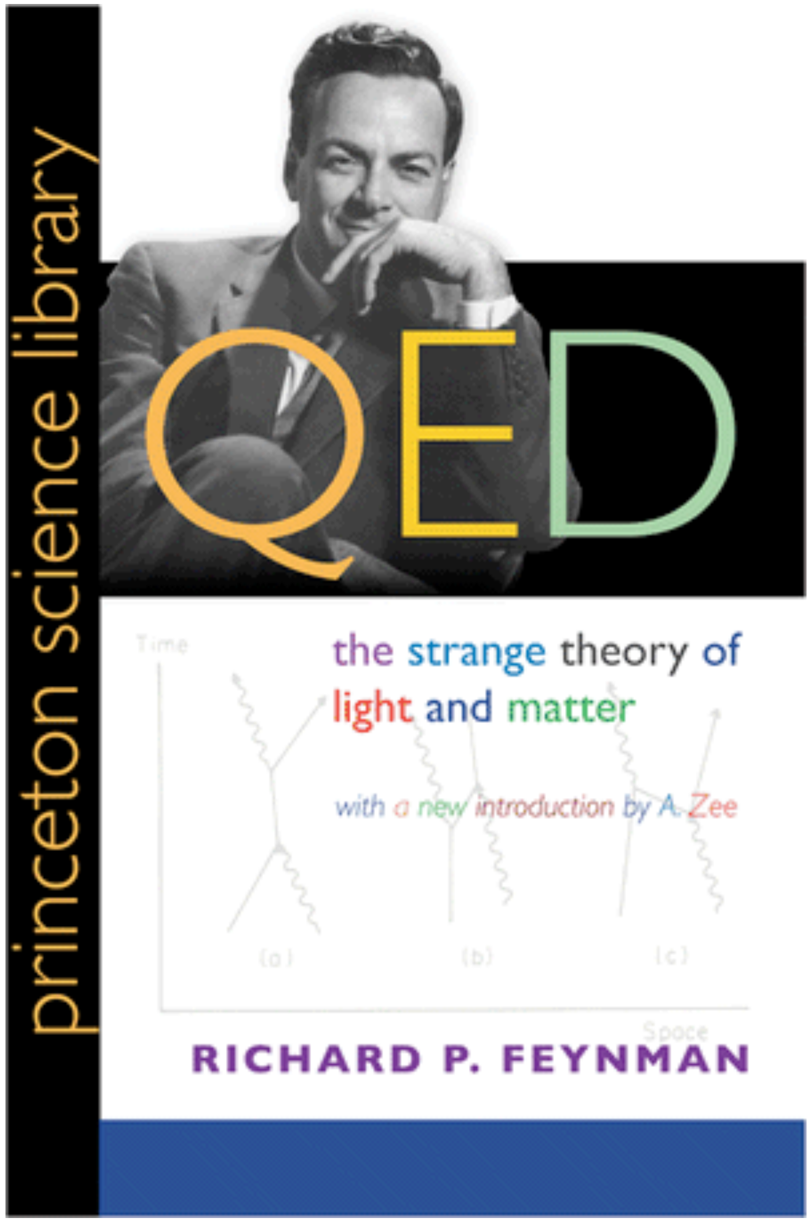
Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 6: Quantum Electrodynamics
The Electromagnetic force, quantised)



- ★$e^-\mu^-\rightarrow e^-\mu^-$ scattering
- ★helicity configurations
- ★higher order diagrams
- ★running coupling and renormalisation

Next Week

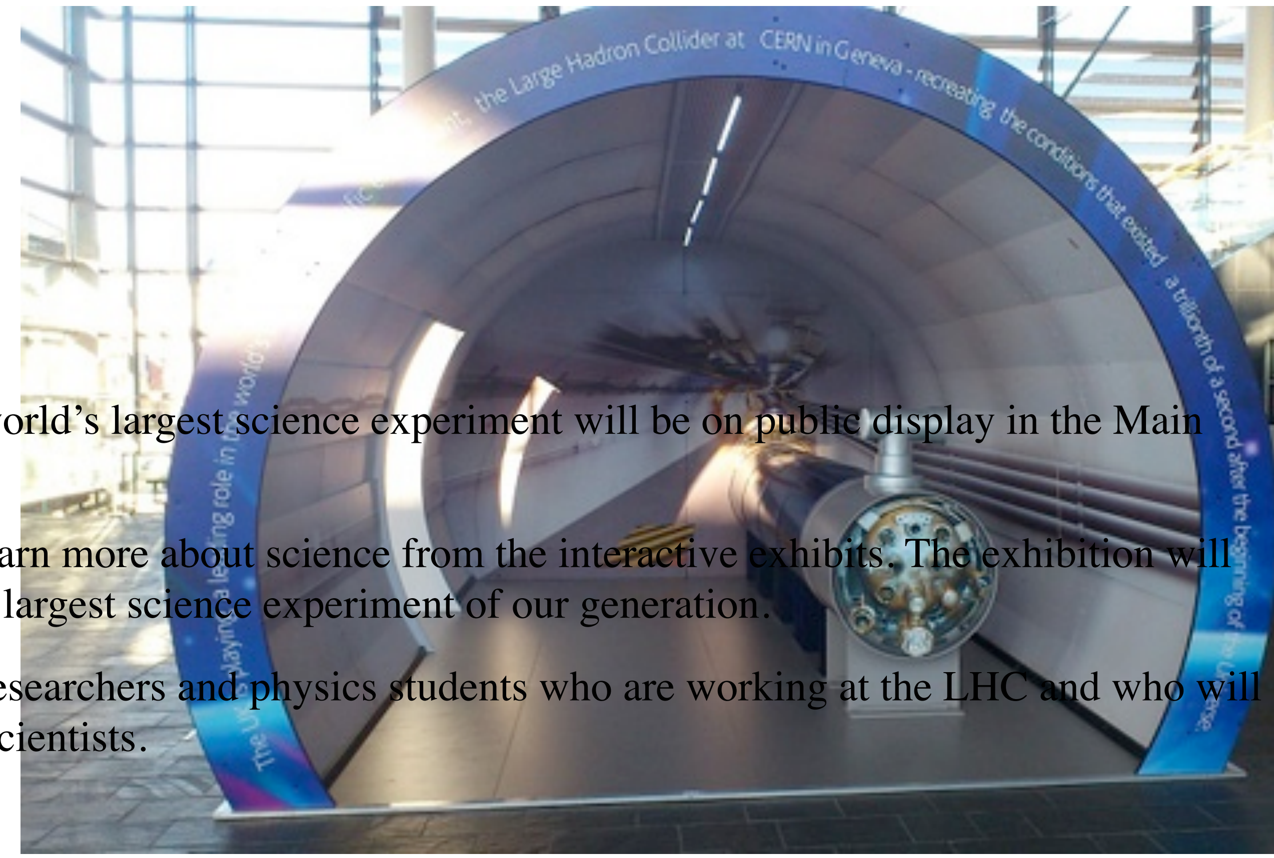
- Lecture on Tuesday only.

