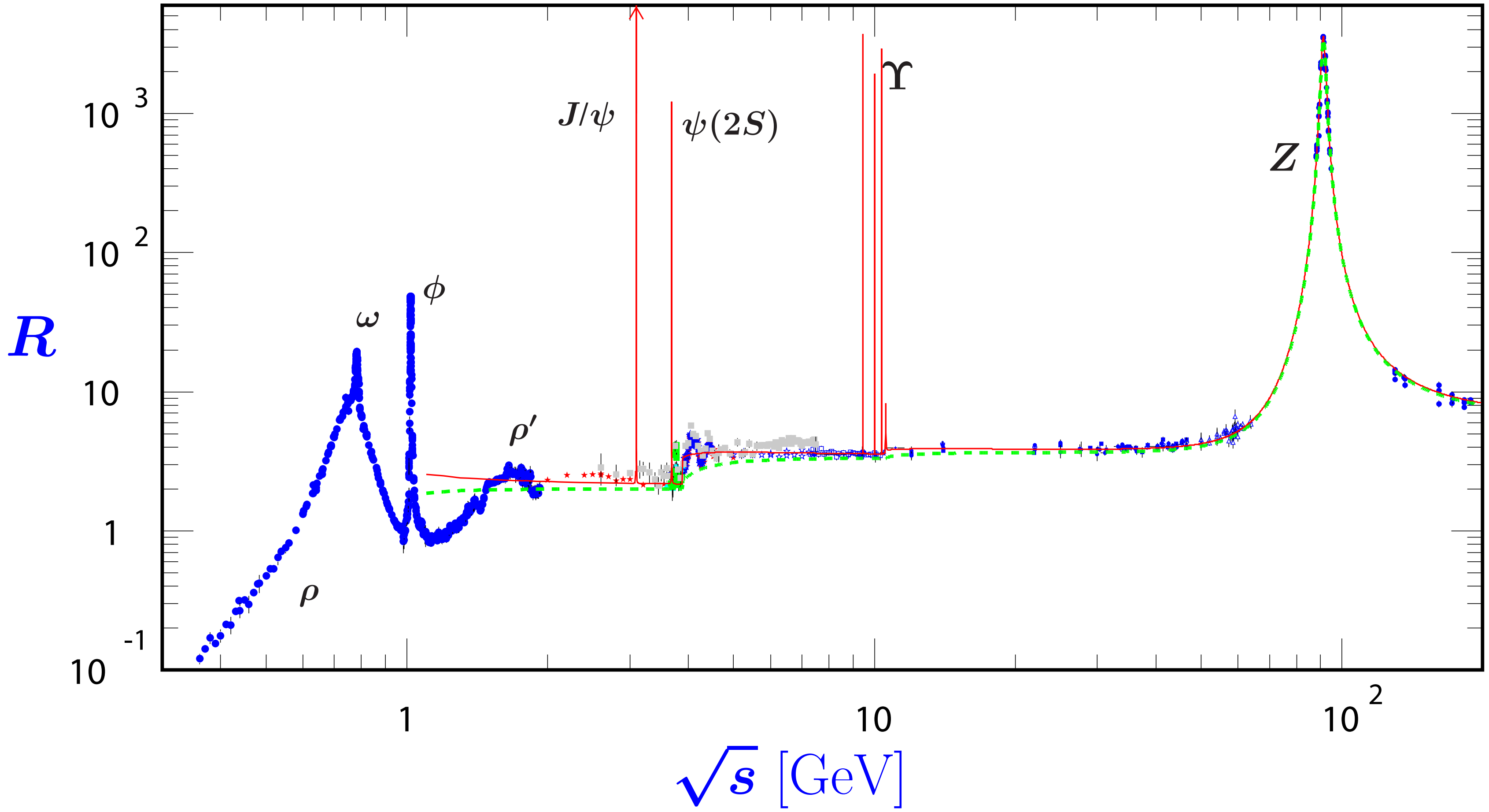
Measurements of R

- Compendium of measurements from many lepton colliders.



- Consistent with N_C=3, this is one of the key pieces of evidence for three quark colours.
- At quark thresholds, $\sqrt{s} \sim 2m_q$ "resonances" occur as bound states of $q\overline{q}$ more easily produced (see next lecture).
- Steps at ~4 and ~10 GeV due to charm and bottom quark threshold
- At $\sqrt{s} \sim 100$ GeV, Z-boson exchange takes over.

Electron Proton Scattering Experiments

- **SLAC-MIT experiment ('67)**
- **Electron beam on liquid hydrogen target**

Won the 1990 Noble prize for: Jerome I. Friedman, Henry Kendall, Richard E. Taylor "for their pioneering investigations concerning **deep inelastic scattering** of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

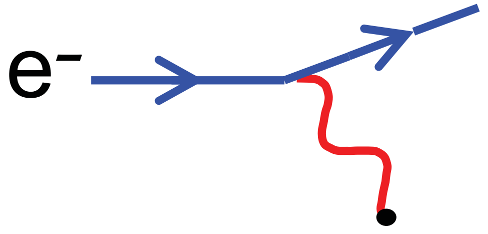
The investigations gave the surprising result that the electrical charge within the proton is concentrated to smaller components of negligible size.

- **DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany**

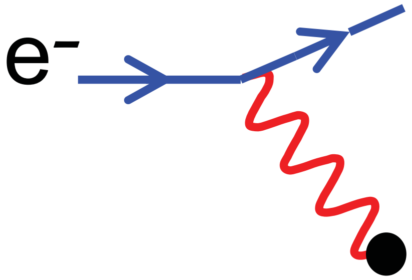
- HERA was the world's only electron - proton collider, ran 1992 - 2007
- **E(e^-) = 30 GeV, E(p) = 820 GeV**
- 6.3 km in circumference
- Three experiments:
  - Two general purpose experiments: ZEUS, H1
    - ➡ Probe proton at very high Q^2 and very low x

Probing the Structure of the Proton

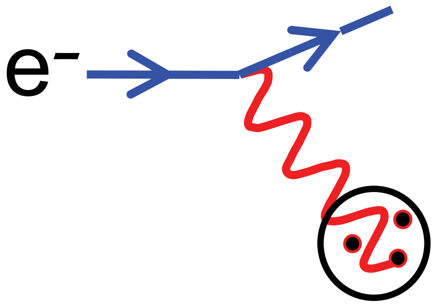
- In $e^-p \rightarrow e^-p$ scattering the nature of the interaction of the virtual photon with the proton depends strongly on wavelength, $\lambda = ch/E$



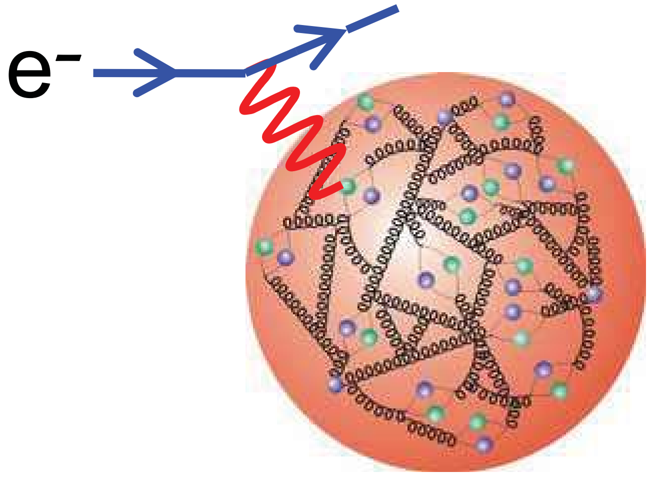
- At very low electron energies $\lambda >> r_p$: the scattering is equivalent to that from a "point-like" spin-less object



- At low electron energies $\lambda \sim r_p$ the scattering is equivalent to that from a extended charged object



- At high electron energies $\lambda < r_p$: the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks



- At very high electron energies $\lambda << r_p$: the proton appears to be a sea of quarks and gluons.

Form Factors

- Extended object - like the proton - have a matter density ***ρ(r)***, normalised to unit volume:

$$\int d^3\vec{p}\ \ (\vec{r}) = 1$$

- Fourier Transform of ***ρ(r)*** is the **form factor, *F(q)***:

$$F(\vec{q}) = \int d^3\vec{r}\, \exp\{i\,\vec{q}\,\vec{r}\}\,\rho(\vec{r}) \Rightarrow F(0) = 1$$

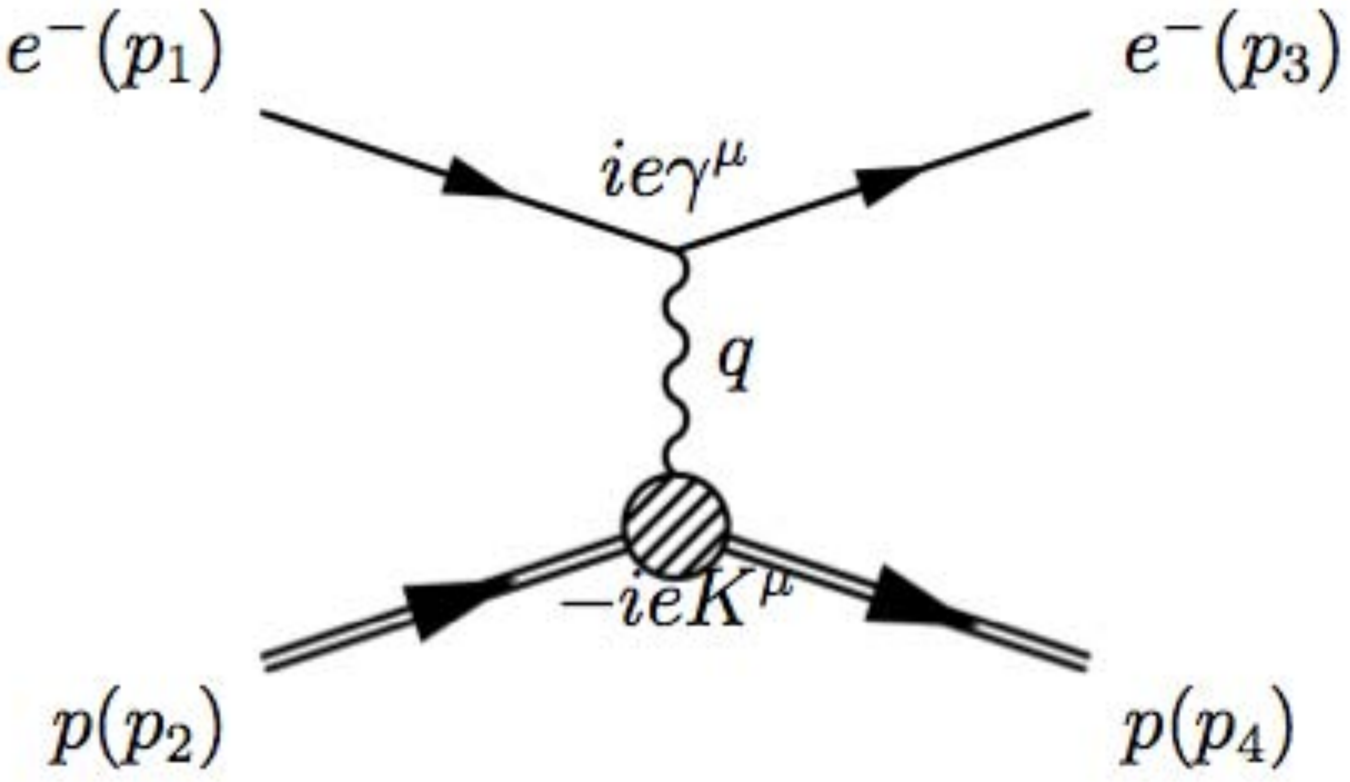
- Cross section are modified by the form factor:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{extended}} \approx \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{point-like}} |F(\vec{q})|^2$$

- For ***ep→ep*** scattering we need two form factors:
  - F_1 to describe the distribution of the electric charge
  - F_2 to describing the recoil of the proton

Elastic Electron Proton Scattering

Scattering of high energy electrons by electromagnetic interactions probes the charge distribution of the proton



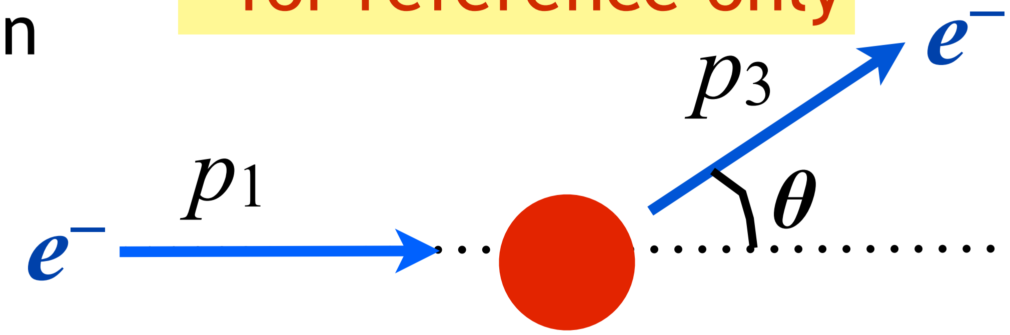
In elastic scattering the proton remains a proton, but the proton current is modified by ***K^μ*** because the proton is not a pointlike particle

$$\mathcal{M}(e^-p \to e^-p) = \frac{e^2}{(p_1 - p_3)^2} (\bar{u}_3 \gamma^\mu u_1) (\bar{u}_4 K_\mu u_2)$$

$$K^\mu = \gamma^\mu F_1(q^2) + \frac{i\kappa_p}{2m_p} F_2(q^2) \sigma^{\mu\nu} q_\nu$$

Low-Energy Scattering *

* for reference only



- Elastic scattering of electron on stationary proton

$$|\vec{p}^*| = |\vec{p}_1| = |\vec{p}_2|$$

- Described by Mott Scattering:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4|\vec{p}^*|^2\beta^2 \sin^4\theta/2}(1 - \beta^2 \sin^2\theta/2)$$

- $\sin^4(\theta/2)$ term due to photon propagator, $1/q^2$

- At very low energies we have Rutherford scattering: Coulomb scattering on the electric charge of proton ($E_K = p^2/2m_e$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E_K^2 \sin^4\theta/2}$$

- At relativistic energies, $\beta \rightarrow 1$, influence of spin-½ nature of proton, need to also account for finite size of proton charge distribution through form factor $F(q^2)$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\theta/2} \cos^2\frac{\theta}{2} |F(\vec{q}^2)|^2$$

Higher Energy Elastic Scattering*

* for reference only

- At higher energies need to account for the recoil of the proton …

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\underbrace{\frac{E_3}{E_1}}_{\text{from proton recoil}}\left(\cos^2\theta/2 - \underbrace{\frac{q^2}{2M^2}\sin^2\theta/2}_{\text{Magnetic interaction due to spin-spin interaction}}\right)$$

- … and finite size effects:

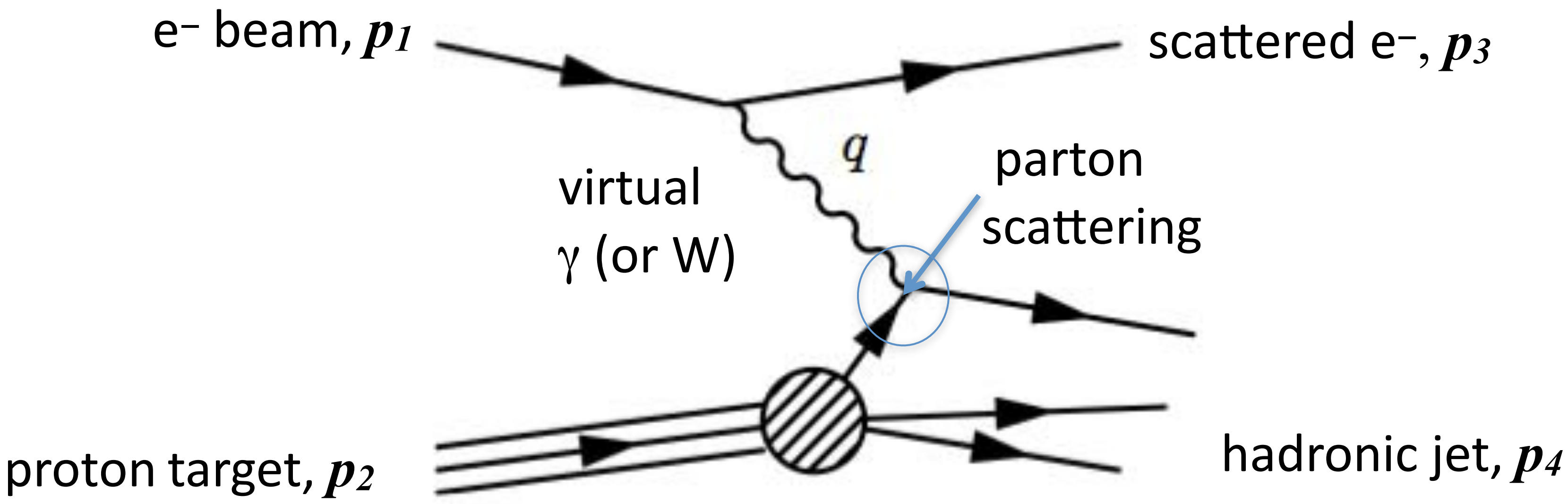
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4m_p^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2}\left(F_1 + \kappa F_2\right)^2 \sin^2\frac{\theta}{2}\right\}$$

- $F_1(q^2)$ and $F_2(q^2)$ are the form factors, which need to be measured.

- Measurement of elastic scattering demonstrate the proton is extended object with rms radius of **~0.8 fm**

Deep Inelastic Scattering (DIS)

- In deep inelastic scattering the proton disintegrates
- The final state hadronic system contains at least one baryon, implying invariant mass of the final state system, $M_X > M_p$



- For deep inelastic scattering introduce new kinematic variables: x, Q^2, ν

$$x \equiv \frac{Q^2}{2p_2 \cdot q} \qquad Q^2 = -q^2 = (p_1 - p_3)^2 > 0 \qquad \nu \equiv \frac{p_2 \cdot q}{M_p}$$

$$M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2 \cdot q + M_p^2$$

$$\Rightarrow \; Q^2 = 2p_2 \cdot q + M_p^2 - M_X^2 \;\Rightarrow Q^2 < 2p_2 \cdot q$$

- inelastic: $0 < x < 1$
- elastic: $x = 1$

DIS: Cross Section

- Assume that the photon is elastically scattering off the individual constituents of the proton.
- Proton constituents are called **partons**.
- x is the fraction of the proton's energy carried by the individual partons
- Cross section for DIS is:

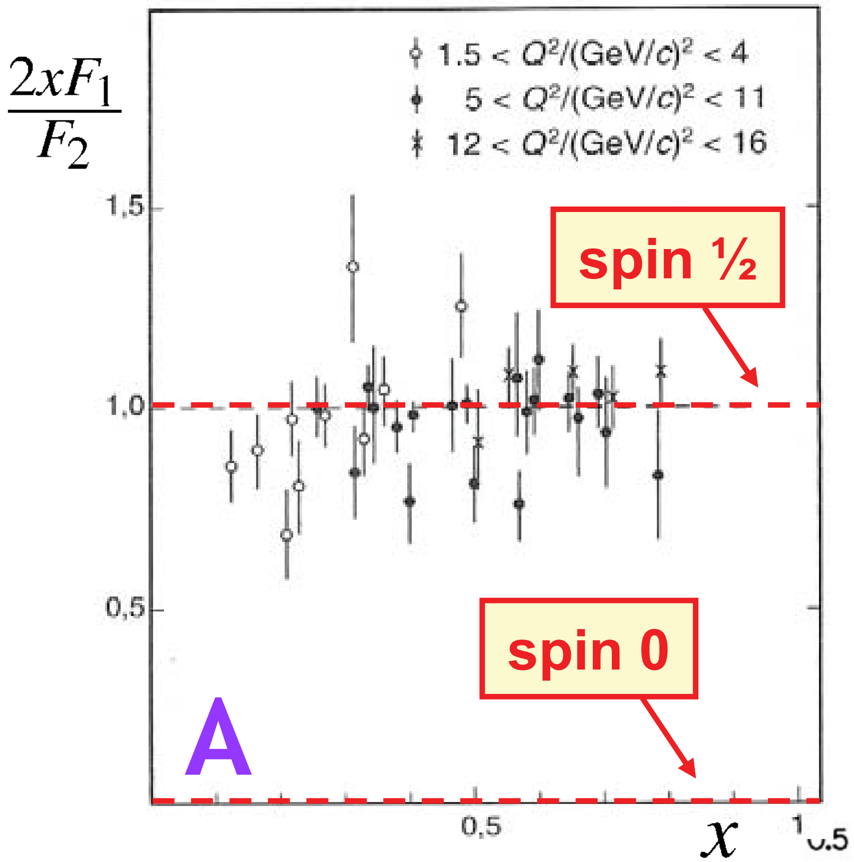
$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \left(\frac{1}{\nu} F_2\underbrace{(x, Q^2)}_{\text{EM structure function}} \cos^2\frac{\theta}{2} + \frac{2}{M} F_1\underbrace{(x, Q^2)}_{\text{Magnetic structure function}} \sin^2\frac{\theta}{2} \right)$$

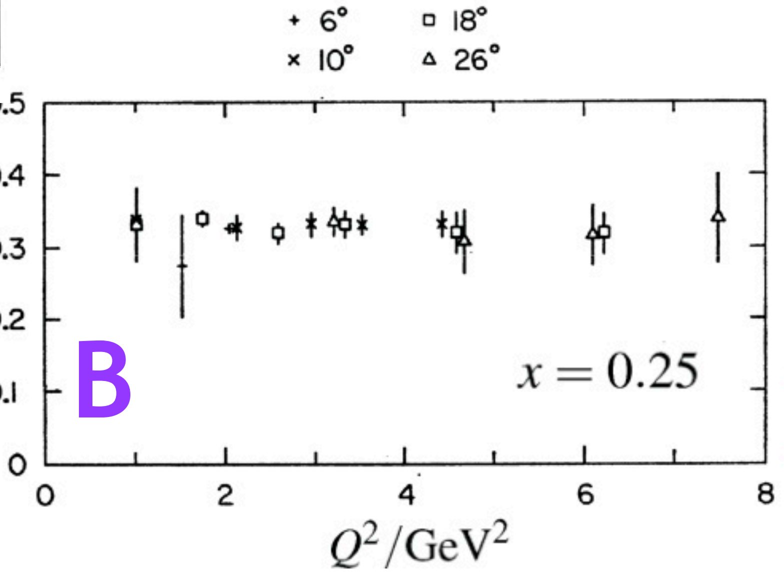
- The structure functions are sums over the charged partons in the proton:

$$2xF_1(x) = F_2(x) = \sum_q x\, e_q^2 \mathrm{q(x)}$$

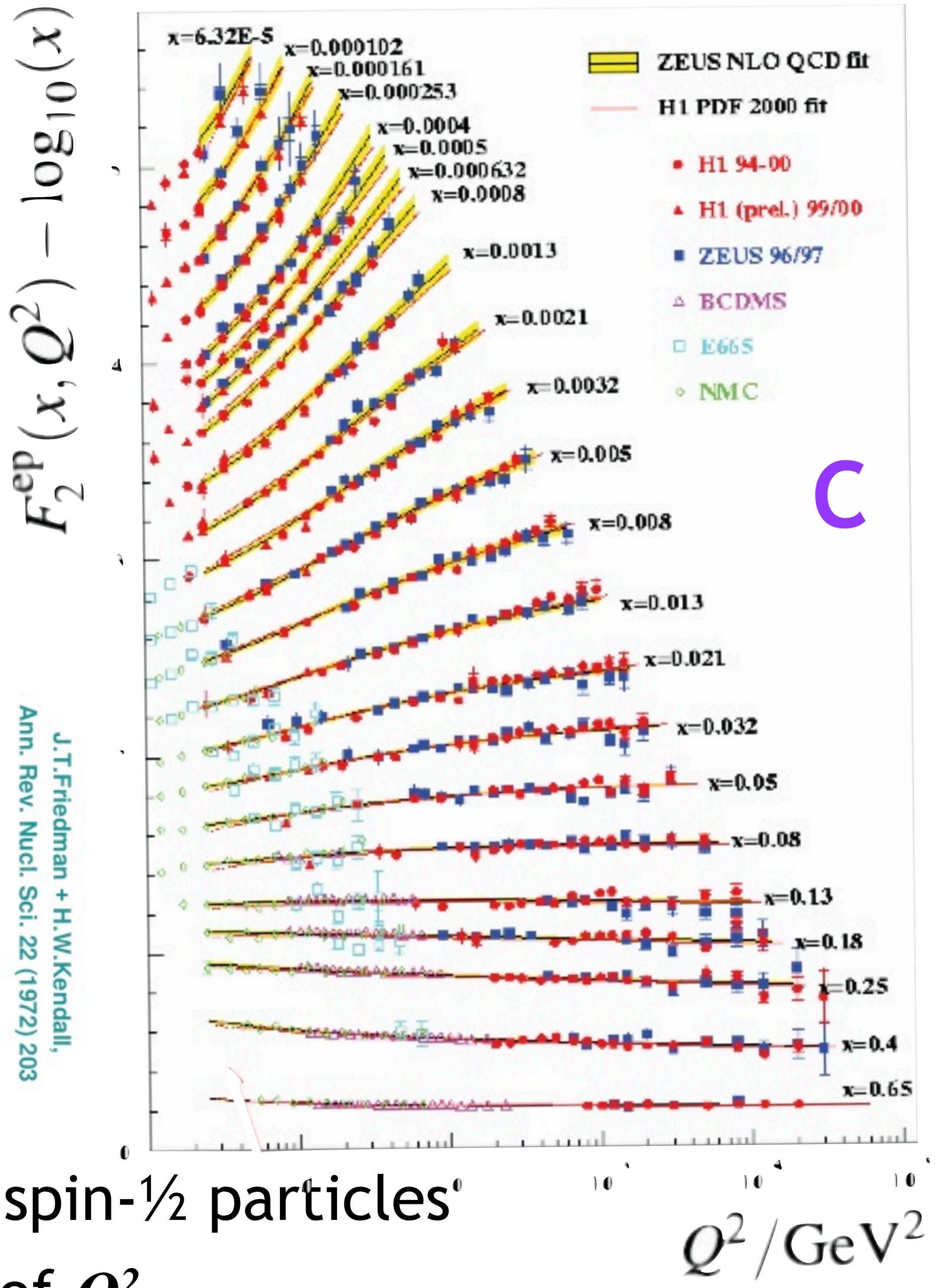
- Partons the proton are:
  - ➡**valance quarks** = **uud**
  - ➡**sea quarks** in quark anti-quark pairs, e.g. $\mathbf{\bar{u}u, \bar{d}d, \bar{s}s, \bar{c}c, ...}$
  - ➡**gluons**, **g**

Experimental Measurements of F_1 and F_2





J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203



A. $2xF_1 = F_2$ is predicted if the constituents are spin-½ particles

B. At low Q^2, F_2 is predicted to be independent of Q^2

C. At very low x, F_2 is not independent of Q^2, as gluons start to take a larger share of the proton momentum.

Summary

- The measurement of

$$\mathbf{R} = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \frac{e_q^2}{e^2}$$

is experimental proof of $N_c=3$, three colours of quarks.

Electron-proton scattering investigates proton substructure

- At lower energies: elastic scattering $e^-p \to e^-p$
  - ➡proton remains intact, scattering can be described by proton form factor.

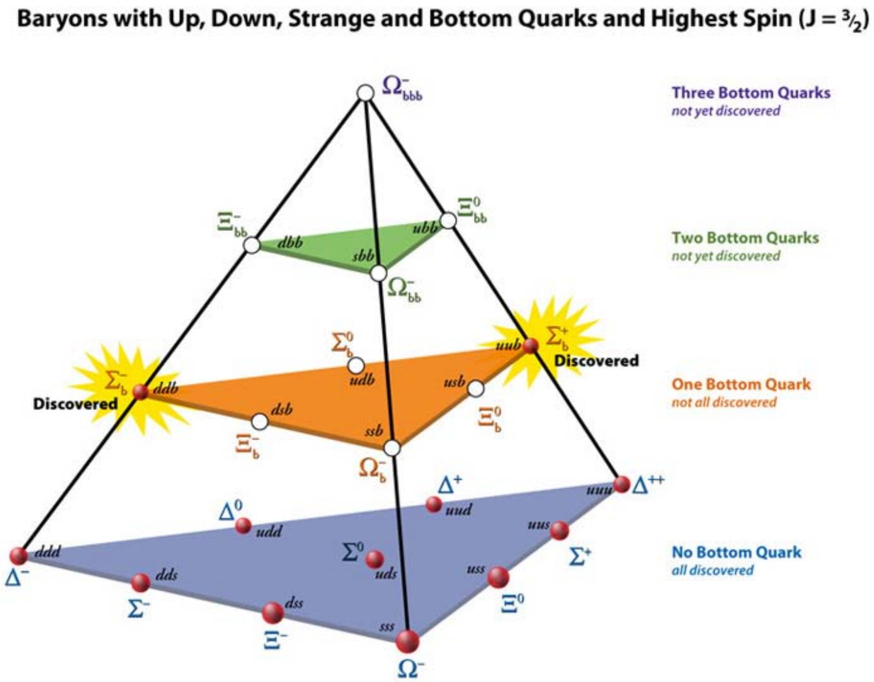
- At higher energies: **Deep inelastic scattering** $e^-p \to e^-X$
  - ➡Scattering from individual quarks within the proton.
  - ➡Each parton carries the momentum fraction x, described by proton structure functions, $\mathbf{q}(x)$
  - ➡Only ~50% proton momentum carried by up and down quarks, remainder carried by gluons, sea quarks.

Particle Physics

Dr Victoria Martin, Prof Steve Playfer

Spring Semester 2013

Lecture 12: Mesons and Baryons



- ★Mesons and baryons
- ★Strong isospin and strong hypercharge
- ★SU(3) flavour symmetry
- ★Heavy quark states

Review from Friday

- Using high energy deep inelastic scattering, $e^- p \to e^- X$, we find out the proton consists of partons:
  - three **valence quarks**: two up quarks, one down quark
  - **gluons** that are continuously exchanged between the quarks
  - **sea quarks:** quark-antiquark pairs produced by gluons
- In today's lecture consider the **valence quark** model of mesons and baryons

Mesons and Baryons

Mesons are quark-antiquark bound states with a **symmetric** colour wavefunction

$$\chi_c = \frac{1}{\sqrt{3}}\left[\mathrm{r\bar{r}} + \mathrm{b\bar{b}} + \mathrm{g\bar{g}}\right]$$

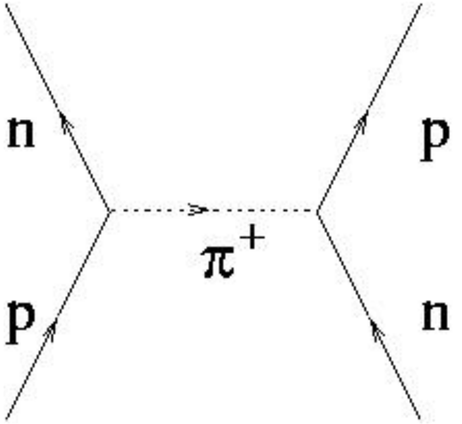
Baryons are three quark bound states with an **antisymmetric** colour wavefunction

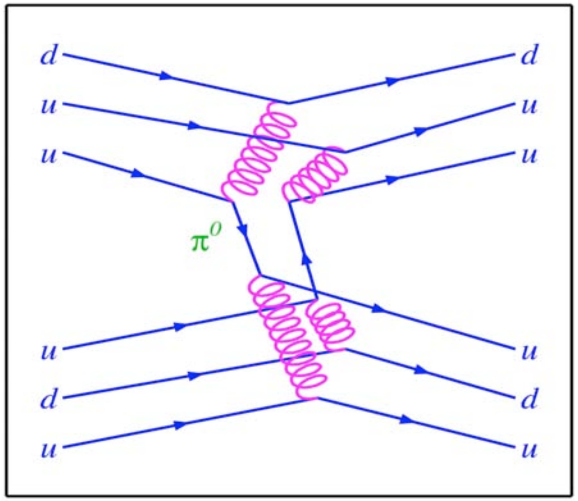
$$\chi_c = \frac{1}{\sqrt{6}}\left[\mathrm{rgb} - \mathrm{rbg} + \mathrm{gbr} - \mathrm{grb} + \mathrm{brg} - \mathrm{bgr}\right]$$

- All these **hadrons** are **colour neutral**
- They do not interact with each other by single gluon exchange
- They couple to each other by hadron exchange, typically through the pion (the lightest meson)
- Yukawa (1935) – the finite range of strong interactions between hadrons is due to the pion mass of **~140 MeV**

Nucleon-nucleon scattering:

a strong interaction as seen at the hadron and quark level





Constituent Quark Masses

- Because of QCD renormalisation, there is an ambiguity in how to define the quark masses. The most commonly used definition is known as the "$\overline{MS}$ scheme":

  m_u= 2.4 MeV, m_d = 4.8 MeV, m_s =104 MeV, m_c =1.27 GeV, m_b =4.20 GeV

- The light quark masses are too small to account for the hadron masses $m(\pi^+)$ =140 MeV is a u-dbar bound state!
- The majority of the mass of hadrons comes from QCD interactions

- The valence quark model introduces constituent quark masses

  m_u= m_d ~ 300 MeV, m_s ~ 500 MeV, m_c ~ 1.5 GeV, m_b ~ 4.7 GeV

- These are an effective model for the observed hadron masses and magnetic moments (but still don't work for the pion!)

Flavour Symmetries: Isospin

- Strong interactions are (approximately) invariant under flavour symmetry rotations.
  Known for hadrons long before quark model was invented (n-n, n-p, p-p the same).

- Assign quantum numbers to characterise these symmetries.
  ➡Strong Isospin (I, I_3): a flavour symmetry between u and d quarks
  $I = 1/2$ doublet with $I_3(\mathrm{u}) = +1/2$ and $I_3(\mathrm{d}) = -1/2$
  (by analogy to S=1/2 with spin states ↑ and ↓)

- Strong interactions are invariant under isospin rotations $\mathbf{u} \leftrightarrow \mathbf{d}$ **or** $p \leftrightarrow n$

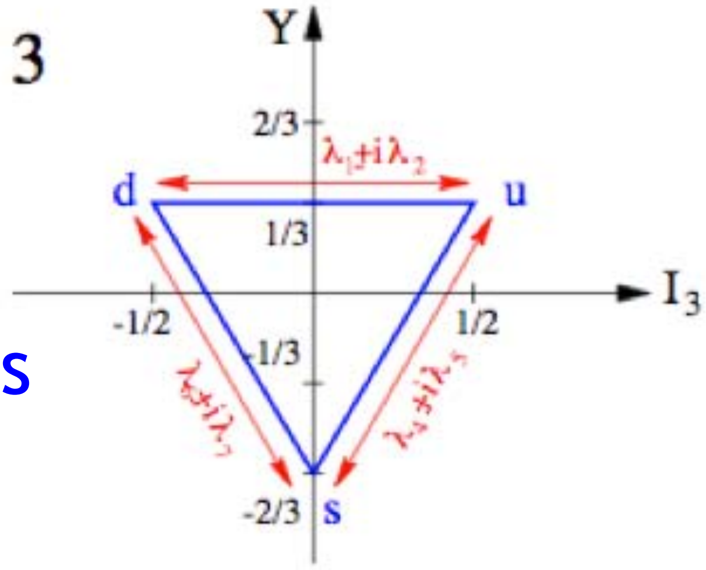
- For heavy quarks introduce quantum numbers for each flavour:
  ➡Strangeness (S), Charm (C), Beauty (B), Truth (T)
  S(s) = -1, S(sbar) = +1, B(b) = -1 , B(bbar) = +1
  C(c) = +1, C(cbar) = -1, T(t) = +1 , T(tbar) = -1

- Strong interactions are (almost) flavour independent, and conserve quark flavour

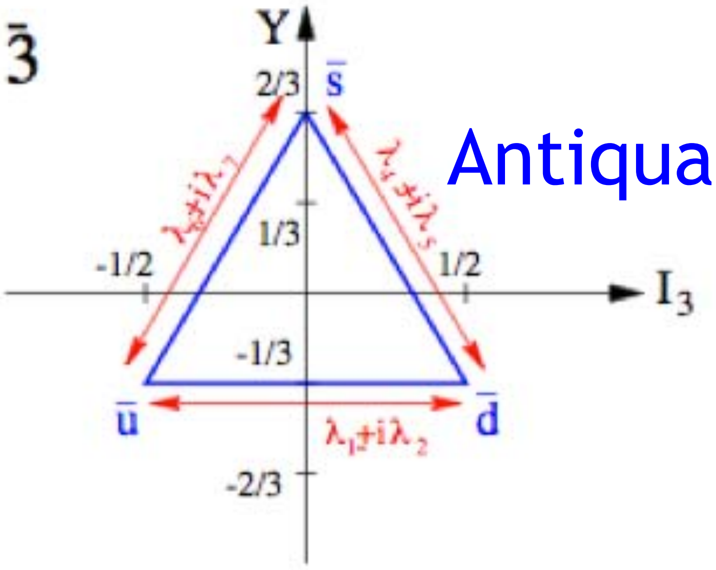
SU(3) Flavour Symmetry

- An SU(3) flavour symmetry is exhibited between **u, d** and **s** quarks
- The symmetry is broken by the **s** quark mass $\mathbf{m_s \sim 100\ MeV \gg m_u, m_d}$

- Classify hadrons in SU(3) multiplets using convenient quantum numbers:
  - Strong Hypercharge $Y = S + \mathcal{B}$ (where $\mathcal{B} = ⅓\,[N(q)-N(\bar{q})]$ is baryon number)
  - Strong Isospin I_3 (note that electric charge $Q = I_3 + Y/2$)



Quarks



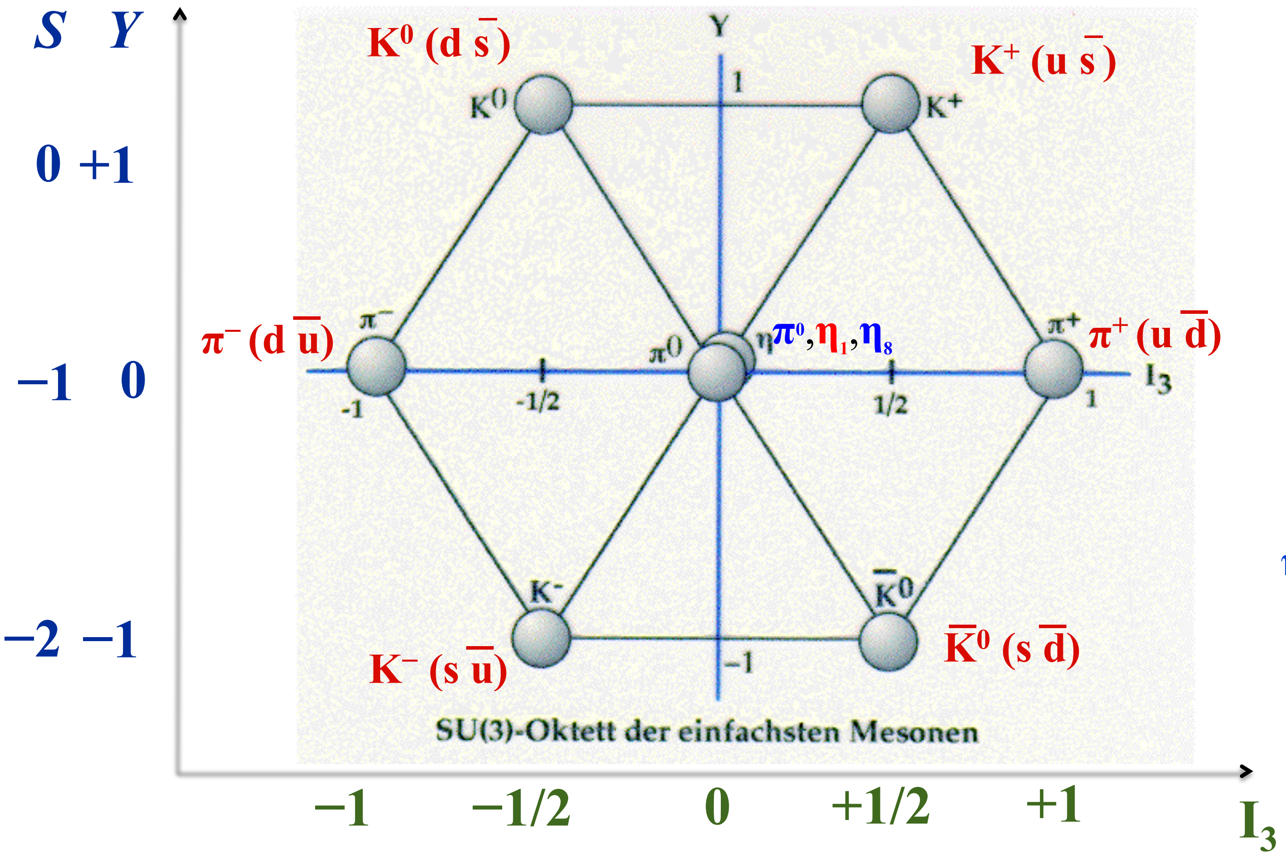
Antiquarks

These are the basic building blocks for constructing mesons ($\mathbf{q\,\bar{q}}$) and baryons ($\mathbf{qqq}$)

The J=0 Pseudoscalar Mesons

- Total angular momentum, J=0: orbital angular momentum, L=0; one spin-up and one spin-down ↑ ↓ quark
- The allowed flavour combinations are given by the Gell Mann λ matrices,

➡same matrices that describe the allowed colour combinations of gluons.



