Particle Physics - Problem Sheet 8

Discussion Questions

D1 The effective Higgs potential can be written as:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

and the vacuum Higgs field is written as a doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

(a) Show that the Higgs potential has a minimum with $\phi^\dagger\phi \neq 0$ only if $\mu^2 > 0$ and $\lambda > 0$. (For the purposes of this discussion you should entertain the possibility that the constant μ^2 could be less than 0.)

(b) Consider an excitation of the field, $h(x)$, about the minimum:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

Write out the expansion in powers of $h(x)$. Which term corresponds to the mass of the Higgs boson?

(c) What are the next two terms in the expansion? What do they represent?

D2 (a) Describe the possible parton-parton scattering processes in proton-proton collisions at the LHC.

(b) How are heavy quark pairs $b\bar{b}$ and $t\bar{t}$ produced?

Standard Questions

S1 In the Higgs mechanism and electroweak theory the following relationships apply:

$$m_W = \frac{v g_W}{2} \qquad e = g'_W \cos\theta_W = g_W \sin\theta_W$$
$$\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g_W'^2}} \qquad m_Z = \frac{1}{2} v \sqrt{g_W^2 + g_W'^2}$$

Use the measured values of $m_Z = 91.2$ GeV, $m_W = 80.4$ GeV and $\alpha = 1/128$ to calculate the sin of the weak mixing angle, the values of the coupling constants and the vacuum expectation value.

S2 *A Classic Question!*

(a) Write down all the allowed final states of Z-boson decay.

(b) The width for each decays is:

$$\Gamma(Z \to f\bar{f}) = \frac{g_W^2[(c_V^f)^2 + (c_A^f)^2]}{48\pi \cos^2\theta_W} m_Z = 322[(c_V^f)^2 + (c_A^f)^2][\text{MeV}]$$

Where $c_V^f = T_3 - 2Q\sin^2\theta_W$ is the vector coupling and $c_A^f = T_3$ is the axial-vector coupling. The measured values of $\sin^2\theta_W$ is

Calculate the total width of the Z-boson and the branching ratios to the following experimentally observed final states:

- Each flavour of charged lepton: $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$.
- Hadrons
- Nothing! (What particles give no signature in the detector?)

S3 *Revision!* Draw Feynman diagrams for each of these and decide whether each of the following decays is the result of a strong, electromagnetic or weak decay:

$$\rho^0 \to \pi^+\pi^- \qquad K^{*+} \to K^0\pi^+ \qquad \eta \to \pi^+\pi^-$$

$$\eta \to \gamma\gamma \qquad \Sigma_0 \to \Lambda\gamma \qquad \Sigma^+ \to n\pi^+$$