- No lecture on Friday 8th February.

- Tutorial next week for catch up: plus a few group activities.

LHC in the Parliament: from tomorrow

http://www.scottish.parliament.uk/visitandlearn/58283.aspx



Saturday 2 February 2013 - Friday 8 February 2013

Main Hall, Free Exhibition

A travelling exhibition showcasing the Large Hadron Collider (LHC), the world's largest science experiment will be on public display in the Main Hall of the Scottish Parliament.

Visitors can walk through a life size model of part of the LHC tunnel and learn more about science from the interactive exhibits. The exhibition will help create a sense of what it's like to be a particle physicist working on the largest science experiment of our generation.

Visitors will also have the opportunity to meet some of the UK's top LHC researchers and physics students who are working at the LHC and who will be available to answer questions and help to inspire the next generation of scientists.

Associated Events

Explore Your Universe - Saturday 2 February

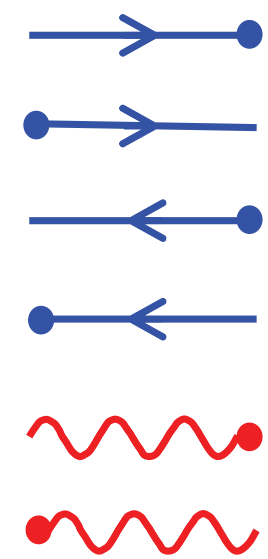
Come along and get involved with the fantastic new family show brought to you by Dynamic Earth as part of the Explore Your Universe project in partnership with the Science and Technologies Facilities Council (STFC) and the Association of Science and Discovery Centres (ASDC). Find out what our world is made of, discover what can be found inside an atom and experiment with making your own electricity! You will also have the chance to see a cloud chamber in action and even make your very own particle to take home with you. Admission is free and will run within normal opening hours.

CERN Lecture - Thursday 7 February

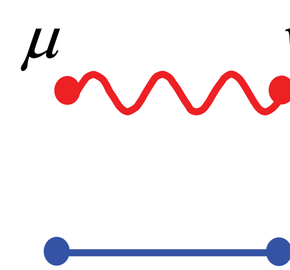
Dr Aidan Robson from the University of Glasgow will be giving a free lecture on CERN, the Large Hadron Collider and his part working on the world's largest scientific experiment. The lecture will begin at 7.00pm. If you wish to take a look at the 'LHC on Tour' exhibition beforehand, please arrive approximately 30 minutes prior to the start.

Review: Feynman Rules for QED

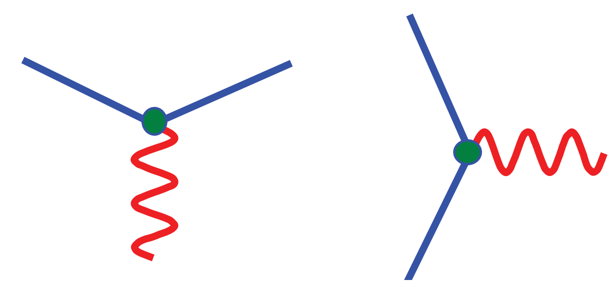
- **External Lines**
  - spin 1/2
    - incoming particle $u(p)$
    - outgoing particle $\overline{u}(p)$
    - incoming antiparticle $\overline{v}(p)$
    - outgoing antiparticle $v(p)$
  - spin 1
    - incoming photon $\varepsilon^{\mu}(p)$
    - outgoing photon $\varepsilon^{\mu}(p)^*$



- **Internal Lines (propagators)**
  - spin 1 photon $\frac{g_{\mu\nu}}{q^2}$
  - spin 1/2 fermion $\frac{(\gamma^{\mu} q_{\mu} + m)}{q^2 - m^2}$