$M(K^0, \bar{K}^0) = 498$ MeV

$M(K^+,K^-) = 494$ MeV

$M(\pi^+,\pi^-) = 140$ MeV

$M(\pi^0)=135$ MeV

$M(\eta)=550$ MeV

$M(\eta')=960$ MeV

$$\pi^0 = 1/\surd 2\ [\, d\,\bar{d} - u\,\bar{u}\,]$$

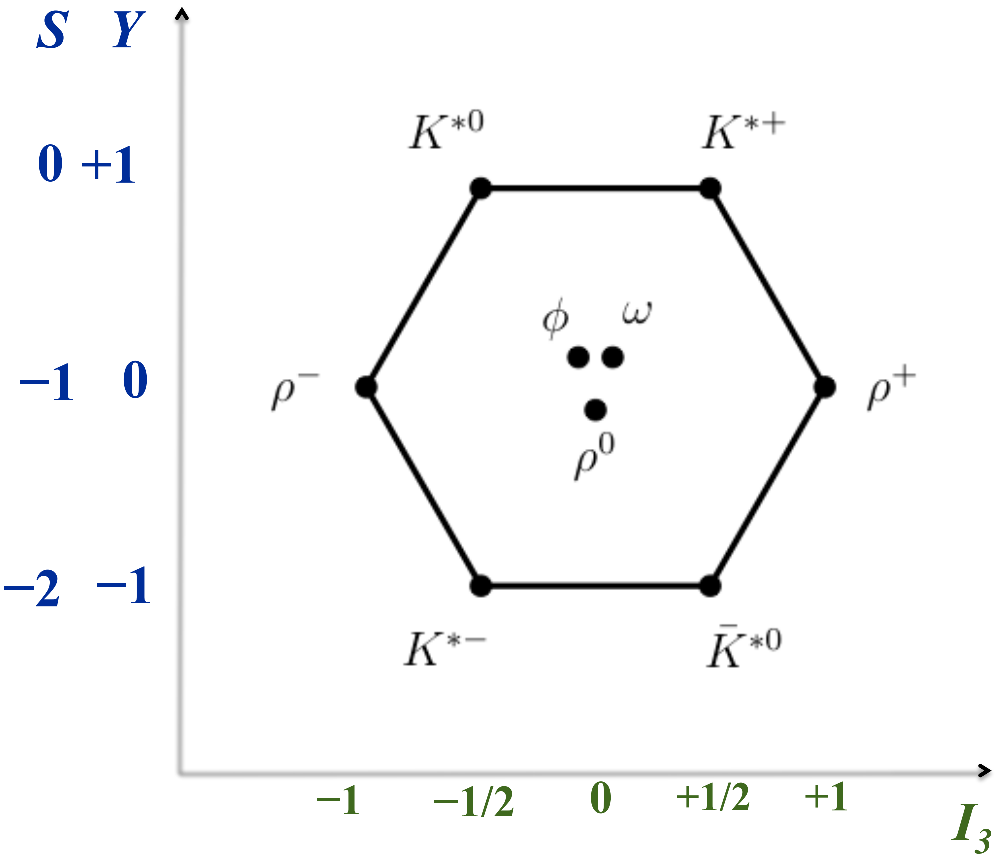
$$\eta_8 = 1/\surd 6\ [\, d\,\bar{d} + u\,\bar{u} - 2\, s\,\bar{s}\,]$$

$$\eta_1 = 1/\surd 3\ [\, d\,\bar{d} + u\,\bar{u} + s\,\bar{s}\,]$$

Observed η,η' mesons are mixtures of η_1 and η_8

The *J=1* Vector Mesons

- Total angular momentum, ***J=1***: ***L=0***, both quarks with same spin ↑ ↑



$M(K^{*+},K^{*-})$ = 892 MeV

$M(K^{*0}, K^{*0})$ = 896 MeV

$M(\rho^+,\rho^-)$ = 776 MeV

$M(\rho^0)$=767 MeV

$M(\omega)$=783 MeV

$M(\phi)$=1019 MeV

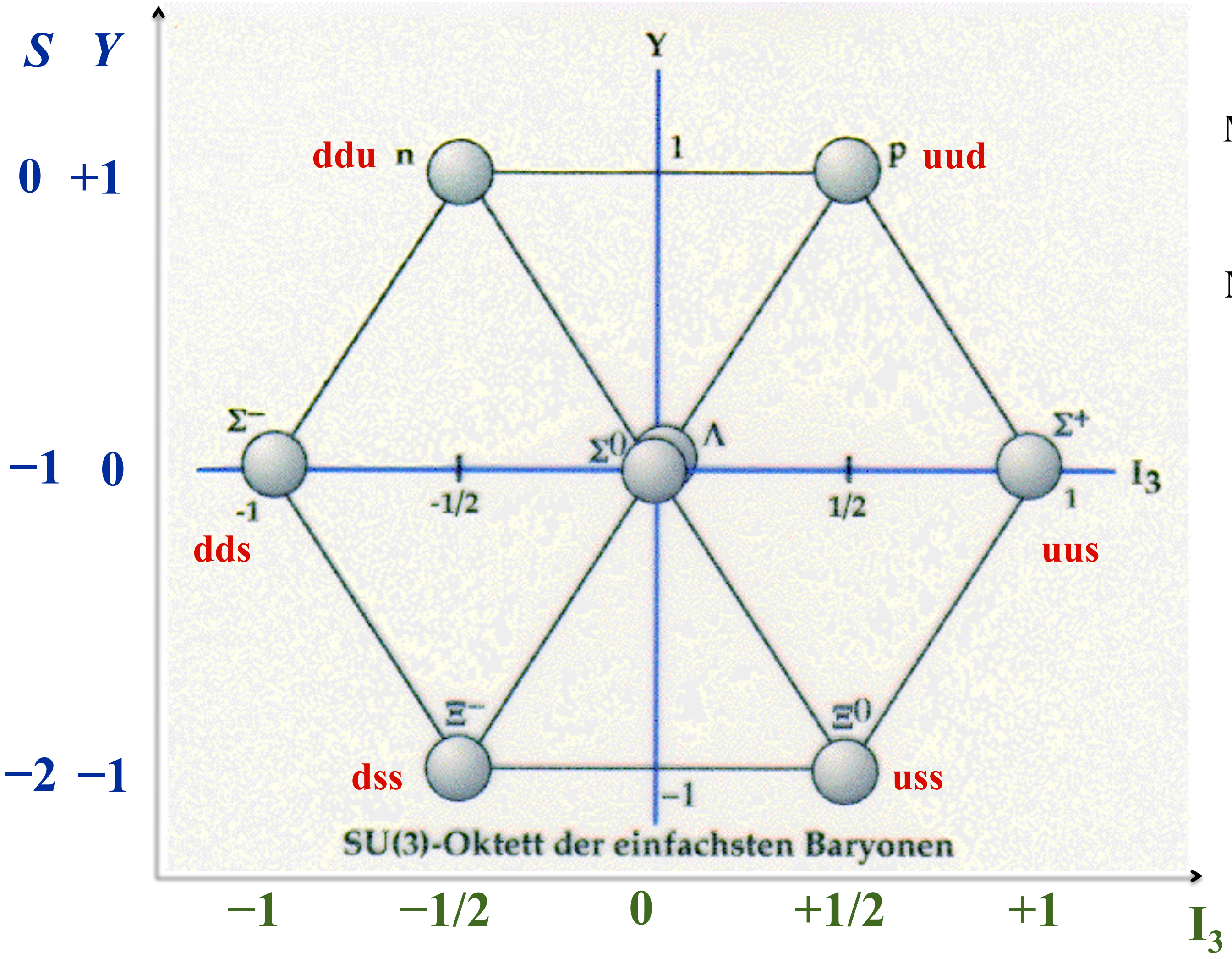
$\omega = 1/\sqrt{2}\ [\, d\,\bar{d} + u\,\bar{u}\,]$

$\phi = s\,\bar{s}$

$\rho^0 = 1/\sqrt{2}\ [\, d\,\bar{d} - u\,\bar{u}\,]$

The J=1/2 Baryon Octet

- $L=0$, Quark spin composition is ↑↑↓



M(n) = 940 MeV

M(p) = 938 MeV

M(Λ) = 1116 MeV

M(Σ) = 1193 MeV

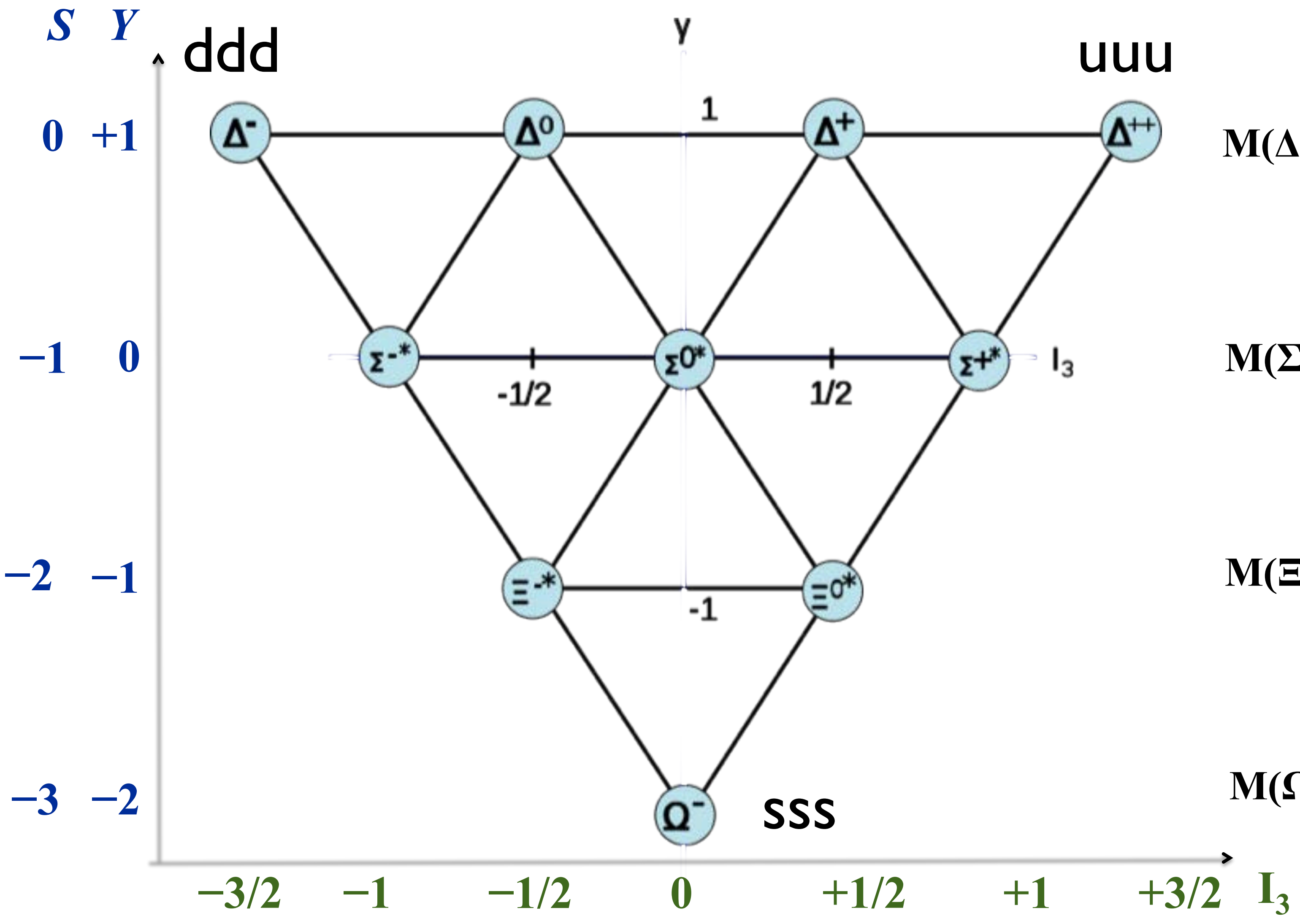
M(Ξ) = 1318 MeV

Λ^0 = [uds] isospin singlet state I=0

Σ^0 = [uds] isospin triplet state I=1

The J=3/2 Baryon Decuplet

- Total angular momentum $J=3/2$: all spins aligned ↑↑↑, $L=0$



M(Δ) = 1232 MeV

M(Σ*)=1383 MeV

M(Ξ*)=1532 MeV

M(Ω)=1672 MeV

Δ^{++} and Baryon Wavefunctions

- The overall wavefunction of a system of identical fermions must be antisymmetric under exchange of any two fermions

$$\psi\,(\Delta^{++}) = \mathrm{u}_\uparrow \mathrm{u}_\uparrow \mathrm{u}_\uparrow = \chi_c\, \chi_f\, \chi_S\, \chi_L$$

➡ The Δ^{++} wavefunction is symmetric in flavour χ_f and spin χ_S (J=3/2)

➡ There is no orbital angular momentum L=0, spatially symmetric χ_L

➡ Hence it must have an antisymmetric colour wavefunction χ_c

- Why are there no J=1/2 **uuu, ddd, sss** baryons?

Baryon	Colour	Flavour	Spin	Spatial	Total
Δ^{++}	A	S	S	S	A
p	A	A or S	A or S	S	A

- Full proton wave function is:

$$\psi(p) = \frac{1}{\sqrt{18}}[\mathrm{u}_\downarrow \mathrm{u}_\uparrow \mathrm{d}_\uparrow + \mathrm{u}_\uparrow \mathrm{u}_\downarrow \mathrm{d}_\uparrow - 2\,\mathrm{u}_\uparrow \mathrm{u}_\uparrow \mathrm{d}_\downarrow + \mathrm{u}_\downarrow \mathrm{d}_\uparrow \mathrm{u}_\uparrow + \mathrm{u}_\uparrow \mathrm{d}_\downarrow \mathrm{u}_\uparrow - 2\,\mathrm{u}_\uparrow \mathrm{d}_\uparrow \mathrm{u}_\downarrow + \mathrm{d}_\downarrow \mathrm{u}_\uparrow \mathrm{u}_\uparrow + \mathrm{d}_\uparrow \mathrm{u}_\downarrow \mathrm{u}_\uparrow - 2\,\mathrm{d}_\uparrow \mathrm{u}_\uparrow \mathrm{u}_\downarrow]$$

Heavy Quark Mesons and Baryons

- Heavy mesons and baryons are obtained by replacing one (or more) of the light **u,d,s** quarks by a heavy **c** or **b** quark
- There are no bound state hadrons containing **t** quarks

Lowest lying charm meson states with $\mathbf{M(D) \sim 1.9\ GeV}$:

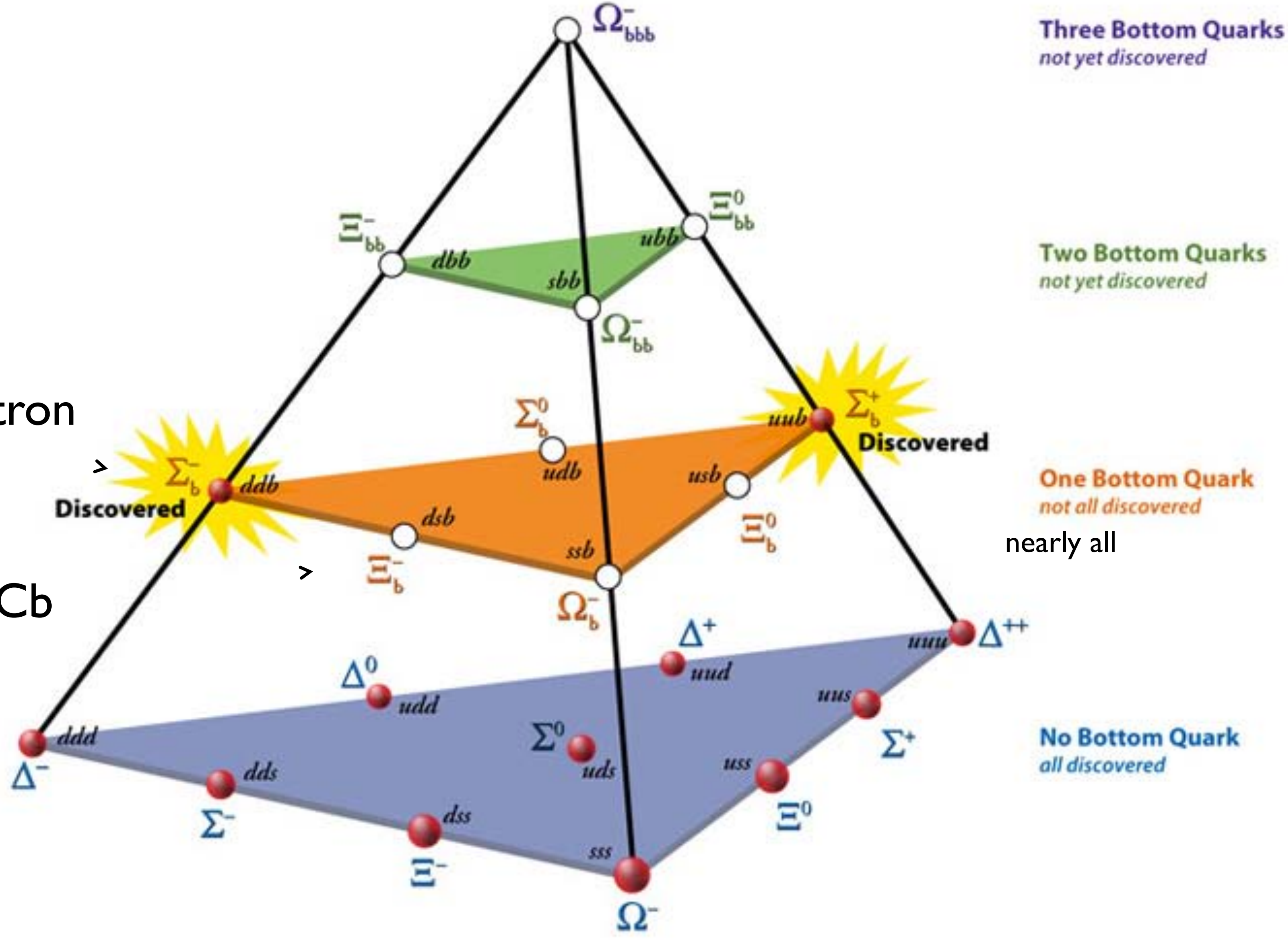
$\mathbf{D^+ (c\,\overline{d}),\ D^- (\overline{c}\,d),\ D^0 (c\,\overline{u}),\ \overline{D}^0 (\overline{c}\,u),\ D_s^+ (c\,s),\ D_s^- (c\,\overline{s})}$

Lowest lying bottom meson states with $\mathbf{M(B) \sim 5.3\ GeV}$

$\mathbf{B^0 (\overline{b}\,d),\ \overline{B}^0 (b\,\overline{d}),\ B^- (b\,\overline{u}),\ B^+ (\overline{b}\,u),\ \overline{B}_S^0 (\overline{b}\,s),\ B_s^0 (b\,\overline{s})}$

- Heavy baryons $\mathbf{\Lambda_c\ (cud)\ ,\ \Lambda_b\ (bud)}$ **...**
- Charmonium $\Psi(\mathbf{c\,\overline{c}})$ and Bottomonium $\Upsilon(\mathbf{b\,\overline{b}})$

Baryons with Up, Down, Strange and Bottom Quarks and Highest Spin ($J = 3/2$)



Tevatron

>

>

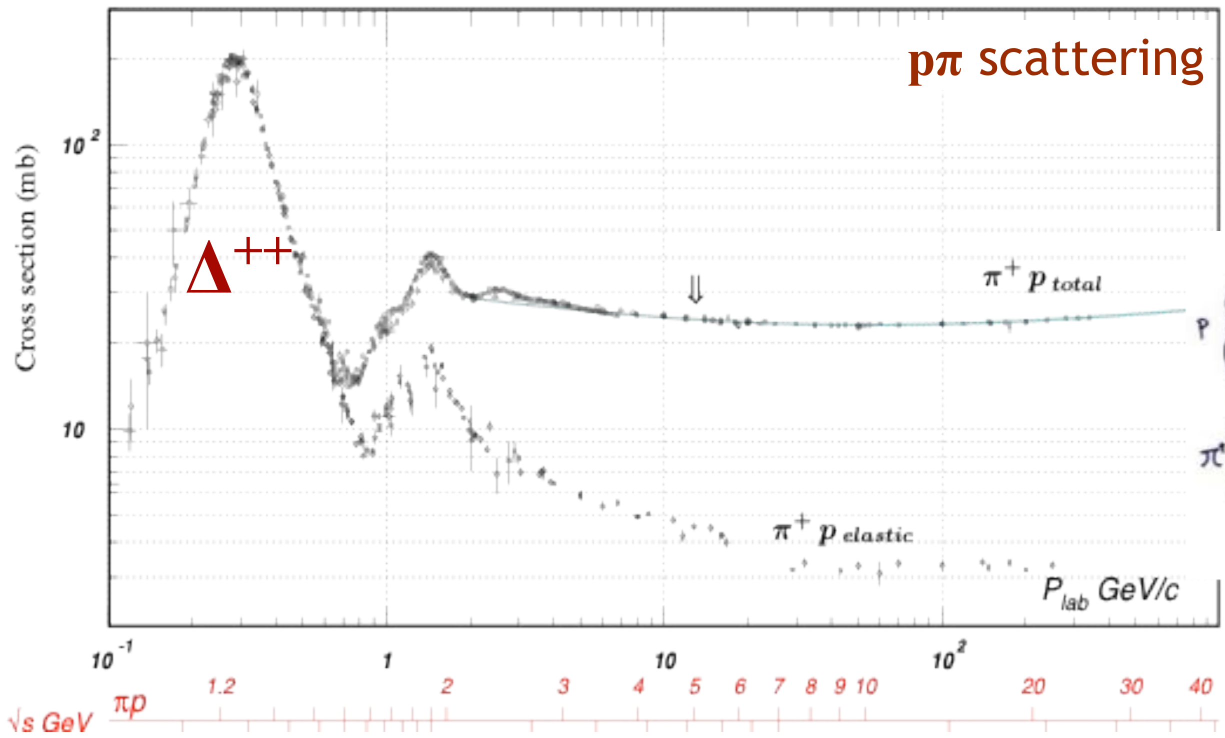
LHCb

nearly all

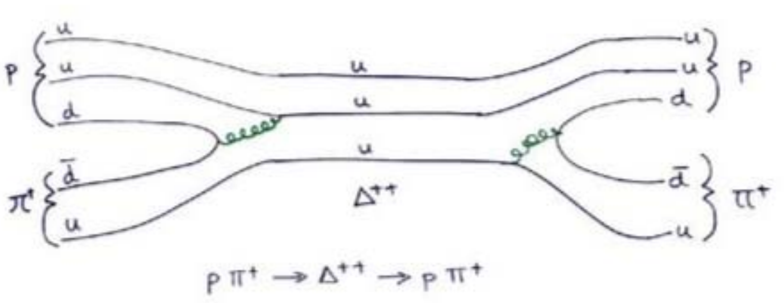
Resonances

- Most hadrons decay due to the strong force, and have very short lifetime $\tau \sim 10^{-24}$ s
- Evidence for the existence of these states are **resonances** in cross-sections
- Shape is Breit-Wigner distribution with width Γ from Fermi's Golden rule, $\Gamma \sim |\mathcal{M}|^2 \rho$

$$\sigma = \sigma_{\max} \frac{\Gamma^2/4}{(E-M)^2 + \Gamma^2/4}$$

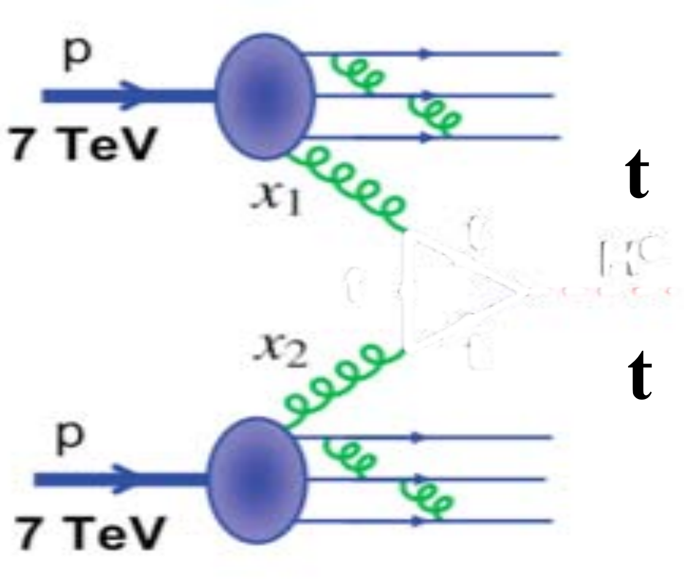


$M(\Delta^{++})$ = 1230 MeV
$\Gamma(\Delta^{++})$ = 120 MeV



Discovery of the Heavy Quarks

- Collider experiments discovered the charm (1974), bottom (1977) and top quarks (1995)
- $e^+e^-\rightarrow \mathbf{c\bar{c}}$ and $e^+e^-\rightarrow \mathbf{b\bar{b}}$ give narrow charmonium and bottomonium resonances near threshold (electromagnetic interaction)
- At higher energies produced in pairs at hadron colliders through gluons (strong interaction)



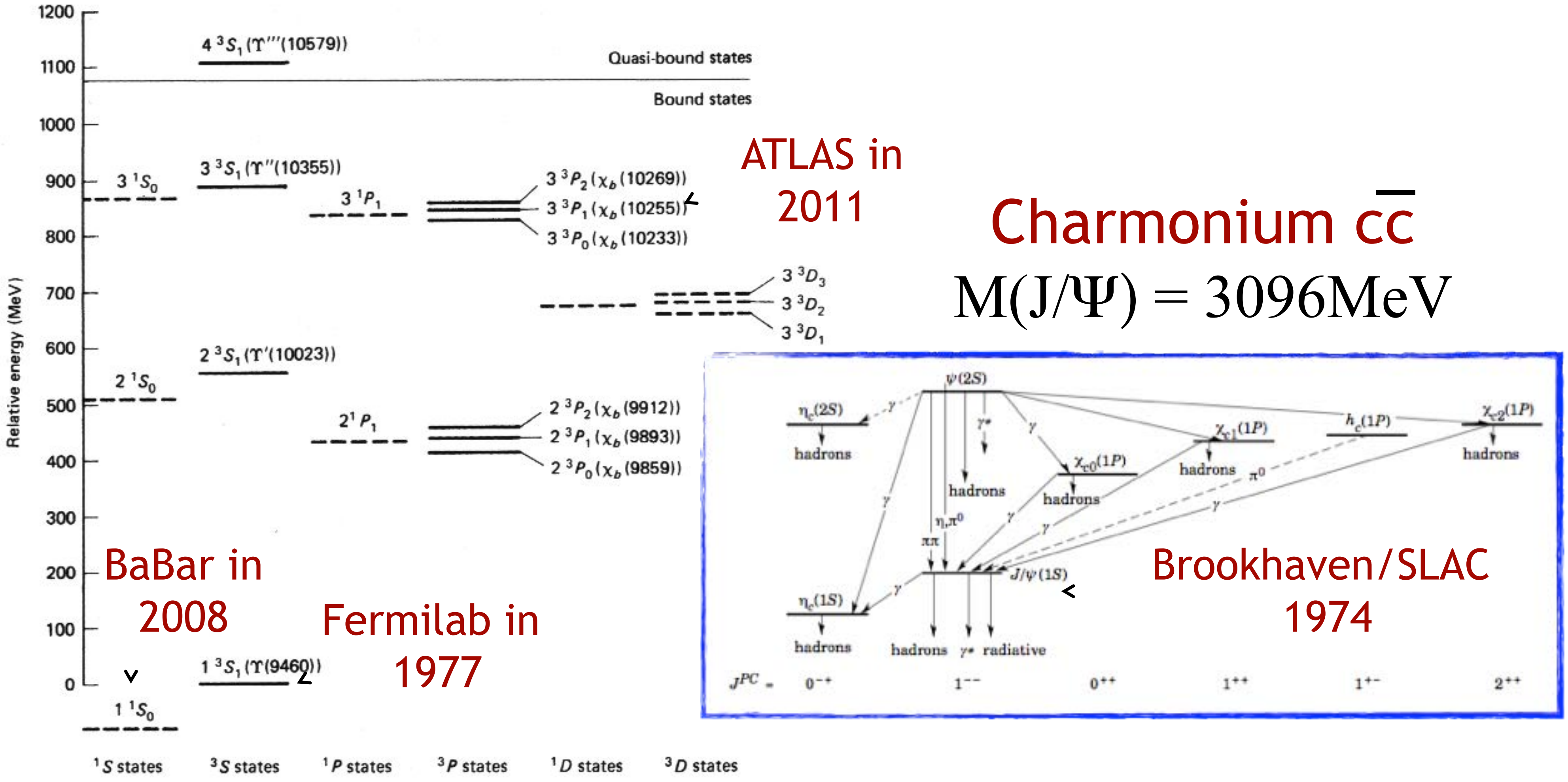
Can identify heavy quark jets by tagging decays of **c** and **b** quarks with lifetimes

$\tau_c \sim 0.4\mathrm{ps}$, $\tau_b \sim 1.5\mathrm{ps}$

- Single heavy quark production needs a W boson (weak interaction)

Heavy Quark bound states

Bottomonium bb $M(\Upsilon(1S)) = 9460\text{MeV}$



ATLAS in 2011

Charmonium $c\bar{c}$

$M(J/\Psi) = 3096\text{MeV}$

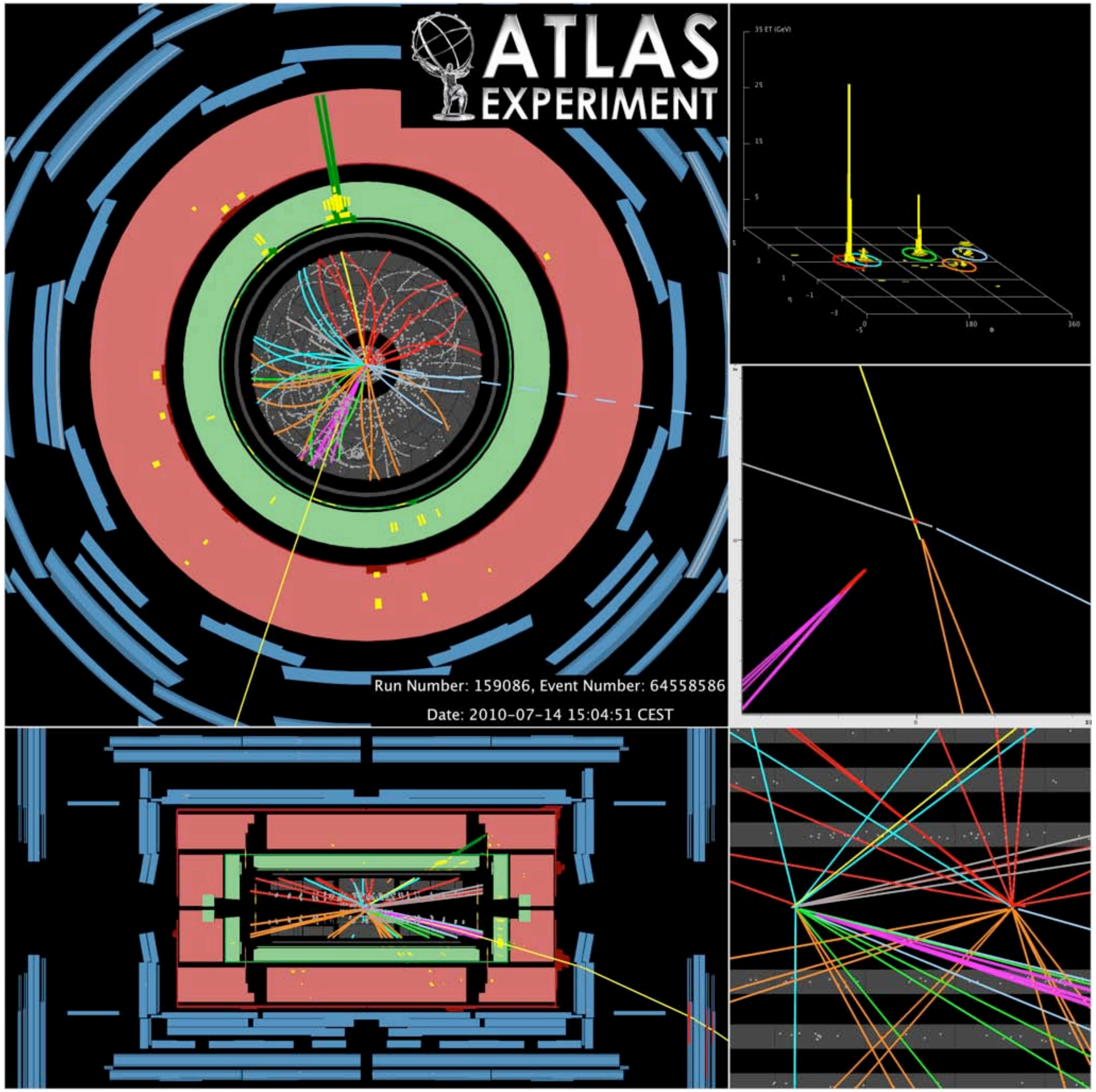
BaBar in 2008

Fermilab in 1977

Brookhaven/SLAC 1974

- Analogous to hydrogen spectroscopy with quark-quark potential $V_{q\bar{q}}(r) = -4/3\,\alpha_S/r + kr$ (see lecture 9)

$t\,\bar{t} \rightarrow W^{+}b\;W^{-}\bar{b}$ candidate event



- Lines are project paths of charged particles through the detector.
- Not all particles originate from collision point.
- Particle produced and travelled short distance before decaying, indicates production of a b-quark!

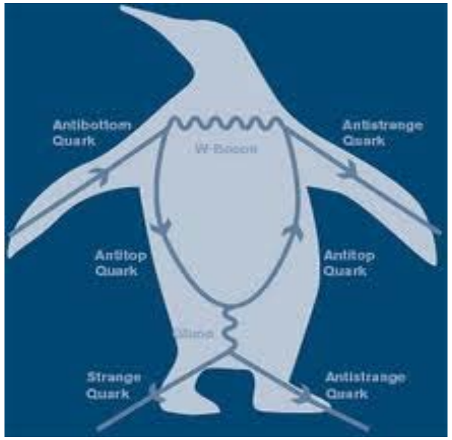
Summary

- Quarks are confined in **colourless** bound states, collectively known as **hadrons**:
  - ➡ **mesons:** quark and anti-quark. Bosons ($J=0$, 1) with symmetric colour wavefunction.
  - ➡ **baryons**: three quarks. Fermions ($J=1/2$, $3/2$) with antisymmetric colour wavefunction.
- The strong force between colourless hadrons is propagated by mesons.
- The lightest mesons & baryons are characterised by strong isospin (I, I_3), strangeness (S) and strong hypercharge Y
  - ➡ strong isospin $I = ½$ for $\mathbf{u}$ and $\mathbf{d}$ quarks with $I_3 = +½$ and $I_3 = -½$
  - ➡ $S = -1$ for strange quarks (and similarly for heavy flavours C,B,T)
  - ➡ strong hypercharge $Y = S + B$ *(Baryon Number)*
  - ➡ charge $Q = I_3 + Y/2$
- Hadrons display SU(3) flavour symmetry between $\mathbf{u}$ $\mathbf{d}$ and $\mathbf{s}$ quarks. The symmetry predicts the allowed meson and baryon states.
- Strong decays of most hadrons cause **resonances** due to very short lifetimes.
- Heavy b and c quarks also form bound states with each other and with the light quarks.
- The t quark does not form bound states, but has been observed at hadron colliders.

Particle Physics

Dr Victoria Martin, Prof Steve Playfer
Spring Semester 2013

Lecture 13: Hadron Decays



- ★Decays of Hadrons
- ★Selection Rules
- ★Weak decays of light hadrons
- ★CKM matrix
- ★Neutral Meson Mixing

Decays of Hadrons

- The proton is the only completely stable hadron
- The free neutron has a weak decay (τ~15 mins)
- **Decay length** of a particle is the distance it travels before decaying $L=\beta\gamma c\tau$

Force	Typical τ (s)
QCD	10^{-20} - 10^{-23}
QED	10^{-20} - 10^{-16}
Weak	10^{-13} - 10^{3}

- $\pi^{\pm}$, $K^{\pm}$, K_L^0 mesons are long-lived (τ~10 ns) and have weak decays
  - ➡ Live long enough to travel outside radii of collider detectors (L~10 m)
- K_S^0 mesons and Λ^0 hyperons are less long-lived (τ~100 ps) and have weak decays with decay lengths of L~ cm which are inside collider detectors

- $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$ are electromagnetic decays, reconstructed from pairs of photons

- ρ, ω, ϕ, K^*, Δ, Σ^*, Ξ^* are resonances with strong decays.
  - ➡Reconstructed as broad structures with widths Γ~100 MeV.

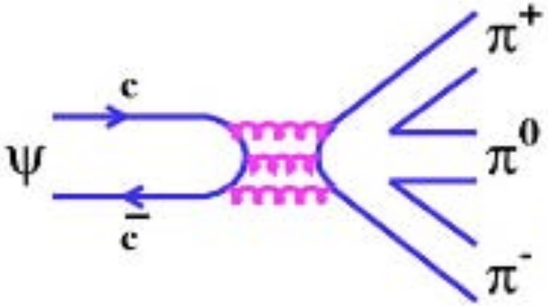
Decay Conservation Laws

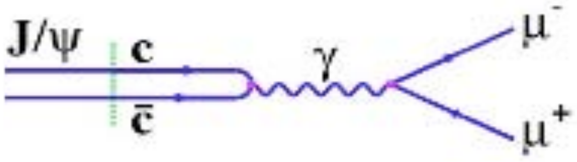
- Relevant quantum numbers are:
  - strong isospin (I, I_3)
  - parity (P)
  - quark flavour: described using strangeness (S=N(s)−N($\overline{s}$)), charm (C=N(c)−N($\overline{c}$)), beauty (B=N(b)−N($\overline{b}$))
  - Baryon number and lepton numbers are always conserved!

	baryon number	Strong Isospin, I	Strong Isospin, I_3	Flavour, S, C, B	Parity, P
Strong	Y	Y	Y	Y	Y
EM	Y	N	Y	Y	Y
Weak	Y	N	N	N	N

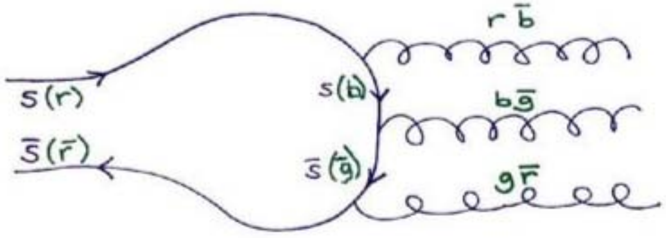
Decays of Charmonium

- The $\mathbf{J/\psi}$ meson is a $\mathbf{c\,\bar{c}}$ state. It must decay to particles without charm quarks as $\mathbf{M(J/\psi) < 2\ M(D)}$.
- Two options: decay via three gluons or one photon.





- Strong rate is suppressed by $\alpha_s^6(q_{\mathrm{gluon}})$. This is comparable to α^2 for EM decay
  - ➡ Both strong and electromagnetic final states have large branching ratios.
- The $\mathbf{J/\psi}$ meson lives for a relatively long time, giving rise to narrow resonance in e.g. $e^+e^- \rightarrow$ **hadrons.**



- Similar phenomena occur in decays of $\mathbf{s\,\bar{s}}$ and $\mathbf{b\bar{b}}$ mesons.

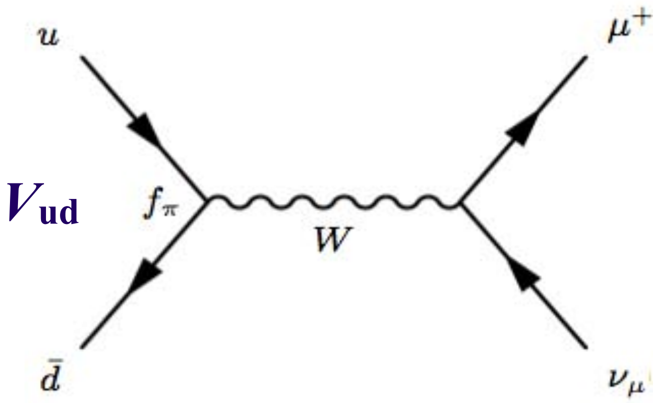
Charged Pion Decay

- See problem sheet 1
- π^+ consists of $\mathbf{u\bar{d}}$, lightest charged meson
- Decays via weak force to change quark flavour u->d

$$\tau(\pi^+) = 26\ \mathrm{ns}$$

➡ CKM matrix element factor V_{ud}.

➡ Hadronic decay constant $f_\pi \sim m_\pi$ to account for finite size of pion



$$\begin{aligned} \mathcal{M} &= \left[\bar{v}(\bar{\mathrm{d}}) g_W V_{\mathrm{ud}} f_\pi \gamma^\mu (1-\gamma^5) u(\mathrm{u})\right] \frac{1}{q^2 - m_W^2} \left[\bar{u}(\nu_\mu) g_W \gamma^\mu (1-\gamma^5) v(\mu^+)\right] \\ &\approx V_{\mathrm{ud}} f_\pi \frac{g_W^2}{m_W^2} \left[\bar{v}(\bar{\mathrm{d}}) \gamma^\mu (1-\gamma^5) u(\mathrm{u})\right] \left[\bar{u}(\nu_\mu) \gamma^\mu (1-\gamma^5) v(\mu^+)\right] \\ |\mathcal{M}|^2 &= 4\, G_F^2 |V_{\mathrm{ud}}|^2 f_\pi^2\, m_\mu^2\, [p_\mu \cdot p_\nu] \end{aligned}$$

μ preferred to e

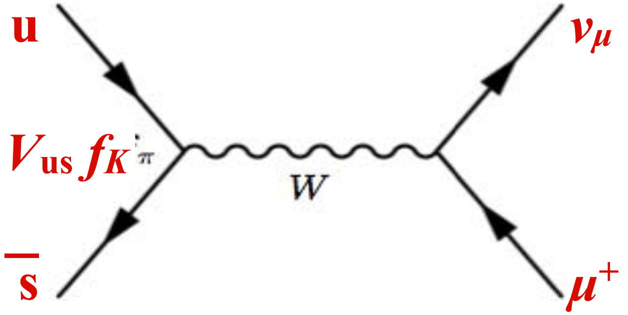
v

$$\Gamma = \frac{|\vec{p}^{\,*}|}{8\pi m_1^2} |\mathcal{M}|^2 = \frac{G_F^2}{8\pi} |V_{\mathrm{ud}}|^2 f_\pi^2\, m_\mu^2 (m_\pi^2 - m_\mu^2)$$

Charged Kaon Decays

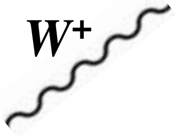
- Charged kaon is $\bar{s}\,u$ with $m_K = 498$ MeV
- lightest mesons containing strange quarks ⇒ must decay by weak force

$\tau(K^{\pm}) = 12$ ns



- Leptonic decays
  - $BR(K^+ \to \mu^+ \nu_\mu) = 63\%$
  - Kaon decay constant, f_K=160 MeV
  - $V_{us} = 0.22$ (Cabibbo angle)

μ^+

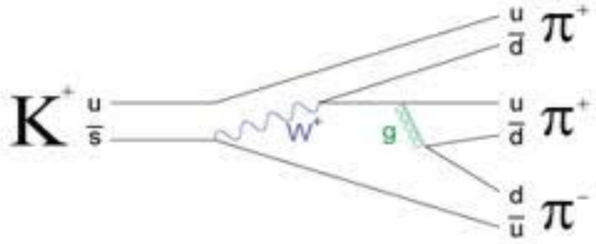


ν_μ

$\bar{s}$ $\bar{d}$

u u

- Semileptonic decays
  - $BR(K^+ \to \pi^0 \mu^+ \nu_\mu) = 3.8\%$
  - $BR(K^+ \to \pi^0 e^+ \nu_e) = 5.1\%$



- Hadronic Decays
  - $BR(K^+ \to \pi^0 \pi^+) = 21\%$
  - $BR(K^+ \to \pi^+ \pi^+ \pi^-) = 5.6\%$

Cabibbo-Kobayashi-Maskawa Matrix

- **Mass eigenstates** and **weak eigenstates** of quarks are not identical.
  - ➡Decay properties measure mass eigenstates with a definite lifetime and decay width
  - ➡The weak force acts on the weak eigenstates.
- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix to mix the down-type quarks:

weak eigenstates

$$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

mass eigenstates

- *e.g.* weak eigenstate of the strange quark is a mixture between down, strange and bottom mass eigenstates

$$\mathrm{s}' = V_{\mathrm{cd}}\mathrm{d} + V_{\mathrm{cs}}\mathrm{s} + V_{\mathrm{cb}}\mathrm{b}$$

- The CKM matrix is unitary, $V_{\mathrm{CKM}}{}^{\dagger} V_{\mathrm{CKM}} = \mathbf{1}$; standard parameterisation in terms of three mixing angles ($\theta_1, \theta_2, \theta_3$) and one complex phase (δ) is:

$$\begin{pmatrix} \cos\theta_1 & \sin\theta_1\cos\theta_3 & \sin\theta_1\sin\theta_3 \\ -\sin\theta_1\cos\theta_3 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2\cos\theta_3 - \cos\theta_2\sin\theta_3 e^{i\delta} & -\cos\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3 e^{i\delta} \end{pmatrix}$$

Nobel Prize in Physics 2008

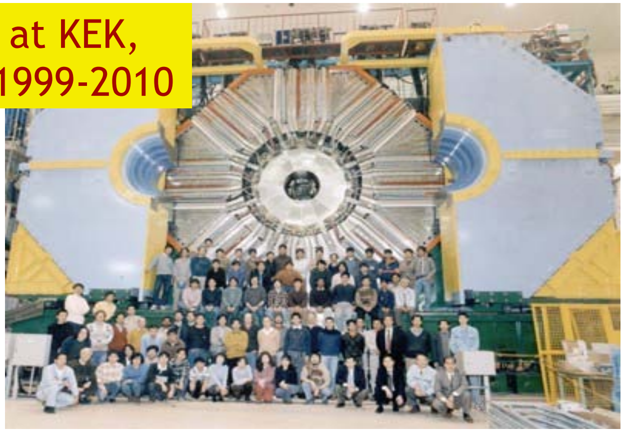
- Awarded to **Makoto Kobayashi**, High Energy Accelerator Research Organization (KEK), Tsukuba, Japan and **Toshihide Maskawa**, Yukawa Institute for Theoretical Physics (YITP), Kyoto University, and Kyoto Sangyo University, Japan

- *"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"*

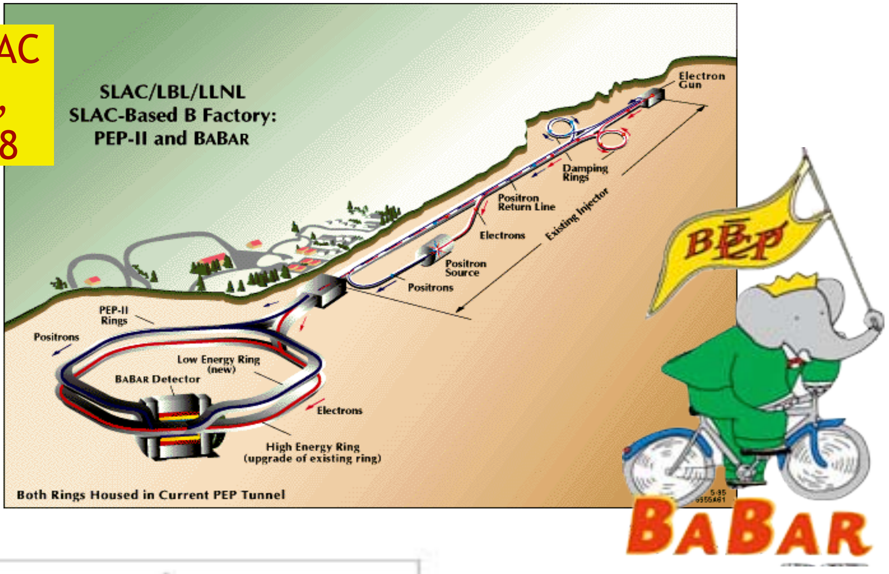
Experimental Measurements of CKM Matrix

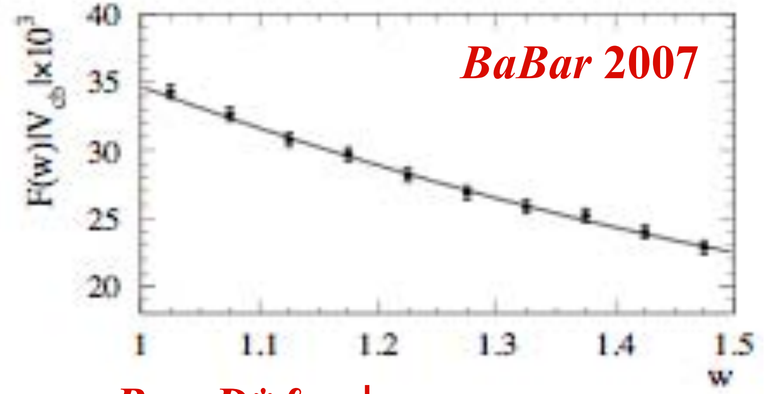
- Many measurements made by the BaBar and Belle experiments.
- Both study $e^+e^- \rightarrow \Upsilon^{(4s)} \rightarrow \mathbf{B^0}\,\mathbf{\overline{B}^0}$ to measure the decays of $\mathbf{b}$ and $\mathbf{c}$ quarks, e.g. V_{cb} and V_{ub}

Belle at KEK,
Japan 1999-2010

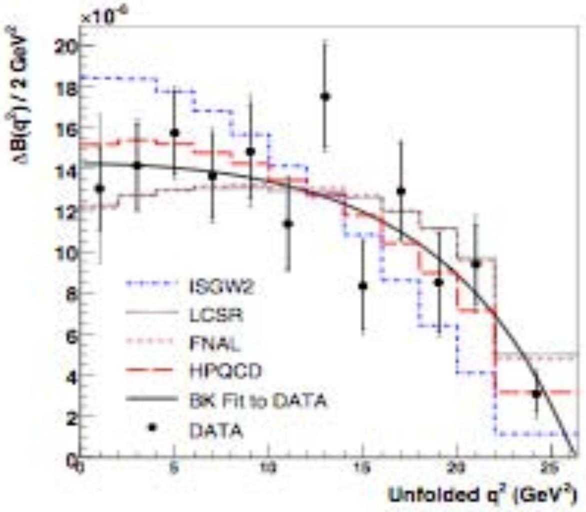


BaBar at SLAC
California,
1999 - 2008





$B \rightarrow D^* \ell\ \nu$ decays
(as function of D^* recoil)
measures $|V_{cb}| = \mathbf{0.0374 \pm 0.0017}$



Exclusive $B \rightarrow \pi\, \ell\ \nu$
(as a function of q^2)
$|V_{ub}| = \mathbf{0.0034(5)}$

The Wolfenstein Parameterisation

- An expansion of the CKM matrix in powers of $\lambda = V_{us} = 0.22$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Parameterisation reflects almost diagonal nature of CKM matrix:
  - ➡The diagonal elements V_{ud}, V_{cs}, V_{tb} are close to **1**
  - ➡Elements $V_{us}, V_{cd} \sim \lambda$ are equal and measure λ
  - ➡Elements $V_{cb}, V_{ts} \sim \lambda^2$ are equal and measure A
  - ➡Elements $V_{ub}, V_{td} \sim \lambda^3$ are very small

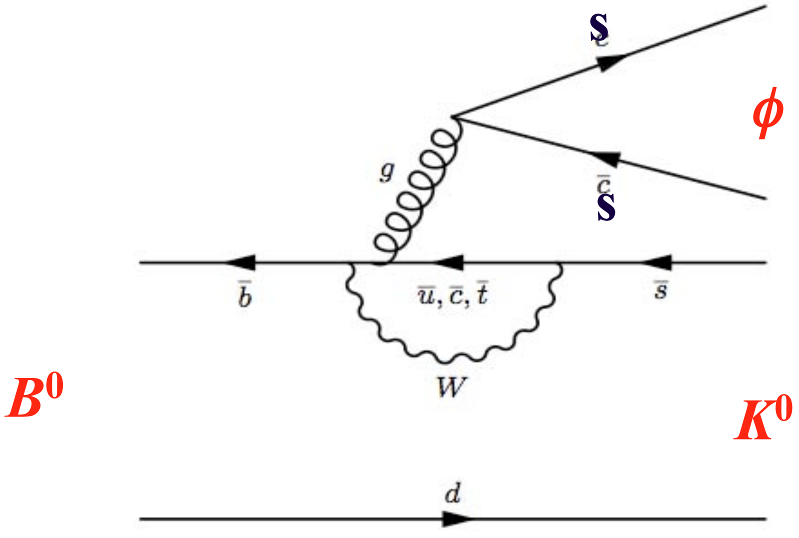
- Note that the parameter ρ and the complex phase η only appear in the very small elements V_{ub} and V_{td}, and are thus hard to measure.

