

Particle Physics - Problem Sheet 7

Discussion Questions

D1 What is the definition of weak isospin and weak hypercharge?

Calculate the weak isospin (T, T_3) and weak hypercharge (Y) for each of the first generation fermions.

D2 In the Electroweak model, the fermion current associated with the photon is:

$$(j^\gamma)^\mu = (j^{W^3})^\mu \sin \theta_W + (j^Y)^\mu \cos \theta_W$$

What does this equation mean? You may wish to discuss:

- What is a *fermion current*?
- What does the μ stand for?
- What is θ_W ?
- What are $(j^{W^3})^\mu$ and $(j^Y)^\mu$?
- What are g_W and g'_W ?

Standard Questions

S1 In the Electroweak model, the fermion current associated with the photon is:

$$(j^\gamma)^\mu = (j^{W^3})^\mu \sin \theta_W + (j^Y)^\mu \cos \theta_W$$

Where:

$$(j^{W^i})^\mu = [g_W T] \bar{\chi}_L \gamma^\mu \tau_i \chi_L \quad (1)$$

and for electrons:

$$j_\mu^Y = \left(\frac{1}{2} g'_W Y_e\right) \bar{e} \gamma^\mu e = \frac{1}{2} g'_W (Y_{eL} \bar{e}_L \gamma^\mu e_L + Y_{eR} \bar{e}_R \gamma^\mu e_R)$$

In these equations e represents the electron spinor and Y_e is the weak hypercharge of the electron.

Substitute in: $\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ and $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and the appropriate values of weak isospin (T) and weak hypercharge (Y).

Show that if the electron charge, $e = g'_W \cos \theta_W = g_W \sin \theta_W$ then the known behaviour of the photon is reproduced.

S2 The W^\pm boson currents are defined as:

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

Using equation (1) and the definition of:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

show W^+ acts as a lowering operator and W^- acts as a raising operator.

You might want to revise the matrix representation of raising and lowering operators here from Quantum Physics lecture 2:

www2.ph.ed.ac.uk/teaching/course-notes/documents/82/1749-lecture02.pdf

What quantity do they raise and lower? *Hint:* Think about the what properties of fermions change when they interact with the W^\pm bosons.