Particle Physics - Problem Sheet 3

Discussion Questions

D1 The International Linear Collider (ILC) is proposed as a new high-energy electron-positron collider. The design of the ILC aims for polarised electron and positron beams: with the electron beams having a polarisation of 80% and positron beams having a polarisation of 30%.

If these design aims are achieved, what will be the helicity composition of the electron and positron beams?

What are the advantages in running with polarised beams?

D2 In lecture 6 we saw that the renormalisation of electric charge led to the following expression:

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} z_f \ln(\frac{q^2}{\mu^2})}$$

where z_f is the charge-squared sum over all accessible fermion flavours:

$$z_f = \sum_f Q_f^2$$

The Landau pole is the scale at which the coupling constant becomes infinite. At what energy, q, does the Landau pole exist for QED? Should this worry us?

Standard Problems

S1 This question is all about electromagnetic electron-muon scattering.

(i) Draw the Feynman diagram for electron-muon scattering $e^-\mu^- \to e^-\mu^-$.

(ii) Write down an expression for the electron current and the muon current: j_e^μ, j_μ^ν in terms of the initial and final state spinors. Note that the subscripts on j indicate the fermion flavour and the superscripts are Lorentz indices.

(iii) Use the muon and electron currents to to write down the matrix element for this process.

(iv) What are the possible initial helicity configurations? In the high energy limit, what are the possible final helicity configurations?

In lecture 5, we found that, at lowest order, high energy $e^-\mu^- \to e^-\mu^-$ scattering has a differential cross section given by:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\pi s}\left(\frac{s^2+u^2}{t^2}\right)$$

Where s, t and u are the Mandelstam variables.

(v) Show that by integrating over the full solid angle $d\Omega = d\phi\, d(\cos\theta)$, over the limits $\phi = -\pi, +\pi$ and $\cos\theta = -1, +1$ that the total cross section in this approximation is infinite. (θ is the scattering angle between the incoming and outgoing electron.) What is the physical interpretation of this result?

You may wish to use:

$$\int \frac{dx}{(1-x)^2} = \frac{1}{1-x} \qquad \int \frac{(1+x)^2}{(1-x)^2} dx = x - \frac{4}{x-1} - 4\log(x-1)$$

(vi) Draw the Feynman diagram for electromagnetic $e^+e^- \to \mu^+\mu^-$.

(vii) The differential cross section for electromagnetic $e^+e^- \to \mu^+\mu^-$ scattering is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

Draw the shape of $d\sigma/d\Omega$ as a function of θ.

(vii) By considering the answer to part (iv), or otherwise, determine which are the helicity configurations dominate the contribution to electromagnetic $e^+e^- \to \mu^+\mu^-$ scattering. Can you find a non-mathematical way to explain why those particular configurations contribute?

(x) There is also a weak force contribution to $e^+e^- \to \mu^+\mu^-$ scattering; draw the Feynman diagram for this process. At what energy scale will this diagram become important?

S2 (i) Draw the Feynman diagram for muon decay: $\mu^- \to e^-\bar{\nu}_e\nu_\mu$.

Write down the matrix element, $\mathcal{M}$ for the process, and show that the decay width for this decay must be proportional G_F^2, where G_F is the Fermi coupling constant:

$$G_F = \frac{\sqrt{2}g_W^2}{8m_W^2}$$

(ii) Explain why this is the only possible decay of a muon.

The full calculation for the decay width gives:

$$\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

The measured mass and lifetime of the muon are:

$$m_\mu = 105.65869 \pm 0.000009 \text{ MeV} \qquad \tau_\mu = 2.19703 \pm 0.00004\ \mu\text{s}$$

(iii) Using these values, determine G_F.

(iv) What is the dimensionless weak coupling constant g_W and how does it compare with e?