Flavour Changing Neutral Currents

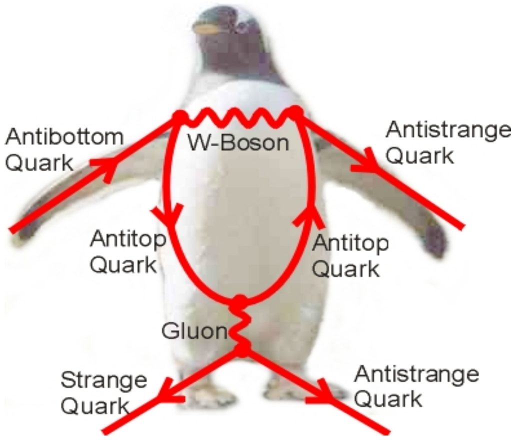
- At 1st order, there are no allowed transitions between quarks of the same charge, e.g. **s↔d, c↔u, b↔s, b↔d**

- Weak neutral current (the Z boson) does not change the flavour of fermions.

- At 2nd order so-called “Penguin Diagrams” can cause transitions such as **b↔s**
  - *e.g.* $\mathbf{b} \rightarrow \mathbf{s}\,\mathbf{s}\,\overline{\mathbf{s}}$, $B^0 \rightarrow \phi\, K^0$



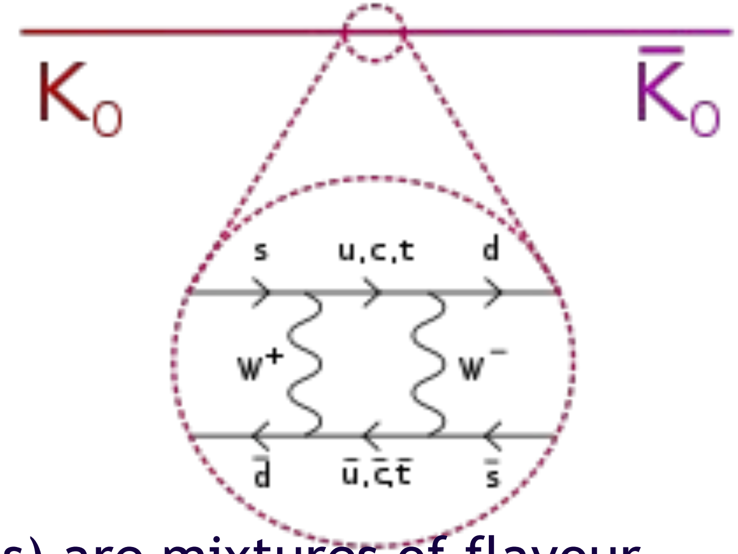
$B^0 \rightarrow J/\psi\; K^0$



Neutral Meson Mixing

- Second order weak interactions mix long-lived neutral mesons with their antiparticles:

  $K^0\ (\bar{s}\ d)$, $D^0\ (\bar{c}\ u)$, $B^0\ (\bar{b}\ d)$, $B_s\ (\bar{b}\ s)$



➡ $K^0 \leftrightarrow \bar{K}^0$ $D^0 \leftrightarrow \bar{D}^0$ $B^0 \leftrightarrow \bar{B}^0$ $B_s \leftrightarrow \bar{B}_s$

Observed particles (weak decay eigenstates) are mixtures of flavour eigenstates:

$K_S = 1/\sqrt{2}\ (K^0 + \bar{K}^0)$ with $\tau_S = 0.09$ ns

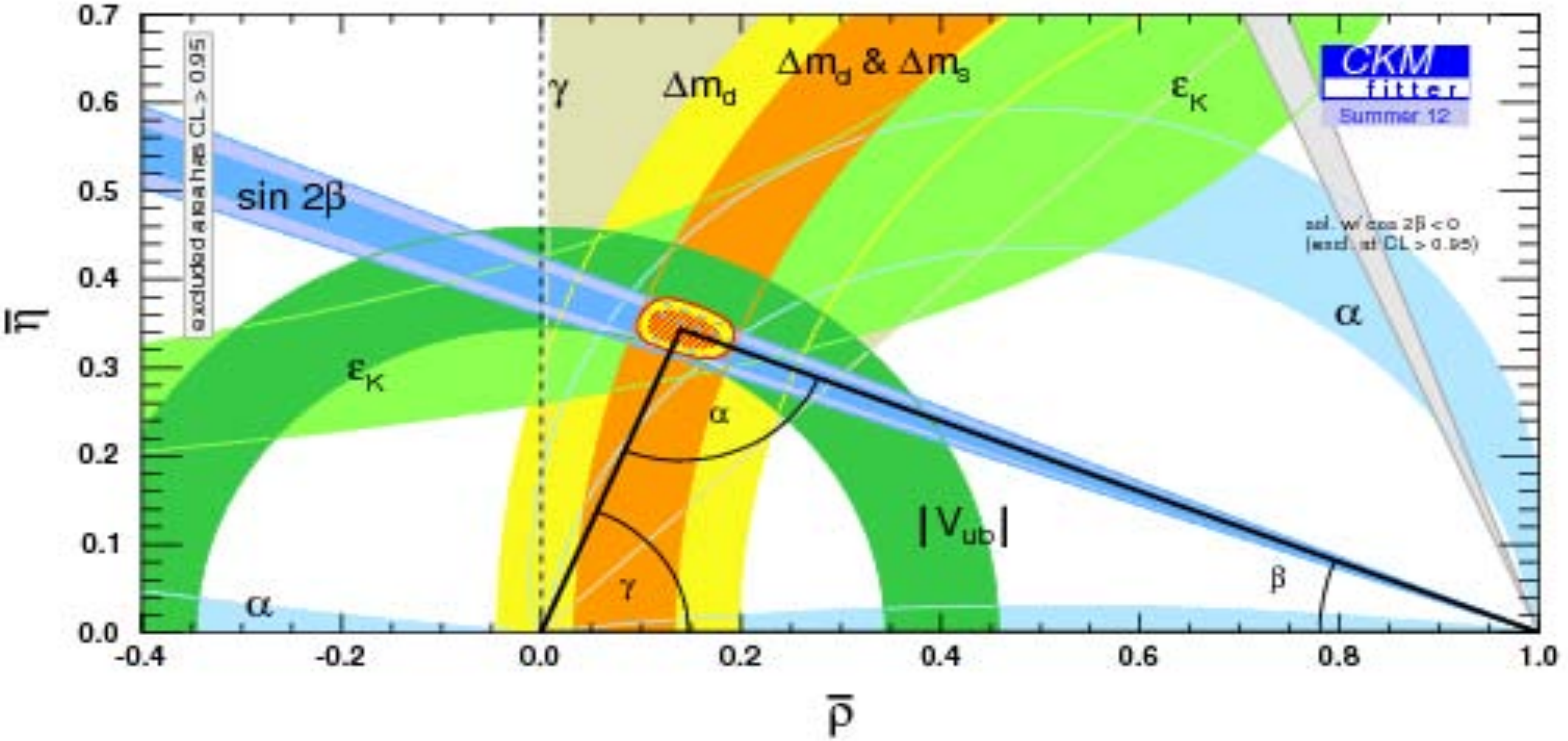
$K_L = 1/\sqrt{2}\ (K^0 - \bar{K}^0)$ with $\tau_L = 51$ ns

Mass difference $\Delta m_K = m_L - m_S = 3.52(1)\ \text{x}10^{-12}$ MeV $= 0.53\ \text{x}\ 10^{-10}\ \text{s}^{-1}$

This is the oscillation frequency of the mixing

More about this next week, when we talk about CP violation

CKM Fit



- Many measurements, including results from BaBar, Belle, Tevatron and LHCb experiments. Semileptonic b->u decays, penguin diagrams, neutral meson mixing and CP violation are used to find best values for **η** and **ρ** parameters in Wolfenstein parameterisation.

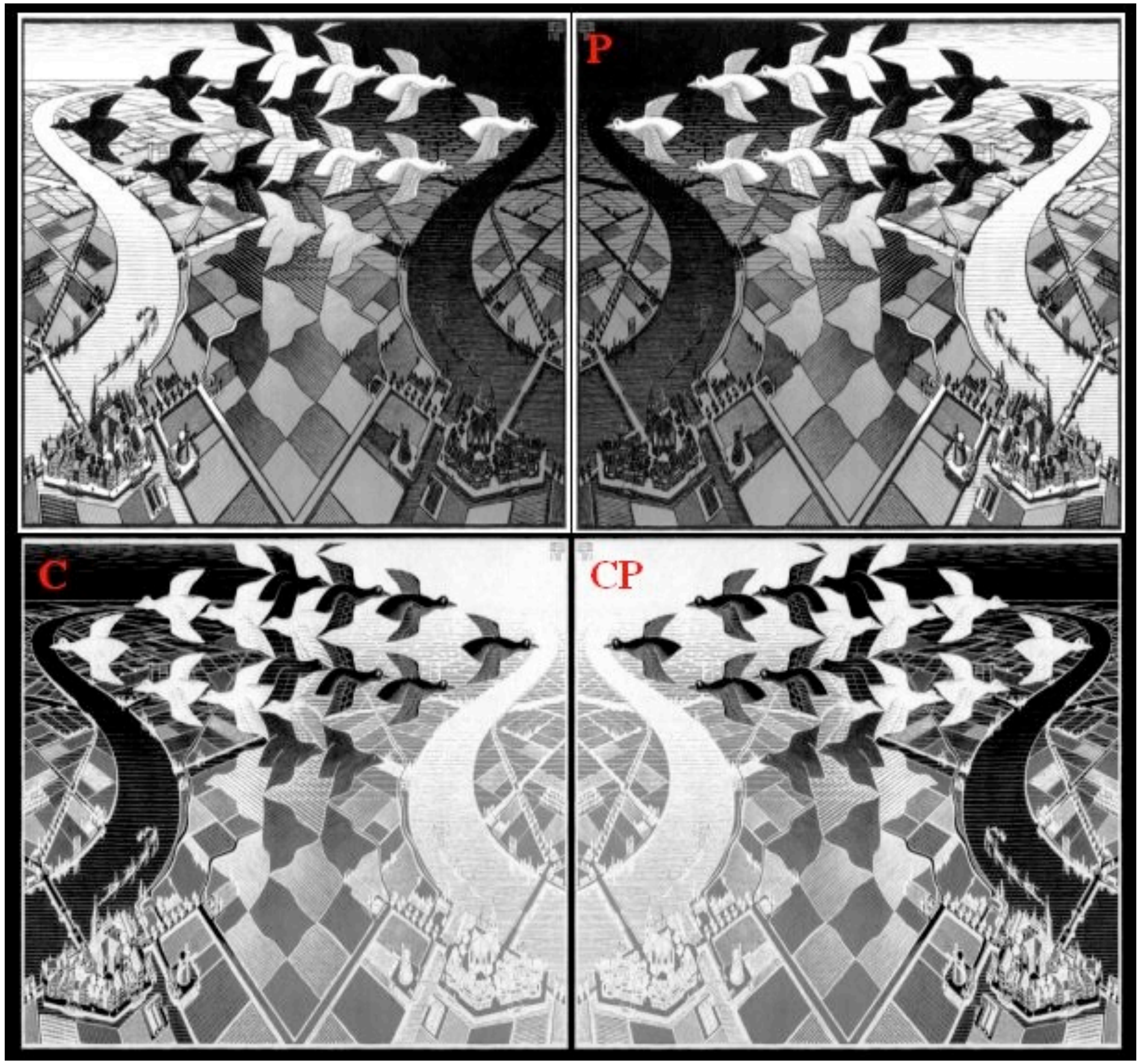
Summary: Decays of Hadrons

- **Strong decays** are characterised by very short lifetimes, $\tau \sim 10^{-20}$ - 10^{-23} s appearing as **resonances** with a large width $\Gamma \sim$ MeV.
  - ➡ Final states are hadronic. All quantum numbers are conserved.
- **Electromagnetic decays** are characterised by $\tau \sim 10^{-20}$ - 10^{-16} s.
  - ➡ Decays containing photons are electromagnetic.
  - ➡ All quantum numbers conserved except total isospin, I.
- **Weak decays** characterised by long lifetimes, $\tau \sim 10^{-13}$ - 10^{3} s.
  - ➡ Only decays that allow change of quark flavour (including s, c, b decays).
  - ➡ Responsible for most light meson and baryon decays.
  - ➡ Particles can live long enough to reach the detector.
  - ➡ Final states may be leptonic, semi-leptonic or hadronic.
  - ➡ Strong Isospin, I, I_3, Parity, P, Flavour quantum numbers not conserved.
- **CKM matrix** relates the quark mass eigenstates to the weak eigenstates
  - ➡ Non-diagonal: mixes quark flavours. Off-diagonal elements get smaller.
  - ➡ Allows higher order penguin diagrams, and neutral meson mixing.
  - ➡ Contains four free parameters, including a complex phase (leads to CP violation).

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 14: Symmetries



- ★Symmetries of QED and QCD
- ★Parity, Charge Conjugation and Time Reversal
- ★Parity Violation in Weak Decay
- ★CP and CPT

Moriond Conference

- I was webcast: http://webcast.in2p3.fr/videos-searches_for_the_beh_boson_into_fermions_at_atlas

Guest Seminars

- From Edinburgh University researchers on their work.

- Next Monday (18th March) in tutorial:
  - Guest seminar from Dr Greig Cowan on B-physics at LHCb

- Following Monday (25th March) in tutorial
  - Guest seminar from Dr Wahid Bhimji on Higgs physicst at ATLAS

Cabibbo-Kobayashi-Maskawa Matrix

- **Mass eigenstates** and **weak eigenstates** of quarks are not identical.

- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix a mixture of the down-type quarks:

weak eigenstates

$$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

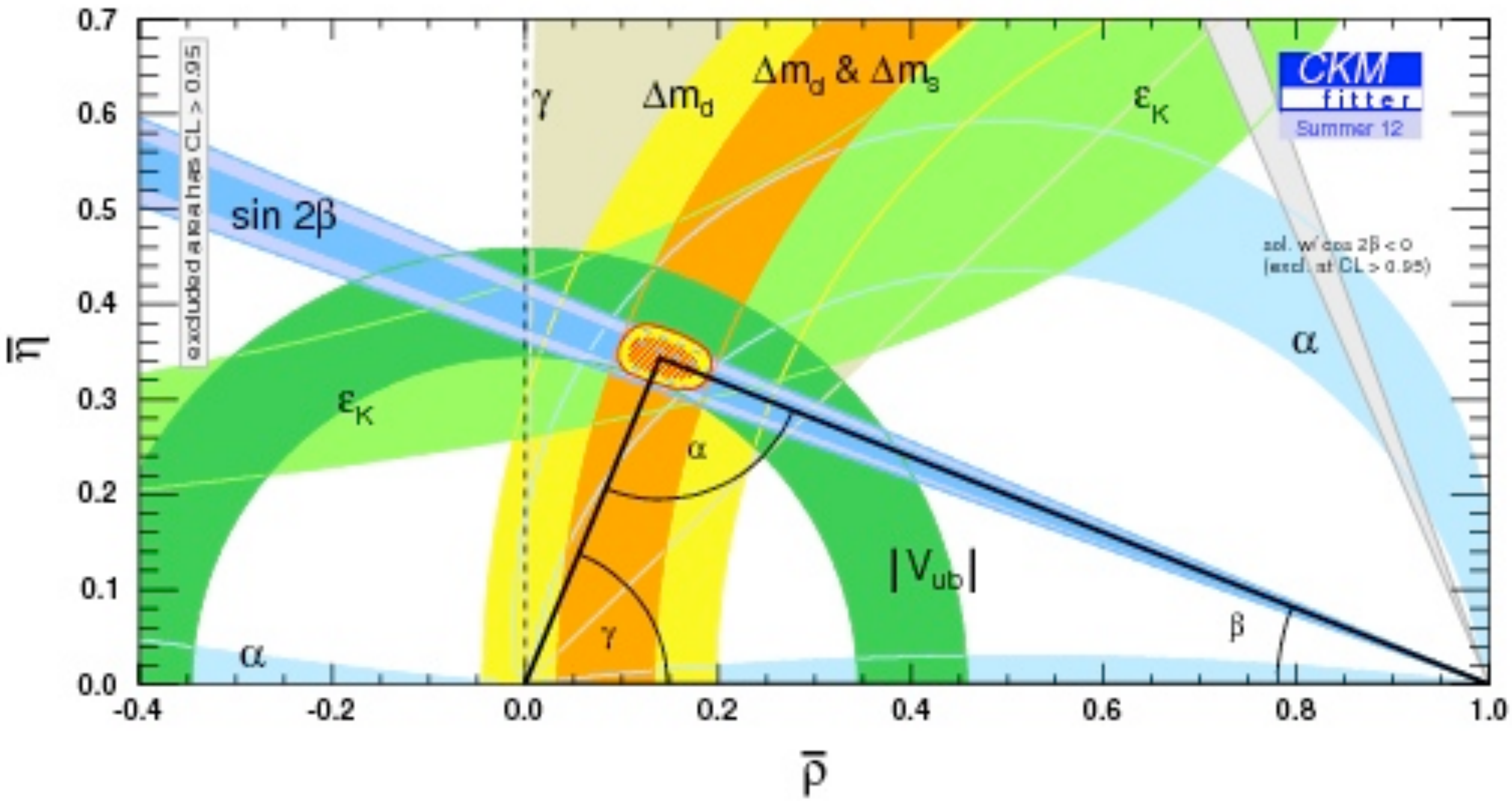
mass eigenstates

- The CKM matrix is unitary, $V_{\mathbf{CKM}}{}^{\dagger} V_{\mathbf{CKM}} = \mathbf{1}$

- **Wolfenstein parameterisation**

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

CKM Fit



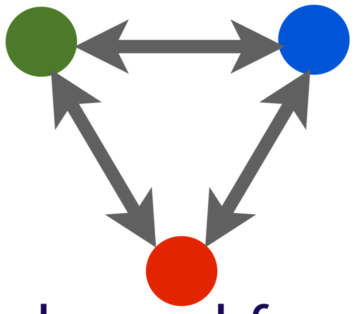
- Results from BaBar, Belle and LHCb experiments used to find best values for **η** and **ρ** parameters in Wolfenstein parameterisation.

Introduction: Symmetries in Particle Physics

- Symmetries play a central role in particle physics.
- Symmetries describe operations which leave a physical system unchanged.

- **Noether's theorem**: every symmetry corresponds to a conservation law.
  - Conservation laws can be experimentally verified

- We use the group theory to describe the symmetries of the three forces:
  - SU(3) symmetry of QCD ⇒ conservation of colour charge
  - SU(2) symmetry of weak force ⇒ conservation of weak isospin
  - U(1) symmetry of QED ⇒ conservation of electric charge
  - Translations in time and space; rotations in space ⇒ conservation of angular and four-momentum



- Some symmetries are observed to only hold under certain conditions, e.g. conservation of quark flavour for strong and electromagnetic interactions

- Breaking of symmetries can be either "dynamical" or "spontaneous" (e.g. Higgs mechanism).

Parity (P)

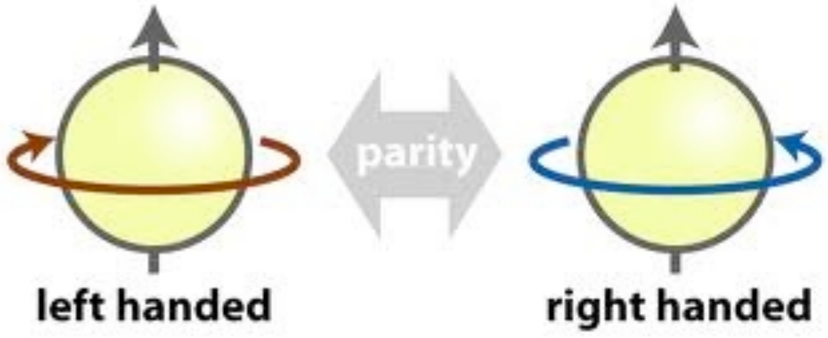
- Parity, P, is a spatial inversion through the origin: $P\psi(\vec{r}) = \psi(-\vec{r})$ (not a mirror reflection in a plane!)

- If you act P twice on a state you get the original state back ⇒ $P^2 = 1$.
  - Eigenvalues of P are either $+1$ (even) or -1 (odd)
    - e.g. $\psi(x) = \sin kx$ is odd $\psi(x) = \cos kx$ is even

- e.g. Hydrogen atom states described by Legendre polynomials: $Y_L{}^m = P_L{}^m(\cos\theta)\, e^{im\phi}$
  - Parity of state described by $Y_L{}^m$ is $(-1)^L$ (depends on orbital angular momentum L)
  - S $(L=0)$ and D $(L=2)$ states are even, P $(L=1)$ states are odd

Intrinsic Parity

- Intrinsic parity is property of the elementary particles, we **define**:
    - Fermions to be eigenstates with $P = +1$ (even)
    - Antifermions to be eignestates w. $P = -1$ (odd)



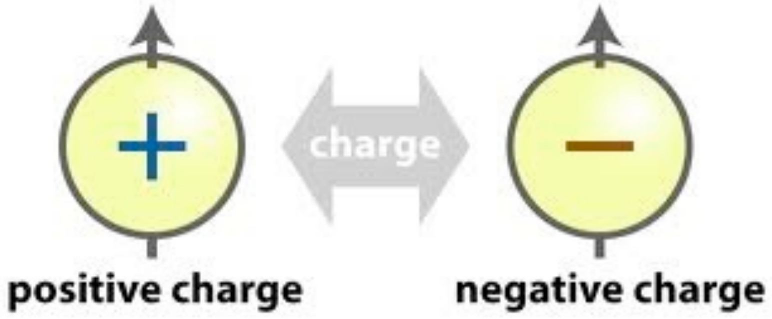
- Parity operator (P) can be represented by $\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix}$
    - Check this by looking at Dirac spinors (on problem sheet)

- Photons have odd parity, $P = -1$
    - Parity operation changes the direction of Electric fields.
    - Check this from transformation of electromagnetic fields (A^μ)

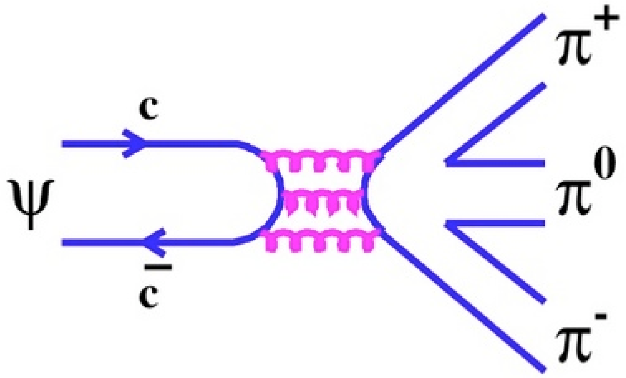
- In general:
    - Scalar ($\overline{\psi}\,\psi$) and axial-vector ($\overline{\psi}\,\gamma^5\,\gamma^\mu\,\psi$) quantities have $P = +1$ (even)
    - Pseudoscalar ($\overline{\psi}\,\gamma^5\psi$) and vector ($\overline{\psi}\,\gamma^\mu\,\psi$) quantities have $P = -1$ (odd)

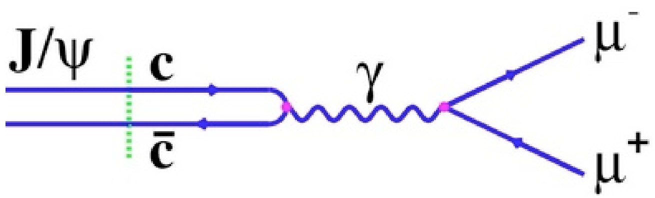
Charge Conjugation (C)

- A change from particle to antiparticle: $C\bar{f} = f$; $Cf = \bar{f}$
- $C^2 = 1$ ⇒ eigenvalues of C are either $+1$ (even) or -1 (odd)
    - (C can be represented by $i\gamma^2$)
- Fermions and antifermions are not eigenstates of C !



- Photons have $C = -1$. C changes sign of the electric charges, and therefore of the electromagnetic field. Similarly gluons have $C = -1$.
- C and P observed to be conserved in electromagnetic and strong interactions.
- Mesons $(q\bar{q})$ have $C = (-1)^{L+S}$ and $P = (-1)^{L+1}$
- Lightest mesons pseudoscalars (↑↓) with $J^{PC} = 0^{-+}$ Second-lightest states are vectors (↑↑ or ↓↓) with $J^{PC}=1^{--}$

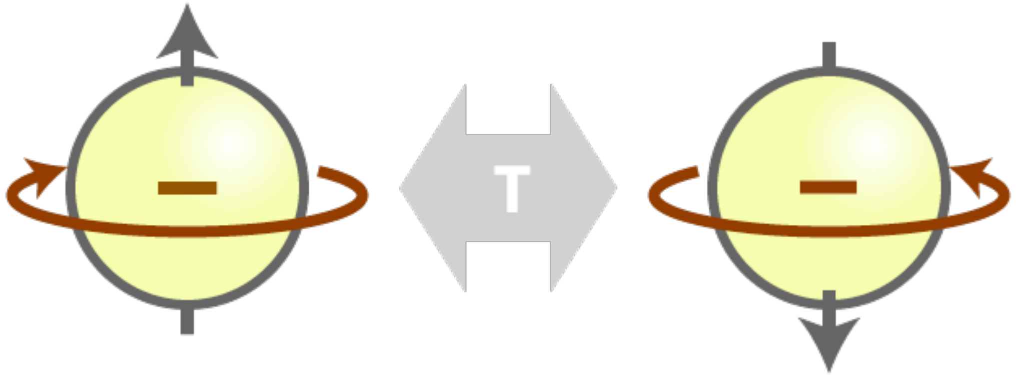




- e.g. J/ψ $(c\bar{c})$ meson has $J^{PC}=1^{--}$. EM and strong decays via odd number of photons or gluons to conserve C and P

Time Reversal (T)

- A reversal of the arrow of time: $T\psi(t) = \psi(-t)$



- $T^2 = 1$ so eigenvalues of T are either $+1$ (even) or -1 (odd)

- T corresponds to interchanging the initial and final states in a Feynman diagram.

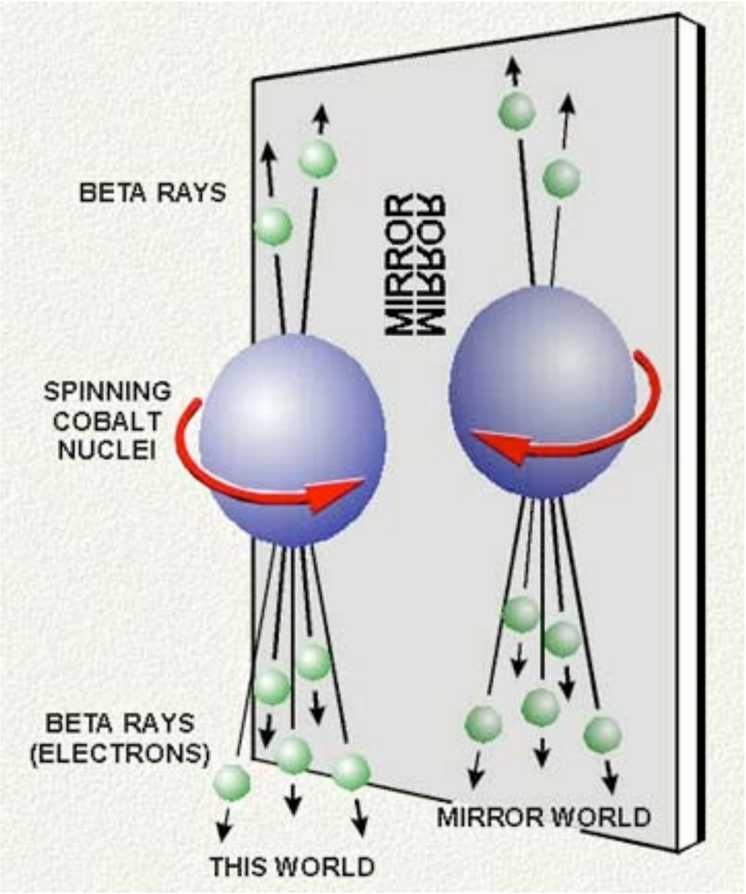
- T reverses the sign of time-dependent variables including momentum and angular momentum (spin)

- It is evident that the Universe is not T symmetric!
  - Big Bang, Hubble expansion, increase in Entropy …

Discrete Symmetries of Physical Quantities

Quantity	Notation	P	C	T
Position	$\vec{r}$	-1	$+1$	$+1$
Momentum (Vector)	$\vec{p}$	-1	$+1$	-1
Spin (Axial Vector)	$\vec{\sigma} = \vec{r} \times \vec{p}$	$+1$	$+1$	-1
Helicity	$\vec{\sigma} \cdot \vec{p}$	-1	$+1$	$+1$
Electric Field	$\vec{E}$	-1	-1	$+1$
Magnetic Field	$\vec{B}$	$+1$	-1	-1
Magnetic Dipole Moment	$\vec{\sigma} \cdot \vec{B}$	$+1$	-1	$+1$
Electric Dipole Moment	$\vec{\sigma} \cdot \vec{E}$	-1	-1	-1
Transverse Polarization	$\vec{\sigma} \cdot (\vec{p_1} \times \vec{p_2})$	$+1$	$+1$	-1

Parity Violation in Weak Decays



- First observed by Chien-Shiung Wu in 1957 through β-decay of polarised ^{60}Co nuclei: $^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^{-} + \overline{\nu}_e$
  - Recall: under parity changes momentum sign but not spin $\vec{p} \rightarrow -\vec{p}$ $\vec{S} \rightarrow \vec{S}$
- Electrons were observed to be emitted to opposite to nuclear spin direction
- Particular direction is space is preferred!

- P is found to be violated maximally in weak decays
- P is a good symmetry in strong and QED.

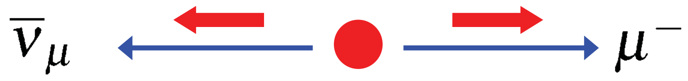
- Recall the vertex term for the weak force is $g_w\, \gamma^\mu(1-\gamma^5)\,/\sqrt{8}$
- Fermion currents are proportional to $\overline{\psi}(e)\,(\gamma^\mu-\gamma^\mu\gamma^5)\,\psi(\nu_e)$ **"vector – axial vector"**
- Under parity, $P\,(\overline{\psi}\,(\gamma^\mu - \gamma^\mu\gamma^5)\,\psi) = \overline{\psi}\,(\gamma^\mu + \gamma^\mu\gamma^5)\,\psi$ (mixture of eigenstates)

- Compare to QED and QCD vertices: $P\,(\,\overline{\psi}\,\gamma^\mu\,\psi\,) = \overline{\psi}\,\gamma^\mu\,\psi$

(pure eigenstates - no P violation)

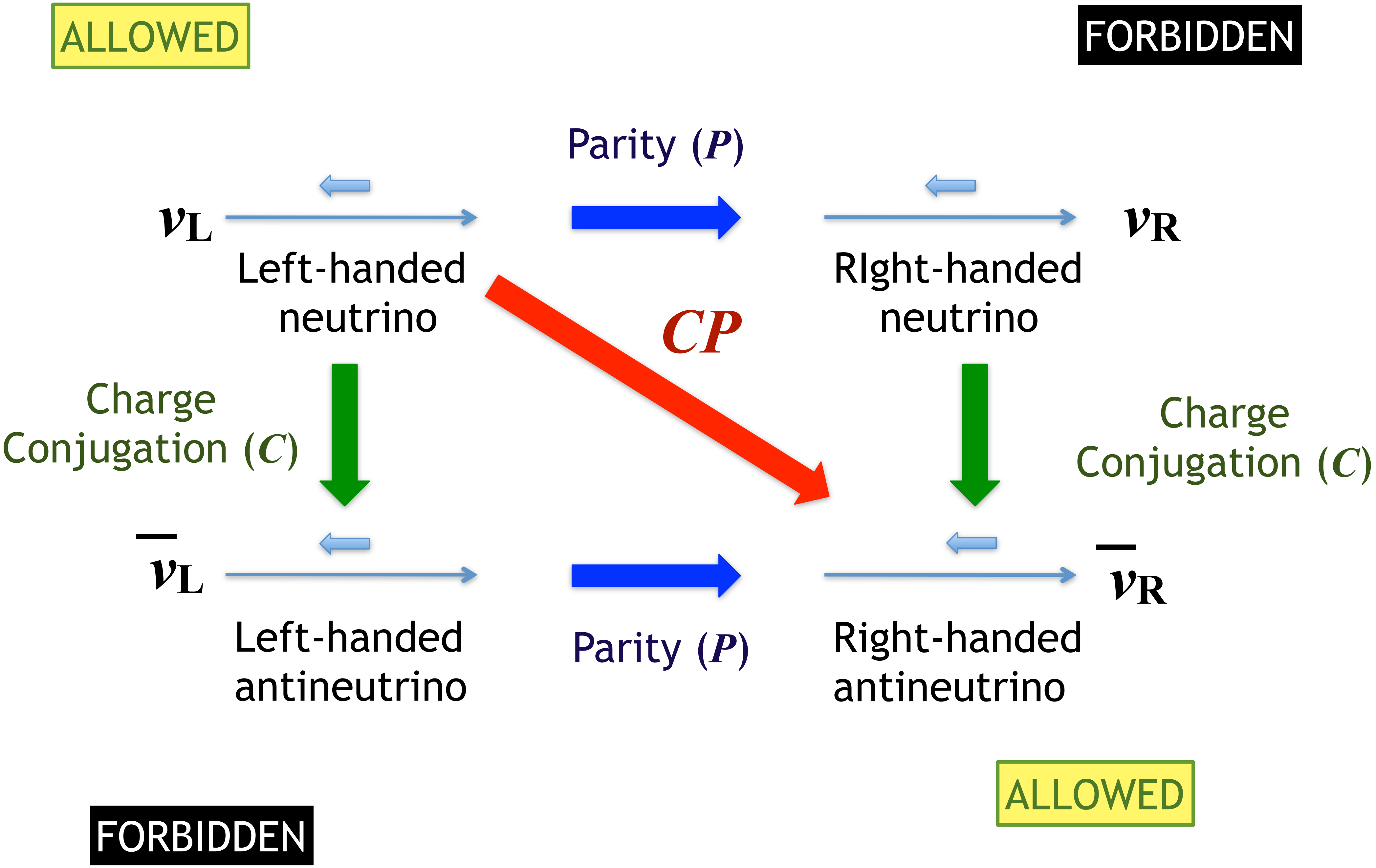
Parity Violation in Pion Decays

- Consider charged pion decay at rest, $\pi^- \to \mu^- + \overline{\nu}_\mu$



- Charged pion has $S = 0 \Rightarrow$ muon and neutrino produced with equal & opposite spin
  - ➡ The muon and neutrino will have identical helicities.
  - ➡ Experiments observe muon helicity is always right-handed
  - ➡ Conclusion: Only right-handed anti-neutrinos exist! (or only right-handed anti-neutrinos interact)
- Similarly for $\pi^+ \to \mu^+ + \nu_\mu$ anti-muon helicity is always left-handed
  - ➡ only left-handed neutrinos exist!
- This observation also explains why $\pi^- \to \mu^- + \overline{\nu}_\mu$ is preferred over $\pi^- \to e^- + \overline{\nu}_e$
  - ➡ The weak interaction only acts on LH **chiral** states, pion must decay to RH **helicity** electron or muon.
  - ➡ The change from LH chiral to RH helicity for lepton (ℓ) mass: $\propto \dfrac{m_\ell}{m_\pi + m_\ell}$

Neutrino States



ALLOWED

FORBIDDEN

Parity (P)

ν_L

Left-handed neutrino

CP

ν_R

RIght-handed neutrino

Charge Conjugation (C)

Charge Conjugation (C)

$\bar{\nu}_L$

Left-handed antineutrino

Parity (P)

$\bar{\nu}_R$

Right-handed antineutrino

ALLOWED

FORBIDDEN

CPT Theorem

- *CPT* is the combination of *C*, *P* and *T*.
  Turns a forward-going particle with LH helicity into backward-going antiparticle with RH helicity.
- *CPT* Theorem states:

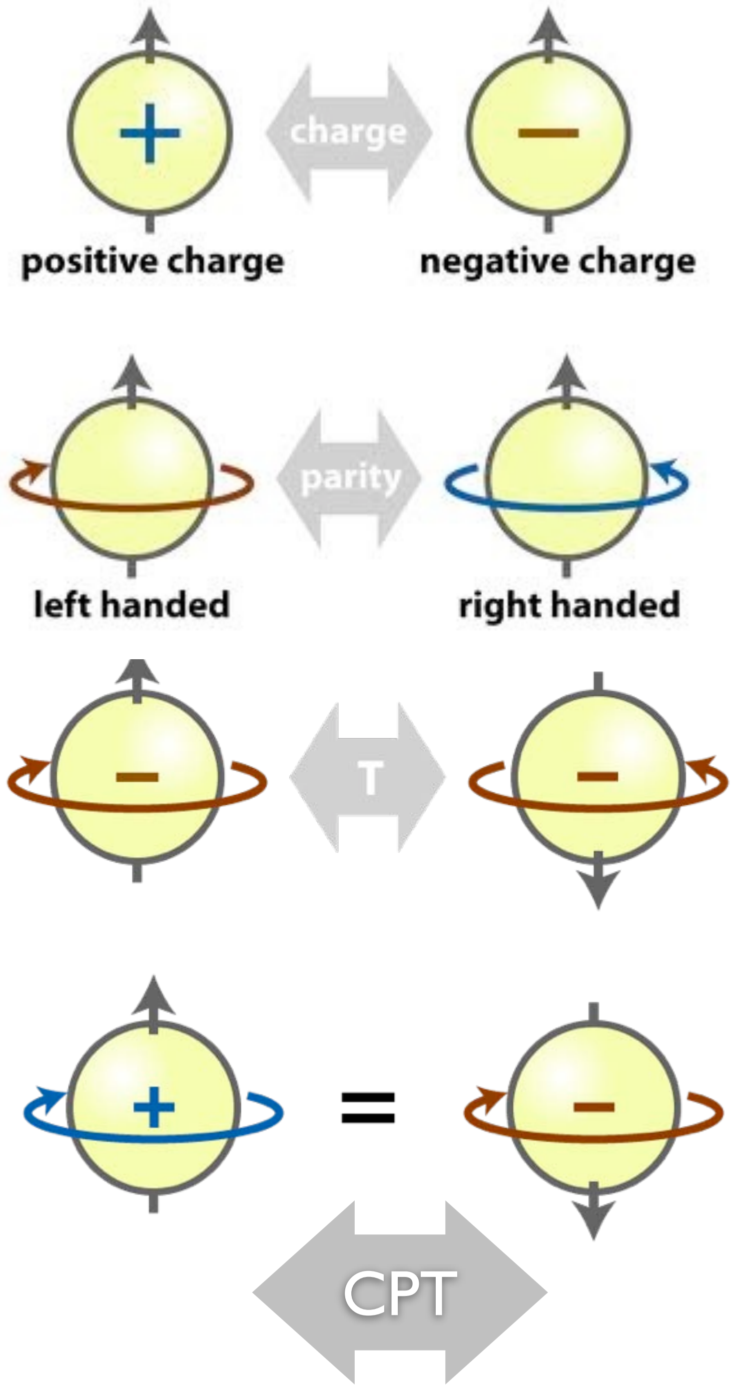
  > "All interactions described by a local Lorentz invariant gauge theory must be invariant under the combined operation *CPT*"

  ➡ *CPT* violation would imply non-locality and/or loss of Lorentz invariance

  ➡ Impossible to write down relativistic quantum field theories?

  ➡ Impossible to describe interactions in terms of Feynman diagrams?