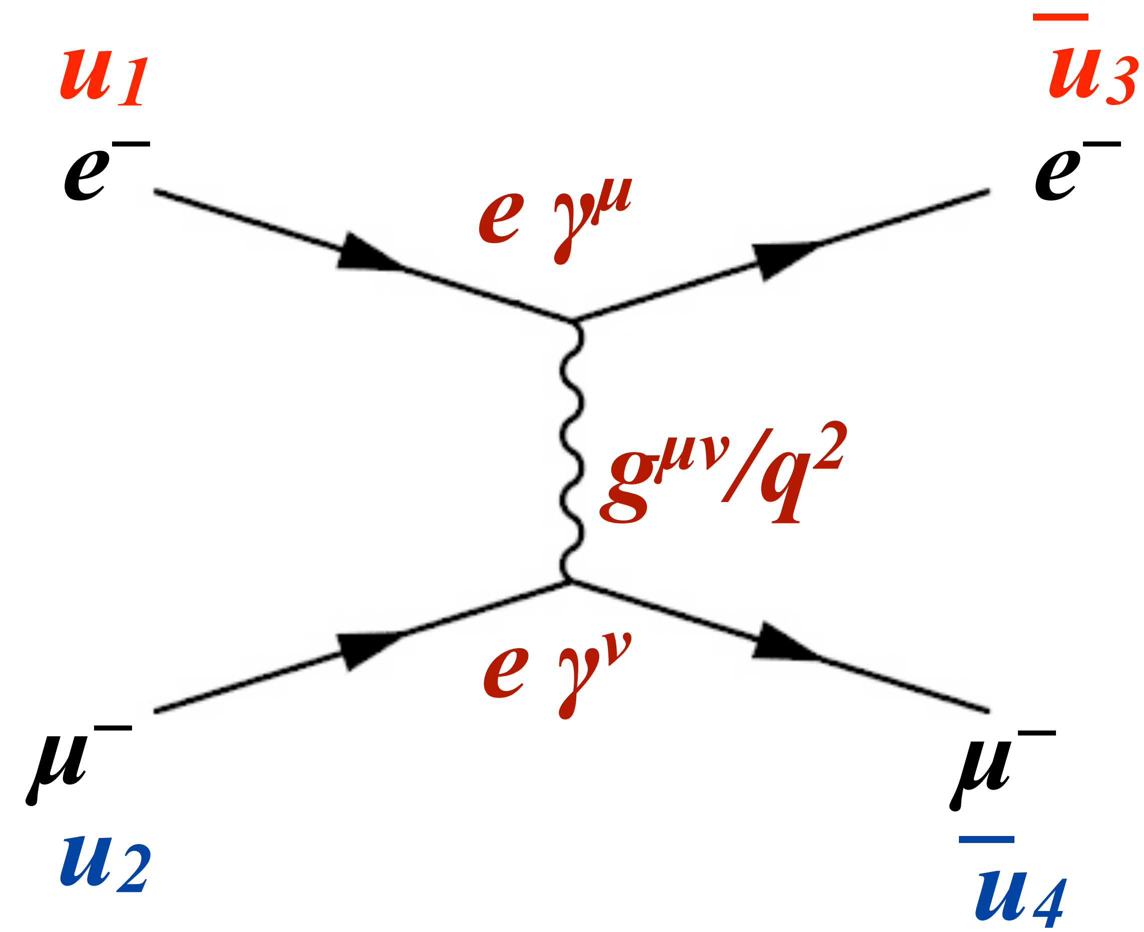
- **Vertex Factors**
  - spin 1/2 fermion (charge -|e|) $e\gamma^{\mu}$



- Matrix element $\mathcal{M}$ is product of all factors
- Integrate over all allowed internal momenta and spins, consistent with momentum conservation

Rewview: Electron-Muon Scattering

- Just one lowest order diagram
- In lecture 3, we considered a similar diagram (but with spinless particles)



$$\mathcal{M} = e^2 \frac{g^{\mu\nu}}{q^2} j^\mu_{13}\, j^\nu_{24}$$

Vertex Couplings

$$\mathcal{M} = e^2 \frac{g^{\mu\nu}}{q^2} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\nu u_2)$$

Photon propagator,

Electron current

Muon current

Squaring: take complex conjugate, use $g^{\mu\nu}$ term to set ν to μ

$$|\mathcal{M}|^2 = \frac{e^4}{q^4} (\bar{u}_3 \gamma^\mu u_1)\,(\bar{u}_3 \gamma^\mu u_1)^* (\bar{u}_4 \gamma^\mu u_2)(\bar{u}_4 \gamma^\mu u_2)^*$$

Spin & Scattering

- Spinors describe fermions with a given helicity (**h=+1 (R)** or **h=−1 (L)**).
  - ⇒ The value of $\mathcal{M}$ depends on the initial and final state helicities

- In reality:
  - ➡ Measure the same process many times (often millions or billions).
  - ➡ The initial helicity (spins) of the fermions are known, either:
    - unpolarised (**50% h=+1; 50% h=−1**)
    - polarised (known fractions of **h=±1**)
  - ➡ Often measure the final state in all outgoing helicity configurations

- To calculate unpolarised cross sections (*i.e.* when initial state is unpolarised):
  - ➡ **average over initial state helicities** and **sum over final state helicities**

