SH/IM Particle Physics - Problem Sheet 1

Discussion Questions

For discussion with your classmates.

D1 At a collider, such as the LHC, how can you measure a cross section? For example the cross section for Higgs bosons decaying into photons $\sigma(pp \to H \to \gamma\gamma)$.

D2 How do you measure a branching ratio of a decay? For example how would you measure the branching ratio of the Z^0 boson into electrons: $Z^0 \to e^+e^-$?

Standard Problems

S1 Draw the lowest order Feynman diagrams, and describe qualitatively how the angular distributions of the following high energy scattering processes differ from each other:

(a) Bhabha scattering $e^+e^- \to e^+e^-$

(b) Muon pair production $e^+e^- \to \mu^+\mu^-$

(c) Moeller scattering $e^-e^- \to e^-e^-$

(d) Electron-muon scattering $e^-\mu^- \to e^-\mu^-$

S2 The Mandelstam variables s, t, u in the scattering process $1+2 \to 3+4$ are defined in terms of the momentum 4-vectors (p^μ) as:

$$\begin{aligned} s &= (p_3^\mu + p_4^\mu)^2 \\ t &= (p_1^\mu - p_3^\mu)^2 \\ u &= (p_1^\mu - p_4^\mu)^2 \end{aligned}$$

(a) Show that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$.

(b) Show that $\sqrt{s}$ is the total energy of the collision in the centre of mass frame.

S3 (a) Draw the Feynman Diagram for elastic scattering of two spinless particles by a photon.

(b) Show that the Matrix element $\mathcal{M}$ for this process can be written as:

$$\mathcal{M} = \frac{\alpha}{q^2}\,(p_1 + p_3)(p_2 + p_4)\,\delta^4(p_1 + p_2 - p_3 - p_4)$$

Where p_1, p_2 are the initial four-momenta and p_3, p_4 are the final momenta of the particles.

The cross section for a 2 particle $\to$ 2 particle scattering process is:

$$\sigma = \frac{S}{64\pi^2 s}\frac{|\vec{p}_f^{\,*}|}{|\vec{p}_i^{\,*}|}\int |\mathcal{M}|^2 d\Omega$$

Where $\vec{p}_f^{\,*}$ is the momentum of the final state particles and $\vec{p}_i^{\,*}$ is the momentum of the initial particles, both in the centre of momentum frame.

(c) What is the differential cross section $d\sigma/d\Omega$ for the process above?

S4 In a 2-body decay, $1 \rightarrow 2+3$ the decay rate is:

$$\Gamma = \frac{S|\vec{p}^*|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2$$

where $|\vec{p}^*|$ is the three-momentum of the final state particles in the centre of momentum frame.

Show that:

$$|\vec{p}^*| = \frac{c}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2}$$

S5 The π^+meson decays almost entirely via the two body decay process $\pi^+ \rightarrow \mu^+\nu_\mu$ with an matrix element given by

$$|\mathcal{M}|^2 = G_F^2 f_\pi^2 m_\mu^2 (m_\pi^2 - m_\mu^2)$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant, and f_π is related to the size of the pion wavefunction (the pion being a composite object).

(a) Obtain a formula for the $\pi^+ \rightarrow \mu^+\nu_\mu$ decay rate, using the equations in question S4.

(b) Assuming $f_\pi \sim m_\pi$, calculate the pion lifetime in natural units and in seconds, and compare to the measured value. Note that $m_\pi = 139.6$ MeV, $m_\mu = 105.7$ MeV, $\tau_\pi = 2.6 \times 10^{-8}$ s.

(c) By replacing m_μ by m_e, show that the rate of $\pi^+ \rightarrow e^+\nu_e$ is 1.28×10^{-4} times smaller than the corresponding decay rate to muons. Show also that, on the basis of phase space alone (i.e. neglecting the factor $|\mathcal{M}|^2$), the decay rate to electrons would be expected to be greater than the rate to muons.