  ➡ *CPT* conservation implies that *CP* violation is equivalent to *T* violation



- The Universe needs *CP* violation for the matter-antimatter asymmetry and it needs *T* violation for the arrow of time

Tests of *CPT* Invariance

- ***CPT*** invariance implies particles and antiparticles must have equal masses:

$$\frac{M(\mathrm{K}^0) - M(\overline{\mathrm{K}}^0)}{\frac{1}{2}[M(\mathrm{K}^0) + M(\overline{\mathrm{K}}^0)]} < 10^{-18}\ \mathrm{GeV}$$

Don't learn all these limits, just for your appreciation

- Particle and antiparticles must have equal lifetimes:

$$\frac{\Gamma(\mathrm{K}^0) - \Gamma(\overline{\mathrm{K}}^0)}{\frac{1}{2}[M(\mathrm{K}^0) + M(\overline{\mathrm{K}}^0)]} < \times 10^{-17}$$

- Particle and antiparticles must have equal and opposite charges and magnetic moments

$$Q(p) + Q(\bar{p}) < 10^{-21} e \qquad \frac{g(e^+) - g(e^-)}{\frac{1}{2}[g(e^+) + g(e^-)]} < 2 \times 10^{-12}$$

- Hydrogen and anti-hydrogen atoms have identical spectra

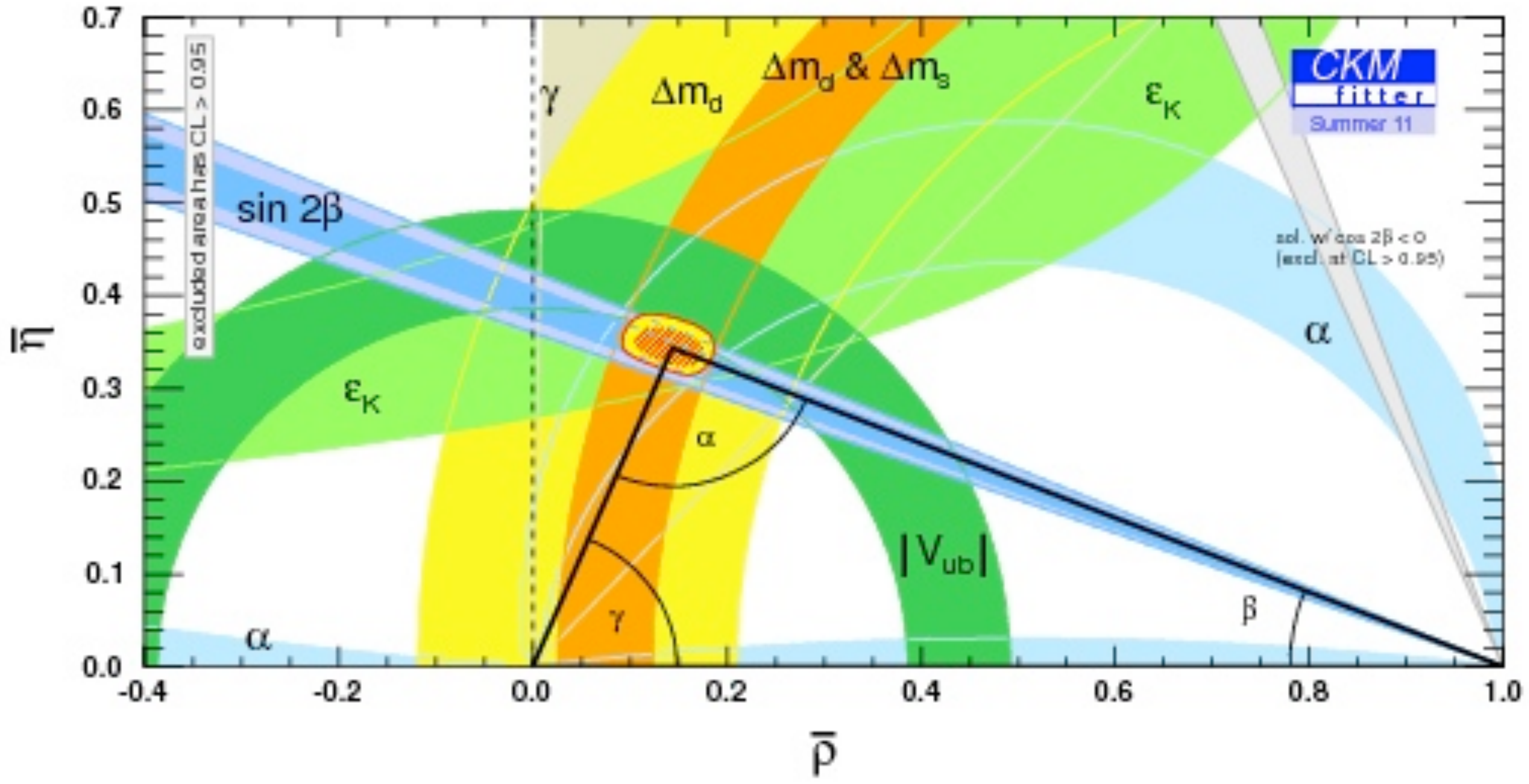
Summary

- Symmetries play a key role in describing interactions in particle physics.
- QED and QCD obey Gauge symmetries in the Lagrangian corresponding to symmetry groups. These lead to conservation of electric and colour charge.
- Three important discreet symmetries: Charge Conjugation (C), Parity (P) and Time reversal (T).
  - C: changes the sign of the charge
  - P: spatial inversion, reserves helicity. Fermions have $P=+1$, antifermions $P=-1$
  - T: changes the initial and final states
  - Gluons and photons have $C=-1$, $P=-1$

- C and P are conserved in QED and QCD, maximally violated in weak
- Only LH neutrinos and RH anti-neutrinos are found in nature.

- CPT is thought to be absolutely conserved.

CKM Fit

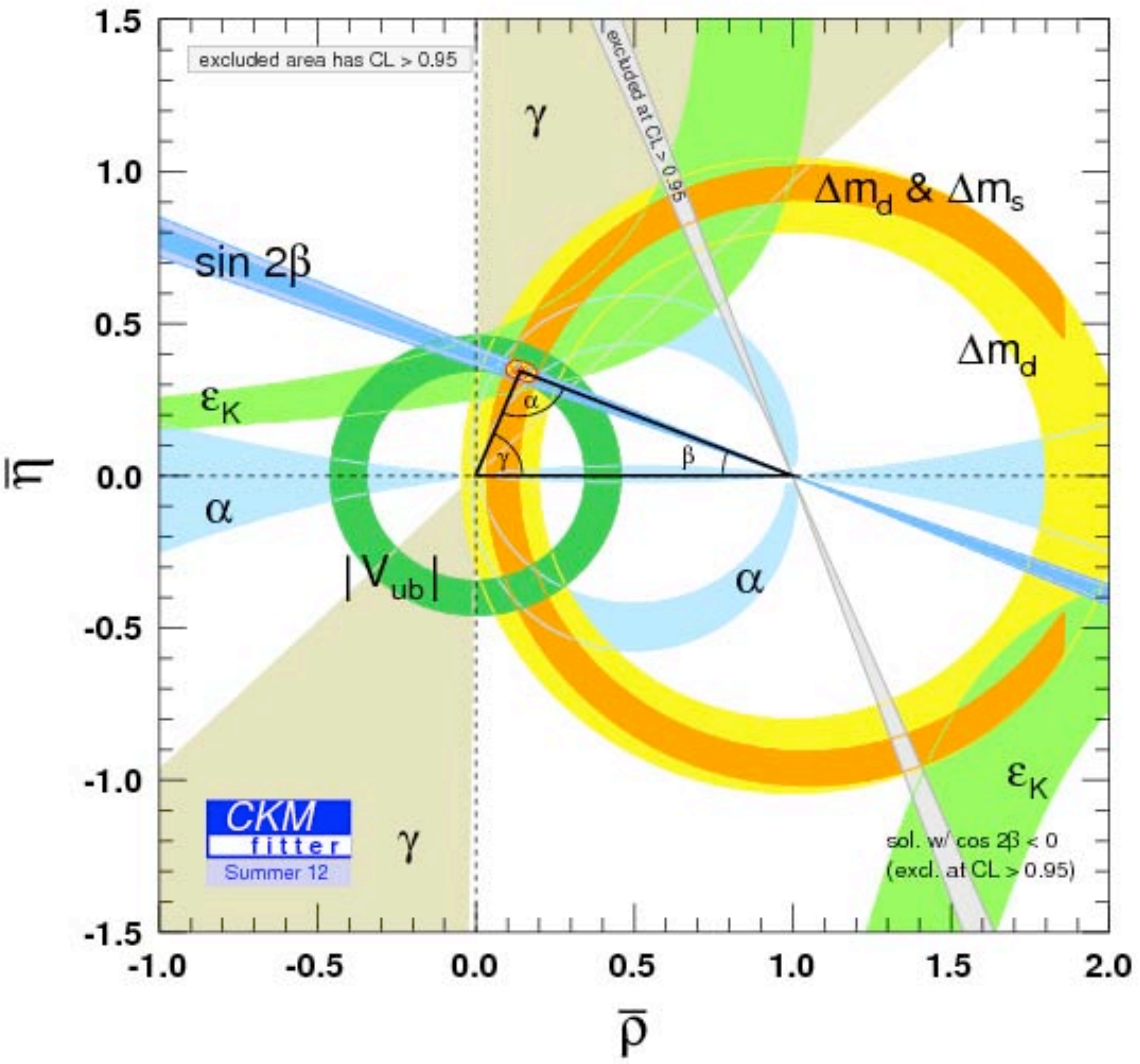


- All measurements, including results from BaBar and Belle experiments, penguin diagrams, neutral kaon mixing used to find
- Best values for η and ρ parameters in Wolfenstein parameterisation.

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 15: Measuring *CP* Violation



★Mixing and decays of kaons

★CKM Matrix revisited

★The unitarity triangle

Reminder: Guest Seminars

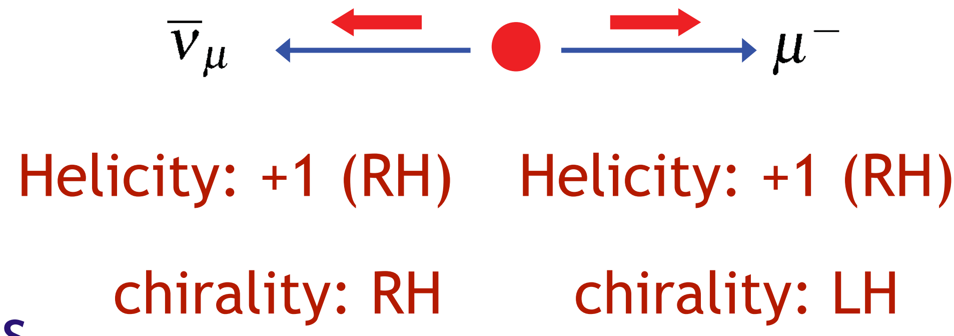
- From Edinburgh University researchers on their work.

- Monday (18th March) in tutorial:
  - Guest seminar from Dr Greig Cowan on B-physics at LHCb

- Following Monday (25th March) in tutorial
  - Guest seminar from Dr Wahid Bhimji on Higgs physics at ATLAS

Parity Violation in Pion Decays

- Consider charged pion decay at rest, $\pi^- \to \mu^- + \overline{\nu}_\mu$
- Charged pion has $S = 0$
  - ➡ muon and neutrino produced with equal & opposite spin
    - ➡ The muon and neutrino will have identical helicities

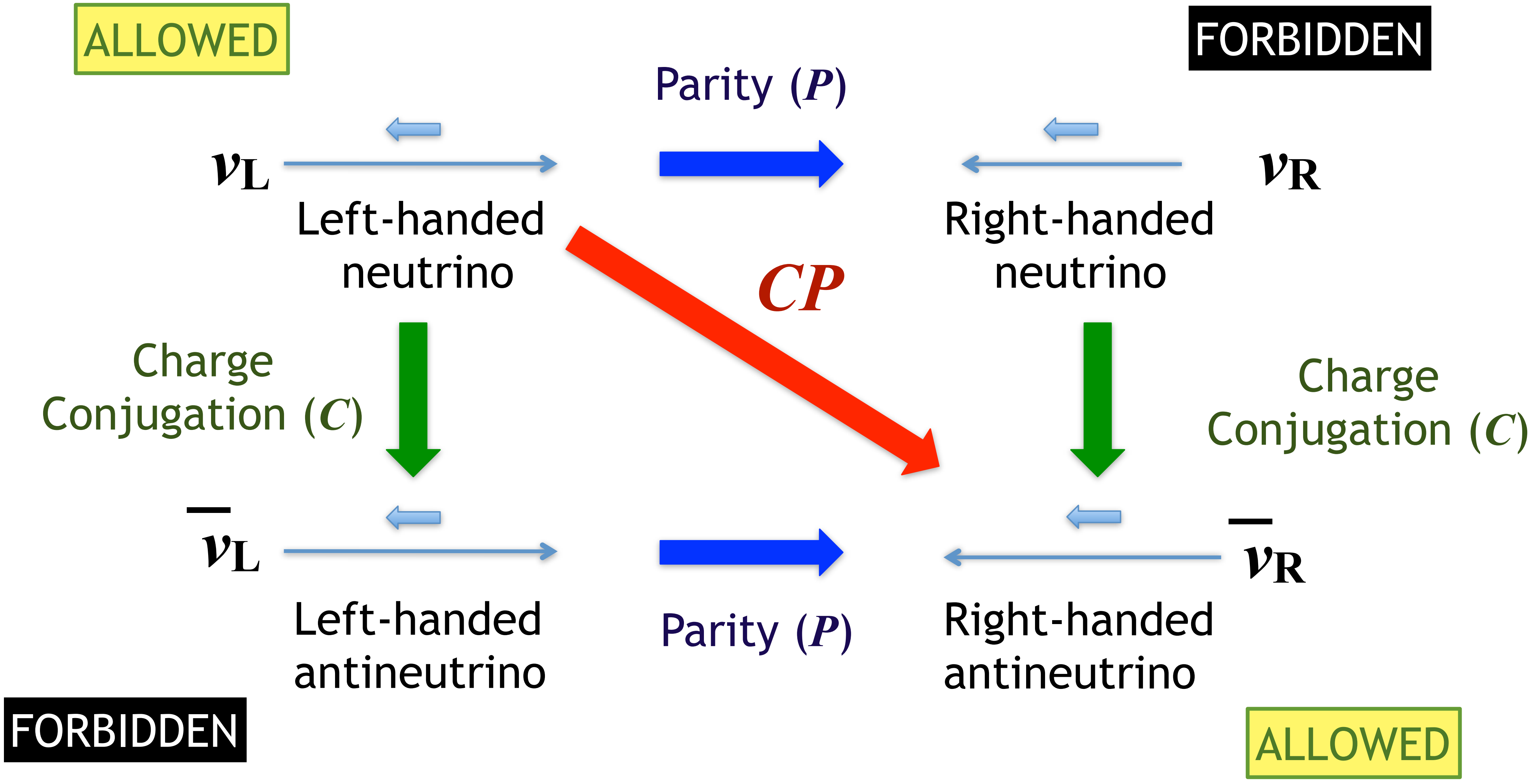


$$\hat{h} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

  - ➡ Experiments observe muon helicity is always right-handed
  - ➡ **Conclusion**: Only right-handed anti-neutrinos exist!

- Similarly for $\pi^+ \to \mu^+ + \nu_\mu$ anti-muon helicity is always left-handed
  - ➡ only left-handed neutrinos exist!
- This observation also explains why $\pi^- \to \mu^- + \overline{\nu}_\mu$ is preferred over $\pi^- \to e^- + \overline{\nu}_e$
  - ➡ Weak force decay produces **particles with left handed chirality** and **antiparticles with right handed chirality**
  - ➡ The amount of LH chiral in a RH helicity state is related to β of the lepton. In terms of lepton (ℓ) mass: $\propto \dfrac{m_\ell}{m_\pi + m_\ell}$

Neutrino States



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Parity (*P*)

ν_L Left-handed neutrino

ν_R Right-handed neutrino

CP

Charge Conjugation (*C*)

Charge Conjugation (*C*)

$\bar{\nu}_L$ Left-handed antineutrino

Parity (*P*)

$\bar{\nu}_R$ Right-handed antineutrino

FORBIDDEN

ALLOWED

Parity changes the direction of momentum, but not spin:
Left handed helicity → right handed helicity

Charge conjugation changes the sign of all charges:
particle → anti-particle

The existence of the ν_L and $\bar{\nu}_R$ suggests that *CP* is a good symmetry in the weak interaction

From Last Lecture: Summary

- Parity P and Charge Conjugation C are maximally violated in weak interactions due to vector – axial vector structure of interaction vertex.
  - Conserved in strong and electromagnetic interactions.
- The combined symmetry CP describes the difference between matter and anti-matter
  - The existence of only LH neutrinos and RH anti-neutrinos suggest CP is good symmetry in the weak force.
- CPT symmetry must be conserved… it’s one of the foundations of QM and field theory!

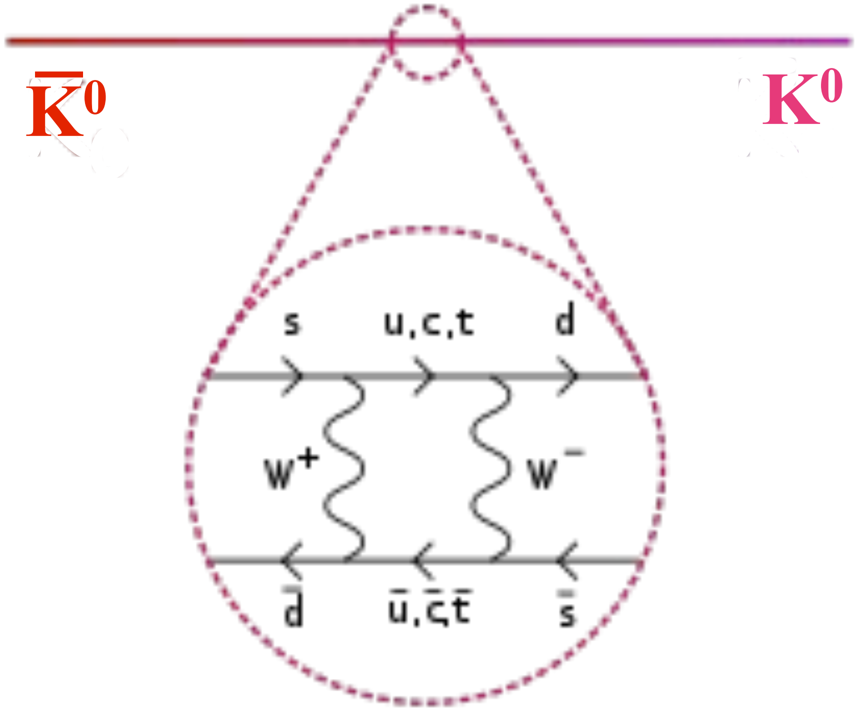
Neutral Meson Mixing

- Second order weak interactions can mix long-lived neutral mesons with their antiparticles:
  - ➡ $K^0(\overline{s}\,d)$, $D^0(\overline{c}\,u)$, $B^0(\overline{b}\,d)$, $B_s(\overline{b}\,s)$ $\quad K^0 \leftrightarrow \overline{K}^0$ $\quad D^0 \leftrightarrow \overline{D}^0$ $\quad B^0 \leftrightarrow \overline{B}^0$ $\quad B_s \leftrightarrow \overline{B}_s$

- e.g. take the neutral kaons K^0 & $\overline{K}^0$ as an example:

$$P\,|K^0\rangle = -|K^0\rangle \qquad P\,|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

$$CP\,|K^0\rangle = -|\overline{K}^0\rangle \qquad CP\,|\overline{K}^0\rangle = -|K^0\rangle$$



  - The *CP* eigenstates are:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \qquad CP = +1$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \qquad CP = -1$$

Neutral Kaon Decay

- Decay eigenstates are (approximately) $\mathbf{K_1}$ (CP=+1) and $\mathbf{K_2}$ (CP=−1)
  - **not** the same as the flavour eigenstates $\mathbf{K^0}$ and $\mathbf{\overline{K}^0}$

- Two common decay modes of kaons 2π and 3π
  - $\pi^0\pi^0$ and $\pi^+\pi^-$ have $CP = +1$
  - $\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$ have $CP = -1$

- If CP is a good symmetry in kaon decay (which it nearly is) we expect:
  - $K_1 \rightarrow \pi^0\pi^0$, $\pi^+\pi^-$ $CP = +1$ conserved
  - $K_2 \rightarrow \pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$ $CP = -1$ conserved

- $K_1 \rightarrow \pi\pi$ has large phase space ⇒ quick decay, travels ~ cm before decay
  - named "**K**-short" or K_S with $\tau_S = 0.09$ **ns**
- Decay $K \rightarrow \pi\pi\pi$ has small phase space ⇒ slow decay, travels ~ 10 m before decay
  - "**K**-long" or K_L with $\tau_L = 51$ **ns**

Neutral Kaons continued

- Because the kaons can mix, a kaon state can be described as a superposition of $\mathbf{K^0}$ and $\mathbf{\overline{K}^0}$:

$$\psi(t) = \left(\begin{array}{c} a(t)|\mathrm{K}^0\rangle \\ b(t)|\overline{\mathrm{K}}^0\rangle \end{array} \right)$$

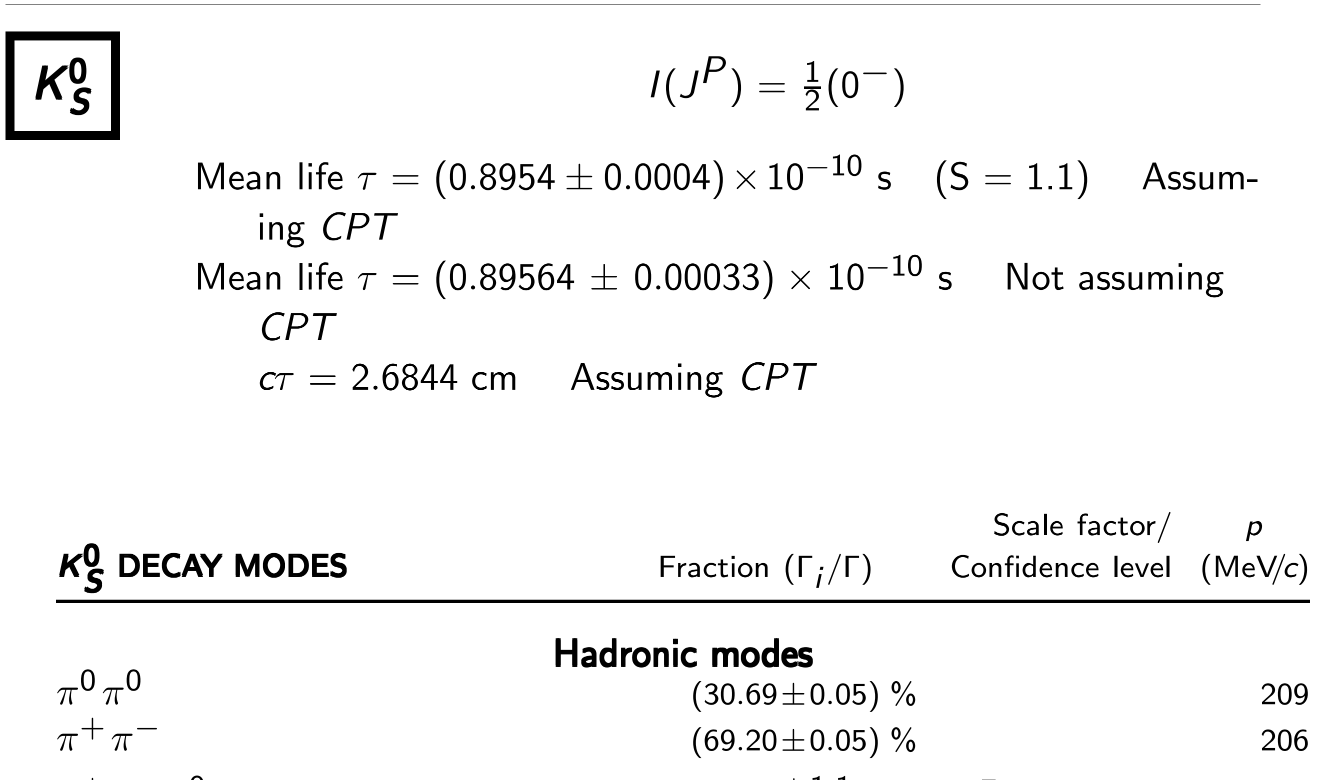
- The hamiltonian will describe both the mixing the decay in terms of two hermitian matrices:

$$i\frac{\partial\psi(t)}{\partial t} = \hat{H}\psi(t) = (\hat{M} - \frac{i}{2}\hat{\Gamma})\psi(t)$$

$$\hat{M} - \frac{i}{2}\hat{\Gamma} = \left(\begin{array}{cc} M_\mathrm{K} & \Delta m_\mathrm{K} \\ (\Delta m_\mathrm{K})^* & M_\mathrm{K} \end{array} \right) - \frac{i}{2}\left(\begin{array}{cc} \Gamma_\mathrm{K} & \Delta\Gamma_\mathrm{K} \\ (\Delta\Gamma_\mathrm{K})^* & \Gamma_\mathrm{K} \end{array} \right)$$

- Mass difference $\mathbf{\Delta} \boldsymbol{m}_\mathbf{K} = \boldsymbol{m}_\mathbf{S} - \boldsymbol{m}_\mathbf{L} = \mathbf{3.52(1)\ x10^{-12}\ MeV = 0.53\ x\ 10^{-10}\ s^{-1}}$ is a measure of the oscillation frequency

Neutral Kaon Properties



K_S^0

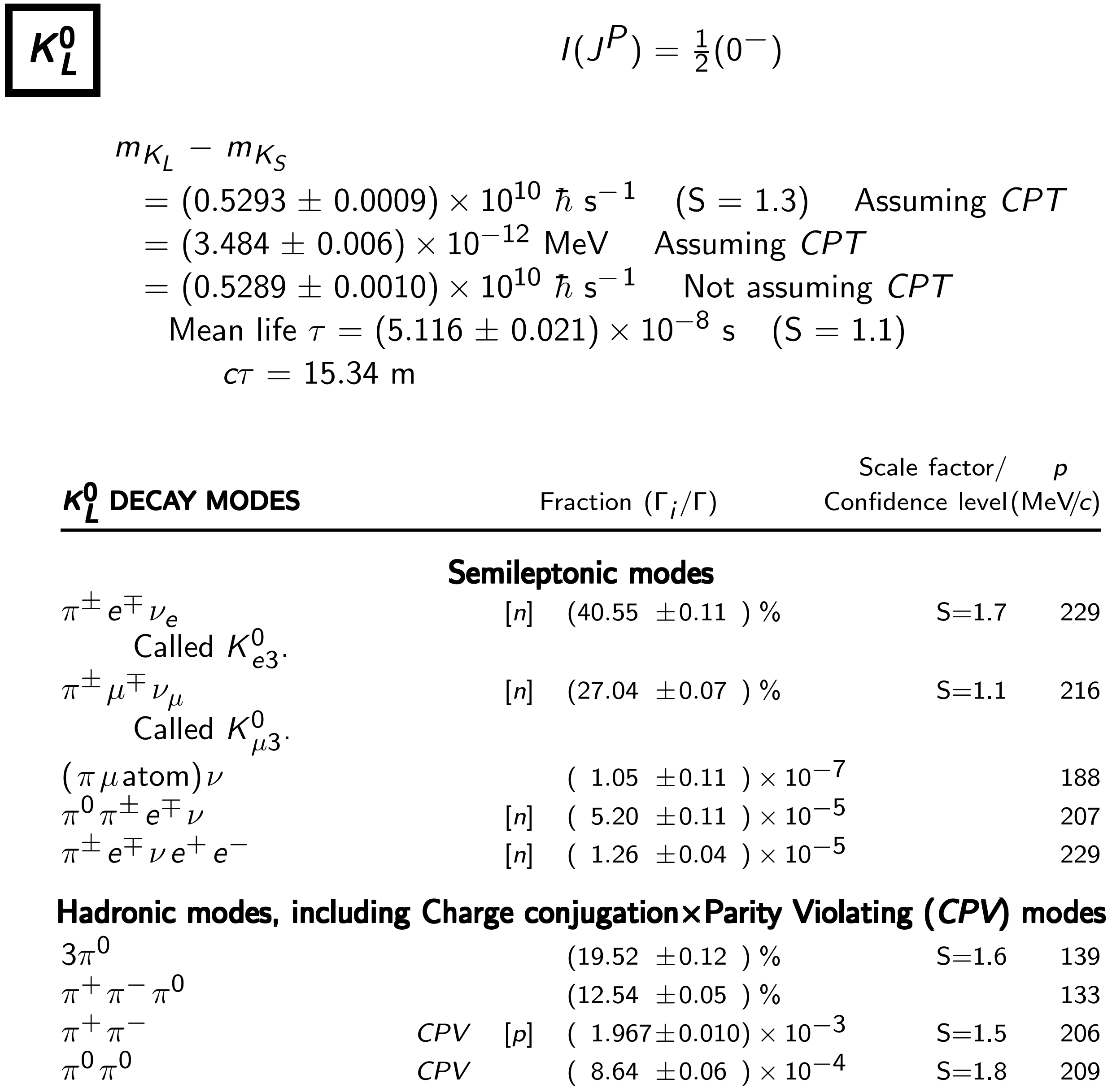
$I(J^P) = \frac{1}{2}(0^-)$

Mean life $\tau = (0.8954 \pm 0.0004) \times 10^{-10}$ s (S = 1.1) Assuming *CPT*

Mean life $\tau = (0.89564 \pm 0.00033) \times 10^{-10}$ s Not assuming *CPT*

$c\tau = 2.6844$ cm Assuming *CPT*

K_S^0 DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Hadronic modes			
$\pi^0\pi^0$	(30.69±0.05) %		209
$\pi^+\pi^-$	(69.20±0.05) %		206



K_L^0

$I(J^P) = \frac{1}{2}(0^-)$

$m_{K_L} - m_{K_S}$

$= (0.5293 \pm 0.0009) \times 10^{10}\ \hbar\ \mathrm{s}^{-1}$ (S = 1.3) Assuming *CPT*

$= (3.484 \pm 0.006) \times 10^{-12}$ MeV Assuming *CPT*

$= (0.5289 \pm 0.0010) \times 10^{10}\ \hbar\ \mathrm{s}^{-1}$ Not assuming *CPT*

Mean life $\tau = (5.116 \pm 0.021) \times 10^{-8}$ s (S = 1.1)

$c\tau = 15.34$ m

K_L^0 DECAY MODES			Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Semileptonic modes					
$\pi^\pm e^\mp \nu_e$ Called K^0_{e3}.		[n]	(40.55 ±0.11) %	S=1.7	229
$\pi^\pm \mu^\mp \nu_\mu$ Called $K^0_{\mu 3}$.		[n]	(27.04 ±0.07) %	S=1.1	216
$(\pi\mu\text{atom})\nu$			(1.05 ±0.11) $\times 10^{-7}$		188
$\pi^0\pi^\pm e^\mp \nu$		[n]	(5.20 ±0.11) $\times 10^{-5}$		207
$\pi^\pm e^\mp \nu e^+ e^-$		[n]	(1.26 ±0.04) $\times 10^{-5}$		229
Hadronic modes, including Charge conjugation×Parity Violating (*CPV*) modes					
$3\pi^0$			(19.52 ±0.12) %	S=1.6	139
$\pi^+\pi^-\pi^0$			(12.54 ±0.05) %		133
$\pi^+\pi^-$	*CPV*	[p]	(1.967±0.010) $\times 10^{-3}$	S=1.5	206
$\pi^0\pi^0$	*CPV*		(8.64 ±0.06) $\times 10^{-4}$	S=1.8	209

http://pdg.lbl.gov/2012/tables/rpp2012-tab-mesons-strange.pdf

Neutral Kaons with *CP* Violation

- The *CP* eigenstates are $\mathbf{K_1}$ and $\mathbf{K_2}$

$$|\mathrm{K}_1\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{K}^0\rangle - |\overline{\mathrm{K}}^0\rangle\right) \qquad CP = +1$$

$$|\mathrm{K}_2\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle\right) \qquad CP = -1$$

- The decay $\mathbf{K_S}$ and $\mathbf{K_L}$ are not quite identical to the *CP* eigenstates; in terms of a (small) parameter $\boldsymbol{\varepsilon}$:

N is an overall normalisation factor

$$|\mathrm{K_S}\rangle = \frac{1}{N}\left((1+\epsilon)|\mathrm{K}^0\rangle - (1-\epsilon)|\overline{\mathrm{K}}^0\rangle\right)$$

$$|\mathrm{K_L}\rangle = \frac{1}{N}\left((1+\epsilon)|\mathrm{K}^0\rangle + (1-\epsilon)|\overline{\mathrm{K}}^0\rangle\right)$$

- Both $\mathbf{K_S}$ and $\mathbf{K_L}$ contain slightly more $\mathbf{K^0}$ (matter) than $\mathbf{\overline{K}^0}$ (antimatter). The decay states contain both $CP = +1$ and $CP = -1$: *CP* is violated in weak force decay

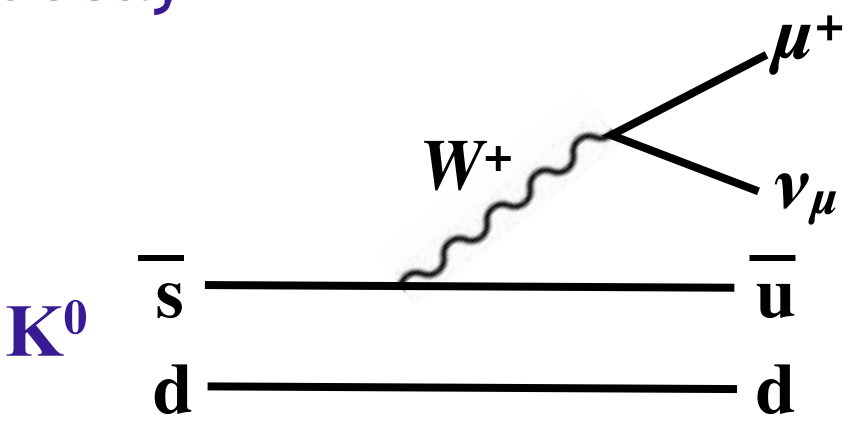
$$|\mathrm{K_S}\rangle = \frac{1}{N}\left(|\mathrm{K}_1\rangle - \epsilon|\mathrm{K}_2\rangle\right)$$

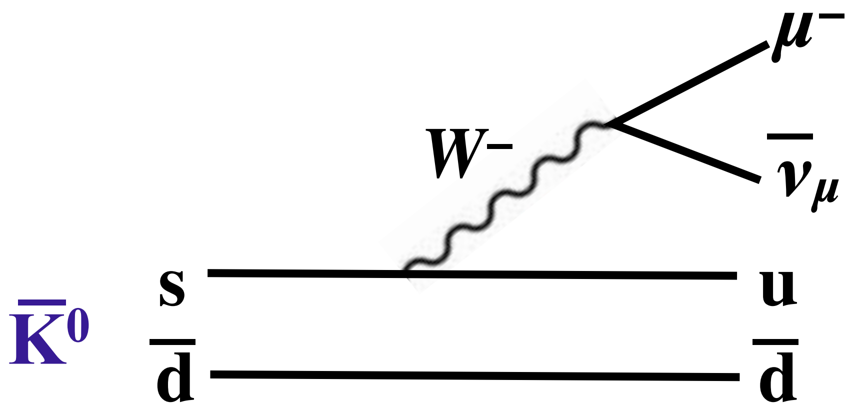
$$|\mathrm{K_L}\rangle = \frac{1}{N}\left(|\mathrm{K}_2\rangle + \epsilon|\mathrm{K}_1\rangle\right)$$

- $\boldsymbol{\varepsilon}$ is measured to be $|\varepsilon| \sim 2 \times 10^{-3}$, the amount of **indirect *CP* violation**

CP and *T* Violation in $K_L \to \pi \ell \nu$

- *CP* violation is also observed in the semileptonic decay $K_L \to \pi \ell \nu$
    - ➡ ℓ stands for e or μ
    - ➡ K^0 can only decay as $K^0 \to \pi^- \ell^+ \nu$
    - ➡ $\bar{K}^0$ can only decay as $\bar{K}^0 \to \pi^+ \ell^- \bar{\nu}$
- $K_L = 1/N\ [\ (1+\varepsilon)\ K^0 + (1-\varepsilon)\ \bar{K}^0\ ]$





- Measure asymmetry in K_L decay rates:

$$\delta = \frac{\Gamma(K_L \to \pi^- \ell^+ \nu) - \Gamma(K_L \to \pi^+ \ell^- \bar{\nu})}{\Gamma(K_L \to \pi^- \ell^+ \nu) + \Gamma(K_L \to \pi^+ \ell^- \bar{\nu})}$$

$$\delta = \frac{(1+\epsilon)^2 - (1-\epsilon)^2}{(1+\epsilon)^2 + (1-\epsilon)^2} = 2\,\mathcal{R}e\,(\epsilon)$$

- Measured to be $\delta = 3.27(12) \times 10^{-3}$
- *CP* violation due to the mixing of the *CP* eigenstates

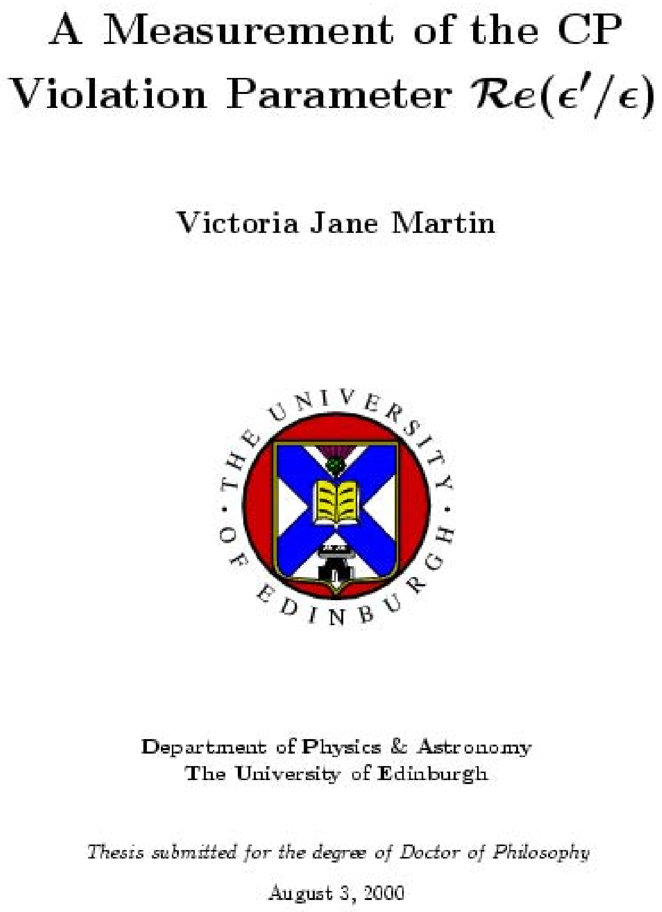
CP Violation in $\mathbf{K \rightarrow \pi\pi}$

- $\mathbf{K_L \sim K_2 + \varepsilon\, K_1}$ mainly $\boldsymbol{CP = -1}$ plus a little $\boldsymbol{CP = +1}$
    - $\mathbf{K_L}$ is observed decay into both $\boldsymbol{\pi\pi\pi}$ ($\boldsymbol{CP = -1}$) and $\boldsymbol{\pi\pi}$ ($\boldsymbol{CP = +1}$)

- Measured rate of $\mathbf{K_L \rightarrow \pi\pi}$ is slightly larger than can be accommodated by $\boldsymbol{\varepsilon}$

- A small amount ($\boldsymbol{\varepsilon'}$) of the $\mathbf{K_2}$ in $\mathbf{K_L}$ decays **directly** to $\boldsymbol{\pi\pi}$.
    - Known as **direct *CP* violation**, measured to be $|\varepsilon'/\varepsilon|$ $= \mathbf{1.65(26) \times 10^{-3}}$

- **It took 40 year's of effort to measure these effects!**
    - (including VJM's PhD thesis →)



A Measurement of the CP Violation Parameter $\mathcal{R}e(\epsilon'/\epsilon)$

Victoria Jane Martin

Department of Physics & Astronomy
The University of Edinburgh

Thesis submitted for the degree of Doctor of Philosophy

August 3, 2000

- The observed amount of *CP* violation in these experiments is very small $\mathbf{\sim 10^{-3}}$
- *CP* is nearly a good symmetry in the weak interaction

Types of *CP* Violation

For reference, three types of *CP* violation are classified:

1. **Direct *CP* violation** in decay amplitudes
   - Can occur in both charged and neutral particle decays
   - e.g. **ε'**, the ***CP*=−1** state decays directly to ***CP*=+1** final state

2. ***CP* violation in neutral meson mixing**
   - e.g. rate for $K^0 \rightarrow \bar{K}^0$ not equal to rate for $\bar{K}^0 \rightarrow K^0$, measured in semileptonic decay by **δ**

3. **Indirect *CP* violation due to interference between mixing and decay**
   - e.g. **ε** measures decay mixing between ***CP*=−1** and ***CP*=+1** states

Summary

- The *CP* symmetry describes the difference between matter and anti-matter - almost a good symmetry in the weak interactions.

- Small amounts of *CP* violation observed in $\mathbf{K^0}$ $\mathbf{B^0}$ $\mathbf{D^0}$ $\mathbf{B_s{}^0}$ through decays and mixing.

- Three types of *CP* violation:
  1. Direct *CP* violation in decay amplitudes
  2. *CP* violation in neutral meson mixing
  3. Indirect *CP* violation due to interference of mixing and decay.

- *CP* violation is accommodated in the Standard Model through a complex phase in the CKM matrix.

- The unitarity triangle of the CKM matrix is used to understand observation of the *CP* violation, and see if measurements are consistent.

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 16: More CP Violation and Introduction to Electroweak Theory


W BOSON *W-, W+*

The **W BOSON** is a messenger particle which communicates the weak force. Unlike the photon and gluon bosons, it has a mass. Like the Z boson, it is one of the most short-lived particles known, with a mere 10^{-25} second lifetime. It can be negatively charged (W-) or positively charged (W+). Luckily you can have both, as the toy is double-sided.

Wool felt with gravel fill for maximum mass.

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Z BOSON *Z*

The **Z BOSON** is a very massive carrier particle for the weak force. Unlike its siblings the W-/W+ particles, the Z is neutrally charged. Living only 10^{-25} second, the Z quickly decays into other particles. Discovered in 1983, the Z has allowed physicists to further study electroweak theory.

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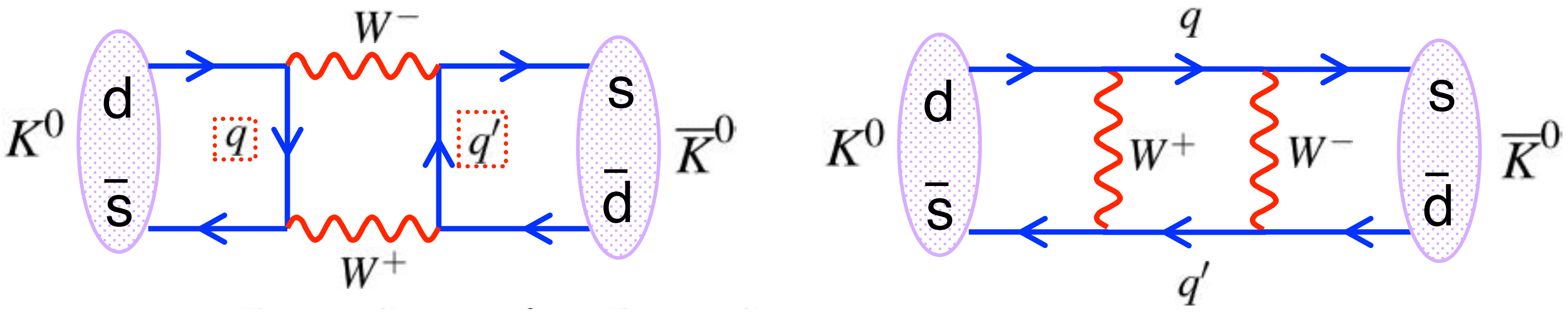
★Kaon mixing revisited

★CKM matrix

★Weak Isospin and Weak Hypercharge

Kaon Mixing Revisited



- Indirect *CP* violation in mixing occurs because the rate between $\mathbf{K^0 \to \overline{K}^0}$ transitions is smaller than the rate between $\mathbf{\overline{K}^0 \to K^0}$ transition.

$$\Gamma(\mathbf{K^0 \to \overline{K}^0}) \neq \Gamma(\mathbf{\overline{K}^0 \to K^0})$$

- Slightly more matter ($\mathbf{K^0}$) is created than anti-matter ($\mathbf{\overline{K}^0}$)
- Therefore both decay eigenstates contain slightly more (ϵ more) matter than anti-matter:

$$|\mathrm{K_S}\rangle = \frac{1}{N}\Big((1+\epsilon)|\mathrm{K}^0\rangle - (1-\epsilon)|\overline{\mathrm{K}}^0\rangle\Big)$$

$$|\mathrm{K_L}\rangle = \frac{1}{N}\Big((1+\epsilon)|\mathrm{K}^0\rangle + (1-\epsilon)|\overline{\mathrm{K}}^0\rangle\Big)$$

CKM elements for kaon mixing

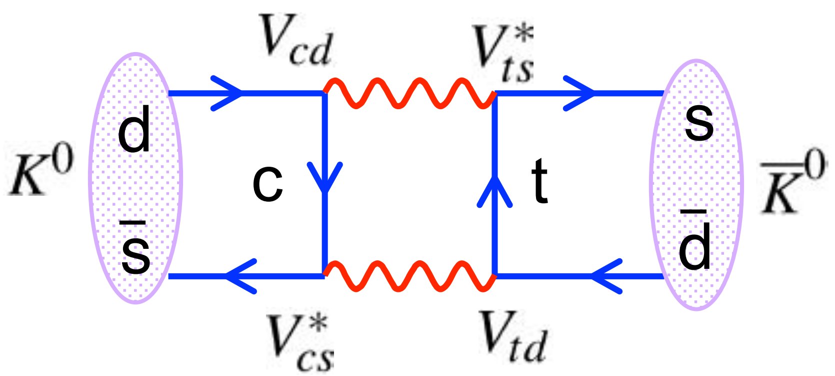
- Calculating $\mathcal{M}$ for this process, we have to consider all possible contributions due to different internal quarks.