High Energy e-μ scattering

- 16 possible helicity configurations:

$$\begin{array}{llll} e_L\mu_L \to e_L\mu_L & e_L\mu_L \to e_L\mu_R & e_L\mu_L \to e_R\mu_L & e_L\mu_L \to e_R\mu_R \\ e_L\mu_R \to e_L\mu_L & e_L\mu_R \to e_L\mu_R & e_L\mu_R \to e_R\mu_L & e_L\mu_R \to e_R\mu_R \\ e_R\mu_L \to e_L\mu_L & e_R\mu_L \to e_L\mu_R & e_R\mu_L \to e_R\mu_L & e_R\mu_L \to e_R\mu_R \\ e_R\mu_R \to e_L\mu_L & e_R\mu_R \to e_L\mu_R & e_R\mu_R \to e_R\mu_L & e_R\mu_R \to e_R\mu_R \end{array}$$

- At high energy helicity is conserved.
- For $e^-\mu^-\to e^-\mu^-$ scattering only four configurations contribute:

$$\mathcal{M}(e_L\mu_L\to e_L\mu_L)\ ,\ \mathcal{M}(e_L\mu_R\to e_L\mu_R)\ ,\ \mathcal{M}(e_R\mu_L\to e_R\mu_L)\ ,\ \mathcal{M}(e_R\mu_R\to e_R\mu_R)$$

➡ Average over initial states: $(2S+1)$ initial states for spin, S

➡ Sum over final states

$$|\mathcal{M}|^2 = \frac{e^4}{q^4}\frac{1}{(2S_1+1)(2S_2+1)}\sum_{S_3,S_4}(\bar{u}_3\gamma^\mu u_1)\,(\bar{u}_3\gamma^\nu u_1)^*(\bar{u}_4\gamma^\mu u_2)(\bar{u}_4\gamma^\nu u_2)^*$$

$$= \frac{e^4}{q^4}\left(\frac{1}{(2S_1+1)}\sum_{S_3}(\bar{u}_3\gamma^\mu u_1)\,(\bar{u}_3\gamma^\nu u_1)^*\right)\left(\frac{1}{(2S_2+1)}\sum_{S_4}(\bar{u}_4\gamma^\mu u_2)(\bar{u}_4\gamma^\nu u_2)^*\right)$$

$$= \frac{e^4}{q^4}\,L_e\,L_\mu$$

Trace Theorems

- The spinor-gamma matrices products in the sum can be evaluated using **trace theorems.** (See details in Griffiths 7.7, equation 7.128)

$$L_e = \frac{1}{(2S_1+1)}\sum_{S_3}(\bar{u}_3\gamma^\mu u_1)(\bar{u}_3\gamma^\nu u_1)^*$$

$$= 2\left[\underbrace{p_3^\mu p_1^\nu}_{\text{matrix}} + \underbrace{p_3^\nu p_1^\mu}_{\text{matrix}} - (p_3\cdot p_1 - m_e^2)\underbrace{g^{\mu\nu}}_{\text{matrix}}\right]$$

- No spinors left! Just matrices, likewise:

$$L_\mu = 2\left[p_4^\mu p_2^\nu + p_4^\nu p_2^\mu - (p_4\cdot p_2 - m_\mu^2)g^{\mu\nu}\right]$$

- If the electron and muon are energetic, $E>>m$, can ignore m_e and m_μ terms

$$|\mathcal{M}|^2 = \frac{e^4}{q^4}L_e L_\mu = \frac{8e^4}{q^4}\left[(p_3\cdot p_4)(p_1\cdot p_2) + (p_3\cdot p_2)(p_1\cdot p_4)\right]$$

- Substitute in Mandelstam variables:

$$|\mathcal{M}|^2 = 2e^4\frac{s^2+u^2}{t^2} = 2e^4\frac{1+4\cos^4(\theta^*/2)}{\sin^4(\theta^*/2)}$$

Cross section for $e^-\mu^- \rightarrow e^-\mu^-$ scattering

- Cross section = $|\mathcal{M}|^2 \rho$, substituting for ρ (see problem sheet 1):

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\vec{p}^{\,*}_f|}{|\vec{p}^{\,*}_i|}$$

- With:
  - ➡ centre of mass energy, $(E_1+E_2)^2=s$
  - ➡ $|p^*_f|=|p^*_i|$
  - ➡ $S=1$ as no identical particles in final state
  - ➡ $\alpha=e^2/4\pi$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\pi s}\left(\frac{s^2+u^2}{t^2}\right)$$

- The different spin configurations give scattering distributions:

$$\mathcal{M}(LL \to LL) = \mathcal{M}(RR \to RR) = e^2\frac{u}{t} = e^2\frac{1+\cos\theta^*}{1-\cos\theta^*}$$

$$\mathcal{M}(LR \to LR) = \mathcal{M}(RL \to RL) = e^2\frac{s}{t} = e^2\frac{2}{1-\cos\theta^*}$$

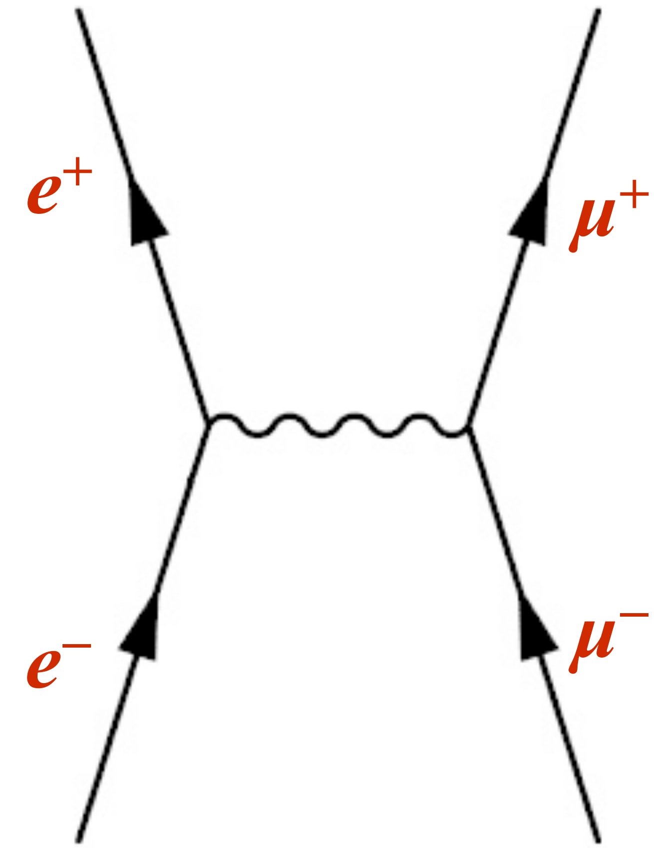
- θ^* is the scattering angle between the incoming and outgoing electron

