Particle Physics - Problem Sheet 4

Discussion Questions

D1 In lecture 8, we calculated the cross section for "inverse muon decay" $\nu_\mu e^- \to \mu^- \nu_e$.

(a) How could this process be measured in reality? What would be the source of the initial state particles, and what signature would be observed?

(b) What are the helicity states of the interacting particles?

(c) What would be the major source of background?

D2 Discuss why there are 8 (and not 9) gluon colour-anticolour states

Standard Problems

S1 (i) Draw the Feynman diagram for muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. Write down the matrix element, $\mathcal{M}$ for the process, and show that the decay width for this decay must be proportional G_F^2, where G_F is the Fermi coupling constant.

(ii) Explain why this is the only possible decay of a muon.

(iii) The full calculation for the decay width gives:

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Determine the value of G_F from muon decay. What is the dimensionless weak coupling constant g_W and how does it compare with e?

S2 Explain how the following measurements demonstrate lepton universality between weak couplings:

$$m_\mu = 105.65869 \pm 0.000009 \text{ MeV} \qquad \tau_\mu = 2.19703 \pm 0.00004\ \mu\text{s}$$

$$m_\tau = 1776.99 \pm 0.26 \text{ MeV} \qquad \tau_\tau = 290.6 \pm 1.0 \text{ fs}$$

$$\text{BR}(\mu \to e\bar{\nu}_e\nu_\mu) = 100\%$$

$$\text{BR}(\tau \to e\bar{\nu}_e\nu_\tau) = 17.85 \pm 0.05\% \qquad \text{BR}(\tau \to \mu\bar{\nu}_\mu\nu_\tau) = 17.36 \pm 0.05\%$$

S3 (a) Draw a Feynman diagram for $q\bar{q} \to q\bar{q}$ scattering in the t-channel at lowest order.

(b) Write down the matrix element for this process and show that the difference main difference between strong and electromagnetic scattering are (i) the strong coupling constant and (ii) the colour factor, $f = \frac{1}{4}\lambda^a_{ji}\lambda^a_{kl}$. What do the values λ, a and i, j, k, l represent in this equation?

(c) What are the possible colour configuration for $q\bar{q} \to q\bar{q}$ scattering? Calculate the value of the colour factors for each configuration.

S4 (a) What are the possible gluon exchanges between the quark and antiquark in a meson which has a color wavefunction:

$$\psi = \frac{1}{\sqrt{3}}(\mathrm{r\bar{r}} + \mathrm{g\bar{g}} + \mathrm{b\bar{b}})$$

(b) Show that the sum of the meson color factors is 4/3. The answers to question S3 maybe be useful here.