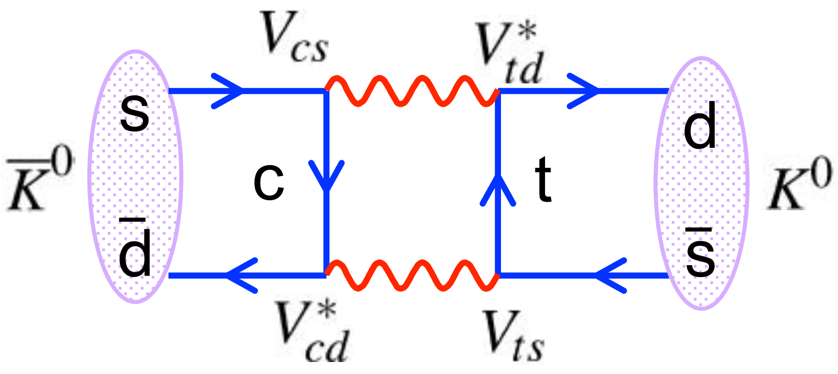
To work out which CKM matrix element, follow the quark line *backwards*:

if **t→s:** use V_{ts}

if **s→t:** then we need V_{st} which doesn't exist, therefore use V^*_{ts}

- For this argument , just consider one contribution:





$$\mathcal{M} \propto V_{\rm cd} V^*_{\rm ts} V_{\rm td} V^*_{\rm cs}$$

$$\mathcal{M}' \propto V_{\rm cs} V^*_{\rm td} V_{\rm ts} V^*_{\rm cd} \propto \mathcal{M}^*$$

$$\Gamma(\mathbf{K^0} \rightarrow \overline{\mathbf{K}}{}^{\mathbf{0}}) - \Gamma(\overline{\mathbf{K}}{}^{\mathbf{0}} \rightarrow \mathbf{K^0}) = \mathcal{M} - \mathcal{M}^* = 2\,\mathcal{I}m(\mathcal{M})$$

- The amount of ***CP*** violation is related to the imaginary parts of the CKM matrix elements

Cabibbo-Kobayashi-Maskawa Matrix

- The CKM matrix is the source of ***CP*** violation in the Standard Model

- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix a mixture of the down-type quarks:

weak eigenstates

$$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$

mass eigenstates

- The CKM matrix is unitary, $\boldsymbol{V}_{\mathbf{CKM}}{}^{\dagger}\boldsymbol{V}_{\mathbf{CKM}} = \mathbf{1}$ implies nine “unitarity relations”

$$\begin{pmatrix} V^*_{\mathrm{ud}} & V^*_{\mathrm{cd}} & V^*_{\mathrm{td}} \\ V^*_{\mathrm{us}} & V^*_{\mathrm{cs}} & V^*_{\mathrm{ts}} \\ V^*_{\mathrm{ub}} & V^*_{\mathrm{cb}} & V^*_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- The most frequently discussed is (1st row × 3rd column):

$$V^*_{\mathrm{ub}}V_{\mathrm{ud}} + V^*_{\mathrm{cb}}V_{\mathrm{cd}} + V^*_{\mathrm{tb}}V_{\mathrm{td}} = 0$$

The Wolfenstein Parameterisation

- An expansion of the CKM matrix in powers of $\lambda = V_{us} = 0.22$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda A\lambda & {}^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

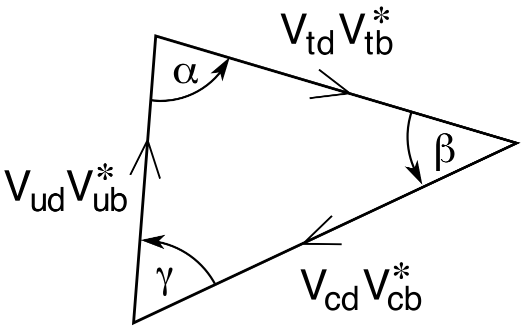
- Parameterisation reflects almost diagonal nature of CKM matrix:
  - ➡The diagonal elements V_{ud}, V_{cs}, V_{tb} are close to 1
  - ➡Elements V_{us}, $V_{cd} \sim \lambda$ are equal
  - ➡Elements V_{cb}, $V_{ts} \sim \lambda^2$ are equal
  - ➡Elements V_{ub}, $V_{td} \sim \lambda^3$ are very small

- Diagonal structure means down quark mass eigenstate is almost equal to down quark weak eigenstate
  - similarly for strange and bottom mass eigenstates
- Note that the complex phase η only appears in the very small elements, and is thus hard to measure.

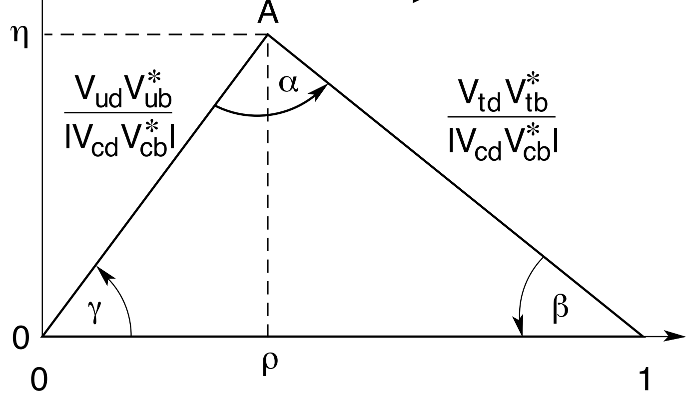
The Unitarity Triangle

$$V_{\mathrm{ud}}V_{\mathrm{ub}}^{*}+V_{\mathrm{td}}V_{\mathrm{tb}}^{*}+V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}=0$$

- Forms a triangle in the complex plane:



- Dividing through by $V_{cd}V_{cb}^{*}$:

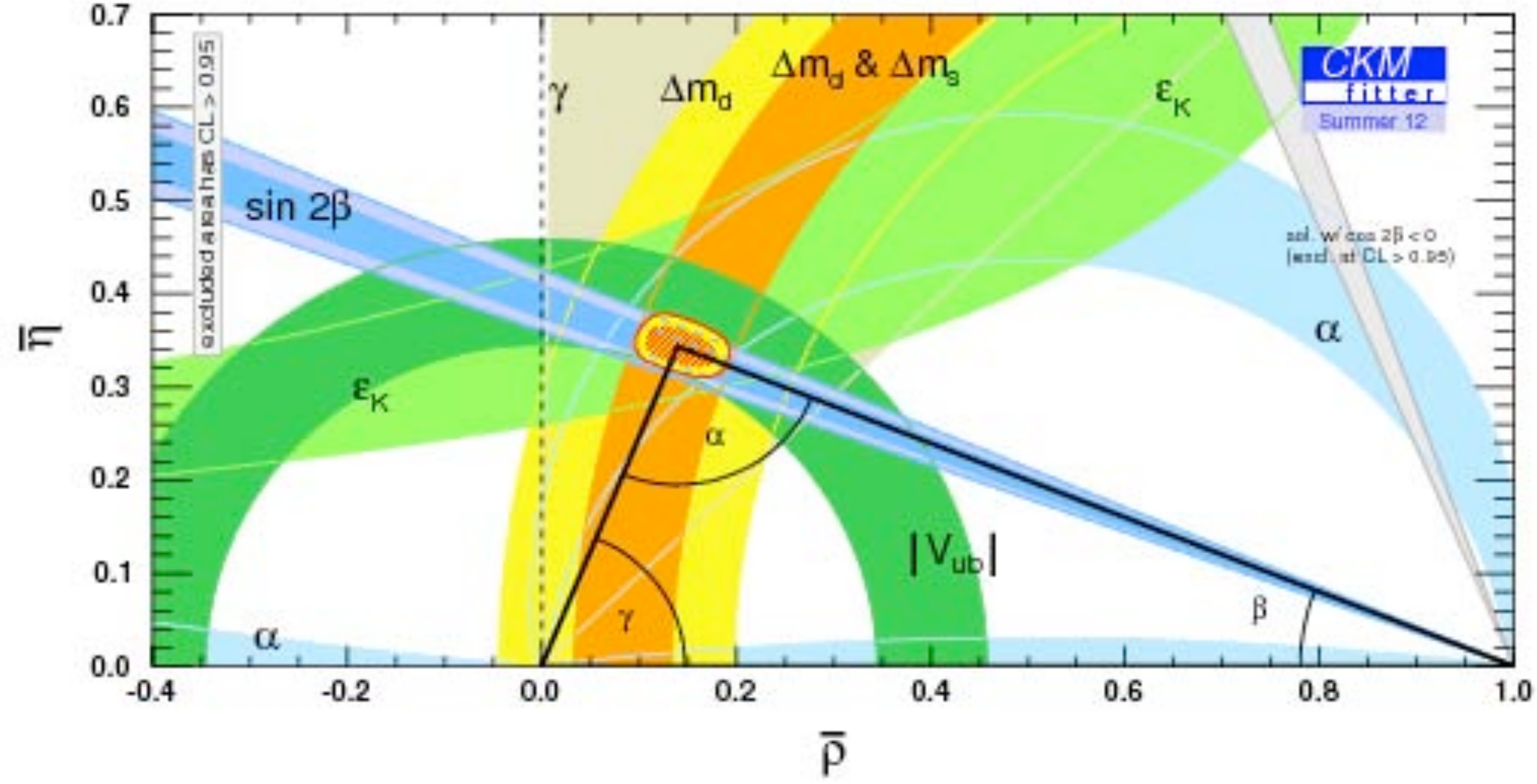


- This **unitarity triangle** is often use to present measurements of CP violation in $\mathbf{B}$-meson decay.
- Lengths and angles of the triangle are: $\left|\frac{V_{\mathrm{ud}}V_{\mathrm{ub}}^{*}}{V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}}\right| \quad \left|\frac{V_{\mathrm{td}}V_{\mathrm{tb}}^{*}}{V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}}\right|$

$$\alpha\equiv\arg\left(-\frac{V_{\mathrm{td}}V_{\mathrm{tb}}^{*}}{V_{\mathrm{ud}}V_{\mathrm{ub}}^{*}}\right)\qquad\beta\equiv\arg\left(-\frac{V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}}{V_{\mathrm{td}}V_{\mathrm{tb}}^{*}}\right)\qquad\gamma\equiv\arg\left(-\frac{V_{\mathrm{ud}}V_{\mathrm{ub}}^{*}}{V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}}\right)$$

- Triangle has a finite area only if relative complex phase between CKM elements

“CKM Fit”



- Experimental measurements used to determine lengths and sides of unitarity triangle.
- Determines best values for η and ρ parameters in Wolfenstein parameterisation.
- Current measurements indicate it is a closed triangle - consistent with only small *CP* violation.

Electroweak Unification

- Electroweak Theory was proposed in 1967 by Glashow, Salam & Weinberg. Unifies the electromagnetic and weak forces (Noble prize 1979)
- In 1970 ‘t Hooft and Veltman showed how to renormalise electroweak theory. (Noble prize 1999)

- At high energies ($E \gtrsim m_Z$) the electromagnetic force and the weak force are unified as a single **electroweak force**.
- At low energies ($E \lesssim m_Z$) the manifestations of the electroweak force are separate weak and electromagnetic forces.

- We will see today:
  1. The coupling constants for weak and electromagnetism are unified:

$$e = g_W \sin\theta_W$$

     ➡ Where $\sin\theta_W$ is the **weak mixing angle**
  2. Electroweak Unification predicts the existence of massive W^+ W^- and Z^0 bosons.

     ➡ Relies on Higgs mechanism to “give mass” to the W and Z bosons.

Review from Lecture 7,8: Charged & Neutral Weak Current

- Neutral Current is the exchange of massive *Z*-bosons.
  - ➡Couples to all quarks and all leptons (including neutrinos)
  - ➡No allowed flavour changes!
  - ➡Neutral weak current for fermion, ***f***:

$$\frac{g_Z}{2}\bar{u}(f)\gamma^\mu(c_V^f - c_A^f\gamma^5)u(f)$$

c^f_V and c^f_A are constants for fermion flavour, ***f***.

Lepton	c^f_V	c^f_A	Quark	c^f_V	c^f_A
ν_e, ν_μ, ν_τ	½	½	**u, c, t**	0.19	½
e, μ, τ	-0.03	−½	**d, s, b**	-0.34	−½

- Charged Current is the exchange of massive *W*-bosons.
  - ➡Couples to all quarks and leptons and changes fermion flavour:
  - ➡Allowed flavour changes are: $e\leftrightarrow\nu_e$, $\mu\leftrightarrow\nu_\mu$, $\tau\leftrightarrow\nu_\tau$, **d’↔u, s’↔c, b’↔t**
  - ➡Acts only on the left-handed components of the fermions: *V−A* structure.

$$g_W\frac{1}{2\sqrt{2}}\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)$$

Weak Isospin and Hypercharge

- QED couples to electric charge; QCD couples to colour charge...
- Electroweak force couples to two “charges”.
  - **Weak Isospin:** total and third component T, T_3. Depends on **chirality**
  - **Weak Hypercharge**, Y In terms of electric charge Q: $Y = 2\,(Q - T_3)$
    - All right-handed fermions have $T=0$, $T_3=0$
    - All left-handed fermions have $T=½$, $T_3=±½$
    - All left-handed antifermions have $T=0$, $T_3=0$
    - All right-handed antifermions have $T=½$, $T_3(\overline{f})=-T_3(f)$

Lepton	T	T_3	Y	Quark	T	T_3	Y
ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$	½	+½	-1	u_L, c_L, t_L	½	+½	⅓
e_L, μ_L, τ_L	½	−½	-1	d_L, s_L, b_L	½	− ½	⅓
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	−⅔

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 17: Electroweak and Higgs

HIGGS BOSON

H

The **HIGGS BOSON** is the theoretical particle of the Higgs mechanism, which physicists believe will reveal how all matter in the universe gets its mass. Many scientists hope that the Large Hadron Collider in Geneva, Switzerland, which collides particles at 99.99% the speed of light, will detect the elusive Higgs Boson

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The PARTICLE ZOO

- ★Weak Isospin and Weak Hypercharge
- ★Weak Isospin and Weak Hypercharge currents
- ★γ $W^{\pm}$ Z^{0} bosons
- ★Spontaneous Symmetry Breaking
- ★The Higgs mechanism and the Higgs boson

Weak Isospin and Hypercharge

- QED couples to electric charge; QCD couples to colour charge...
- Electroweak force couples to two "charges".
    - **Weak Isospin:** total and third component T, T_3. Depends on **chirality**
    - **Weak Hypercharge,** Y In terms of electric charge Q: $Y = 2\,(Q - T_3)$
        - All right-handed fermions have $T=0$, $T_3=0$
        - All left-handed fermions have $T=½$, $T_3=±½$
        - All left-handed antifermions have $T=0$, $T_3=0$
        - All right-handed antifermions have $T=½$, $T_3(\overline{f})=-T_3(f)$

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	½	+½	-1	u_L, c_L, t_L	½	+½	⅓
e_L, μ_L, τ_L	½	−½	-1	d_L, s_L, b_L	½	− ½	⅓
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	−⅔

Weak Isospin Doublets

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{e\mathrm{L}}$, $\nu_{\mu\mathrm{L}}$, $\nu_{\tau\mathrm{L}}$	½	+½	-1	$\mathrm{u_L}$, $\mathrm{c_L}$, $\mathrm{t_L}$	½	+½	⅓
e_L, μ_L, τ_L	½	−½	-1	$\mathrm{d_L}$, $\mathrm{s_L}$, $\mathrm{b_L}$	½	− ½	⅓
ν_R	0	0	0	$\mathrm{u_R}$, $\mathrm{c_R}$, $\mathrm{t_R}$	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	$\mathrm{d_R}$, $\mathrm{s_R}$, $\mathrm{b_R}$	0	0	−⅔

- Neutrinos and left-handed charged leptons from a "**weak isospin doublet**":

$$\chi_\mathrm{L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_\mathrm{L} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_\mathrm{L} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_\mathrm{L} \quad T = 1/2; \begin{array}{l} T_3 = +1/2 \\ T_3 = -1/2 \end{array}$$

- Doublet consists of "charged current flavour change pair".
  - ➡They have the same total weak isospin T=½.
  - ➡They are differentiated by the third component T_3=±½.
- Left-handed up-type quarks and left-handed down-type quarks also form isospin doublets

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}_\mathrm{L} \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix}_\mathrm{L} \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix}_\mathrm{L} \qquad T = 1/2; \begin{array}{l} T_3 = +1/2 \\ T_3 = -1/2 \end{array}$$

Weak Isospin Currents

- Weak Isospin and Weak Hypercharge couple to a different set of bosons.

- Weak isospin doublets χ_L couple to a set of **three** W-bosons: W^1, W^2, W^3, with SU(2) symmetry described by the 3 Pauli matrices:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- The W-bosons current is:

$$(j^{W i})^\mu = [g_W T] \; \overline{\chi_\mathrm{L}} \; \gamma^\mu \; \tau_i \; \chi_\mathrm{L}$$

$\tau_{1,2,3}$: Pauli Matrix
χ_L: weak isospin doublet column vector spinors
$\overline{\chi_L}$: weak isospin doublet row vectors spinors
T : weak isospin charge of the doublet
g_W: weak coupling constant

- e.g for the W^1 boson and the electron doublet:

$$(j^{W1})^\mu = [g_W T] \; (\nu_e \; e^-)_\mathrm{L} \; \gamma^\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_\mathrm{L}$$

- Strength of the fermion interaction with W-bosons is: $g_W T$

Weak Hypercharge Current

- Particles with weak hypercharge couple to **one** ***B***-boson: ***B*⁰** with U(1) symmetry.

- Use electron as an example:

$$j_\mu^Y = (\frac{1}{2}g'_W\,Y_e)\,\bar{e}\gamma^\mu e = \frac{1}{2}g'_W(Y_{e\mathrm{L}}\bar{e_\mathrm{L}}\gamma^\mu e_\mathrm{L} + Y_{e\mathrm{R}}\bar{e_\mathrm{R}}\gamma^\mu e_\mathrm{R})$$

Y_e : weak hypercharge of electron
$Y_{e\mathrm{L}}$: weak hypercharge of left-handed electron
$Y_{e\mathrm{R}}$: weak hypercharge of right-handed electron
e: Electron spinor
e_L: Left-handed electron spinor (u)
e_R: Right-handed electron spinor (u)
$\bar{e_\mathrm{L}}$: Left-handed electron spinor ($\bar{u}$)
$\bar{e_\mathrm{R}}$: Right-handed electron spinor ($\bar{u}$)
g'_W: coupling constant

- Strength of the fermion interaction with bosons is: $g'_W\,Y/2$

Physical Bosons

- The physical W^+, W^-, Z^0, γ bosons are linear superpositions of the W^1, W^2, W^3 and B^0 bosons.
  - Use $\cos\theta_W$ and $\sin\theta_W$ to ensure the states are properly normalised

$$W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2) \qquad W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2)$$

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W \qquad \gamma = W^3 \sin\theta_W + B^0 \cos\theta_W$$

- The coupling of the W^+, W^- bosons are

$$\tfrac{1}{\sqrt{2}}(g_W T) = \tfrac{1}{2\sqrt{2}} g_W$$

  - No $(1-\gamma^5)$ term: it integrated into the definition of the χ_L doublet.

The Photon

$$\gamma = W^3 \sin\theta_W + B^0 \cos\theta_W$$

- The electron current associated with the γ is:

$$\begin{aligned}
&(j^{W3})^\mu \sin\theta_W + (j^Y)^\mu \cos\theta_W \\
&\quad = \left[g_W T \sin\theta_W\right] \overline{\chi_{\mathrm{L}}}\gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \chi_{\mathrm{L}} + \left[\tfrac{1}{2} g'_W Y_e \cos\theta_W\right] \overline{e}\gamma^\mu e \\
&\quad = -\left[\tfrac{1}{2} g_W \sin\theta_W\right] \overline{e_{\mathrm{L}}}\gamma^\mu e_{\mathrm{L}} + \left[\tfrac{1}{2} g'_W \cos\theta_W\right] \left(-\overline{e_{\mathrm{L}}}\gamma^\mu e_{\mathrm{L}} - 2\overline{e_{\mathrm{R}}}\gamma^\mu e_{\mathrm{R}}\right) \\
&\quad = -\left[\underbrace{\tfrac{1}{2} g_W \sin\theta_W + \tfrac{1}{2} g'_W \cos\theta_W}_{=e}\right] \overline{e_{\mathrm{L}}}\gamma^\mu e_{\mathrm{L}} - \left[\underbrace{g'_W \cos\theta_W}_{=e}\right] \overline{e_{\mathrm{R}}}\gamma^\mu e_{\mathrm{R}}
\end{aligned}$$

- Consistent with the photon coupling if $e = g'_W \cos\theta_W = g_W \sin\theta_W$

$\sin^2\theta_W$ and Z-boson couplings

- The mixing angle between g_W and g'_W is not a prediction of the model, it must be measured experimentally.

$$\sin^2\theta_W = \frac{g_W'^2}{g_W^2 + g_W'^2} \approx 0.23$$

- The Z-boson the orthogonal mixture to the γ:

$$Z^0 = W^3\cos\theta_W - B^0\sin\theta_W$$

- predicts the couplings of the Z^0 boson in terms of T_3 and $Y = 2\,(Q - T_3)$
- e.g. for electron:

$$\begin{aligned}(j^Z)^\mu &= \frac{g_W}{\cos\theta_W}\left[(T_3 - Q\sin^2\theta_W)(\overline{e_\mathrm{L}}\gamma^\mu e_\mathrm{L}) - (Q\sin^2\theta_W)(\overline{e_\mathrm{R}}\gamma^\mu e_\mathrm{R})\right] \\ &= \frac{g_Z}{2}\,\bar{e}\,\gamma^\mu\,(c_V^e - c_A^e\gamma^5)\,e\end{aligned}$$

- if:

$$g_Z = \frac{g_W}{\cos\theta_W} \qquad c_V = T_3 - 2Q\sin^2\theta_W \qquad c_A = T_3$$

Summary of Electroweak Unification

- We have recovered the behaviour of the $W^\pm$, Z and γ

  - ➡We introduced an SU(2) symmetry (3 bosons) coupling to weak isospin with a coupling constant g_W
  - ➡We introduced a U(1) symmetry (1 boson) coupling to weak hypercharge with a coupling constant g'_W
  - ➡Together predicts four bosons we identify with W^+, W^-, Z and γ
  - ➡Electroweak Theory is often called **SU(2) ⊗ U(1)** model

- All of the properties of electroweak interactions described by:
  - the intrinsic charges of the fermions
  - the SU(2) ⊗ U(1) symmetry
  - g_W and g'_W: free parameters that need to be measured

- Along with QCD, Electroweak Theory is the Standard Model.

The Higgs Mechanism: Introduction

- The Higgs Mechanism was proposed in 1964 separately by Higgs and Brout & Englert.
- It introduces an extra field, ϕ, which interacts with the electroweak currents. The potential of the field is:

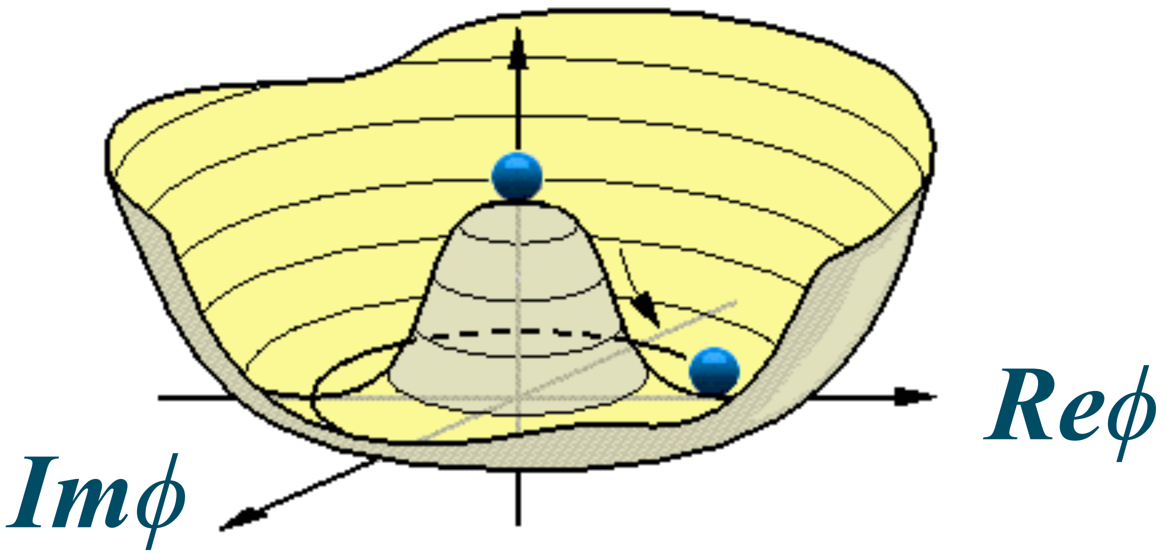
$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \qquad \text{with } \mu^2 > 0, \lambda > 0$$

- The Higgs mechanism allows the ***W*** and ***Z*** bosons to have a mass. (Otherwise forbidden by the external symmetries.)

- Provides an explanation for fermion masses (***e***, ***μ***, ***τ***, **u**, **d**, **s**, **c**, **t**, **b**).

- P.W. Higgs pointed out that a further consequence would be the existence of a spin-0 boson: the Higgs boson, ***H***.

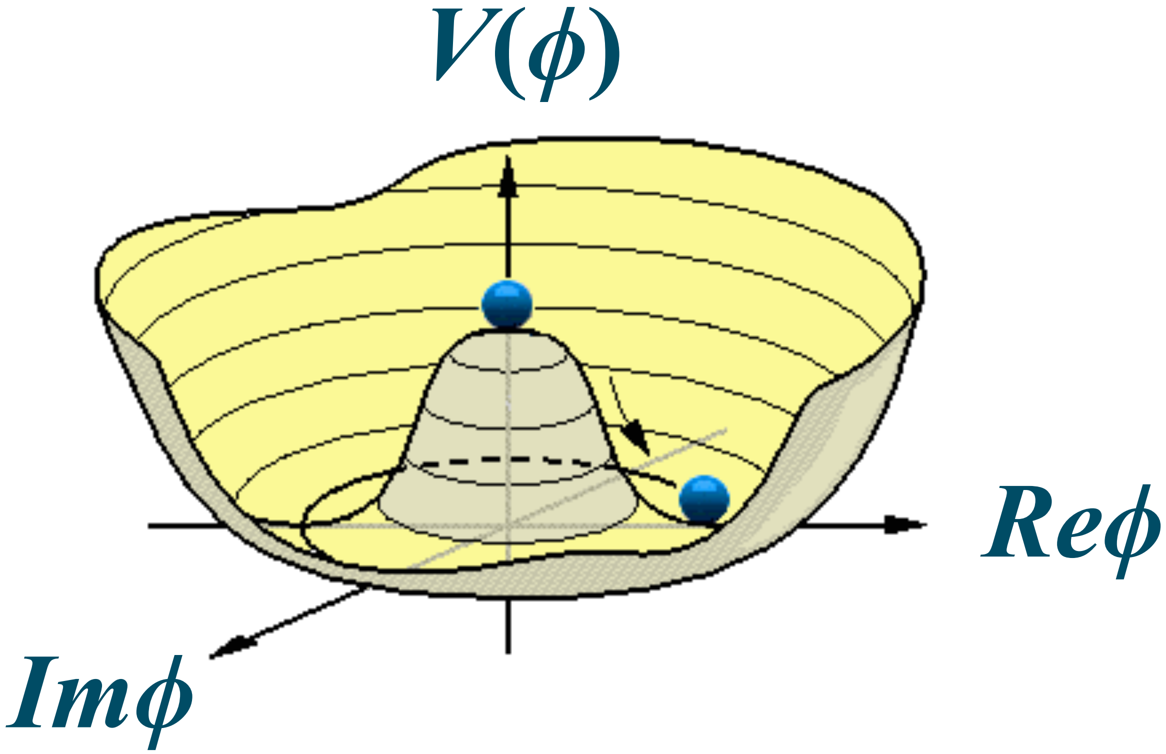
Spontaneous Symmetry Breaking



- Start with a system that has an intrinsic symmetry
  - ➡ Choosing a particular ground state configuration the symmetry is broken
  - ➡ If the choice is arbitrary, i.e. no external agent is responsible for the choice, then the symmetry is “spontaneously” broken
- Everyday example: A circle of people are sitting at a dining table with napkins between them. The first person who picks up a napkin, either with their left or right hand spontaneously breaks the L/R symmetry. All the others must do the same if everyone is to end up with a napkin.
- Physics example: In a domain inside a ferromagnet all the spins align in a particular direction. If the choice of direction is random, the underlying theory has a rotational symmetry which is spontaneously broken. The presence of an external magnetic explicitly breaks the symmetry and defines a preferred direction.

Higgs Potential

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$



- ϕ is complex function.
- $V(\phi)$ is symmetric: the maximum symmetry occurs at $\phi=0$.
- A circle of values minimise the potential at $\phi=\phi_0 \equiv -v/\sqrt{2}$ with $|\phi_0| = \frac{\mu}{\sqrt{2\lambda}}$
  - Any coordinate around the circle minimise the potential: $\arg(\phi_0) = [0,2\pi)$
- The choice of which complex value of ϕ_0 is chosen spontaneously breaks the symmetry.
- The value of v, related to the value of $|\phi|$ at the minimum of the potential known as the vacuum expectation value. Measured to be $v = 246$ GeV

Standard Model Higgs Field

- In the Standard Model, the Higgs field is a complex isospin doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad T = 1; \begin{array}{l} T_3 = +1 \\ T_3 = \ \ 0 \end{array}$$

ϕ^+ : +ve charged field

ϕ^0 : neutral field

ϕ_0 : minimum of field

- Higgs field has four degrees of freedom.

- In the Higgs mechanism (when the symmetry is spontaneously broken) three of these degrees of freedom are used to give mass to W^+, W^-, Z^0.
- This fixes three of the degrees of freedom: two charged and one neutral.

- The minimum of the potential ϕ_0 for ground state can then be written in terms of the remaining free parameter:

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Where v is related to the value of ϕ which minimises V: $v = \frac{\mu}{\sqrt{2\lambda}}$

Introducing the Higgs Boson

- Consider a fluctuation of the Higgs field about its minimum:

$$\phi(x) = \phi_0 + h(x) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + h(x) \end{array} \right)$$

- Substitute $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$ into ***V(ϕ)*** and expand to second order in ***h(x):***

$$V(\phi) = -\mu^2 \left(\frac{v+h(x)}{\sqrt{2}}\right)^2 + \lambda \left(\frac{v+h(x)}{\sqrt{2}}\right)^4 = \ldots = V(\phi_0) + \underbrace{\lambda v^2 h^2}_{= \frac{1}{2} m_H^2} + \mathcal{O}(h(x)^3)$$

- In quantum field theory a term quadratic in the field describes a particle's mass.
- This fluctuation around the minimum of the potential describes a spin-0 particle with a mass $m = \sqrt{2\lambda} v$

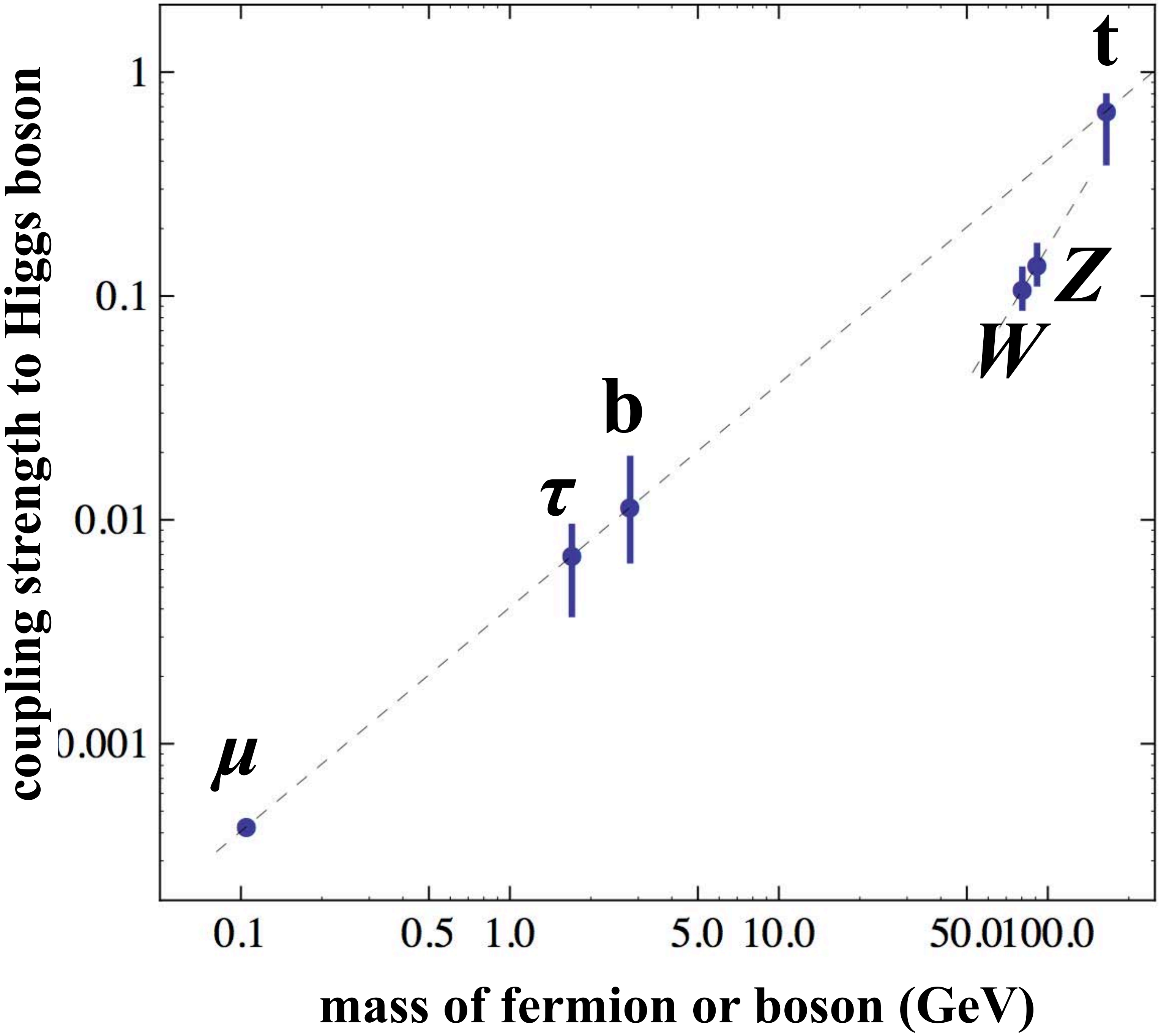
- **The Higgs boson!**

Higgs Couplings

- The Higgs mechanism predicts that the Higgs boson interacts with the *W* and *Z* bosons and massive fermions, in proportion to their mass.

- The dotted line is the prediction.

- The points are the measured values from the CMS collaboration at the LHC.



Particle Physics

Dr Victoria Martin, Spring Semester 2013

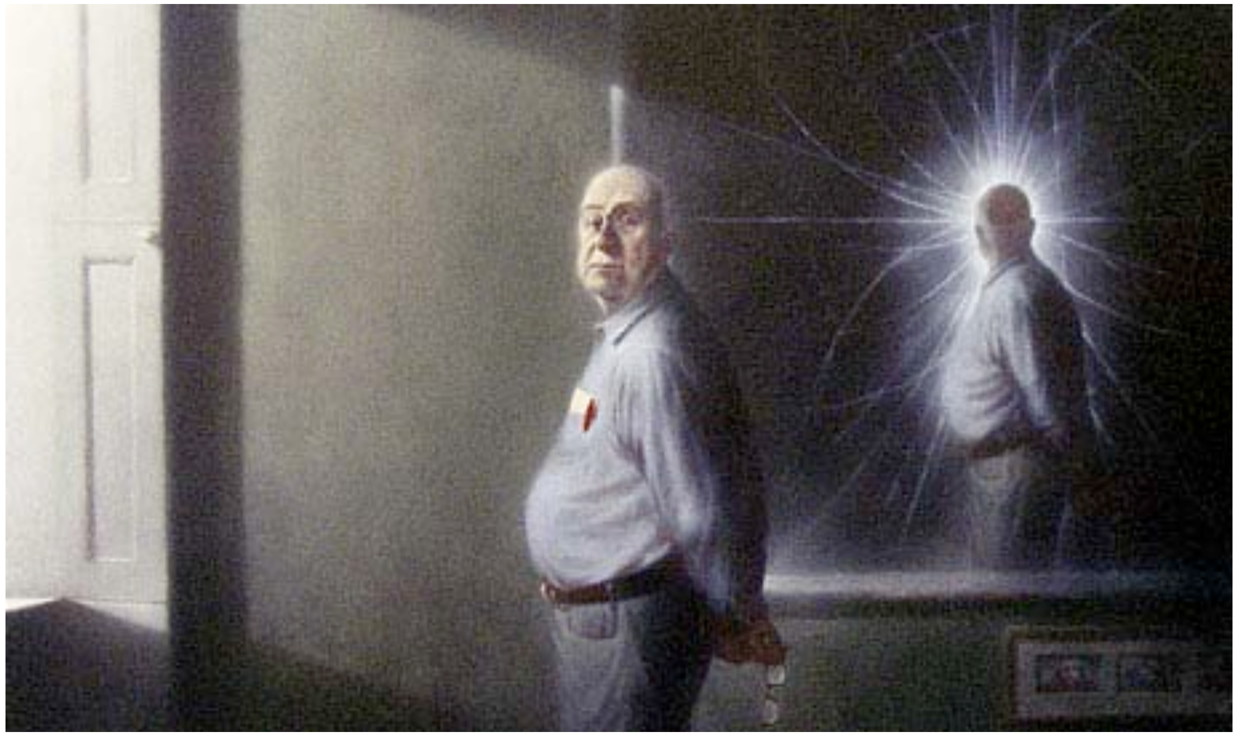
Lecture 18: Higgs

★Review of electroweak theory

★Spontaneous Symmetry Breaking

★W and Z bosons

★The Higgs mechanism and the Higgs boson



Weak Isospin and Hypercharge

- QED couples to electric charge; QCD couples to colour charge...
- Electroweak force couples to two "charges".
  - **Weak Isospin:** total and third component T, T_3. Depends on **chirality**
  - **Weak Hypercharge,** Y In terms of electric charge Q: $Y = 2\,(Q - T_3)$
    - All right-handed fermions have $T=0$, $T_3=0$
    - All left-handed fermions have $T=½$, $T_3=±½$
    - All left-handed antifermions have $T=0$, $T_3=0$
    - All right-handed antifermions have $T=½$, $T_3(\overline{f})=-T_3(f)$

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	½	+½	-1	u_L, c_L, t_L	½	+½	⅓
e_L, μ_L, τ_L	½	−½	-1	d_L, s_L, b_L	½	− ½	⅓
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	−⅔

Summary of Electroweak Unification

- **Weak Isospin:** total and third component T, T_3. Depends on **chirality**
- **Weak Hypercharge,** Y In terms of electric charge Q: $Y = 2\,(Q - T_3)$
  - ➡ We introduced an SU(2) symmetry which has three boson W^1, W^2, W^3 coupling to weak isospin with a coupling constant g_W
  - ➡ We introduced a U(1) symmetry which has one boson B^0 coupling to weak hypercharge with a coupling constant g'_W
  - ➡ Bosons mix to give the physical W^+, W^-, Z and γ bosons

$$W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2) \qquad W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2)$$

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W \qquad \gamma = W^3 \sin\theta_W + B^0 \cos\theta_W$$

  - ➡ Consistent with electromagnetism if $e = g'_W \cos\theta_W = g_W \sin\theta_W$

$$\sin^2\theta_W = \frac{g'^2_W}{g^2_W + g'^2_W}$$

All of the properties of electroweak interactions described by:

- the intrinsic charges (isospin and hypercharge) of the fermions
- the SU(2) ⊗ U(1) symmetry
- g_W and g'_W: free parameters that need to be measured

The Higgs Mechanism: Introduction

- The Higgs Mechanism was proposed in 1964 separately by Higgs and Brout & Englert.
- It introduces an extra field, ϕ, which interacts with the electroweak currents. The potential of the field is:

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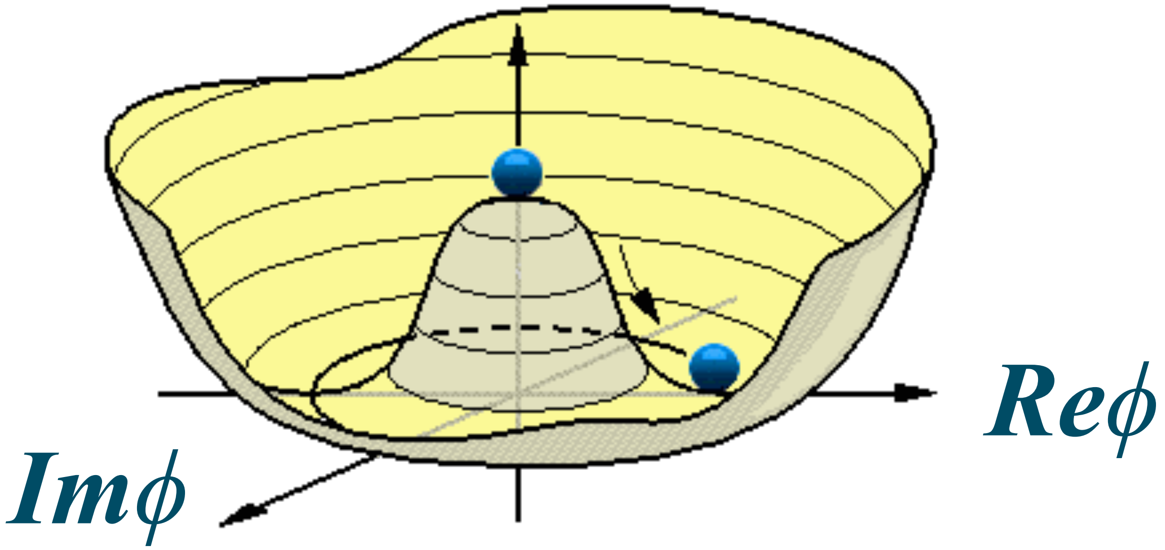
- Provides an explanation for fermion masses **(*e*, *μ*, *τ*, u, d, s, c, t, b).**
- P.W. Higgs pointed out that a further consequence would be the existence of a spin-0 boson: the Higgs boson, ***H***.