$e^- e^+ \to \mu^- \mu^+$ Scattering

- Related to $e\mu \to e\mu$ scattering by exchanging $t \leftrightarrow s$

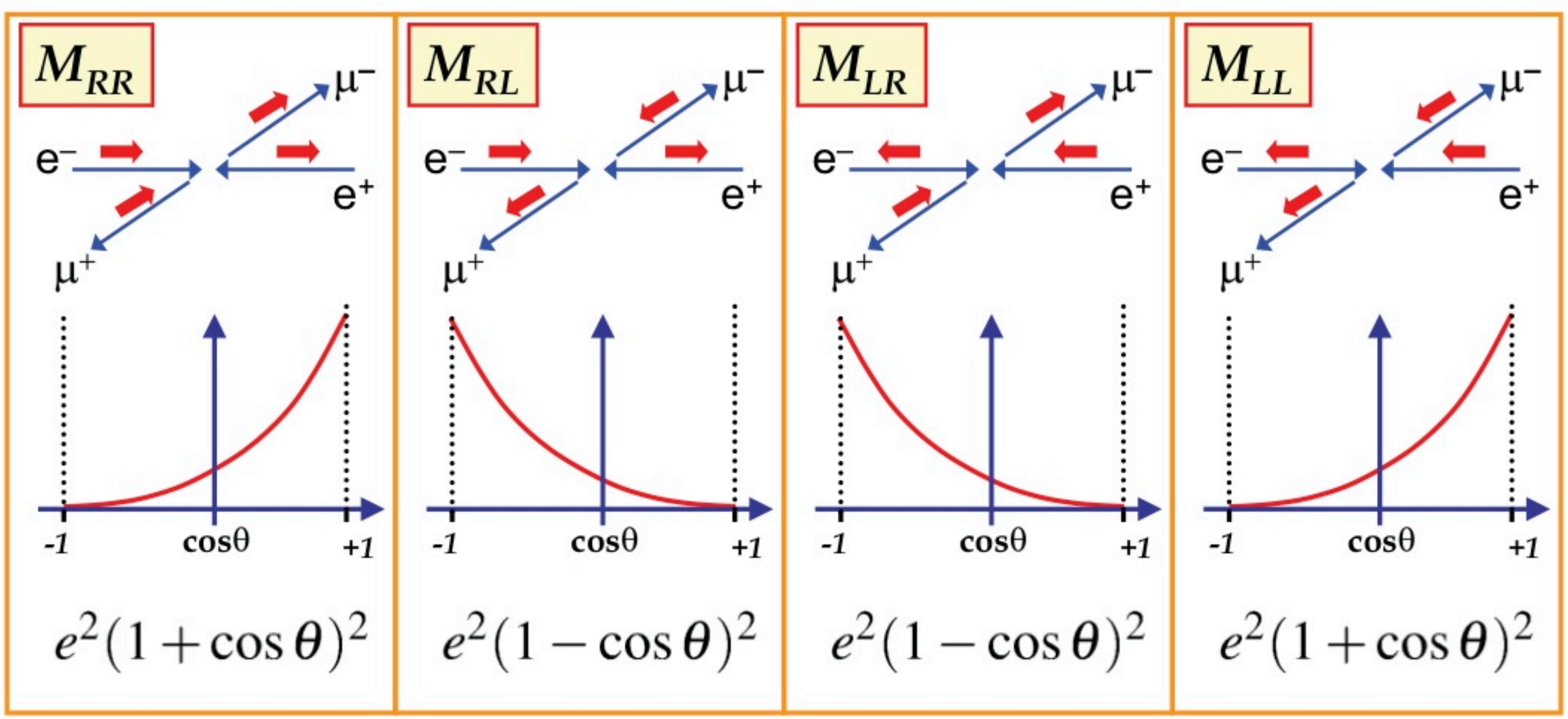
$$|\mathcal{M}|^2 = 2e^4 \frac{(t^2+u^2)}{s^2} = e^4(1+\cos^2\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4\pi s}(1+\cos^2\theta)$$



In the high energy limit, there are 4 contributions:
RL→RL ($\mathcal{M}_{RR}$), RL→LR ($\mathcal{M}_{RL}$), LR→RL ($\mathcal{M}_{LR}$), LR→LR ($\mathcal{M}_{LL}$)

We have averaged over initial states and summed over final states to get unpolarised cross section:



$e^-e^+ \to \mu^-\mu^+$ Total Cross Section

- Total cross section, integrate over solid angle:

$$
\begin{aligned}
\sigma &= \int \frac{d\sigma}{d\Omega}\, d\Omega \\
&= \frac{\alpha^2}{4\pi s} \int (1+\cos^2\theta) d\cos\theta d\phi \\
&= \frac{\alpha^2}{4\pi s} [\phi]_{-\pi}^{\pi} \left[\cos\theta + \frac{1}{3}\cos^3\theta\right]_{\cos\theta=-1}^{\cos\theta=+1} \\
&= \frac{4\alpha^2}{3s}
\end{aligned}
$$

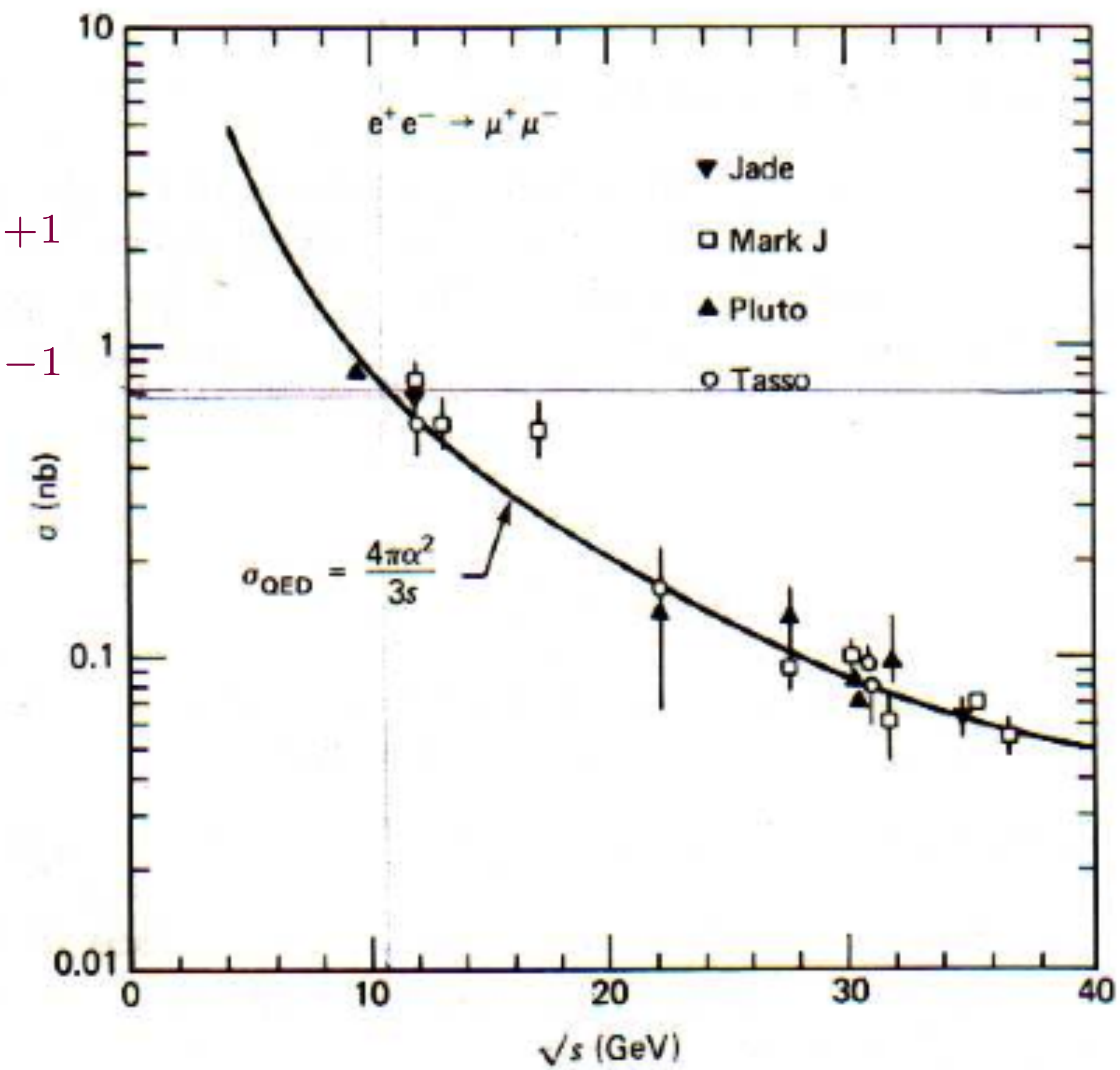


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

- Comparison prediction to measurement. Pretty good for a 1st order calculation!

Higher Order QED

- We have been drawing and calculating 1st order Feynman diagrams with one boson exchanged
- There are more diagrams with higher numbers of vertices.
- We should sum them all to obtain the total value for $\mathcal{M}$