Spontaneous Symmetry Breaking



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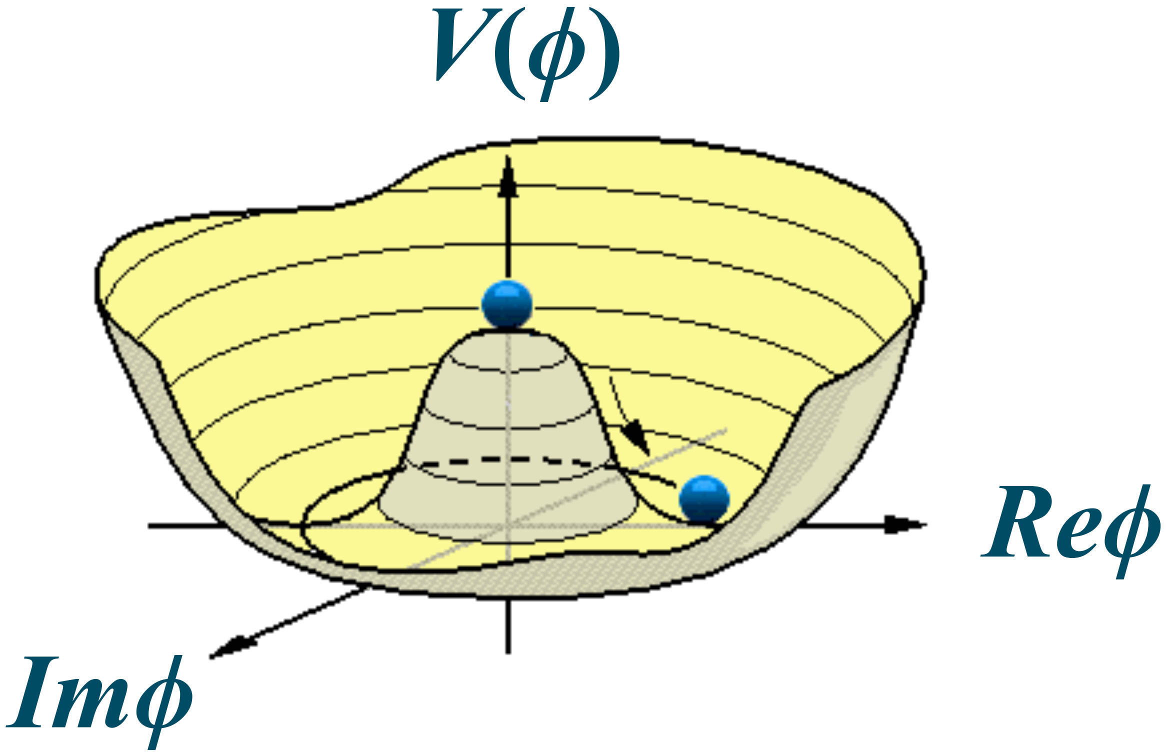

- ϕ is complex function.
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- Where v is related to the value of ϕ which minimises V: $v = \dfrac{\mu}{\sqrt{2\lambda}}$

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- Substitute $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$ into ***V(ϕ)*** and expand to second order in ***h(x):***

$$V(\phi) = -\mu^2 \left(\frac{v + h(x)}{\sqrt{2}}\right)^2 + \lambda \left(\frac{v + h(x)}{\sqrt{2}}\right)^4 = \ldots = V(\phi_0) + \underbrace{\lambda v^2}_{= \frac{1}{2} m_H^2} h^2 + \mathcal{O}(h(x)^3)$$

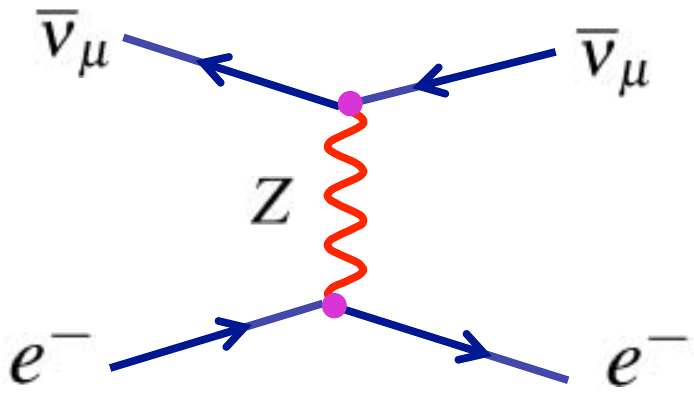
- In quantum field theory a term quadratic in the field describes a particle's mass.
- This fluctuation around the minimum of the potential describes a spin-0 particle with a mass $m = \sqrt{2\lambda} v$

- **The Higgs boson!**

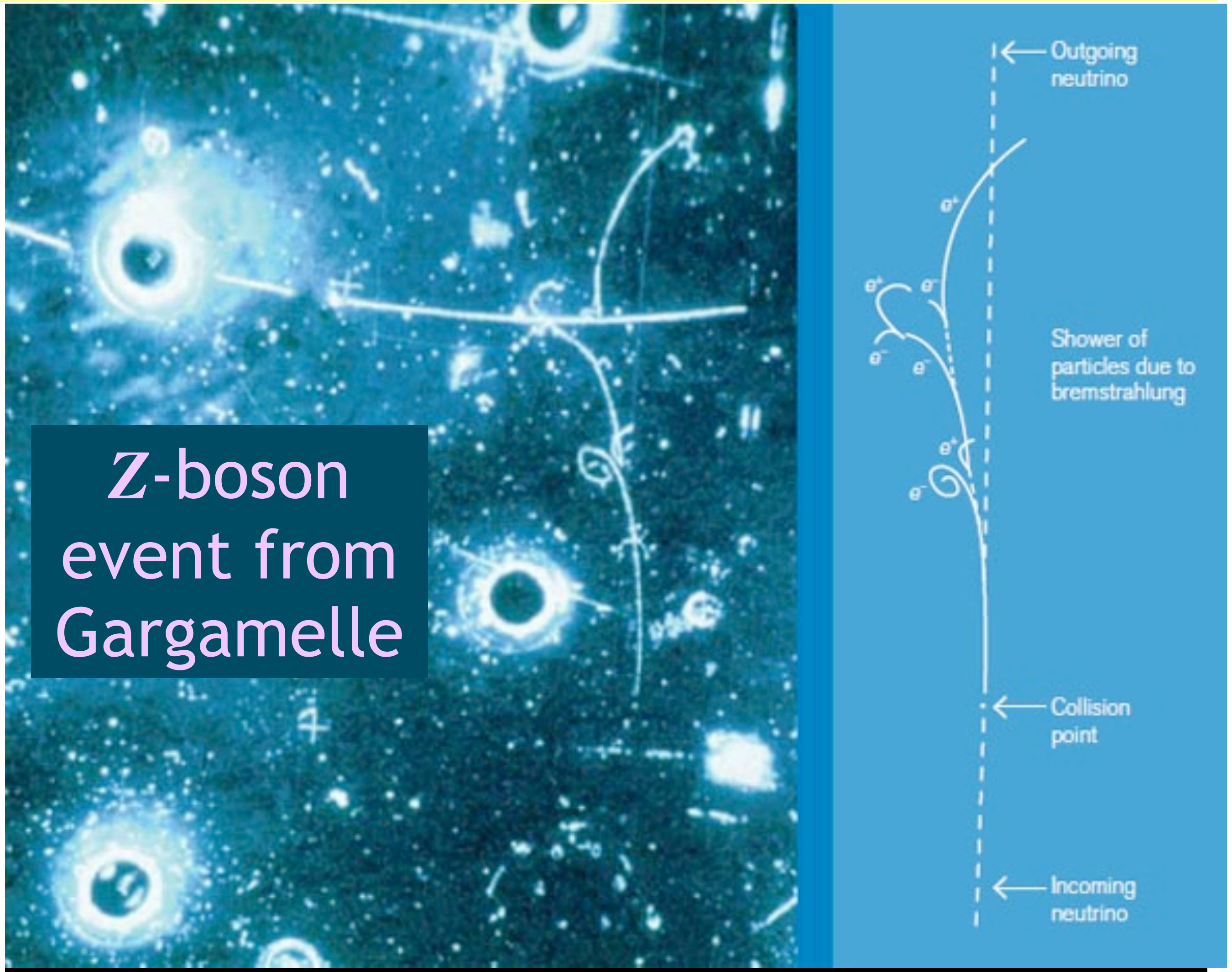
W and *Z* bosons

- *W*-boson interactions already been observed in beta decay

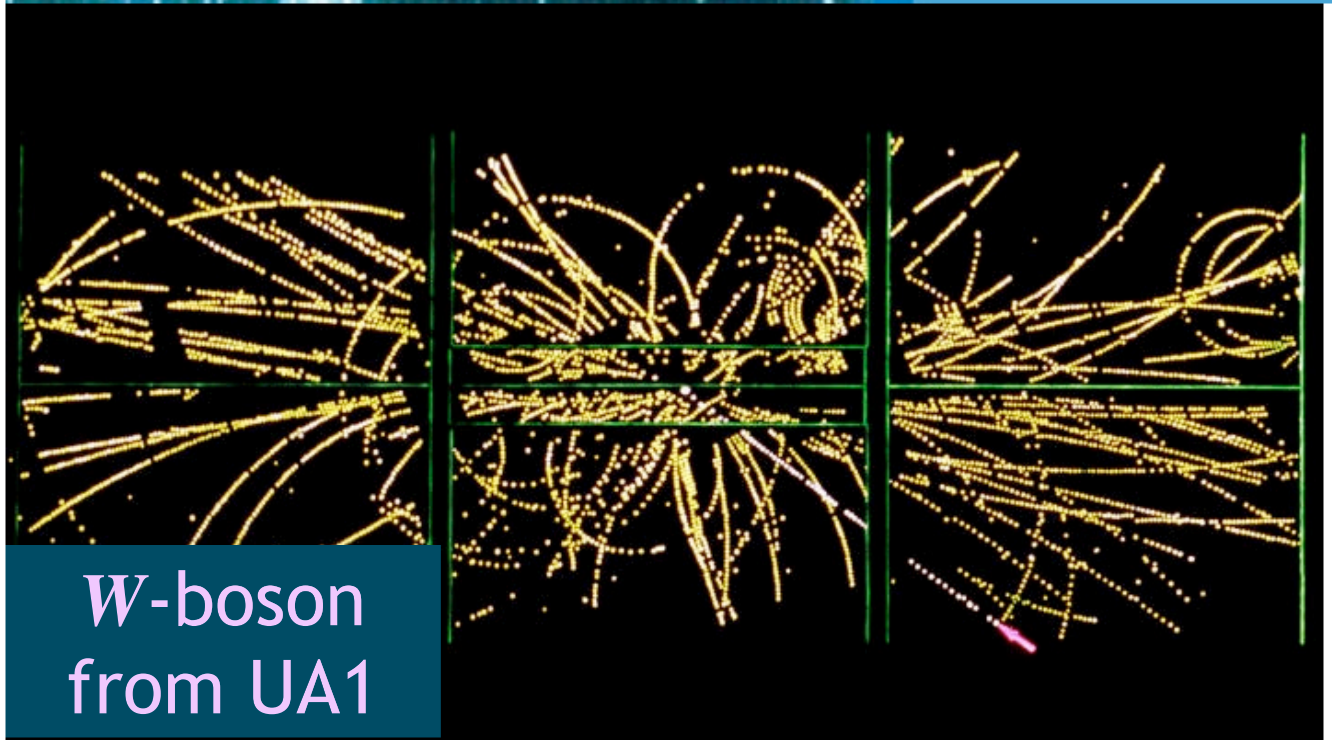
- *Z*-boson interactions were first observed 1973 at CERN Gargamelle experiment through $\overline{\nu}_\mu e^- \rightarrow \overline{\nu}_\mu e^-$



- Real (as opposed to virtual) *W* and *Z* bosons were first observed 1983 at UA1 and UA2 experiments at CERN in $p\overline{p} \rightarrow WX$, $p\overline{p} \rightarrow ZX$ (Noble prize 1984)

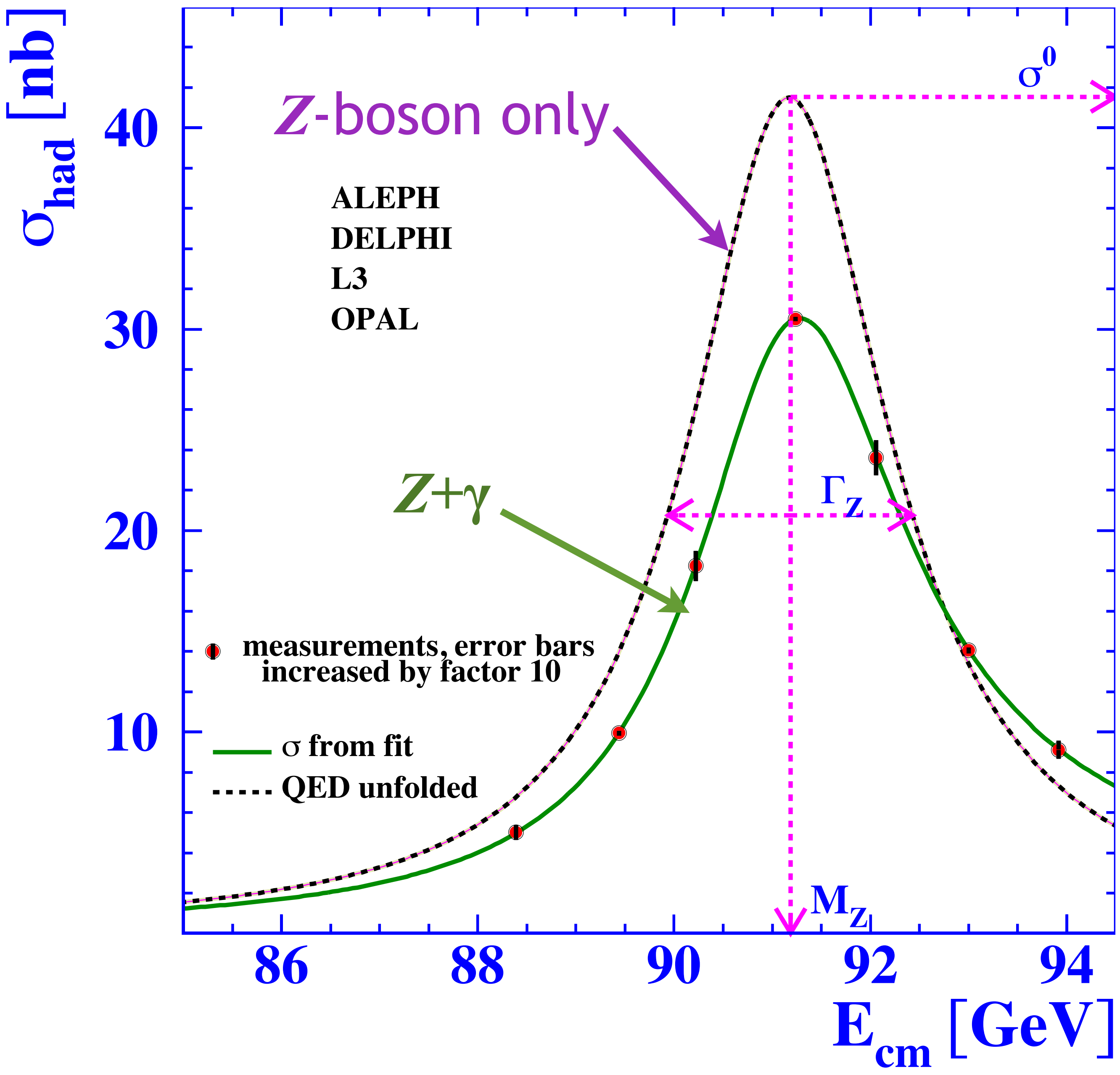


Z-boson event from Gargamelle



W-boson from UA1

W and *Z*-boson measurements



- Z^0 bosons studied at Large Electron Positron (LEP) Collider at CERN.
- Approximately ~1M events at $\sqrt{s}\sim m_Z$: $e^+e^-\rightarrow Z^0\rightarrow f\overline{f}$
- Measured mass and total width:
  - $m_Z = 91.1876 \pm 0.0021$ GeV
  - $\Gamma_Z = 2.4952(23)$ GeV

- *W* bosons studied at LEP and Tevatron collider at Fermilab
- Measured mass and total width:
  - $m_W = 80.385 \pm 0.015$ GeV
  - $\Gamma_W = 2.085 \pm 0.042$ GeV

Higgs Coupling to Bosons

- Non-rigorous arguments for the W and Z boson masses.
- Consider the interactions between the minimum of Higgs field and W and B bosons:

$$
\begin{aligned}
(g_W W^a \tau^a + g'_W B^0)\begin{pmatrix} 0 \\ v \end{pmatrix} &= \left(g_W \begin{pmatrix} W^3 & W^1 - iW^2 \\ W^1 + iW^2 & -W^3 \end{pmatrix} + g'_W B^0 \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= v g_W (W^1 - iW^2) + v(-g_W W^3 + g'_W B^0) \\
&= \sqrt{2} v g_W \left(\frac{W^1 - iW^2}{\sqrt{2}} \right) - v\sqrt{g_W^2 + g'^2_W}\,(W^3 \cos\theta_W - B^0 \sin\theta_W) \\
&= \underbrace{\sqrt{2} v g_W}_{=2\sqrt{2}\,m_W}\, W^+ - \underbrace{v\sqrt{g_W^2 + g'^2_W}}_{=2m_Z}\, Z^0
\end{aligned}
$$

τ^a: three Pauli matrices

Extra factor of √2 for W-bosons as there's two of them: W^+, W^-

$$m_W = \frac{v\, g_W}{2} \qquad m_Z = \frac{1}{2} v \sqrt{g_W^2 + g'^2_W}$$

- Measured masses are $m_W = 80.385 \pm 0.015$ **GeV** and $m_Z = 91.1876 \pm 0.0021$ **GeV**.
- Implies: $\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g'^2_W}} = \frac{m_W}{m_Z}$

$\alpha \sim 1/128$ at $E \sim m_Z$

- Using $e = g'_W \cos\theta_W = g_W \sin\theta_W = \sqrt{4\pi\alpha} \sim \sqrt{\frac{4\pi}{128}}$ gives **v=246 GeV**

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 19: Physics at the Large Hadron Collider

- ★Higgs couplings to Fermions
- ★Higgs branching ratios
- ★Hadron Collider physics
- ★Higgs boson discovery

Electroweak & Higgs Mechanism

- Three parameters are needed to describe all electroweak and Higgs physics phenomena.
  - e.g. Coupling constants: g_W, g'_W and the vacuum expectation value **v**
- The Higgs mechanism gives mass to the ***W*** and ***Z*** bosons through their couplings with the Higgs field

$$m_W = \frac{v\,g_W}{2} \qquad m_Z = \frac{1}{2} v \sqrt{g_W^2 + g_W'^2} \qquad e = g'_W \cos\theta_W = g_W \sin\theta_W$$

- Other useful combinations:

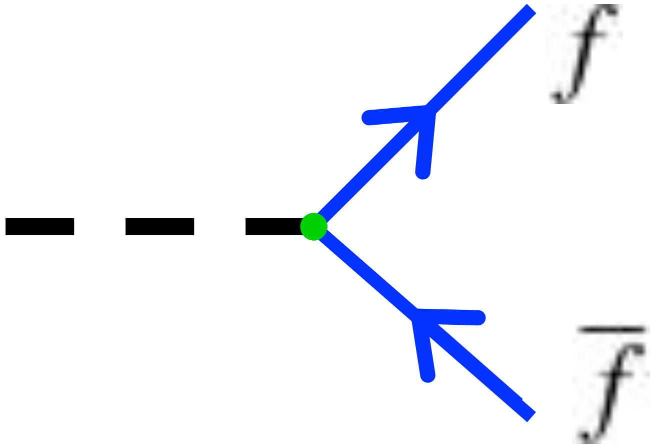
$$\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g_W'^2}} = \frac{m_W}{m_Z} \qquad G_F = \frac{\sqrt{2}}{8}\frac{g_W^2}{m_W^2} = \frac{1}{\sqrt{2}\,v^2}$$

Higgs Couplings to Fermions

- The Higgs field also couples to all of the fermions, ***f***. Induces a term coupling incoming and outgoing fermions spinors to **v = 246 GeV**.

$$g_f(\bar{f}f)v = g_f(\bar{f}_{\rm L}f_{\rm R} + \bar{f}_{\rm R}f_{\rm L})v$$

- g_f is a unknown coupling constant, and can be different for each fermion.

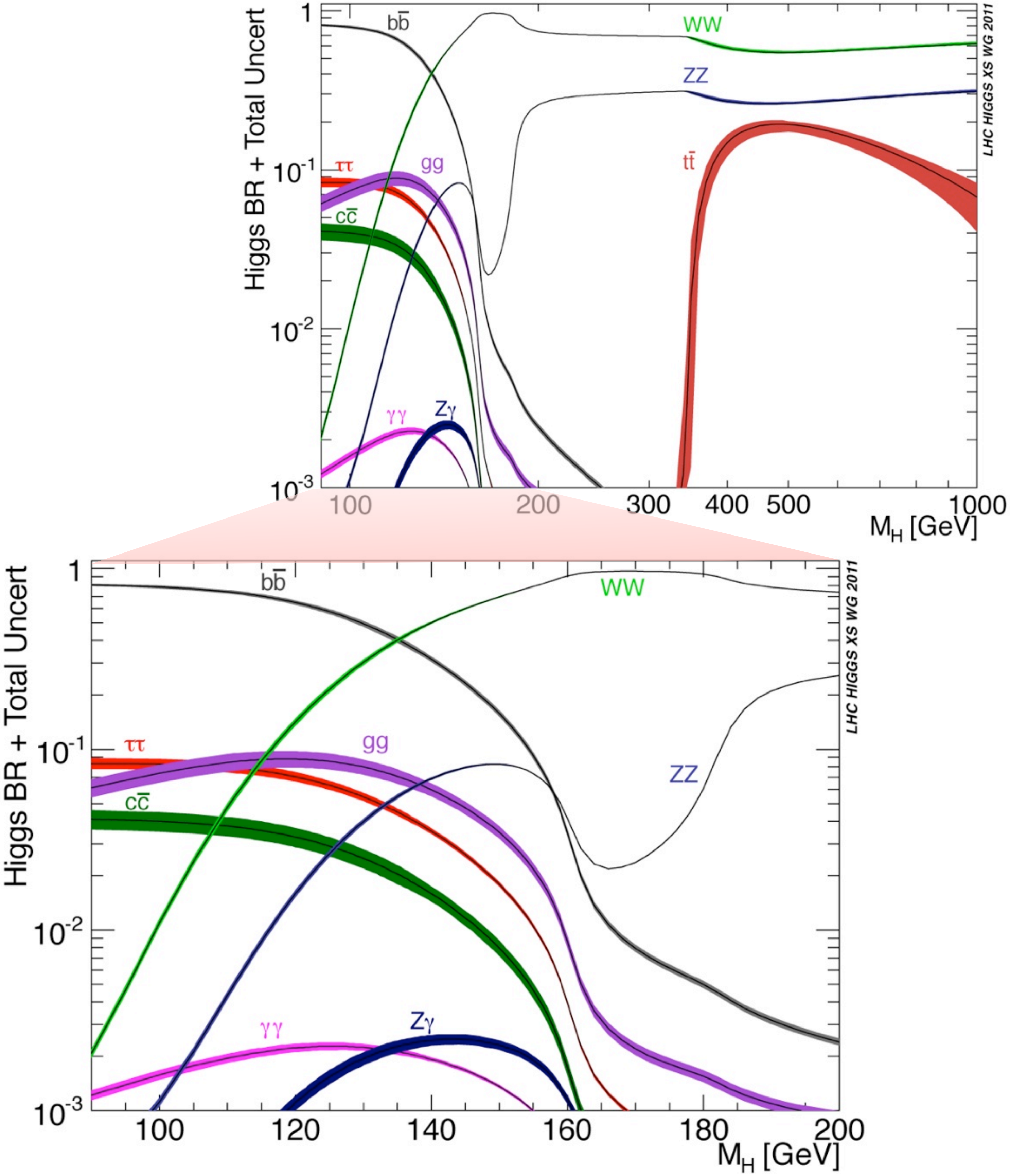


- Fermion wavefunctions are coupled to **v**, a quantity with mass dimension.
- We interpret (g_f **v**) as the mass of the fermion.
- The Higgs mechanism explains the masses of all the quarks and charged leptons! (Maybe neutrinos too!?)

Higgs Boson Decay

- In the Standard Model the Higgs boson decays preferentially to the heaviest thing it can.
- (Factor of ½ if the final state particles are identical e.g. ZZ)

- for $m_H = 125\ \mathrm{GeV}/c^2$
  - $\mathrm{BR}(H\to b\bar{b}) = 57.7\%$
  - $\mathrm{BR}(H\to W^+W^-) = 22.3\%$
  - $\mathrm{BR}(H\to \tau^+\tau^-) = 6.3\%$
  - $\mathrm{BR}(H\to c\bar{c}) = 2.9\ \%$
  - $\mathrm{BR}(H\to ZZ) = 2.6\%$
  - $\mathrm{BR}(H\to \gamma\gamma) = 0.23\%$
  - $\mathrm{BR}(H\to Z\gamma) = 0.15\%$
  - $\mathrm{BR}(H\to \mu^+\mu^-) = 0.022\%$



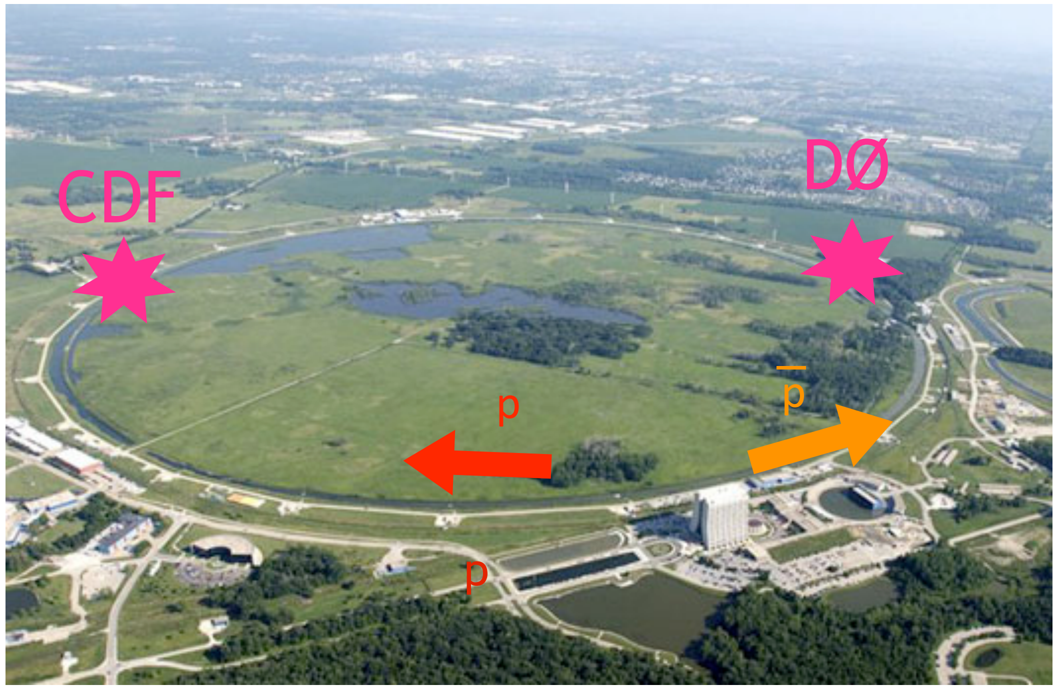
Hadron Colliders

- **Sp$\bar{p}$S**: Super Proton anti-Proton Synchrotron at CERN
- 1981 - 1984, 6.9 km, $\sqrt{s} = 400$ GeV
- Two experiments: UA1 and UA2
- Tunnel now used for pre-acceleration for LHC

Nobel Prize for Physics 1984

Carlo Rubbia and Simon van der Meer, from CERN

“For their decisive contributions to large projects, which led to the discovery of the field particles W and Z, communicators of the weak interaction.”



- **TeVatron** at Fermilab, near Chicago
- Proton anti-proton collider, 6.3 km
- Run 1: 1987 - 1995 $\sqrt{s}= 1.80$ TeV
- Run 2: 2000 - 2011 $\sqrt{s} = 1.96$ TeV
- Two experiments: CDF and DØ
- Highlight: discovery of the top quark!

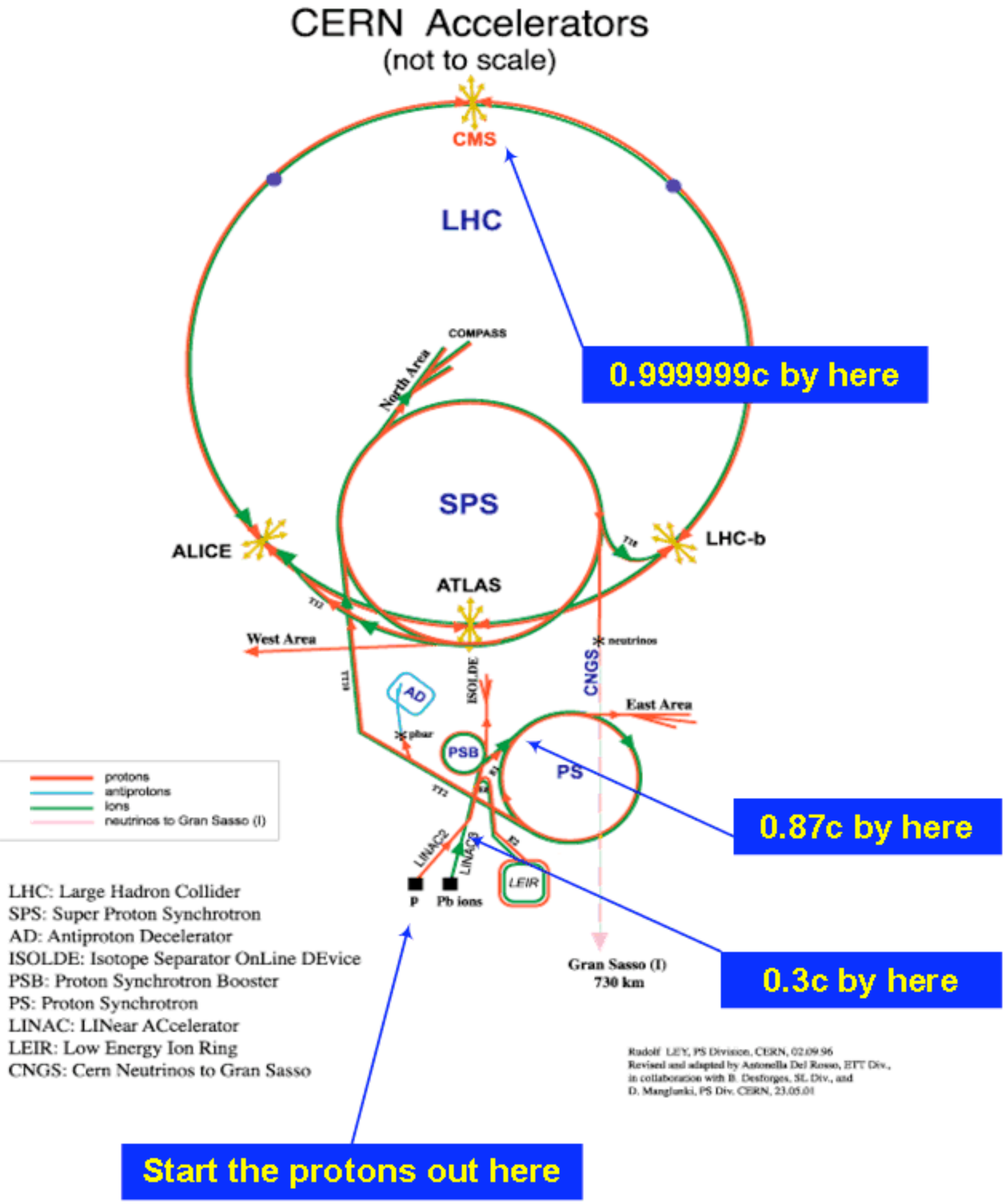
The Large Hadron Collider (LHC)

- At CERN
- Proton-proton collider, $\sqrt{s} = 7$ to 14 TeV
- 2009 – 202X

- Relies on network of accelerators

- Four collision points: ATLAS, CMS, LHCb, ALICE

- CMS & ATLAS: general purpose detectors: observation of highest energy collisions
- LHCb: specialist experiment looking at b-hadrons
- ALICE: specialist experiment looking at Pb ion collisions



CERN Accelerators
(not to scale)

LHC: Large Hadron Collider
SPS: Super Proton Synchrotron
AD: Antiproton Decelerator
ISOLDE: Isotope Separator OnLine DEvice
PSB: Proton Synchrotron Booster
PS: Proton Synchrotron
LINAC: LINear ACcelerator
LEIR: Low Energy Ion Ring
CNGS: Cern Neutrinos to Gran Sasso

Rudolf LEY, PS Division, CERN, 02.09.96
Revised and adapted by Antonella Del Rosso, ETT Div.,
in collaboration with B. Desforges, SL Div., and
D. Manglunki, PS Div. CERN, 23.05.01

0.999999c by here

0.87c by here

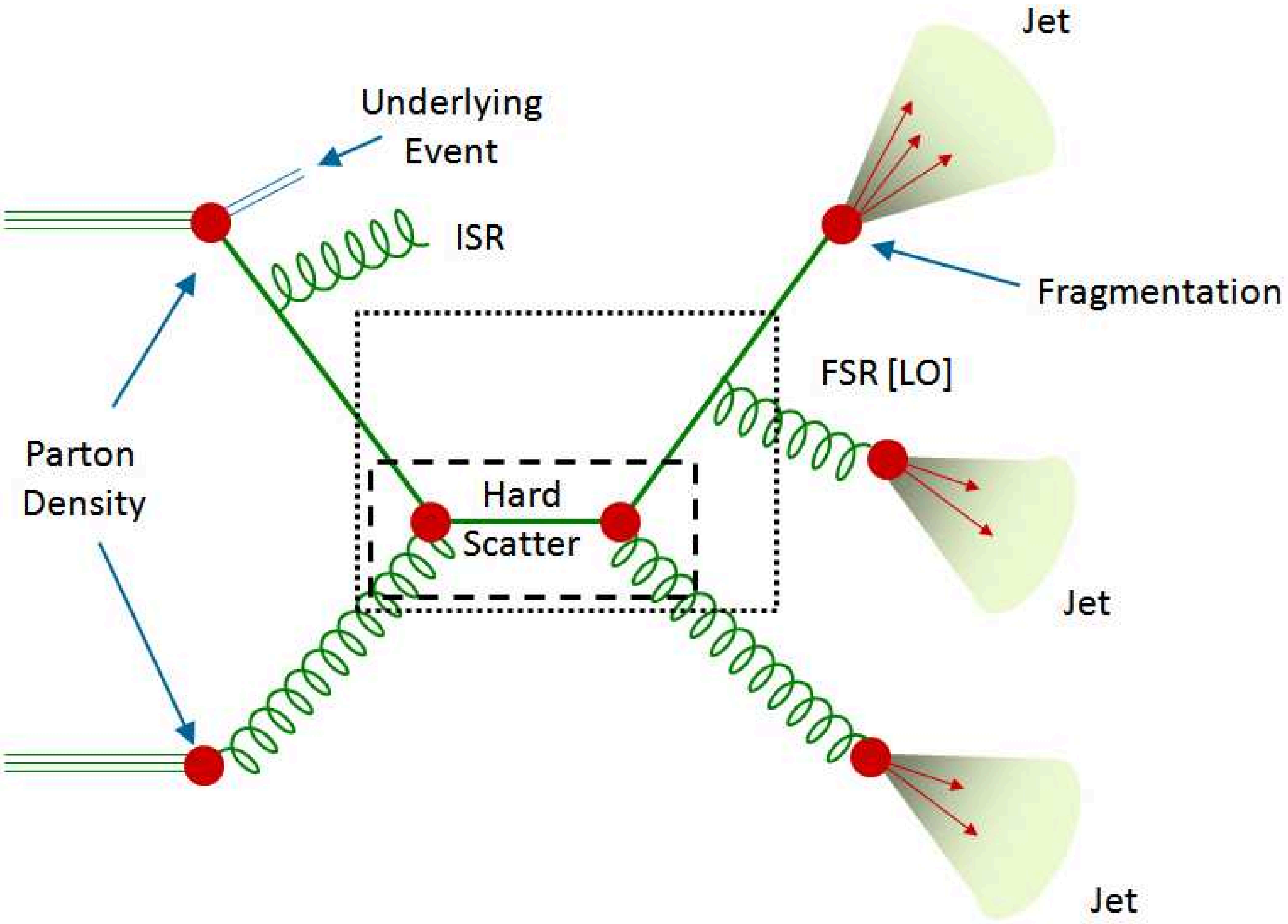
0.3c by here

Start the protons out here

The Large Hadron Collider: Facts

- 27km circumference
- 50 to 175 m underground
- 9300 magnets used to keep the beam in orbit
- 1232 dipole magnets
- Ultra-high vacuum of 10^{-10} Torr, equivalent to 1,000 km above earth
- Each dipole magnet: 14.3 m long, runs at 1.9 K, provides 8.3 Tesla, cost 500 kCHF
- four 400 MHz power supplies accelerate the beams from 450 GeV to collision energy
- Design energy: 7 TeV per beam
- Currently running at 4 TeV per beam

QCD production at Hadron Colliders



- Much more complicated due initial state hadrons not being fundamental particles
- Every object is colour charged: all object can interact with each other.
- QCD is very strong
- Not able to use perturbation theory to describe the interactions with low four momentum transfer q.

Hadron Collider Dictionary

- The **hard scatter** is an initial scattering at high q^2 between partons (gluons, quarks, antiquarks).
- The **underlying event** is the interactions of what is left of the protons after parton scattering.
- **Initial and final state radiation** (ISR and FSR) are high energy gluon emissions from the scattering partons.
- A hadronic **jet** is a collimated cone of particles associated with a final state parton, produced through fragmentation.
- Transverse quantities are measured transverse to the beam direction.

- An event with high **transverse momentum** (p_T) jets or isolated leptons, is a signature for the production of high mass particles ($W,Z,H,\mathbf{t}$).
- An event with **missing transverse energy** (E_T) is a signature for neutrinos, or other missing neutral particles.

Measuring Jets

- A jet has a four-momentum $E = \sum_i E_i \quad \vec{p} = \sum_i \vec{p}_i$
  - ➡ Where the constituents (*i*) are hadrons detected as charged tracks and neutral energy deposits.

- Transverse momentum of jet:

$$p_T^{\mathrm{JET}} = \sqrt{p_x^2 + p_y^2}$$

- Position in the detector in two coordinates:
  - ➡ ***Pseudorapidity*** of jet (*η*) $\eta^{\mathrm{JET}} = -\ln\left(\tan\frac{\theta}{2}\right)$

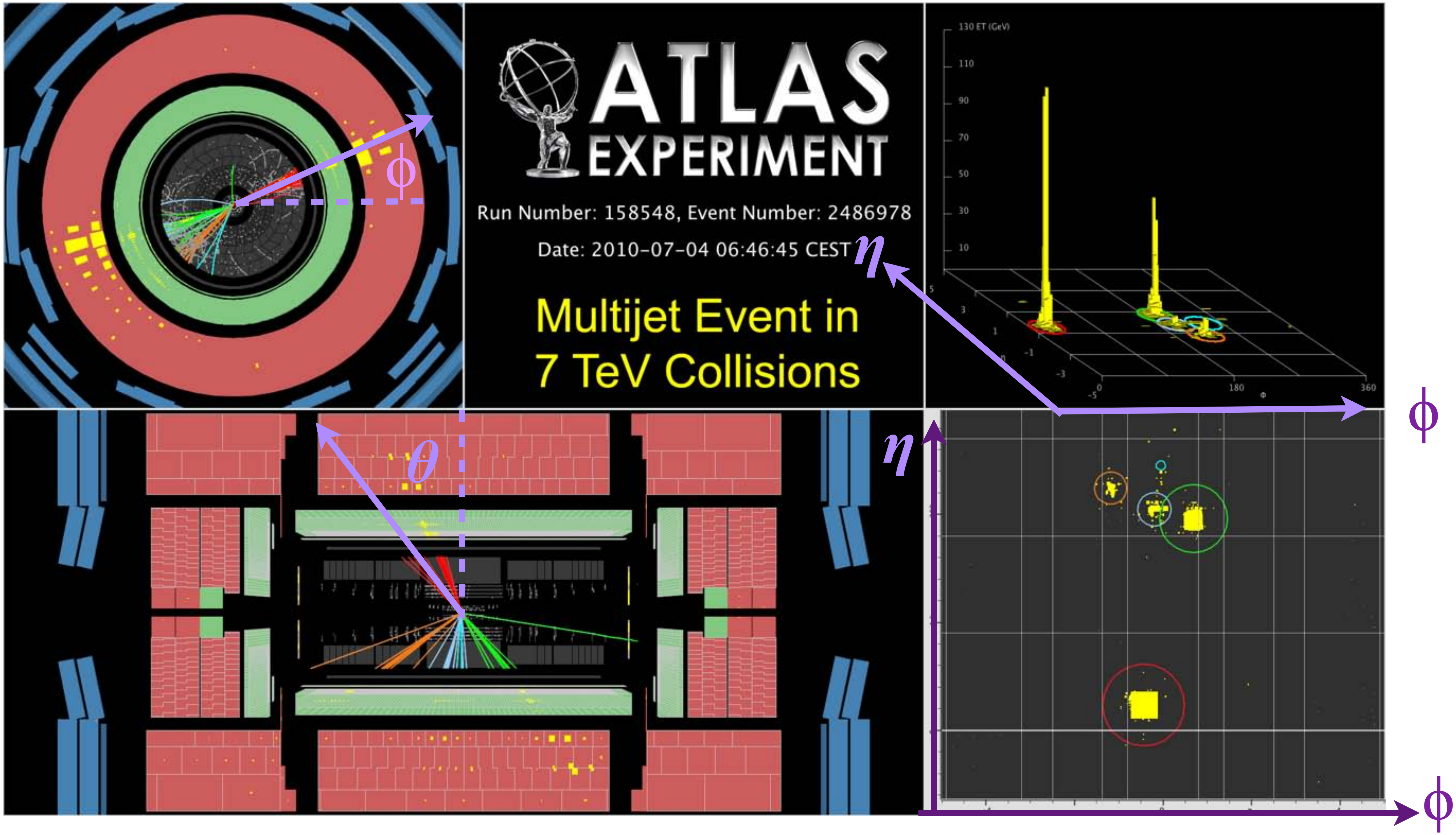
    with polar angle, *θ* $\cos\theta = \frac{\sqrt{p_x^2+p_y^2}}{p_z}$
  - ➡ **Azimuthal angle** of jet (ϕ) $\phi^{\mathrm{JET}} = \tan^{-1}\left(\frac{p_y}{p_x}\right)$

- To assign individual constituents to the jet, simplest algorithm is to define a **cone** around a central value: η^{JET}, ϕ^{JET}.

$$R^2 = (\eta_i - \eta^{\mathrm{JET}})^2 + (\phi_i - \phi^{\mathrm{JET}})^2$$

- All objects with *R* less than a given value (typically 0.4 or 0.7) are assigned to the jet
- Many sophisticated jet clustering algorithms exist which take into account QCD effects.

ATLAS Multijet Event



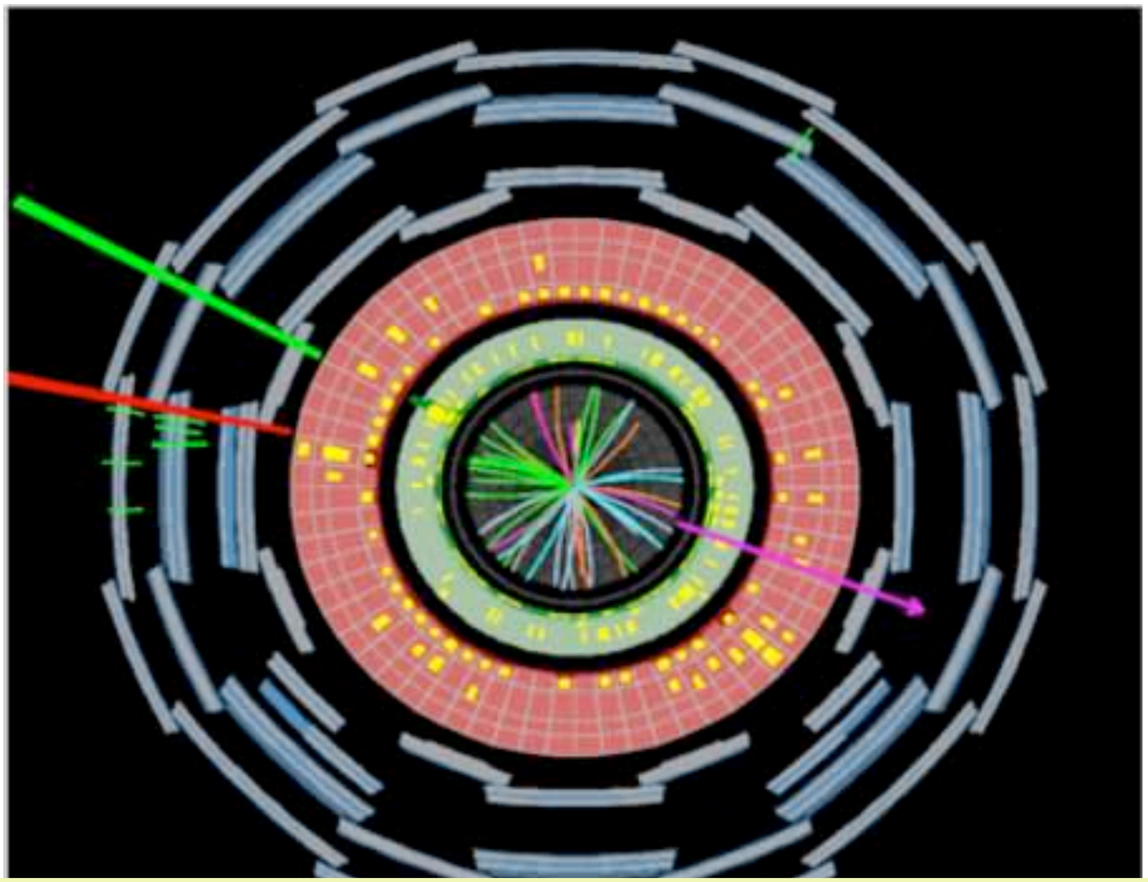
- η and ϕ act as map of activity in the detector

Detecting Neutrinos

- Neutrinos do not interact with any of the detector material.
- Their presence in the detector can be inferred from the lack of momentum balance transverse to the beam, known as **missing transverse energy** (E_T).

$$\sum_{\text{initial}} p_x = \sum_{\text{final}} p_x = 0 \qquad \sum_{\text{initial}} p_y = \sum_{\text{final}} p_y = 0$$

- In an LHC collision, the momentum of the interacting partons is along the beam direction (e.g. the z-direction)
- If measured p_x and p_y don't sum to zero, the presence of a neutrino is inferred.
- The Σp_x and Σp_y of the total number of neutrinos in an event is measured.
  - Very sensitive to measurements effects, such as detector inefficiency.
- This doesn't work in the beam direction, as the momentum of the individual partons colliding is not known for each event.