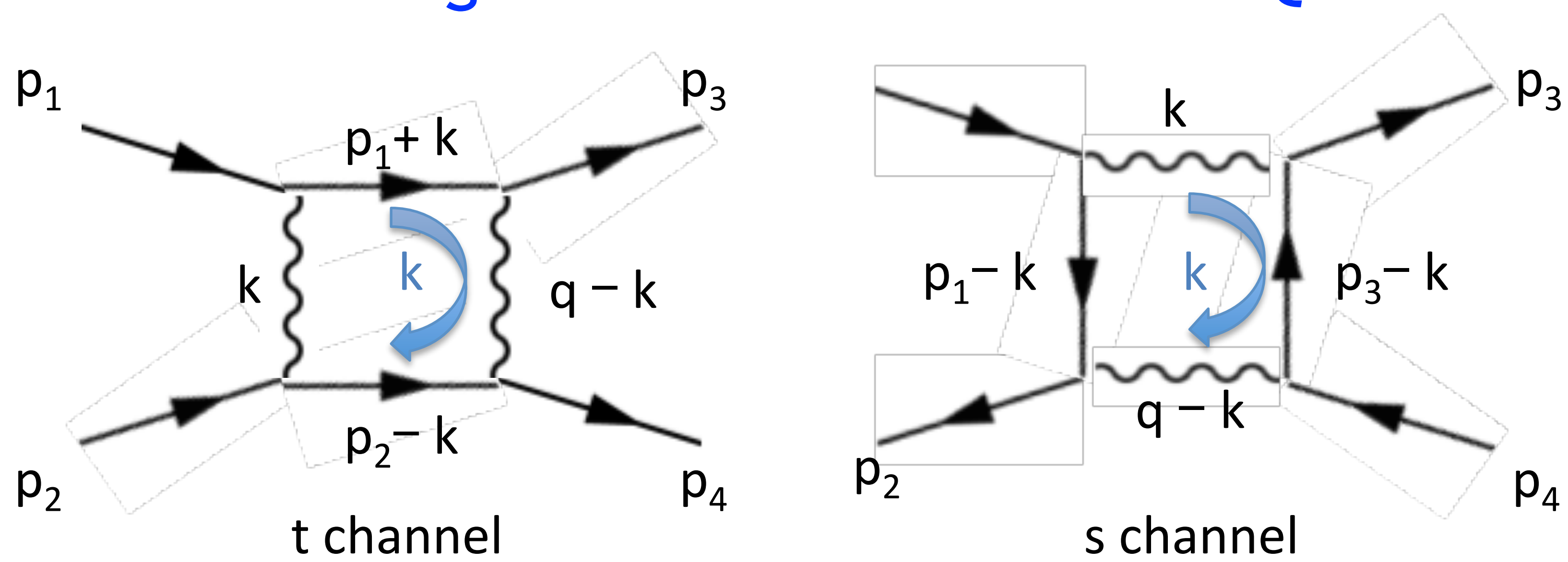
$$\mathcal{M}_{\text{tot}} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 \ldots$$

➡ … but for every two vertices you have a suppression factor of $\alpha = 1/137$

- The most precise QED calculations go up to $O(\alpha^5)$ diagrams

Higher Order QED

e.g. Two photon "box" diagrams also contribute to QED scattering



Two extra vertices ⇒ contribution is suppress by a factor of $\alpha = 1/137$

- The four momentum must be conserved at each vertex.
- However, four momentum k flowing round the loop can be anything!

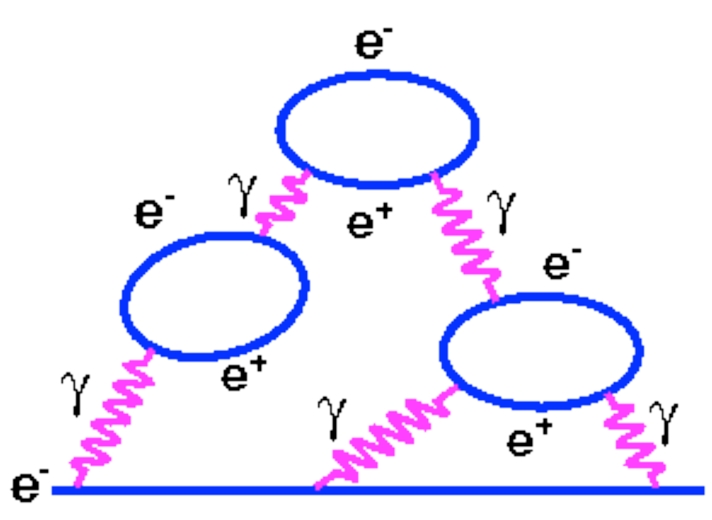
- In calculating $\mathcal{M}$ integrate over all possible allowed momentum configurations: $\int f(k)\, d^4k \sim \ln(k)$ leads to a divergent integral!
- This is solved by **renormalisation** in which the infinities are "miraculously swept up into redefinitions of mass and charge" (Aitchison & Hey P.51)

Renormalisation

- Impose a "cutoff" mass M, do not allow the loop four momentum to be larger than M. Use $M^2 >> q^2$, the momentum transferred between initial and final state.
  - ➡ This can be interpreted as a limit on the shortest range of the interaction
  - ➡ Or interpreted as possible substructure in pointlike fermions
  - ➡ Physical amplitudes should not depend on choice of M
- Find that $\ln(M^2)$ terms appear in the $\mathcal{M}$
- Absorb $\ln(M^2)$ into **redefining fermion masses and vertex couplings**
  - ➡ Masses $m(q^2)$ and couplings $\alpha(q^2)$ are now functions of q^2

- *e.g.* Renormalisation of electric charge (considering only effects from one type of fermion):

$$e_R = e\sqrt{1 - \frac{e^2}{12\pi^2}\ln\left(\frac{M^2}{q^2}\right)}$$



- Can be interpreted as a "screening" correction due to the production of electron/positron pairs in a region round the primary vertex
- e_R is the effective charge we actually measure!

Running Coupling Constant

- Renormalise α, and correct for all possible fermion types in the loop:

$$\alpha(q^2) = \alpha(0)\left(1 + \frac{\alpha(0)}{3\pi} z_f \ln(\frac{-q^2}{M^2})\right)$$

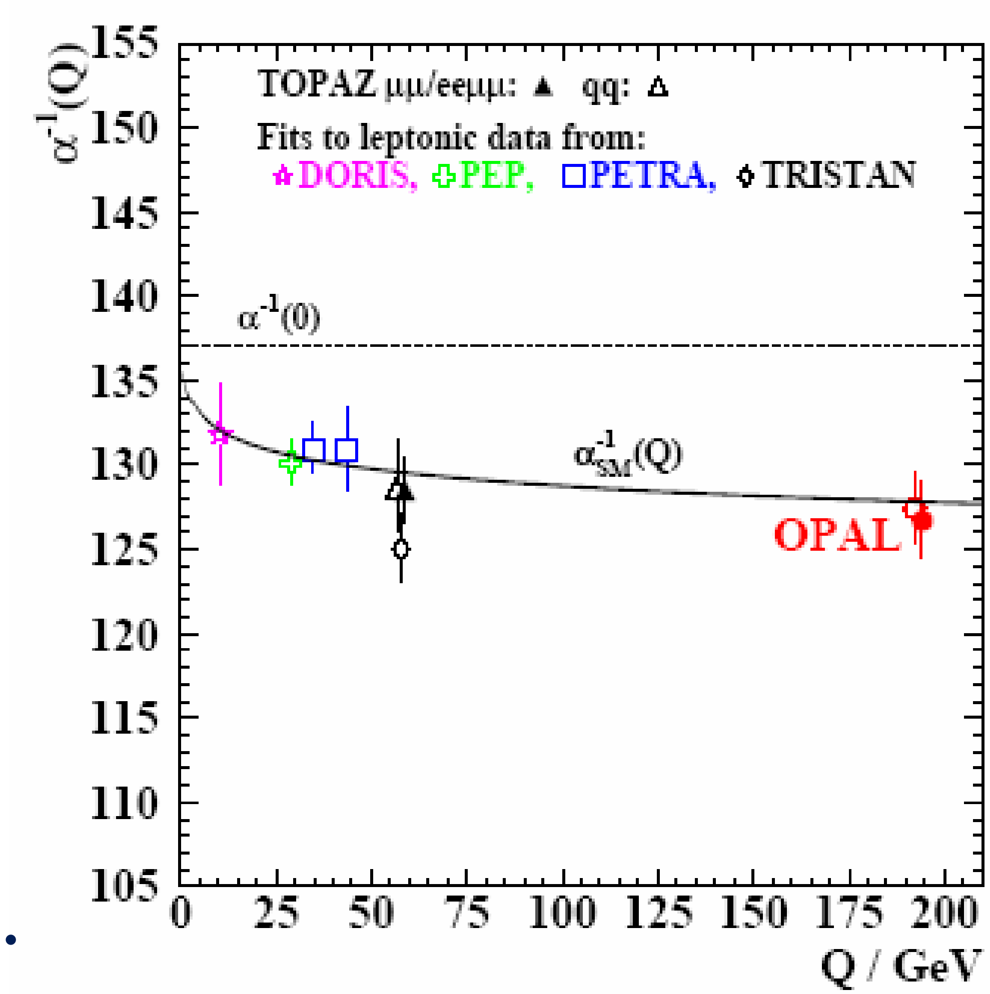
- z_f is the sum of charges over all possible fermions in the loop
  - ➡ At $q^2 \sim 1$ **MeV** only electron, $z_f = 1$
  - ➡ At $q^2 \sim 100$ **GeV**, $f = e, \mu, \tau, u, d, s, c, b$ $z_f = 60/3$

$$z_f = \sum_f Q_f^2$$

- Instead of using M^2 dependence, replace with a reference value μ^2:

$$\alpha(q^2) = \alpha(\mu^2)\left(1 - \frac{\alpha(\mu^2)}{3\pi} z_f \ln(\frac{q^2}{\mu^2})\right)^{-1}$$

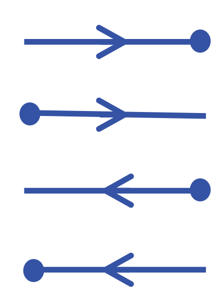
- Usual choices for μ are **1 MeV** or $m_Z \sim$ **91 GeV.**
  - ➡ $\alpha(\mu^2 = 1\ \mathrm{MeV}^2) = 1/137$
  - ➡ $\alpha(\mu^2 = (91\ \mathrm{GeV})^2) = 1/128$
- We choose a value of μ where make a initial measurement of α, but once we do the evolution of the values of α are determined by the above eqn.



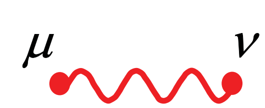
QED Summary

- For full QED calculations use spinors to define fermion current in Feynman diagrams.
- Fermion current are $\bar{\psi}\gamma^{\mu}\psi$ where $\bar{\psi}$ is the adjoint spinor: $\bar{\psi} \equiv \psi^{\dagger}\gamma^{0}$
- Spin-1 bosons are described by polarisation vectors, $\varepsilon^{\mu}(s)$
- To calculate the cross section for an unpolarised process need to average over initial helicities and sum over all possible final states.
- Fermion masses, charges and the coupling constant α evolve as a function of momentum transfer.

spin 1/2
- incoming particle $u(p)$
- outgoing particle $\bar{u}(p)$
- incoming antiparticle $\bar{v}(p)$
- outgoing antiparticle $v(p)$



spin 1 photon $\frac{g_{\mu\nu}}{q^2}$



- **Vertex Factors**

spin 1/2 fermion (charge -|e|) $e\gamma^{\mu}$