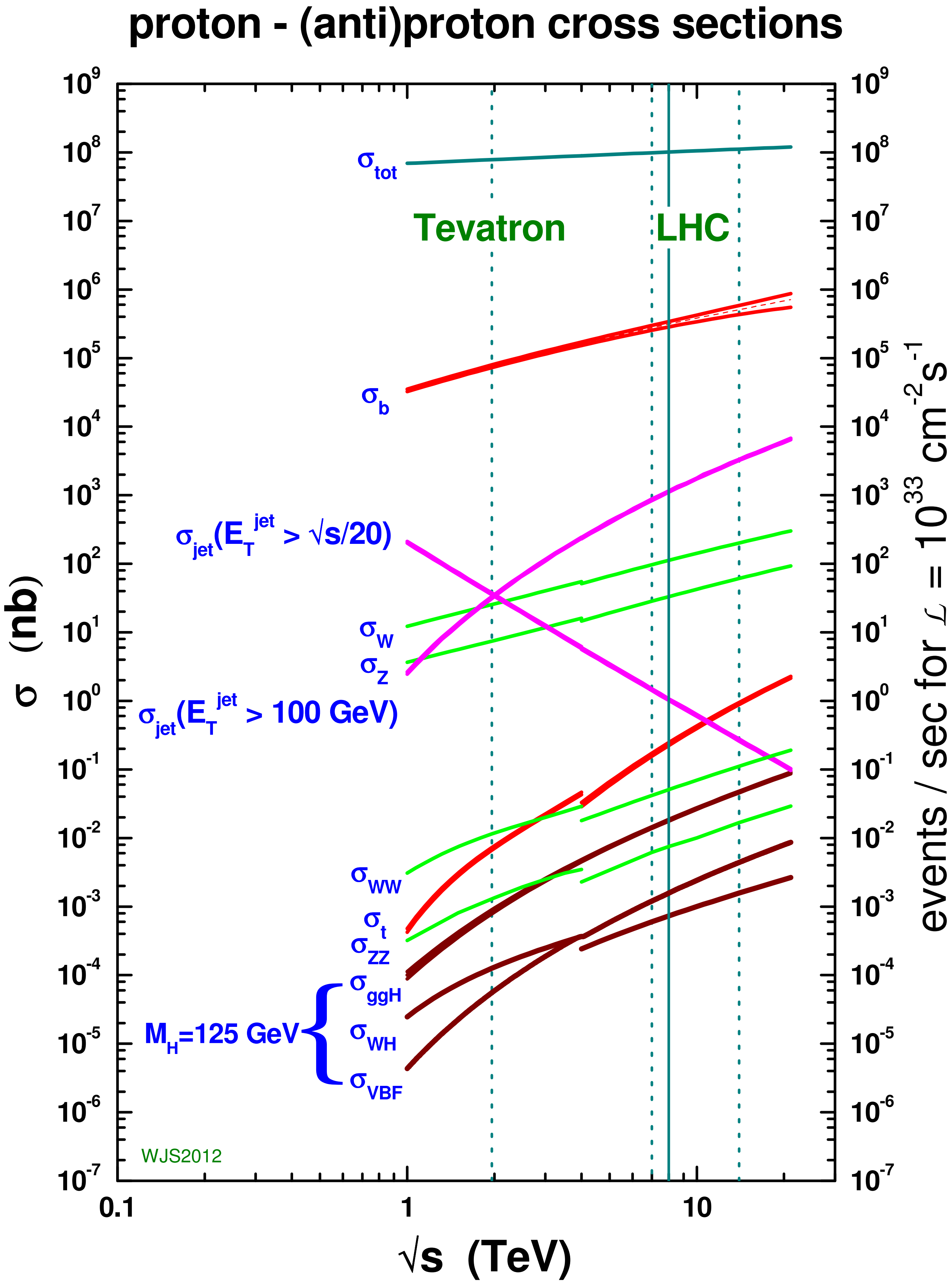
Red and green are reconstructed electron and muon

Pink is the inferred direction of the ν_e and ν_μ

LHC Collisions

- In 2012 the LHC run at **4 TeV** per beam
- 10^{11} protons in each bunch
- two bunches collide every **50 ns**

- Large backgrounds from non-Higgs processes



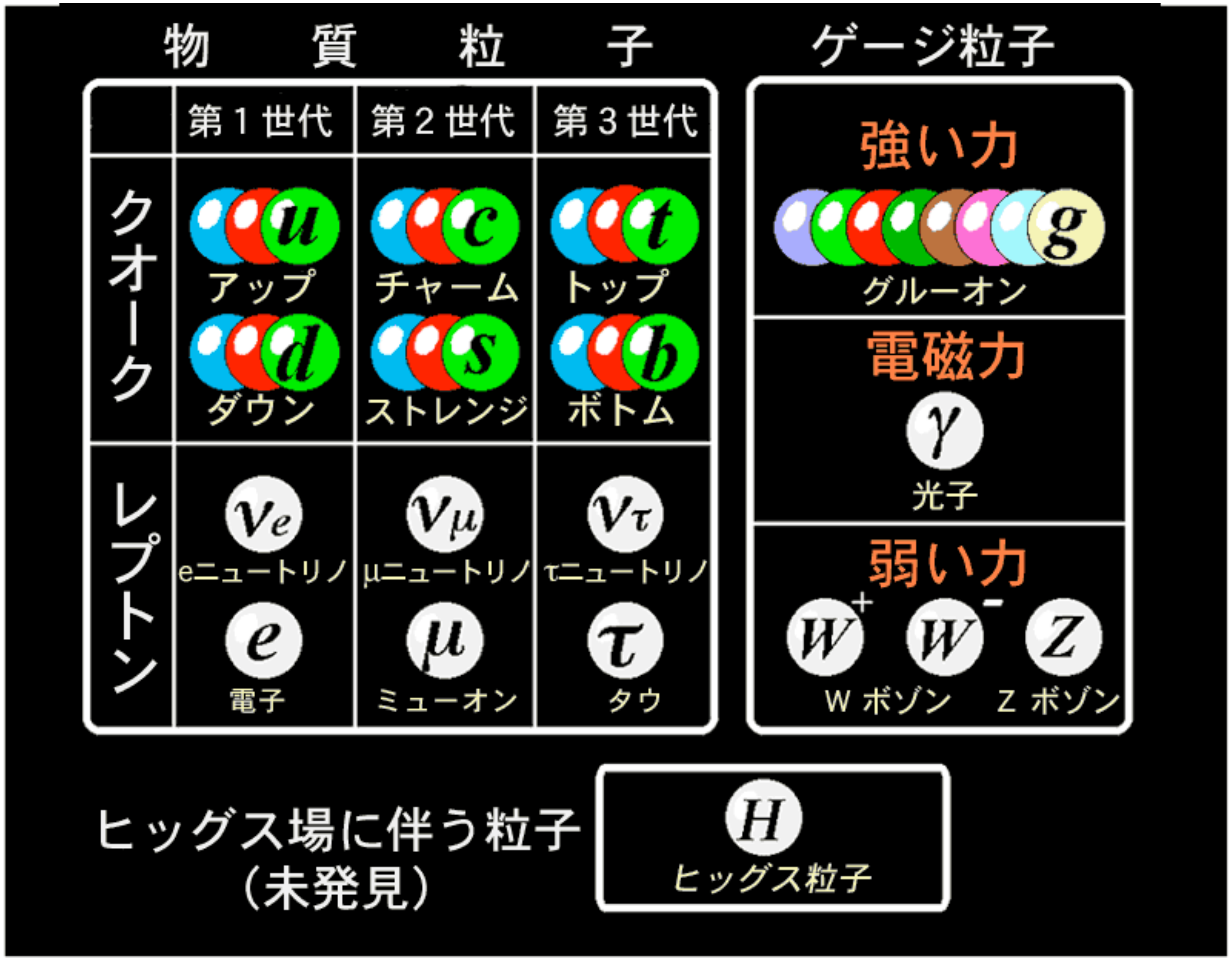
https://twiki.cern.ch/twiki/pub/AtlasPublic/HiggsPublicResults//Hgg-FixedScale-Short2.gif

https://twiki.cern.ch/twiki/pub/AtlasPublic/HiggsPublicResults//4l-FixedScale-NoMuProf2.gif

Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 20: The Final Lecture



- ★Higgs boson discovery
- ★Test of the Standard Model
- ★Supersymmetry...

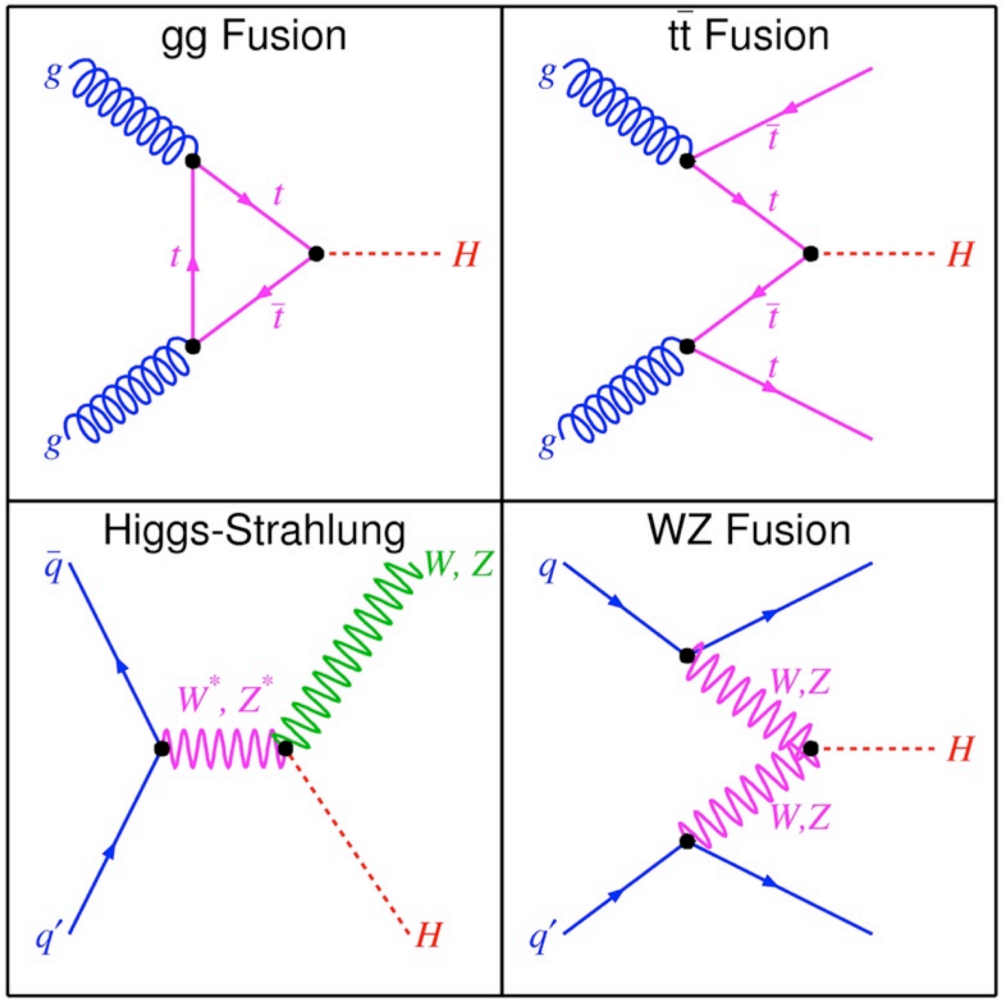
- Today is the last lecture

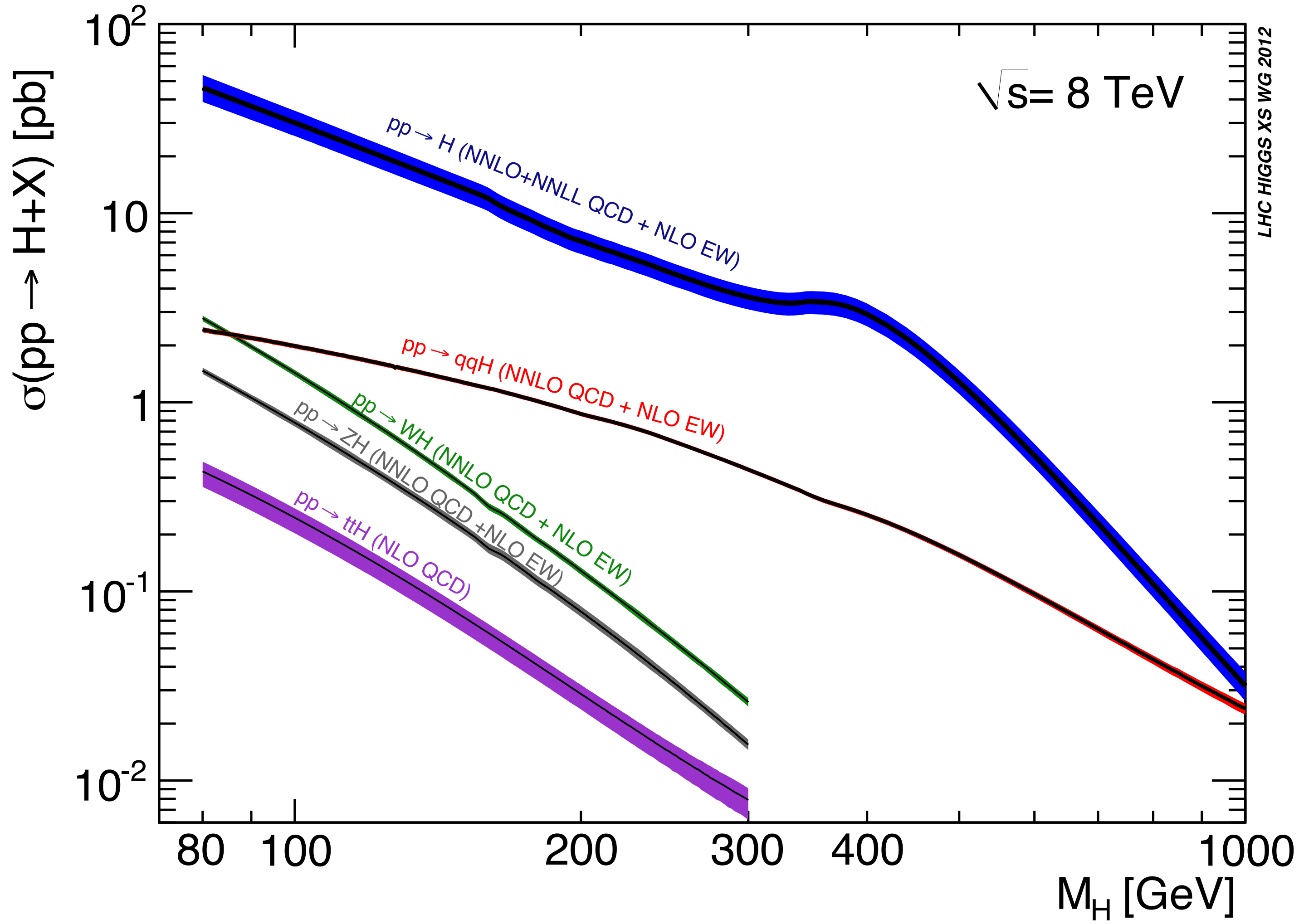
- Revision Session:
    - With me: week of 22 - 26 April??
    - With a tutor week of 13 - 17 May??
    - Both??

- I will make a complete, corrected copy of the notes and study guide for the webpage, and email to let you know it's ready.

Higgs Boson Production at the LHC

- Four main production modes at the LHC:
  - 2 induced by (virtual) top quarks
  - 2 induced by W or Z bosons
- Different production modes have different backgrounds and are used in different searches





Higgs Boson Discovery

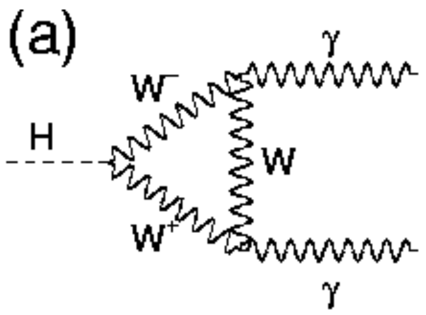
- The Higgs boson was discovered by ATLAS and CMS in July 2012 with a mass, $m_H \sim 125\ \mathrm{GeV}$
- Main search modes:
  - $H \rightarrow b\bar{b}$ $H \rightarrow W^+W^-$ $H \rightarrow \tau^+\tau^-$ $H \rightarrow ZZ$ $H \rightarrow \gamma\gamma$
- The W and Z bosons are searched for through their decays to leptons:
  - $Z \rightarrow \mu\mu$ (3.3%) $Z \rightarrow ee$ (3.3%)
  - $W \rightarrow \mu\nu$ (10%) $W \rightarrow e\nu$ (10%)
- The invariant mass of the decay products is used to look Higgs boson production. Expect a ~gaussian bump at m_H
- The following results are all taken from the ATLAS experiment. The CMS experiment have similar results.

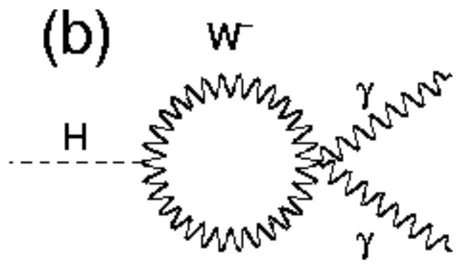
http://atlas.ch/

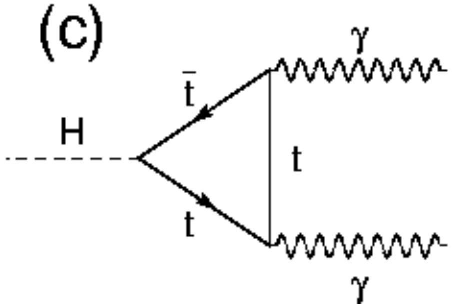
http://cms.web.cern.ch/

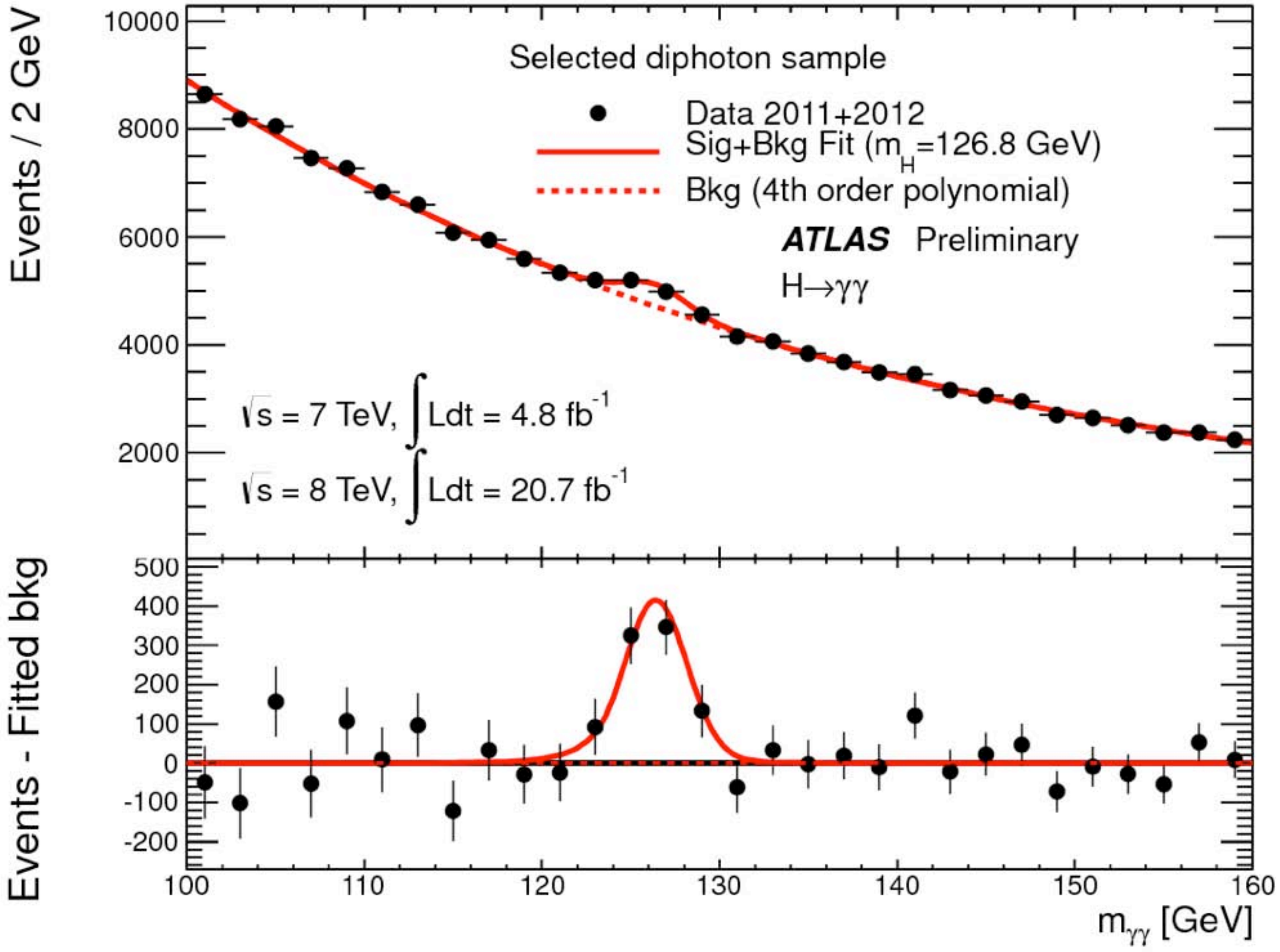
Higgs Boson Searches: $H \rightarrow \gamma\gamma$

- $H \rightarrow \gamma\gamma$ proceeds through loop diagrams
- The search uses all Higgs boson production modes: $gg \rightarrow H$ is the largest
- Look for diphoton invariant mass bump

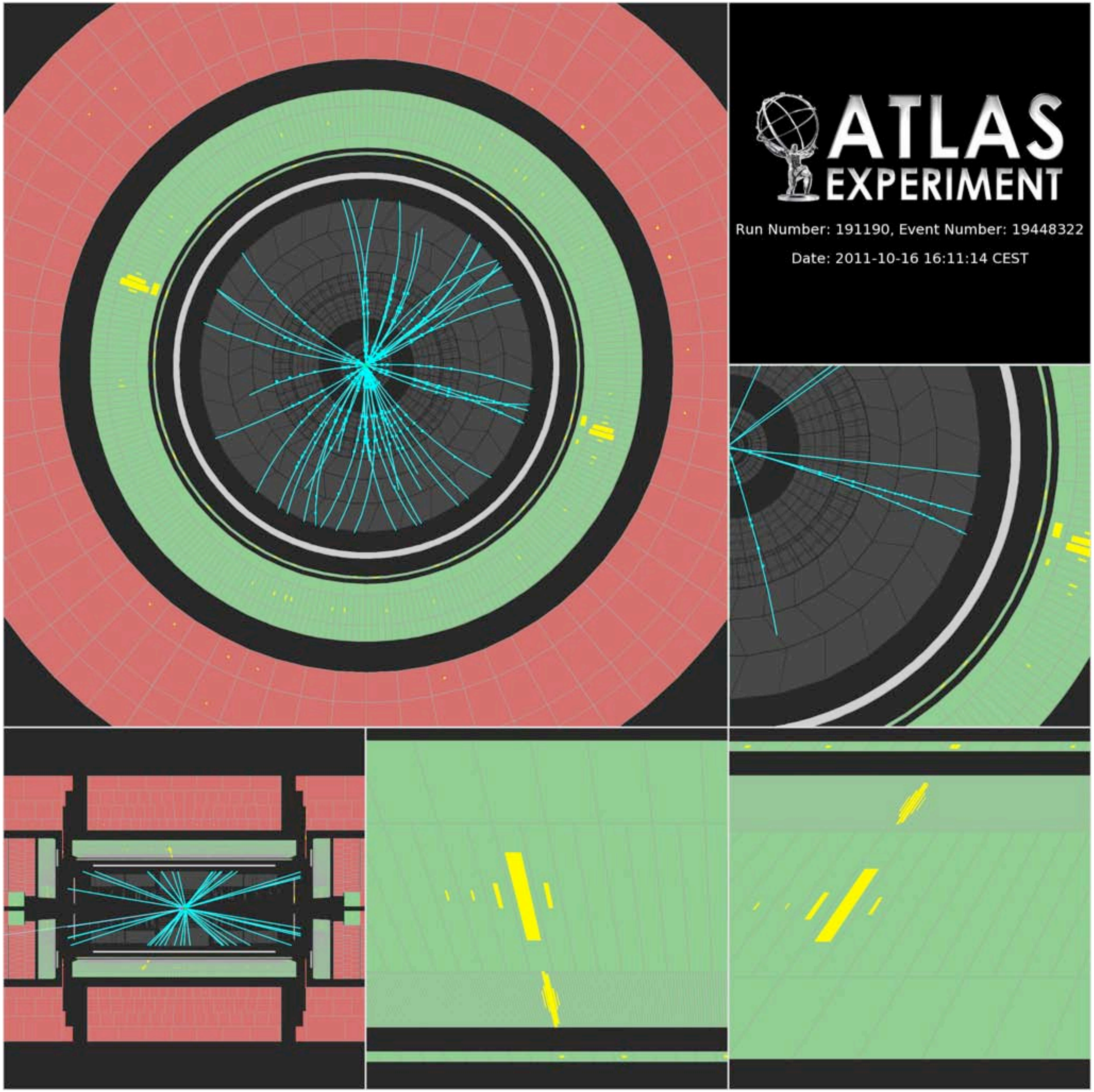






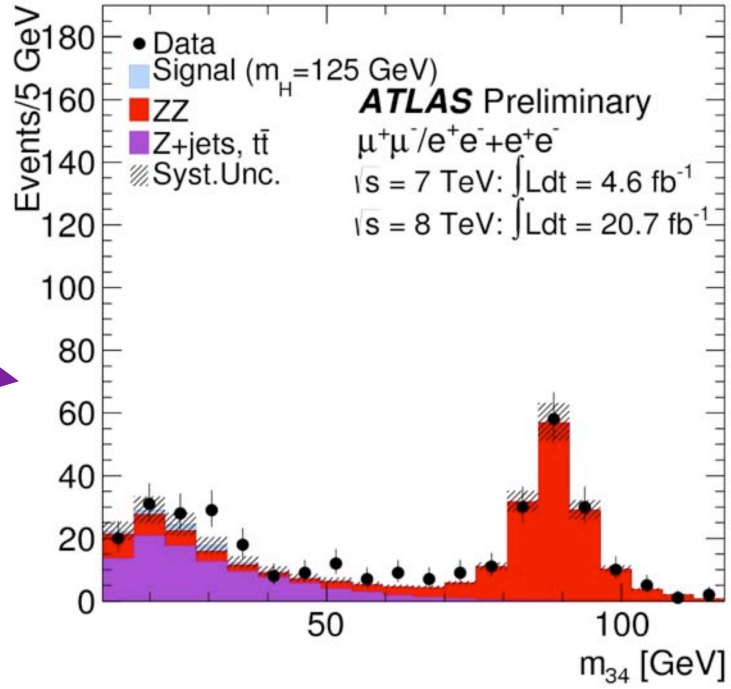


$H \rightarrow \gamma\gamma$ Candidate Event

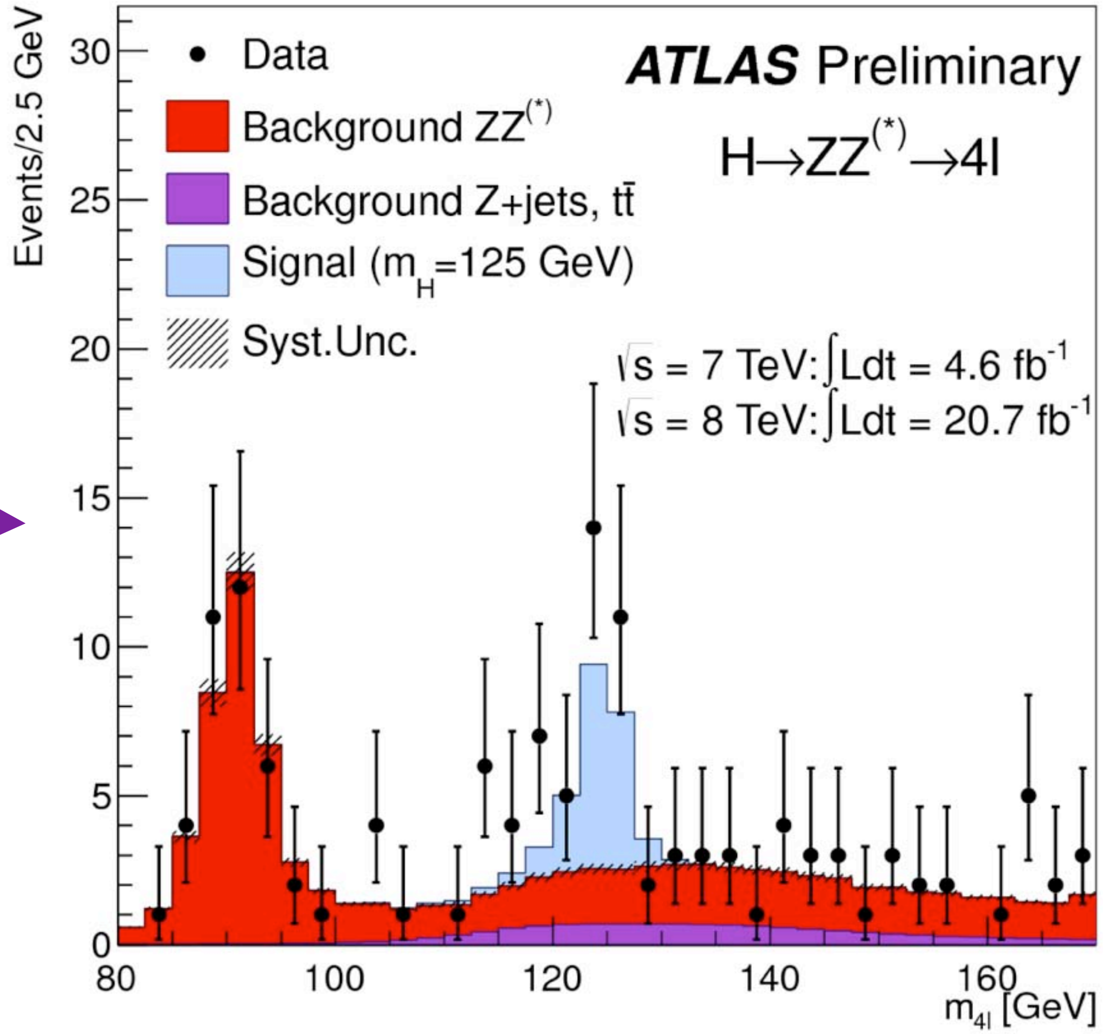


Higgs Boson Searches: $H \to ZZ^*$

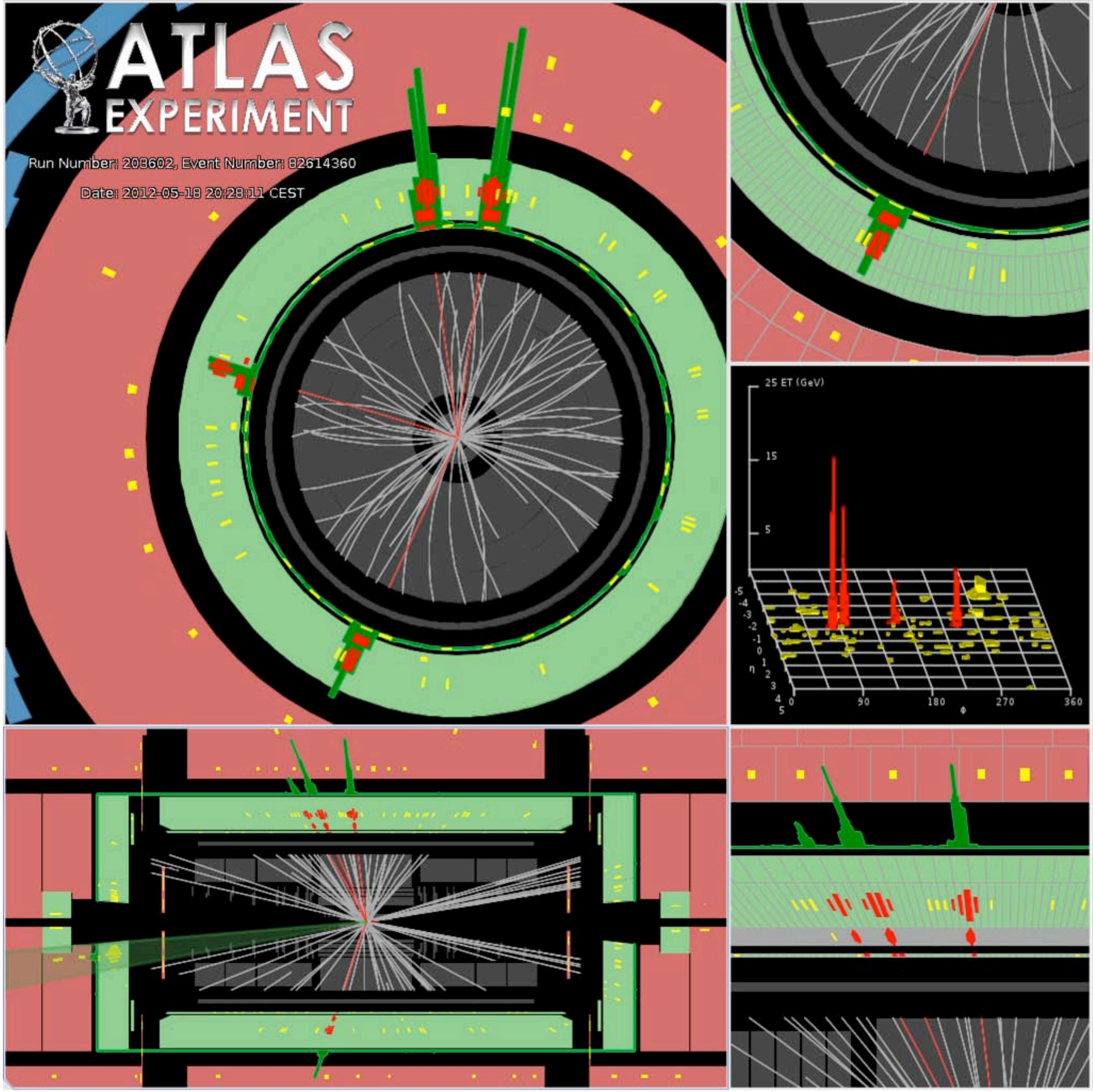
- At $\mathbf{m_H \sim 125\ GeV}$ at least one of the Z bosons must be virtual.



- Z boson are searched for through their decays to leptons:
  - ➡ $Z\to\mu\mu$ (3.3%) $Z\to ee$ (3.3%)
- Four final states: $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, $\mu^+\mu^-\mu^+\mu^-$
- Look for bump in m(**4 leptons**) invariant mass distribution

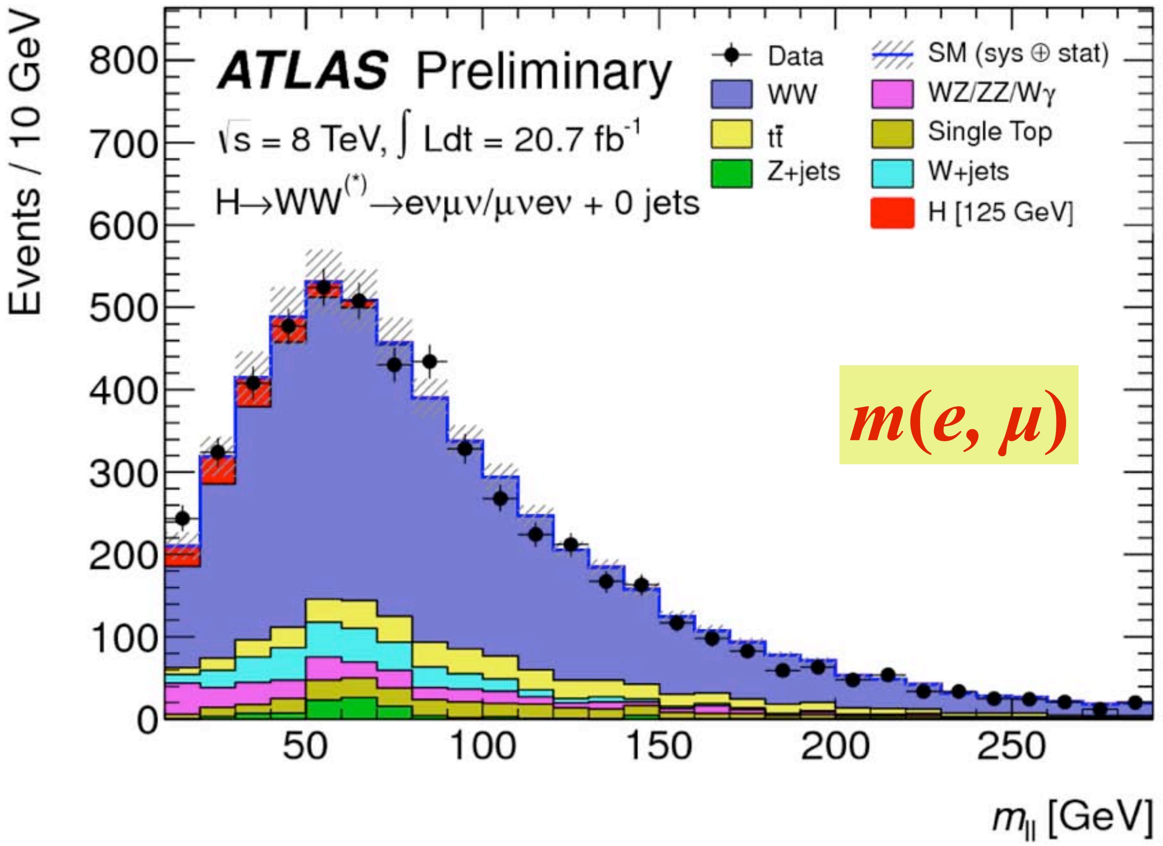


$H\rightarrow ZZ^*\rightarrow e^+e^-e^+e^-$ candidate event



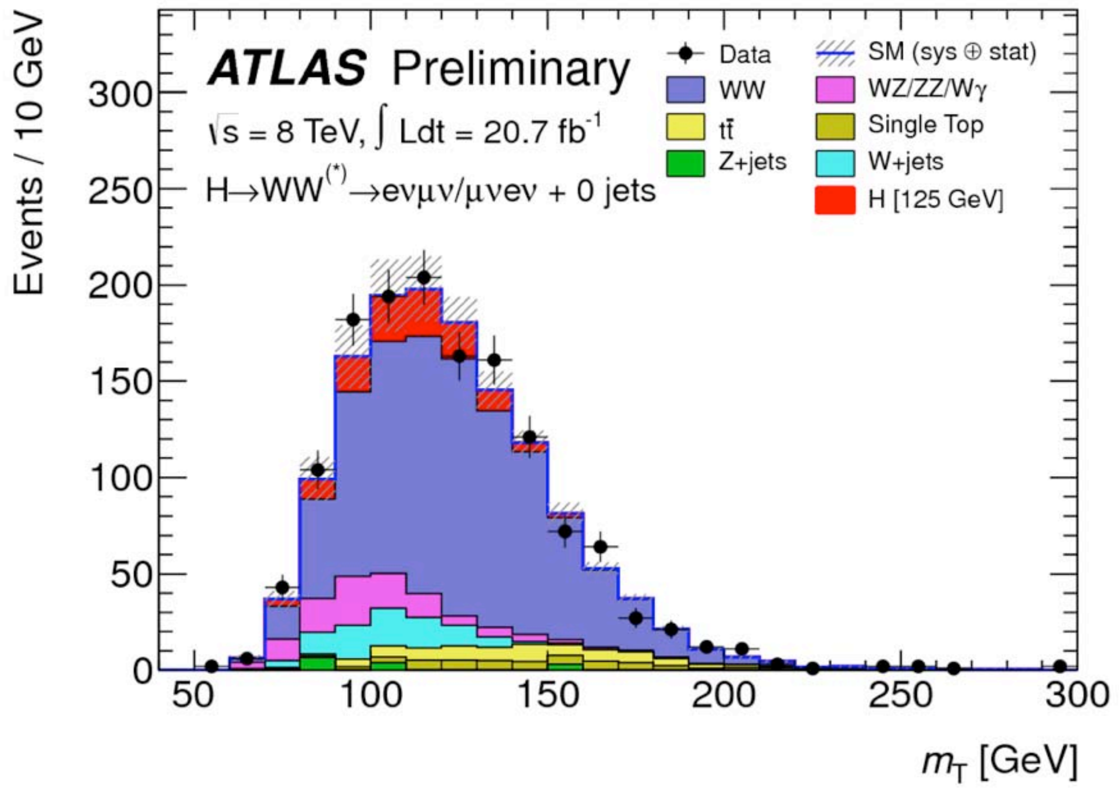
Higgs Boson Searches: $H \to W^+W^-$

- W bosons are searched for through their decays to leptons:
    - $W \to \mu\nu$ (10%) $W \to e\nu$ (10%)
- The individual momentum of the two neutrinos cannot be measured: cannot reconstruct the mass of the Higgs boson.
- Instead examine other combinations of the measured momentum of the charged leptons (e, μ) and the missing transverse energy.

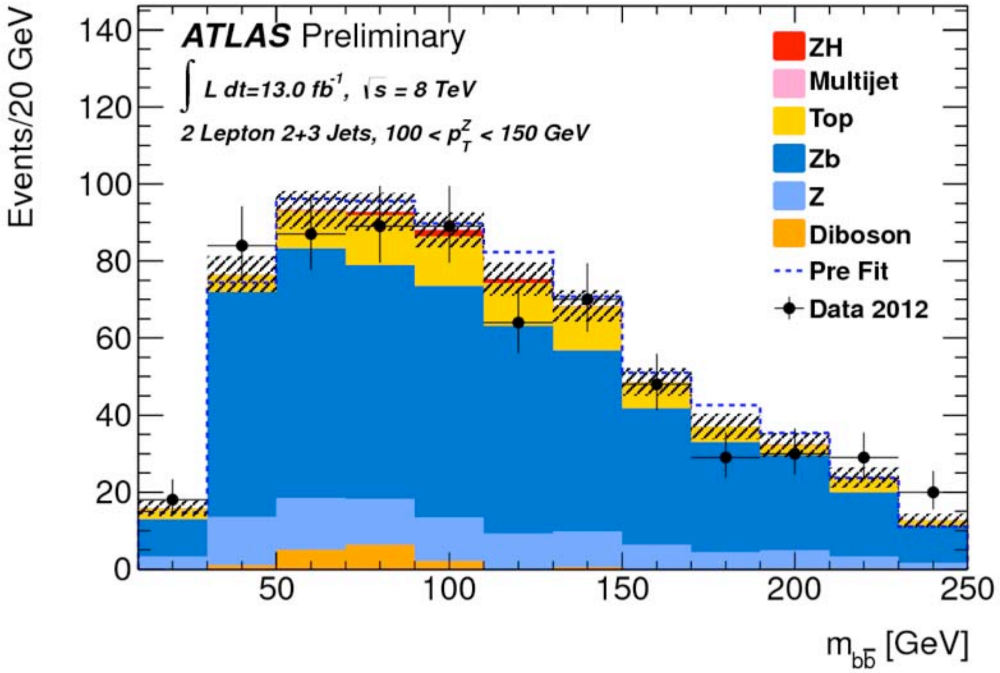


$m(e, \mu)$

$\sqrt{\{(p_x^{(e,\,\mu,\,MET)})^2 + (p_y^{(e,\,\mu,\,MET)})^2\}}$



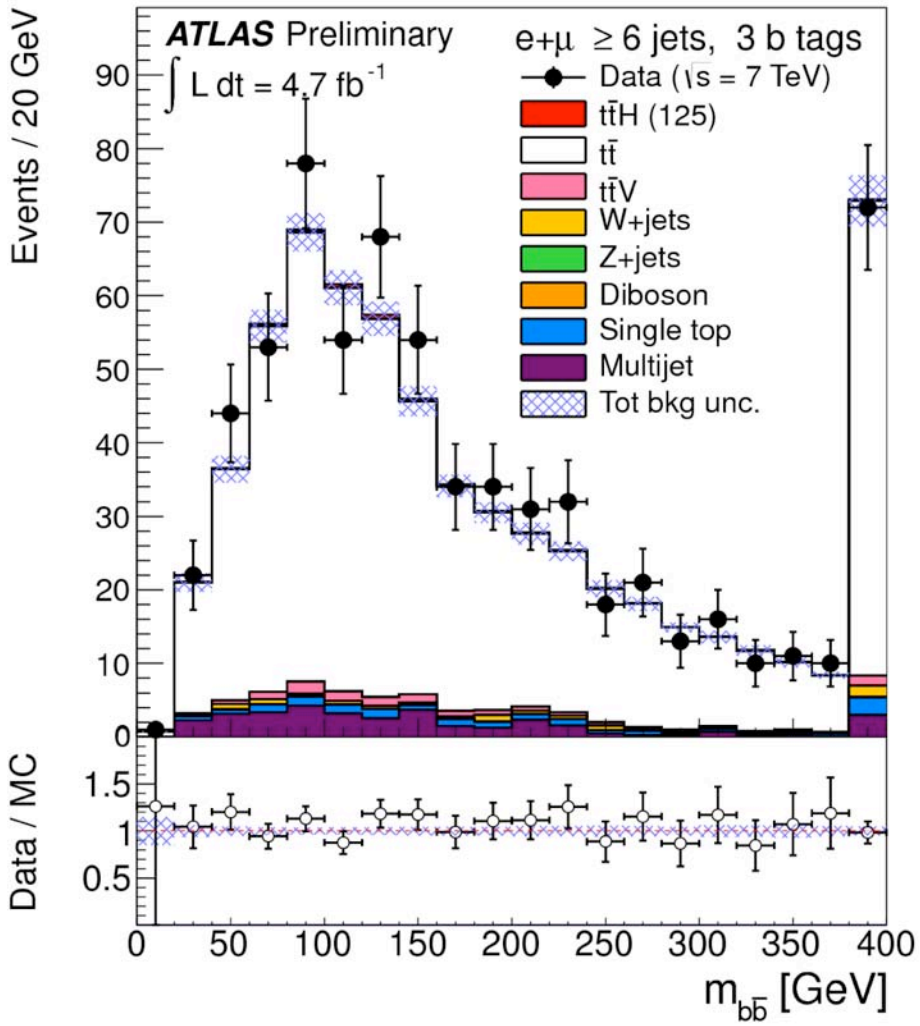
Higgs Boson Searches: $H \to b\bar{b}$



- The background from QCD processes producing b-quarks at the LHC is huge!

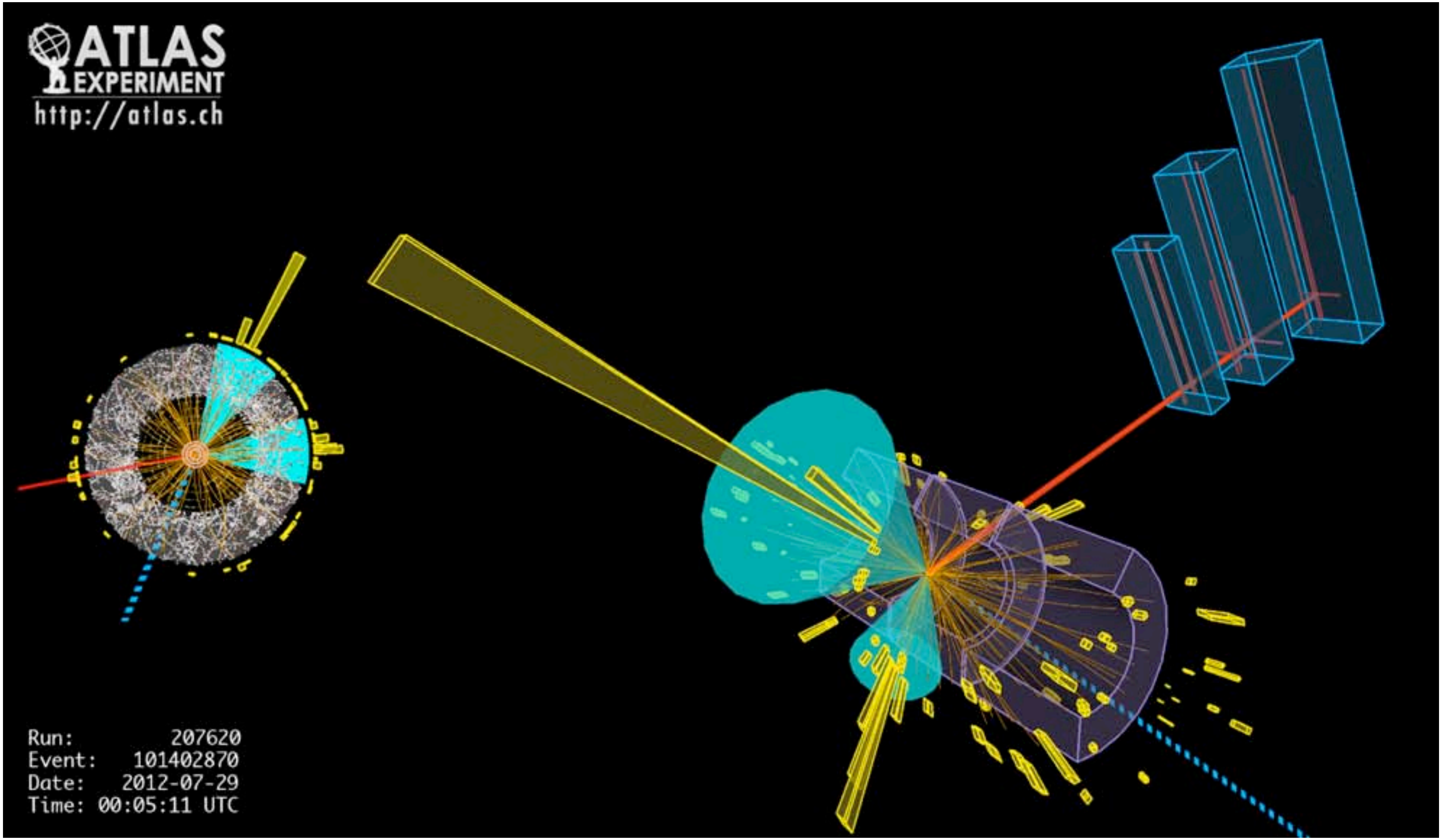
- Impossible to see $H \to b\bar{b}$ above this QCD background.
  - Look for a $H \to b\bar{b}$ in the other production modes: WH, ZH, $t\,\bar{t}\,H$.

- Look for a bump in $m(b\bar{b})$ distribution



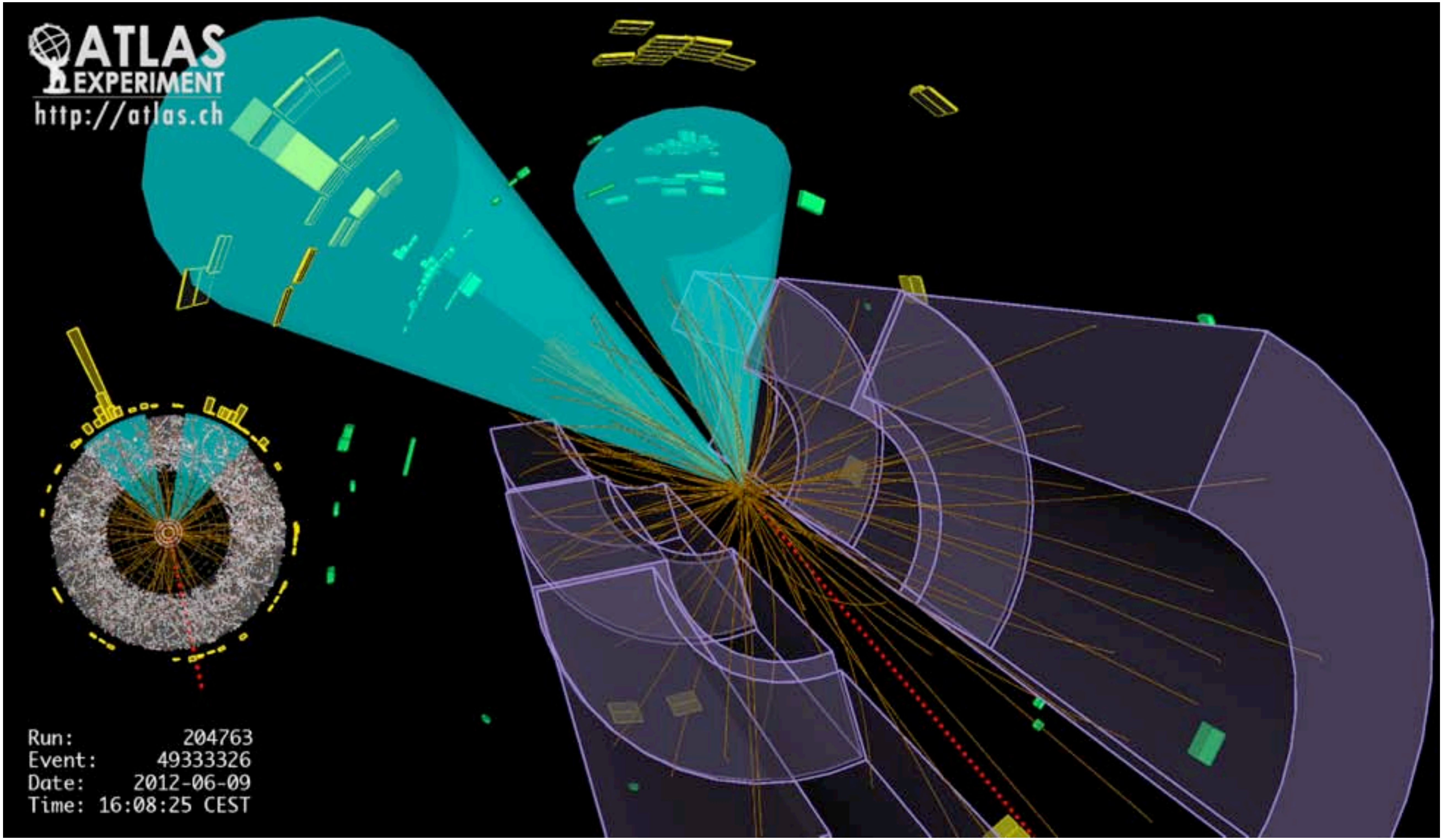
$WH \to \mu\nu\, b\bar{b}$ candidate event

- $m_{b\bar{b}} = 109$ GeV, $E_T^{\text{miss}} = 139$ GeV



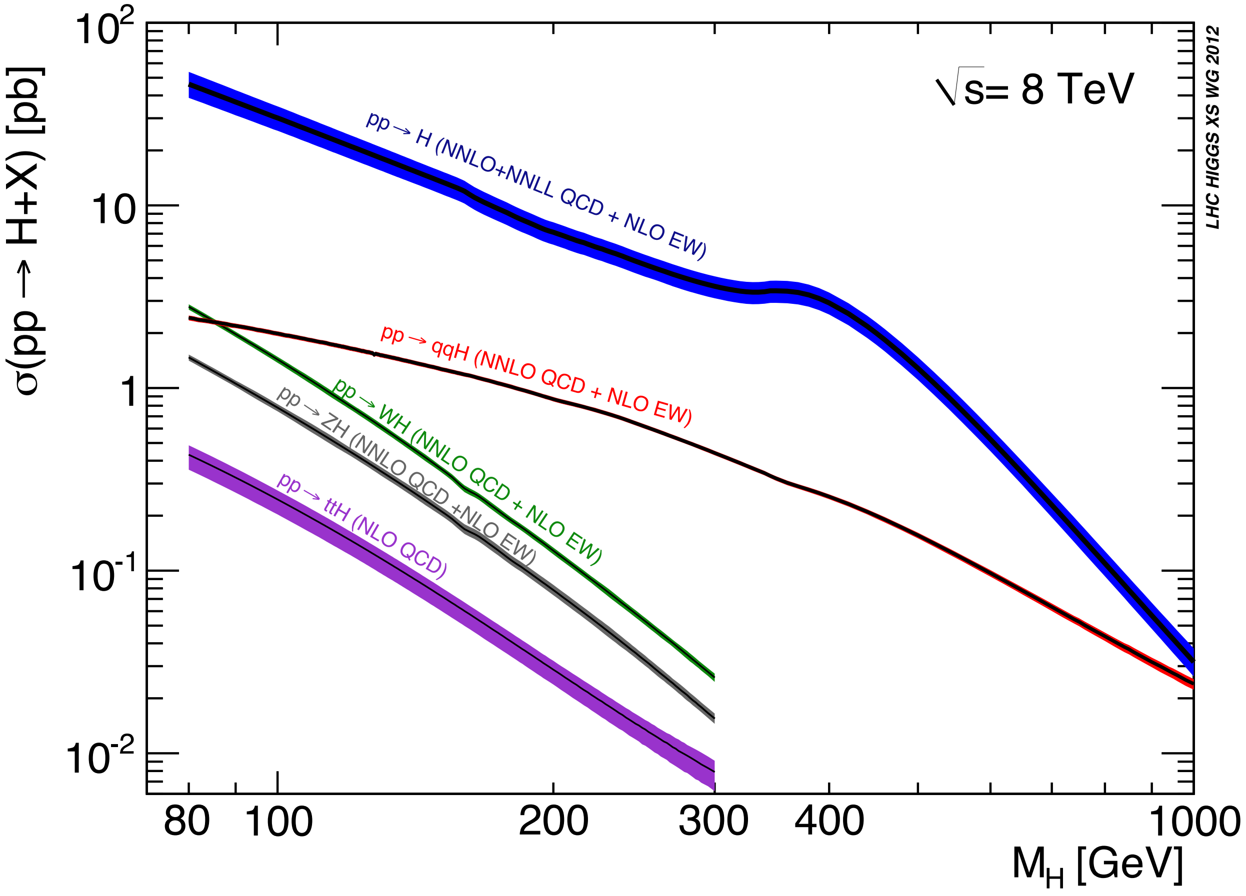
$ZH \rightarrow \nu\bar{\nu}\, b\bar{b}$ candidate event

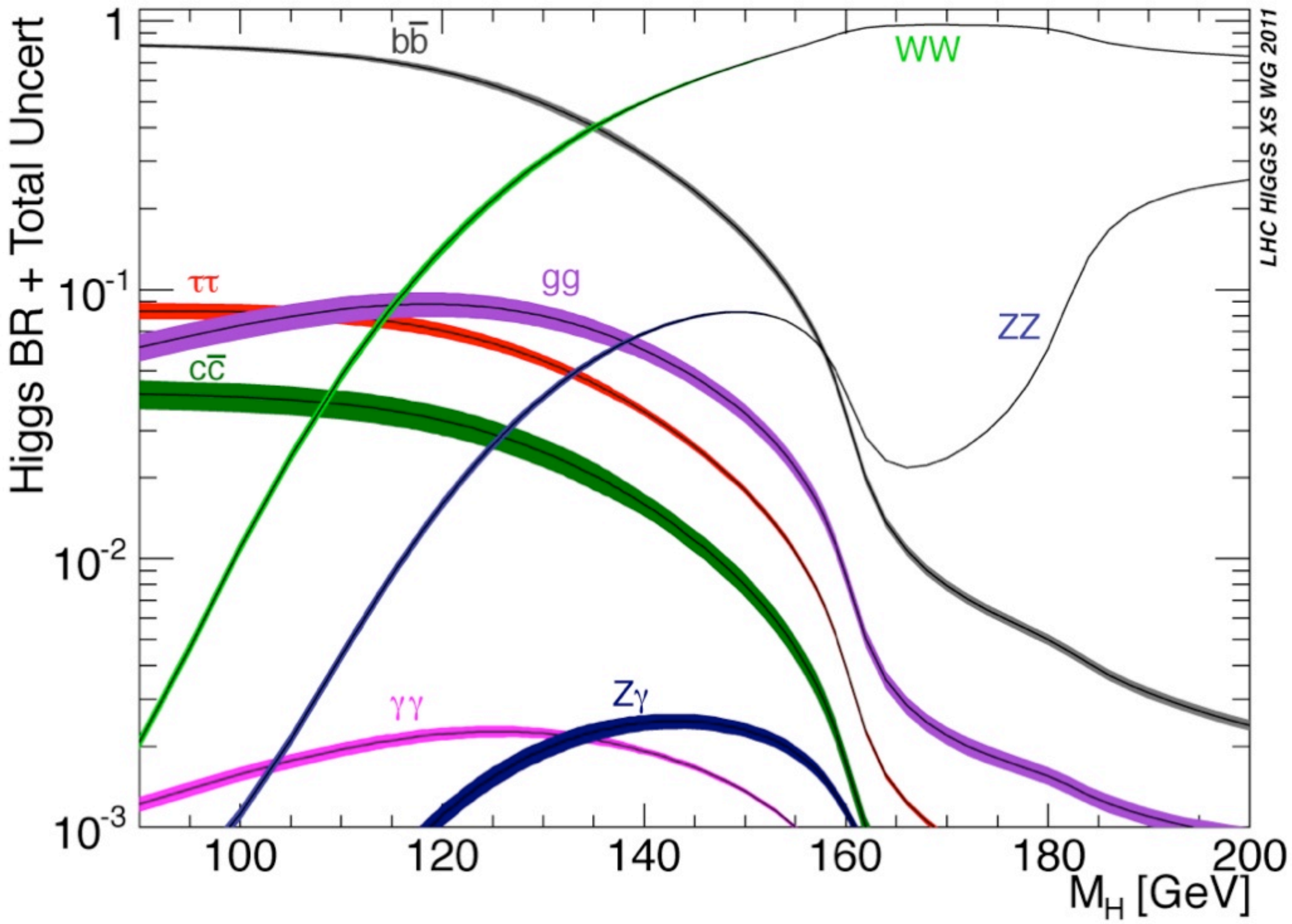
- $m_{b\bar{b}} = 123$ GeV $E_T^{miss} = 271$ GeV



Reminder: Higgs Boson Properties

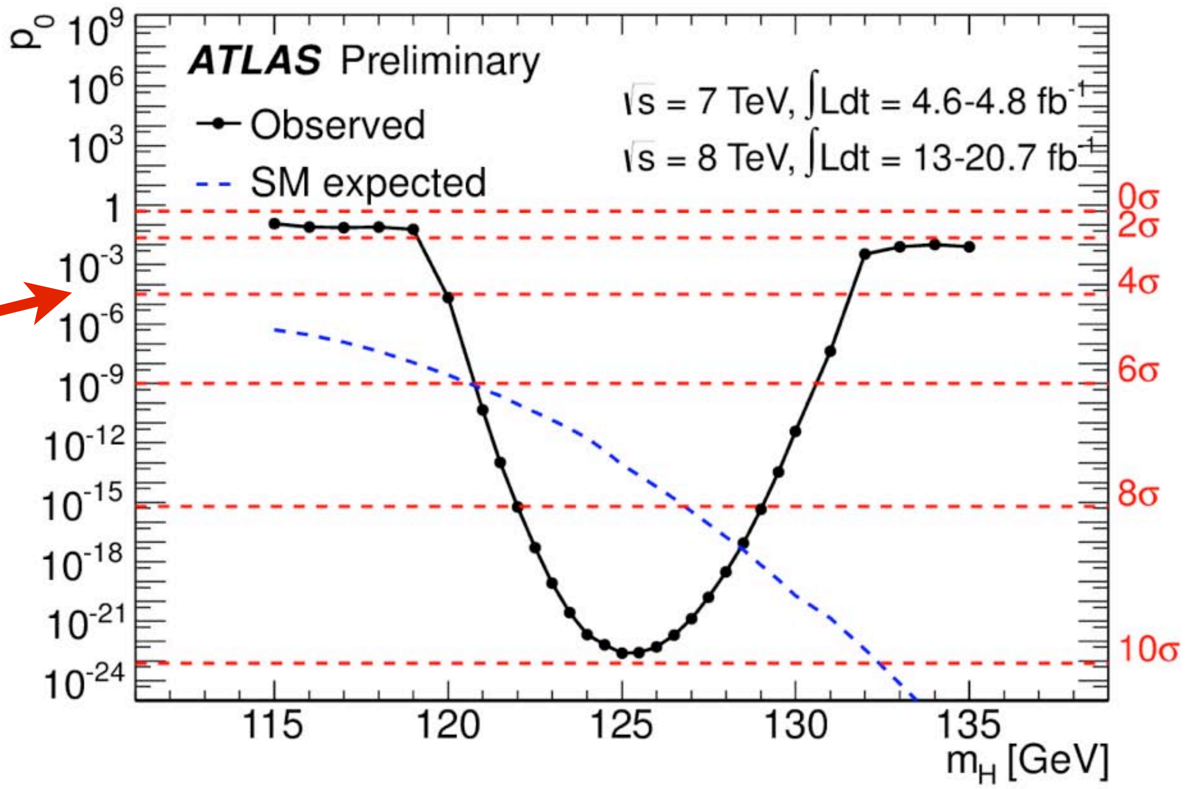
- In the Standard Model the mass of the Higgs boson is not predicted (λ is not predicted) $m_H = \sqrt{2\lambda}v$
  - m_H must be measured:
- Once m_H is known, the values of the Higgs boson production cross section and the branching ratios are predicted:

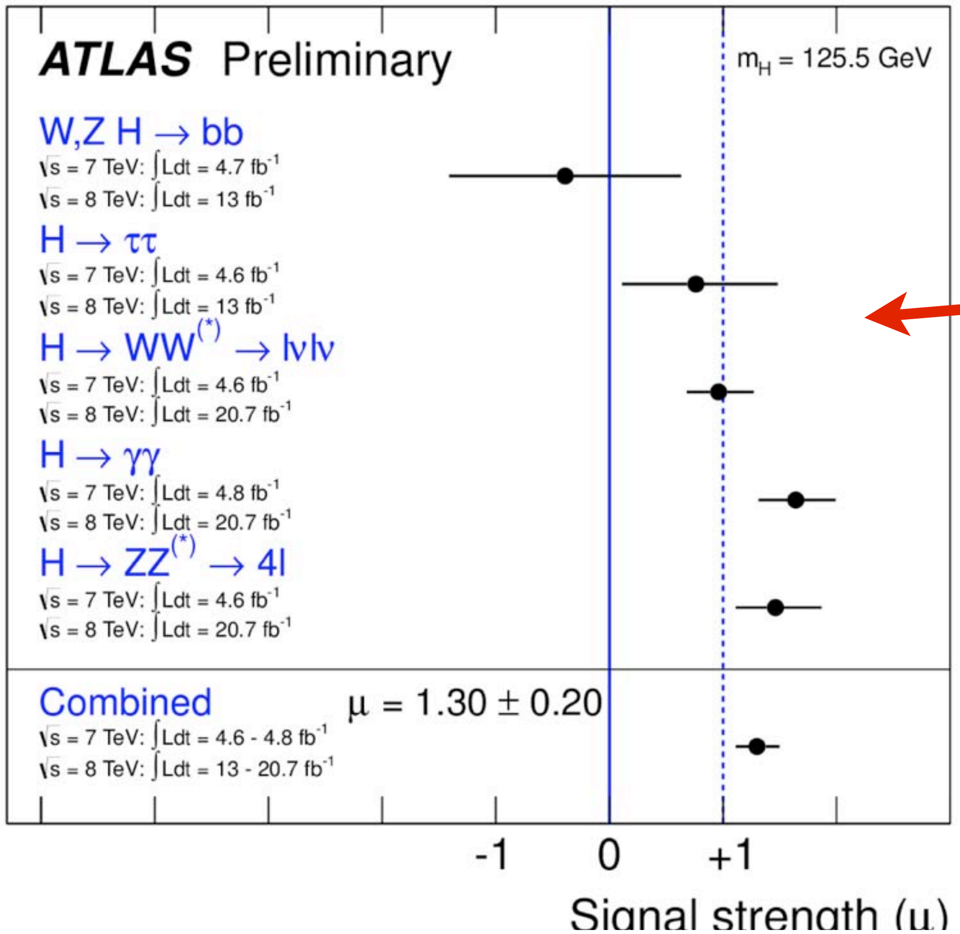




Measured Higgs boson properties

- The ZZ^* and $\gamma\gamma$ channels provide the best measurement of the Higgs boson mass.
    - Measured to be $m_H = 125.5 \pm 0.6$ GeV
    - The significance of the observation is ~10 standard deviations.





Ratio between the predicted and measured **cross section * branching ratio**

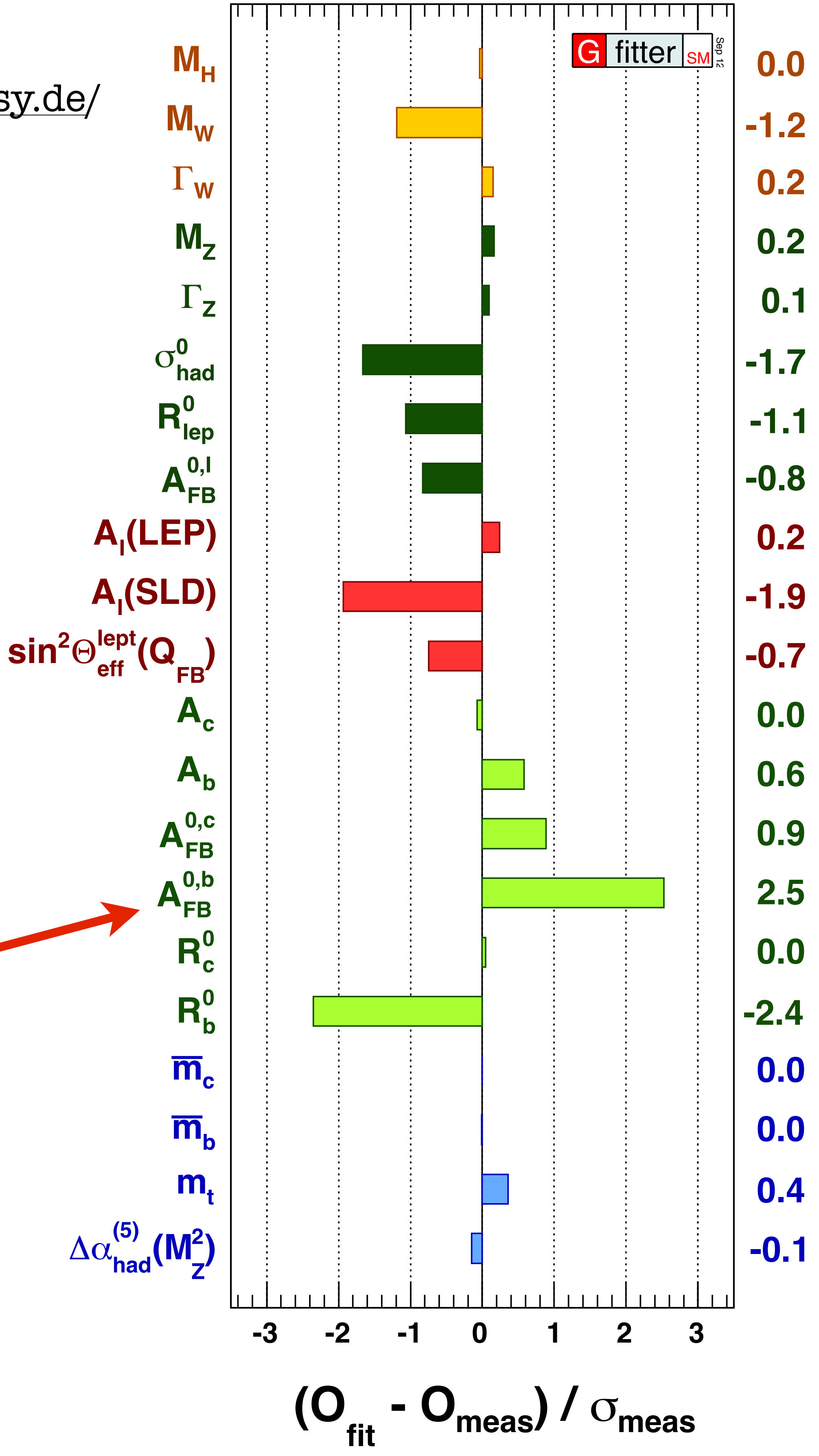
- More data is required before we are sure that what we observe is the Standard Model Higgs boson.

Electroweak Precision Tests

http://gfitter.desy.de/

- Tests can be run to test the consistency of Electroweak theory.
- Check if it is really described by three parameters g_W, g'_W v
  - Plus m_H and the fermion masses

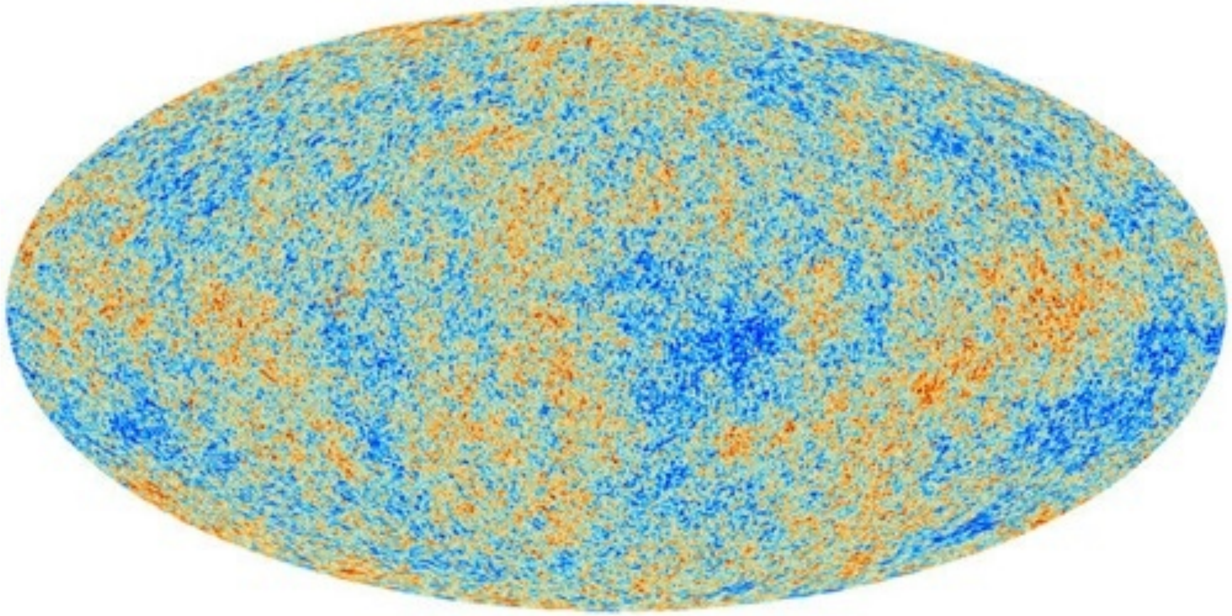
Difference between the predicted and measured **parameter** divided by the **error**

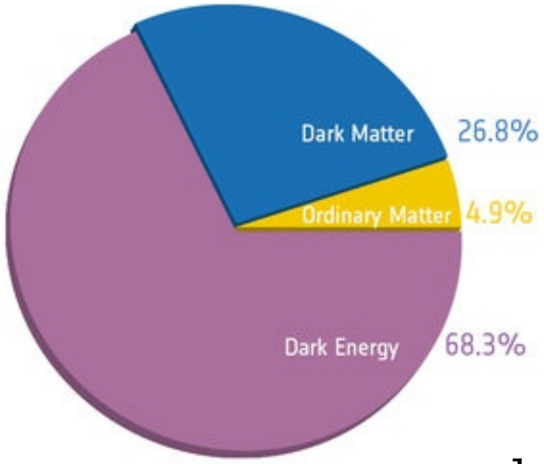


- Electroweak model is consistent within errors

For Fun:

- Finding the Higgs boson doesn’t solve everything. For example:
  - The amount of **CP** violation observed in the Standard Model is not consistent with cosmological models.
  - Recent measurements of the cosmic microwave background radiation from the Planck satellite suggest that only 4.9% of the universe is described by Standard Model particles.



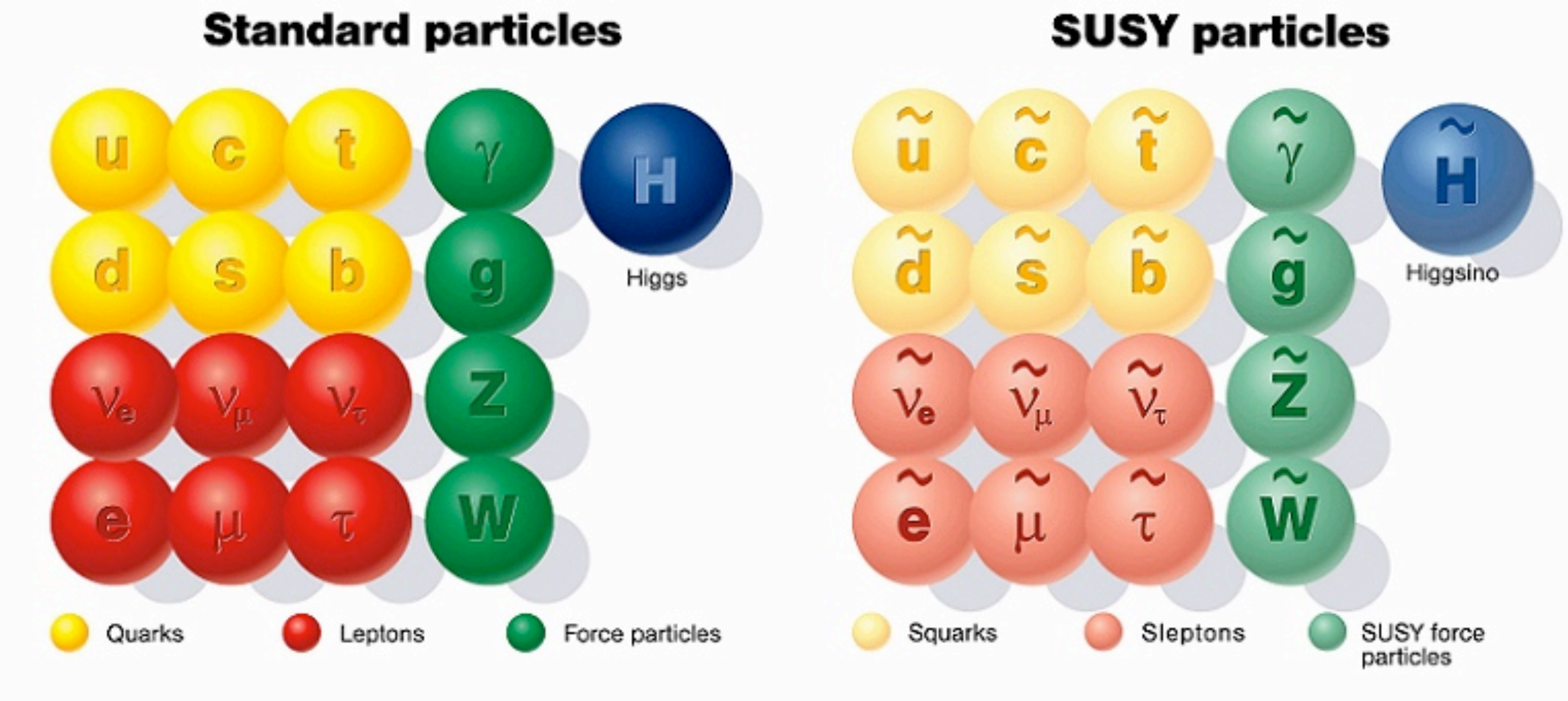


planck.esa.int

  - We have no idea why there are three generations of quarks and leptons
  - No nice theoretical way to give a mass to the neutrinos.
  - Gravity doesn't fit into the Standard Model at all!

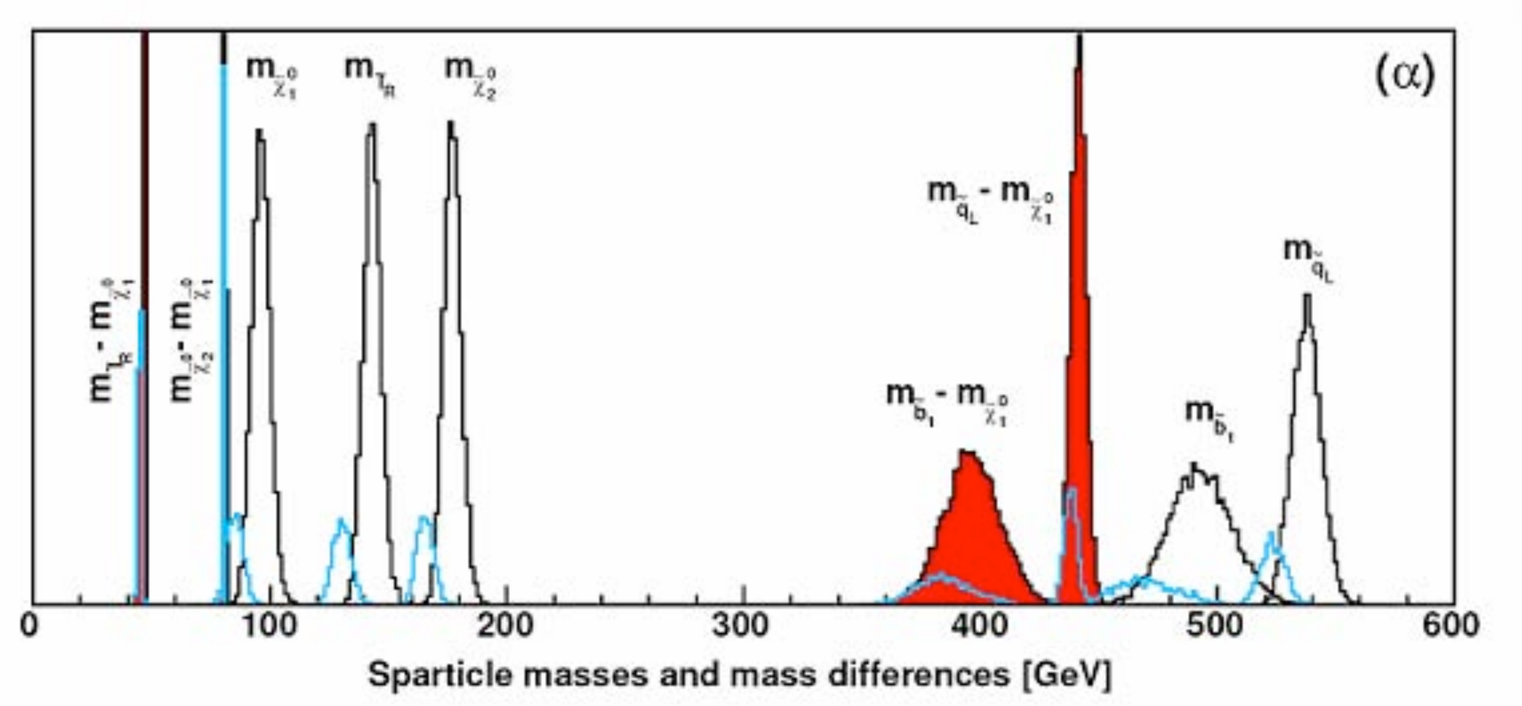
Supersymmetry (SUSY)

- Supersymmetry is a proposed additional symmetry of nature between fermions and bosons.
  - Every fermion will have a bosonic partner
  - Every boson will have a fermionic partner
- The partners should have identical properties (charge, mass) except for the spin.
- This cannot be true: otherwise we would have seen the SUSY particles already. The supersymmetry must be broken.

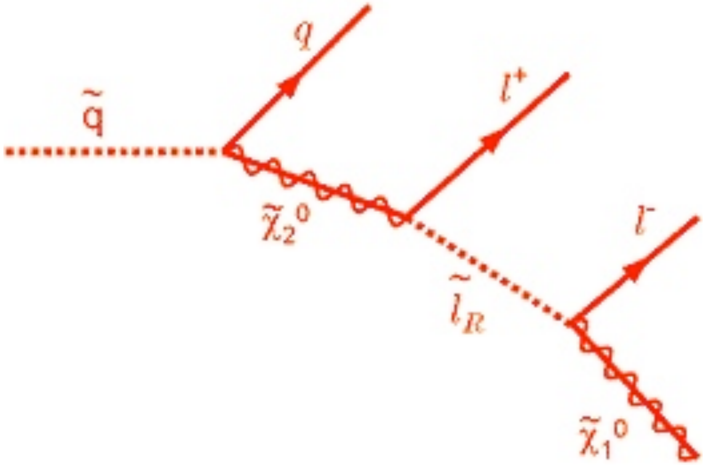


SUSY at the LHC

Measuring mass differences



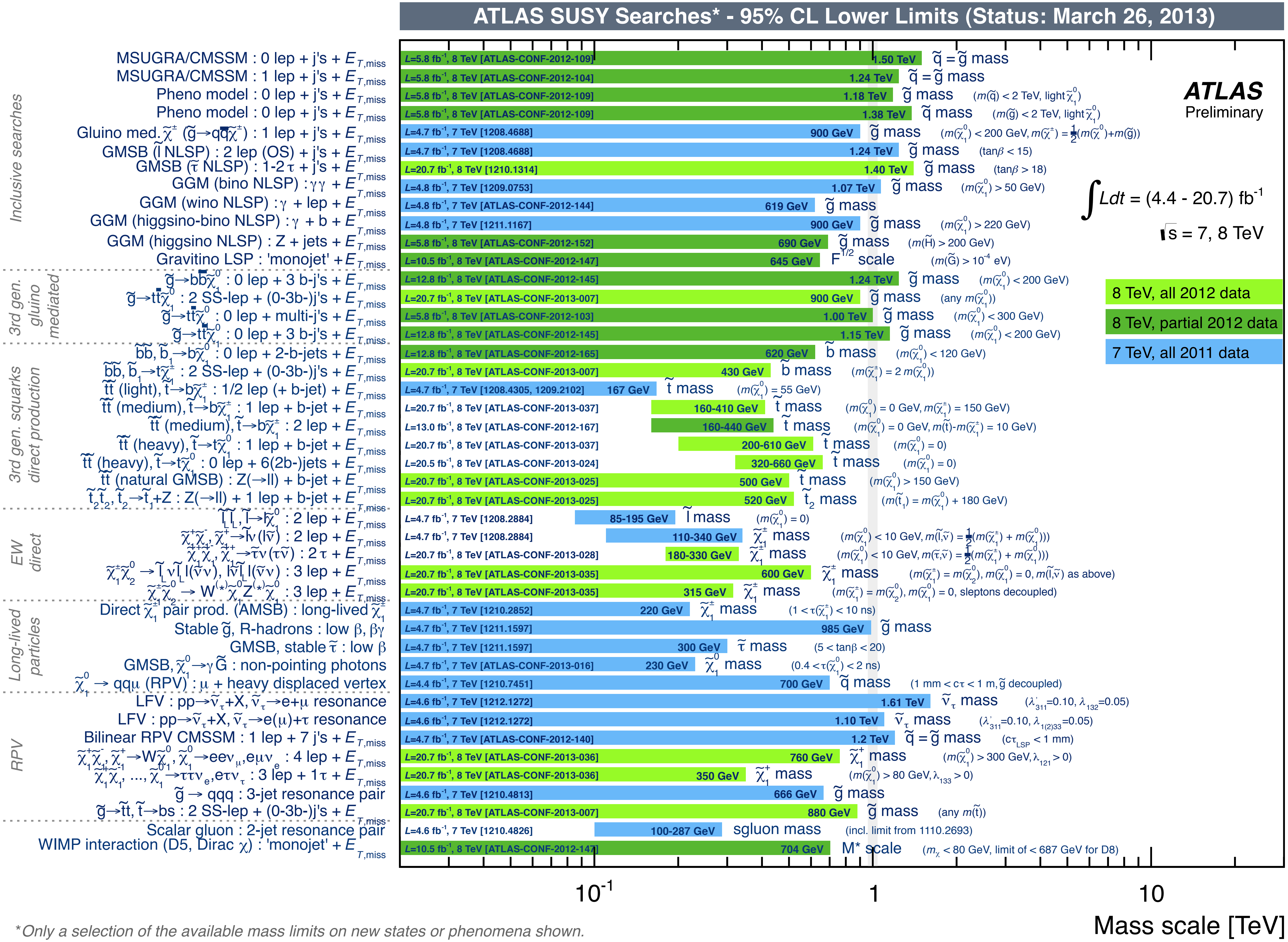
http://www.physics.gla.ac.uk/ppt/bsm.htm



Typical decay of a squark

- The lightest supersymmetric particle is though to be neutral and stable.
- It would appear in an LHC experiment as missing transverse energy.
- Signatures such as these have been searched for at ATLAS and CMS, but nothing has (yet) been observed.

ATLAS (null) results on Supersymmetry



*Only a selection of the available mass limits on new states or phenomena shown.
All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.